Optimal Stopping within an Equity Momentum Model

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Abstract

Patterns, whether recognizable or falsely-recognized, are present in our financial markets. So-called 'Market Wizards' around the world have explored countless avenues to 'crack' the code but the universe and its solutions are endless. One phenomenon that has held the test of time is the theory of momentum. Momentum in physics was defined by Sir Isaac Newton in his laws of motion, stating that an object has a tendency to continue in the same direction unless the object is acted on by an outside force.

1 Introduction

In this paper, I adapt Andreas Cleanow's Momentum Model into my own equity momentum model. I've previously revamped this model to adjust for trend, macroeconomic optimization, and portfolio construction. This is intended to research optimal stopping of a Brownian Bridge which would be the optimal selling point of a position.

2 Implementing the Momentum Model

Our momentum factor is calculated by multiplying the annualized exponential regression slope by the R^2 coefficient for every stock in our selected universe. In layman's terms, we want to find the speed at which every stock in our selected universe is moving (exponential regression), then confirm the accuracy of that speed (using the R^2 coefficient).

Convert all stock returns to log-returns, $r_T = \ln\left(\frac{S_T}{S_0}\right)$, for a daily-interval $[0, T]$ (1)

Get the annualized exponential regression slope, $y = a^x$, for the interval $[0, m]$ (2)

Multiply annualized exponential regression slope by R^2 coefficient (3)

The universe we will be using will consist of the S&P 500 Index derived from Wikipedia's database. This universe consists of large-cap United States companies which tend to outperform most other equity classes for two main reasons: (1) Small and mid-cap companies have higher risk of revenue / cash-flow deficits due to market competition and (2) Investing in foreign companies not only has historic sub-par returns but also poses a risk of the foreign governments interfering with their 'free' market economies.

For portfolio construction we have many choices: equal weighting, mean-variance optimization, efficient-frontier, and inverse volatility. Given that this momentum model is considered high-beta to the underlying index, we will use inverse volatility weighting for portfolio construction. To exemplify this construction method, let's say we have a portfolio that contains AAPL and TSLA. AAPL has a volatility of 30 and TSLA has a volatility of 70. Using an inverse volatility weighting approach, we would construct our portfolio with a 70% allocation to AAPL and a 30% allocation to TSLA.

3 Introduction of Trend Filters and Macro Optimization

Since we will be using the S&P 500 Index as our universe, it will also be used as a trend filter. A trend filter is used by investors to determine whether or not their universe is investible. When our trend filter indicates a bull market, we will build and rebalance existing positions to get to our optimal portfolio (the top stocks and their respective portfolio weights). When our trend filter indicates a bear market, then we will sell all of our holdings. Since the information we are using is available to our competitor investors, our only edge is to sell first during downturns. This is the main reason for selling all of our holdings when our universe in not investible.

To fully understand bear markets, we have to take into consideration where the underlying economy in the US is heading and how that will affect our bear market universe: Gold, Cash, and Treasuries. Is Cash King? Will the FED need to lower rates (increasing market values for Treasuries)? Will demand increase for Gold if the Dollar crashes? These are all important questions to consider, but the full macro-exploration will not be included in this paper.

4 Backtesting Methodology

The systems we will be using for backtesting is a combination of multiple technologies: Python 3 – a programming language for scripting and object-oriented programming, Zipline – a python-based back-testing engine, and YFinance – a python package that pulls yahoo finance data for equities in our universe. The performance metrics that we will take into consideration are Sharpe Ratio, Drawdown, and Total Return which will all be provided by the Python package – PyFolio. The backtesting timeline will be August 1st, 1994 to August 18th, 2022 given our data limitations.

5 Application of Optimal Stopping

Erik Ekström and Henrik Wanntorp laid a framework that we will use as an addition to this momentum model called Optimal Stopping of a Brownian Bridge. Each stock in our universe will be assumed to have standard Brownian motion. In Definition 2.1, we let $W = \{Wt\}t \geq 0$ denote a standard Brownian motion, where a continuous process $\{Xs\}t \leq s \leq 1$ satisfying

$$\begin{cases} dXs = -\frac{(Xs)}{(1-s)}ds + dWs, \ t \leq s < 1, \\ \qquad\qquad Xt = x, \end{cases}$$

will represent a Brownian Bridge starting from x at time $t \geq 0$ and ending at 0.

We consider the SDE:

$$dXt = b - Xt1 - tdt + dWt, X0 = a, \quad for\ t \in [0,1]\ with\ a, b \in R \tag{1}$$

To solve for this SDE we use the constant variation method[fill] where we consider the ODE:

$$x'(t) = \frac{b - x(t)}{(1-t)} + f(t),\ x(0) = a,\ for\ t \in [0,1]\ with\ a, b \in R \tag{2}$$

The solution to this ODE is:

$$x(t) = a(1-t) + bt + (1-t)\int_0^t \frac{f(s)}{1-s} ds \tag{3}$$

Using a stochastic integration by parts, we obtain:

$$Xt = a(1-t) + bt + Wt - (1-t)\int_o^t \frac{Ws}{(1-s)^2} ds \tag{4}$$

Then, considering $Yt = Xt - Wt$, we get:

$$dYt = \frac{b - Yt}{1-t} dt - \frac{Wt}{1-t} dt \tag{5}$$

Lastly, using Ito's isometry we obtain:

$$\mathbb{E}\{X_t^2\} = [a(1-t) + bt]^2 + (1-t)^2 \int_0^t \frac{1}{(1-s)^2} ds \tag{6}$$
$$= [a(1-t) + bt]^2 + t(1-t) \xrightarrow[t \to 1]{} b^2 \tag{7}$$

We will use python to create a Brownian Bridge on the interval $[0, m]$ for consistency with the original momentum model:

$$\mathbb{E}\{X_m^2\} = [a(1-m) + bm]^2 + (1-m)^2 \int_0^m \frac{1}{(1-s)^2} ds \tag{8}$$
$$= [a(1-m) + bm]^2 + t(1-m) \xrightarrow[m \to 1]{} b^2 \tag{9}$$

Then, we assume that the percentage change of the securities in our portfolio will return to the same level. If the returns within our volatility window exceed the Brownian Bridge, we assume that we have made enough profit and sell out of the positive. If the returns are lower, then we will continue with the assumptions that we expect returns in the future to be greater.

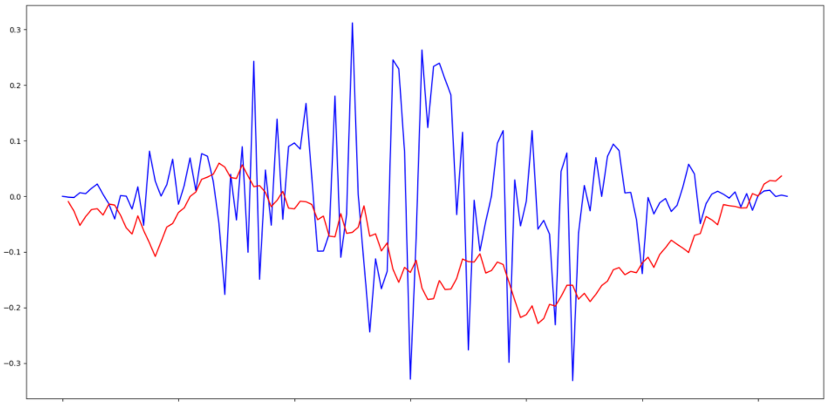

Figure 1: Example creation of a Brownian Bridge within Python, AAPL (Red), Brownian Bridge (Blue)

6 Performance Assessment

To begin the assessment of both of our models: Sam's Equity Momentum (SEM) and Sam's Equity Momentum with Optimal Stopping (SEMOS), we will explore the total return annualized and downdraws. EM has an annualized total return of 15.73% with a max drawdown of -42.08%. This drawdown was fueled by the dotcom bubble which led to a vicious market crash but overall this algorithm will beat the benchmark S&P 500. EMOS has an annualized return of 5.02% with a max drawdown of -18.33%. This drawdown was also fueled by the dotcom bubble but optimal stopping mitigated a lot of the risk to the downside.

Moving forward, SEM shows an average 6-month rolling Sharpe ratio of ~1.02 while SEMOS shows ~0.63. From a volatility and return perspective SEM still outperforms SEMOS. SEMOS carries far less long exposure in equites than SEM over the course of time with ~45% long exposure in comparison to ~88%. Relatively small percentage of long exposure with a high portfolio turnover rate can lead to lower expected returns and even execution risk. Furthermore, SEMOS does outperform SEM during the dotcom bubble.

Fundamentally, the positions in SEMOS reach optimal stopping very frequently which lowers the upside of the portfolio. Given the assessment of the backtested results, the preferred choice is SEM with potential improvement such as using a quantitative macroeconomic framework that will protect against all market cycles including the dotcom bubble.

7 Conclusion

The use of Optimal Stopping of a Brownian Bridge within a momentum model may not be the best choice but may be helpful for mean-reversion algorithms in financial markets. We can see that it is beneficial for protecting the portfolio but this drastically limits the expected return. Overall when constructing a model to exploit patterns in markets it's important to know who your competitors are, where you may have an edge, and the mathematics behind your process.

References

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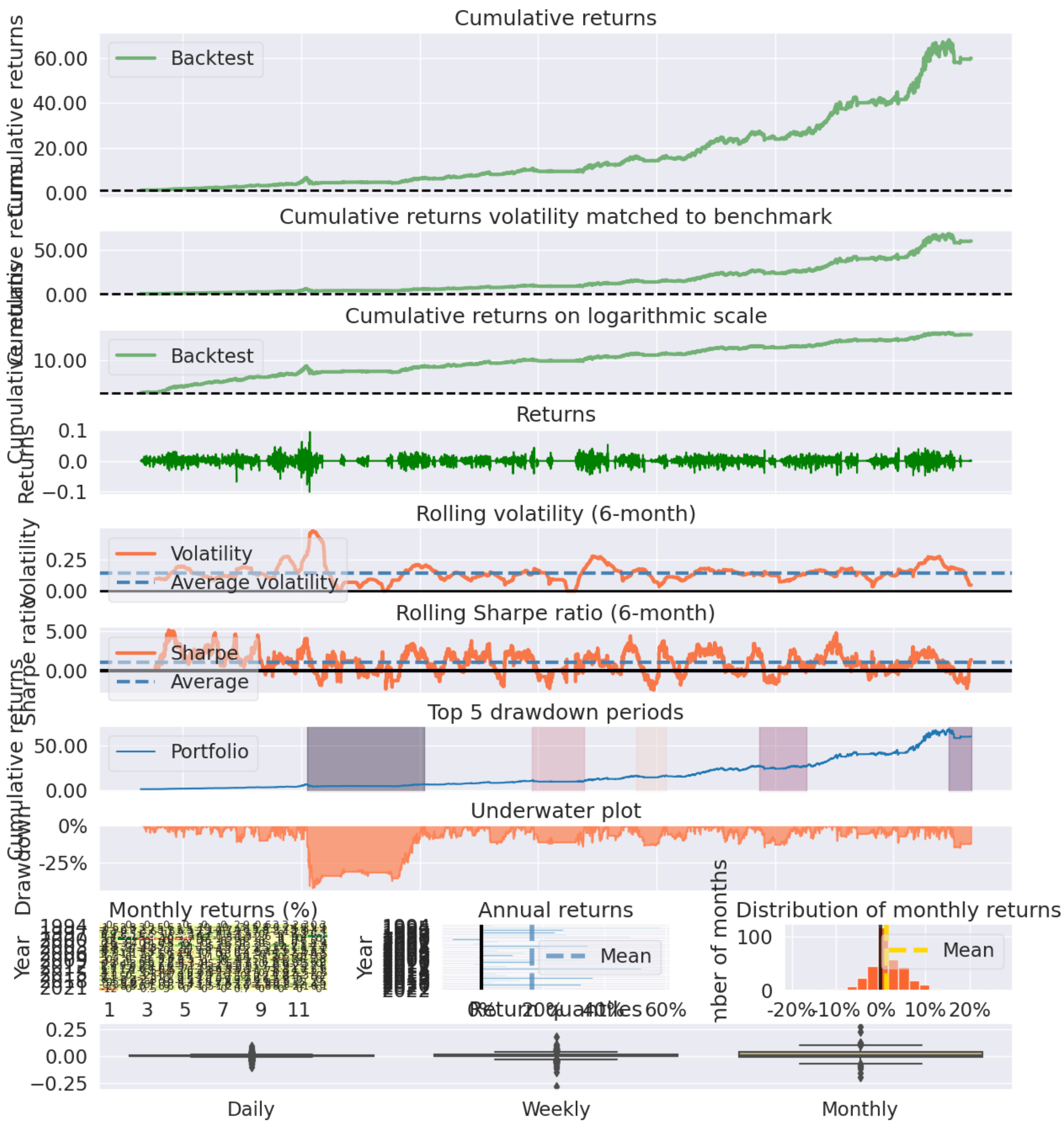
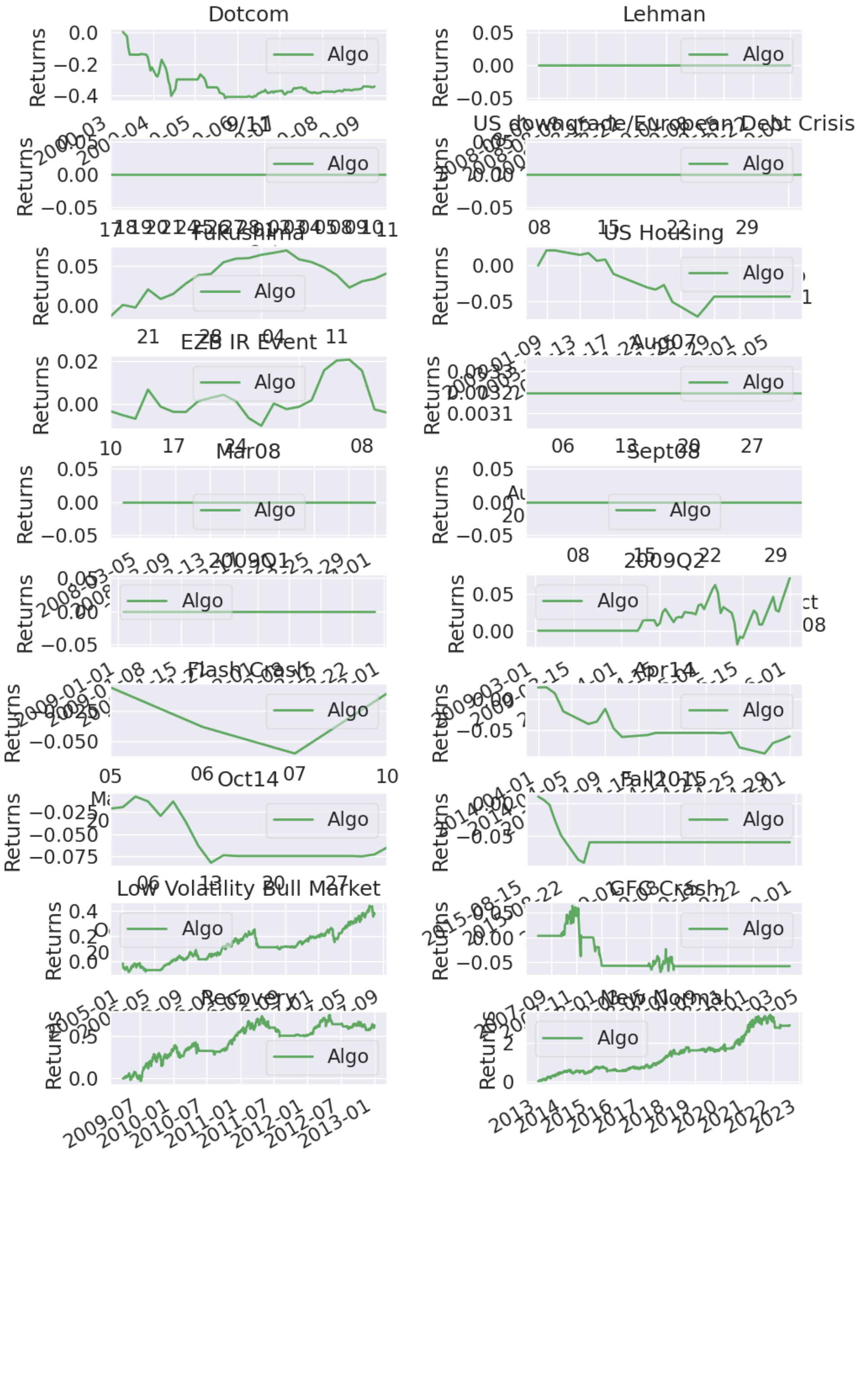
Appendix

A. Sam's Equity Momentum Model's (SEM) – PyFolio Tear Sheet

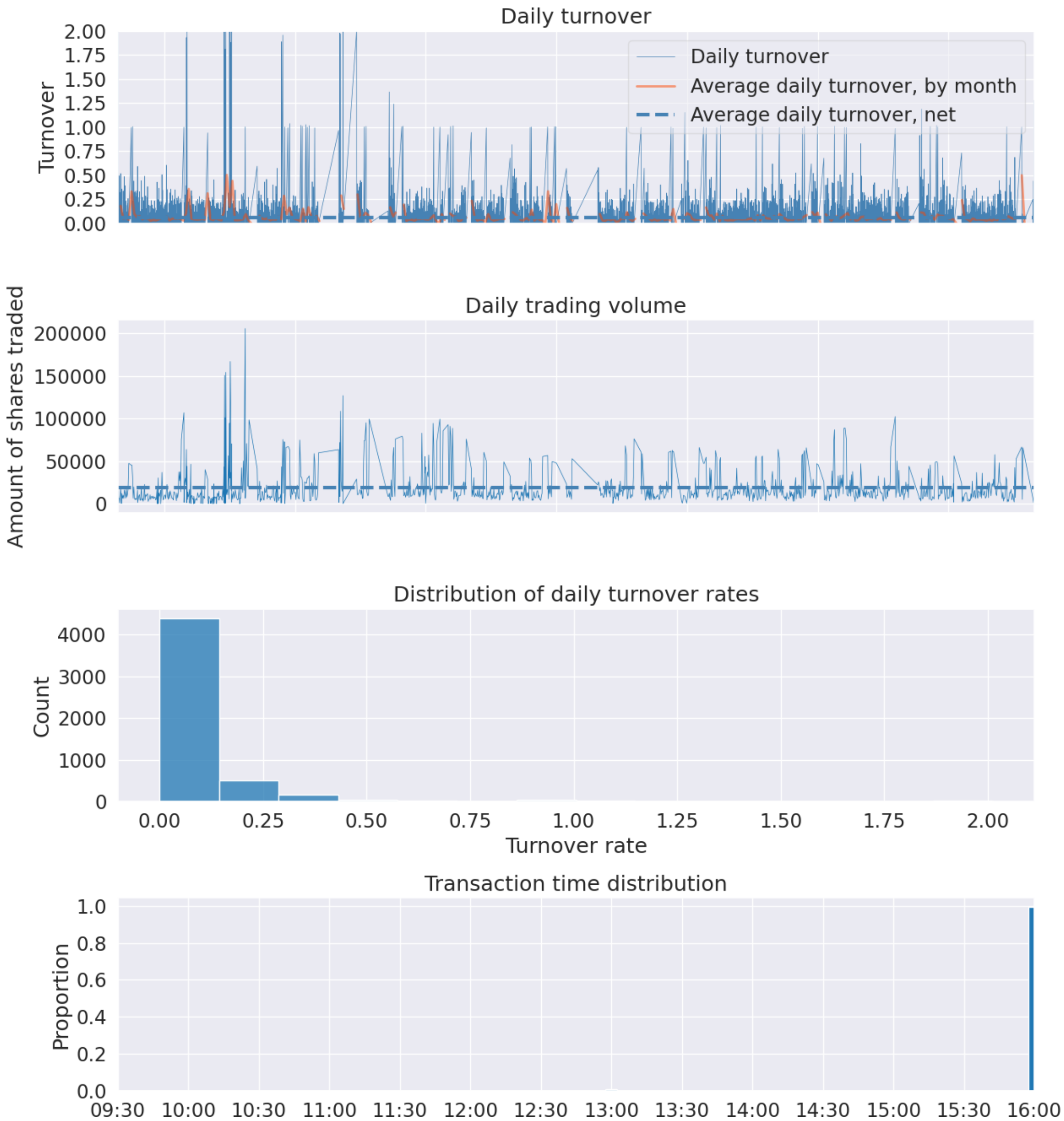
i.



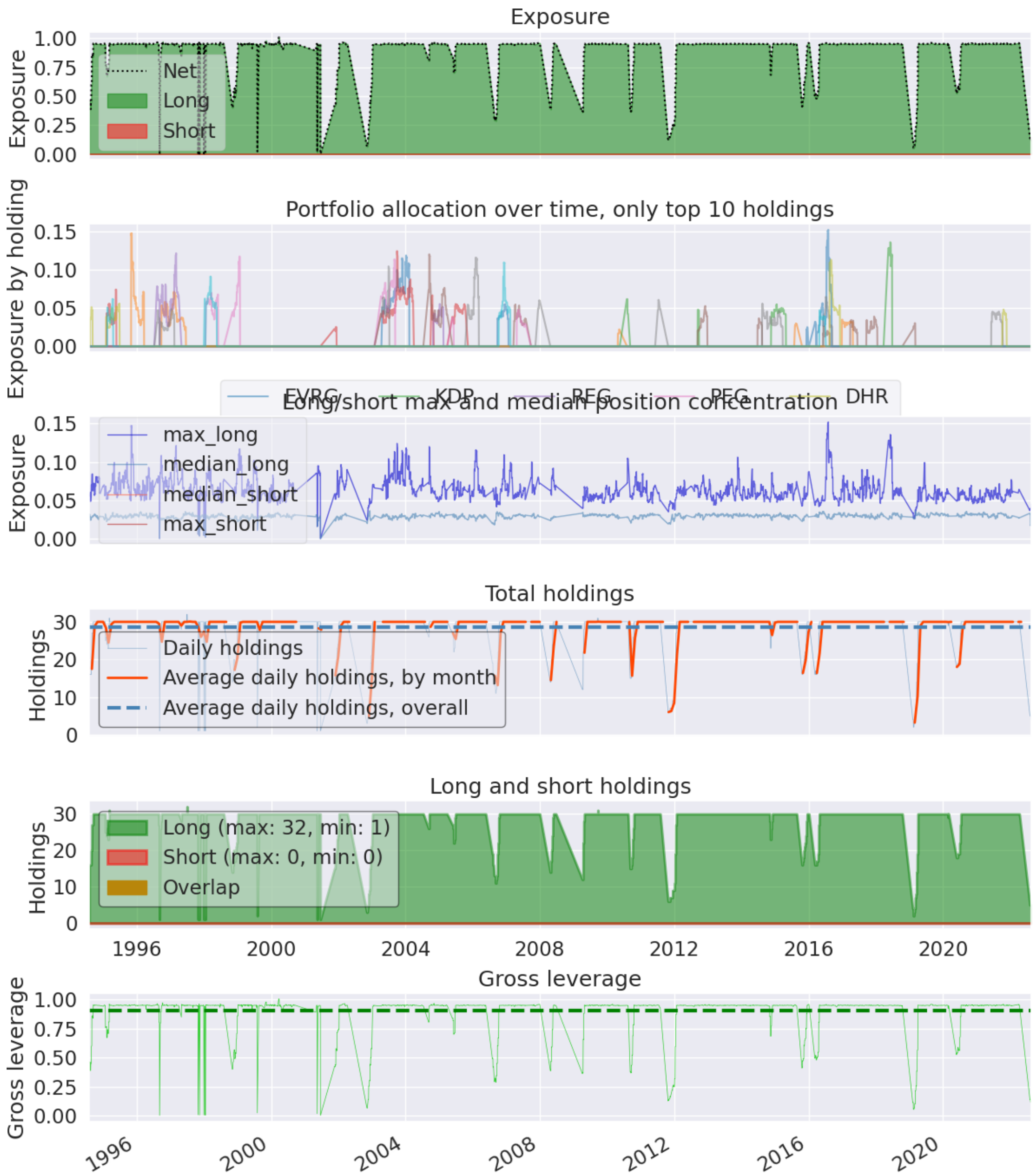
ii.


Dotcom
Lehman
9/11
US downgrade/European Debt Crisis
Fukushima
US Housing
EZB IR Event
Aug07
Mar08
Sept08
2009Q1
2009Q2
Flash Crash
Apr14
Oct14
Fall2015
Low Volatility Bull Market
GFC Crash
Recovery
New Normal
Returns
Algo

iii.

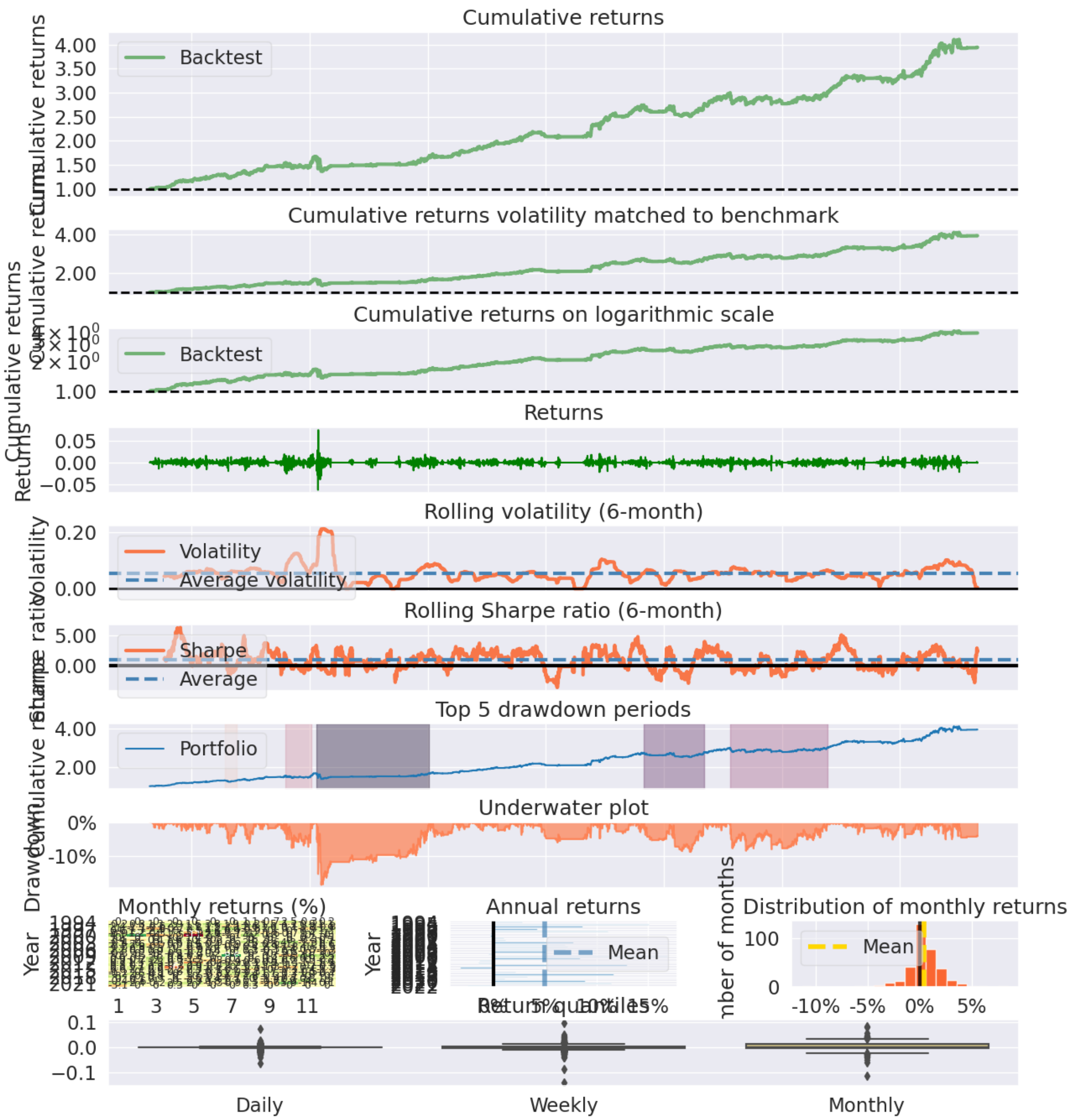

Daily turnover
Turnover
Daily turnover
Average daily turnover, by month
Average daily turnover, net
Daily trading volume
Amount of shares traded
Distribution of daily turnover rates
Count
Turnover rate
Transaction time distribution
Proportion

iv.

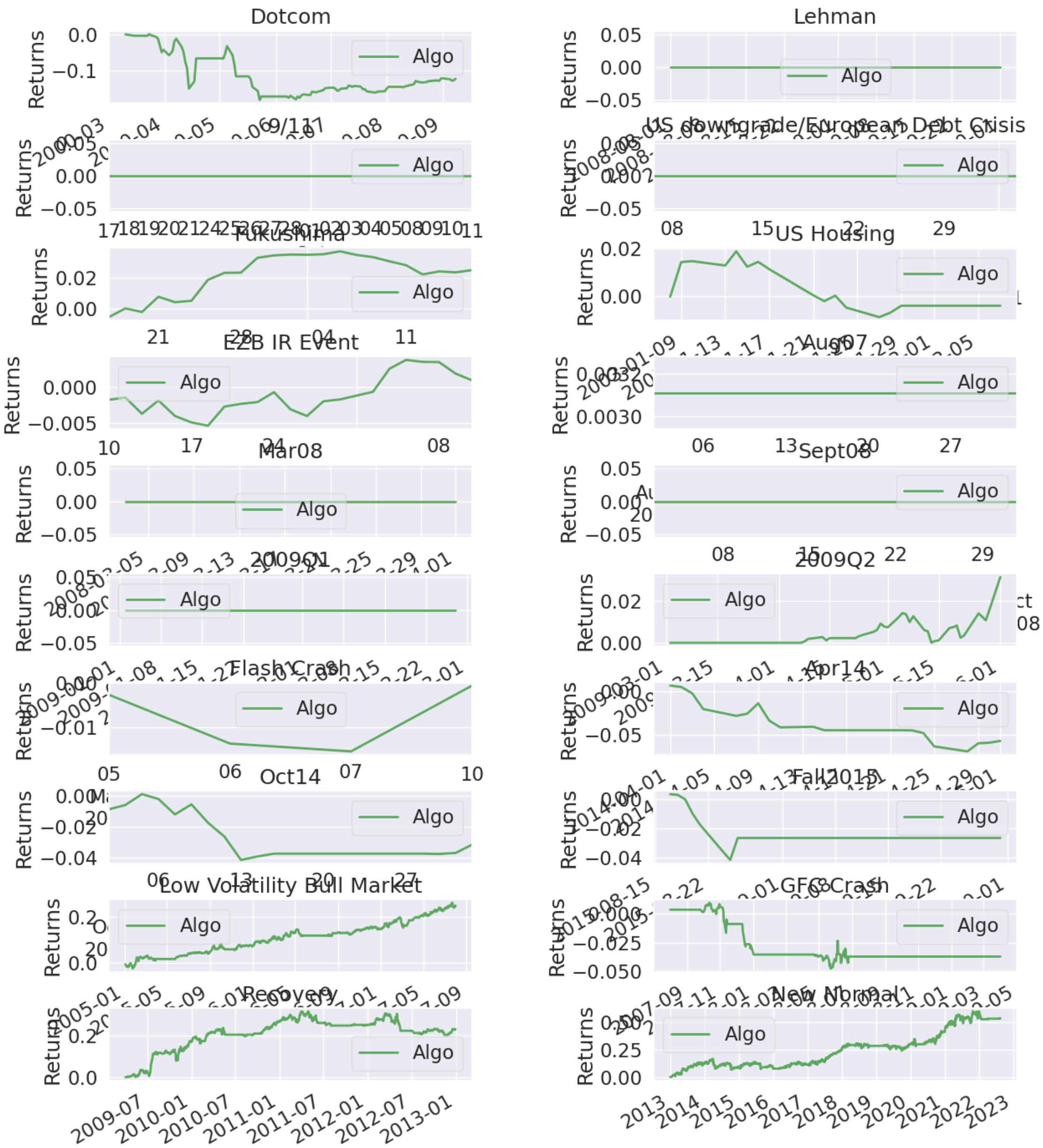

Exposure
Net
Long
Short
Portfolio allocation over time, only top 10 holdings
Exposure by holding
EVRG
KDP
REG
PEG
DHR
Long/short max and median position concentration
max_long
median_long
median_short
max_short
Total holdings
Holdings
Daily holdings
Average daily holdings, by month
Average daily holdings, overall
Long and short holdings
Long (max: 32, min: 1)
Short (max: 0, min: 0)
Overlap
Gross leverage
1996
2000
2004
2008
2012
2016
2020

B. Sam's Equity Momentum Optimal Stopping Model's (SEMOS) – PyFolio Tear Sheet

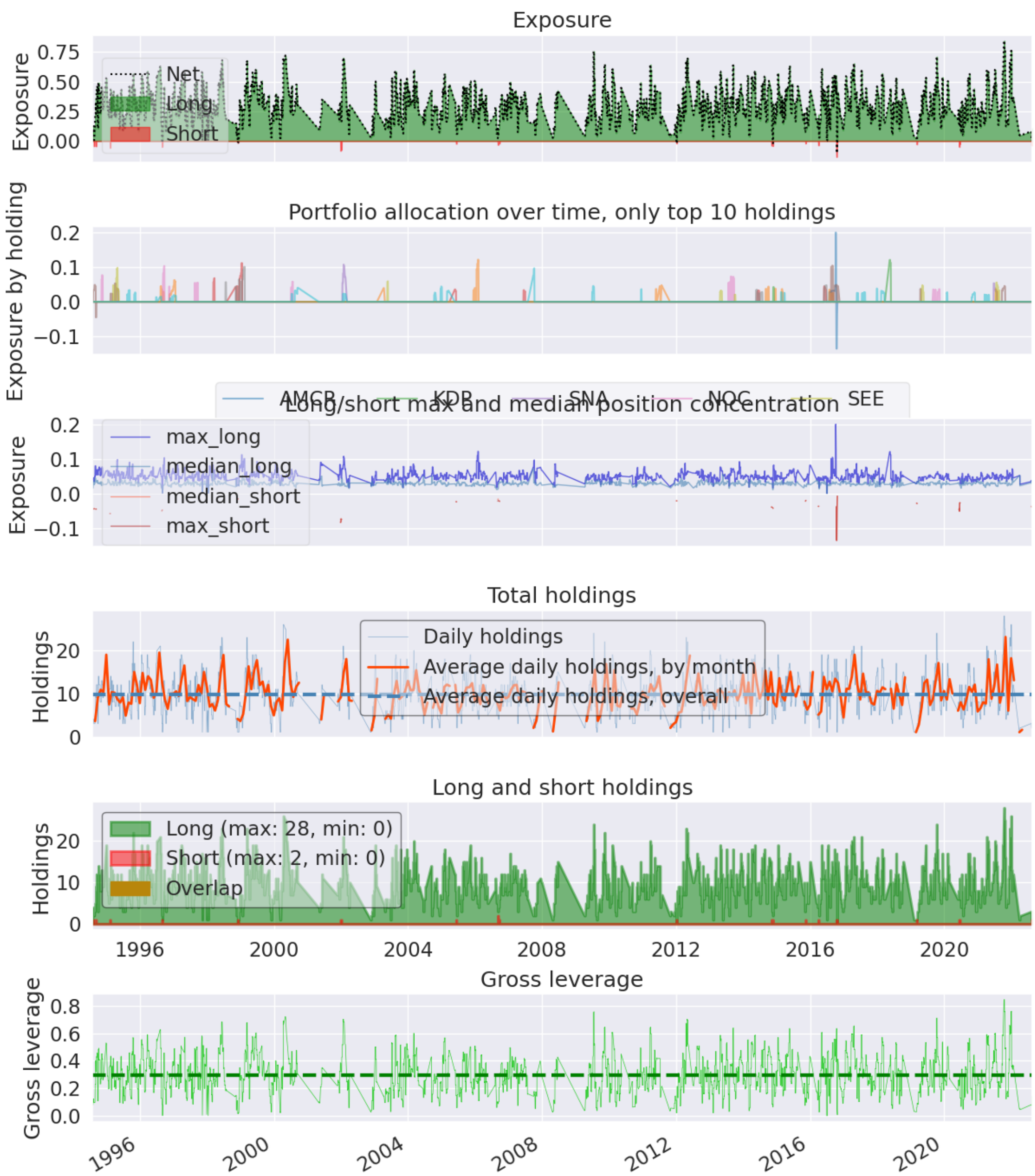
i.



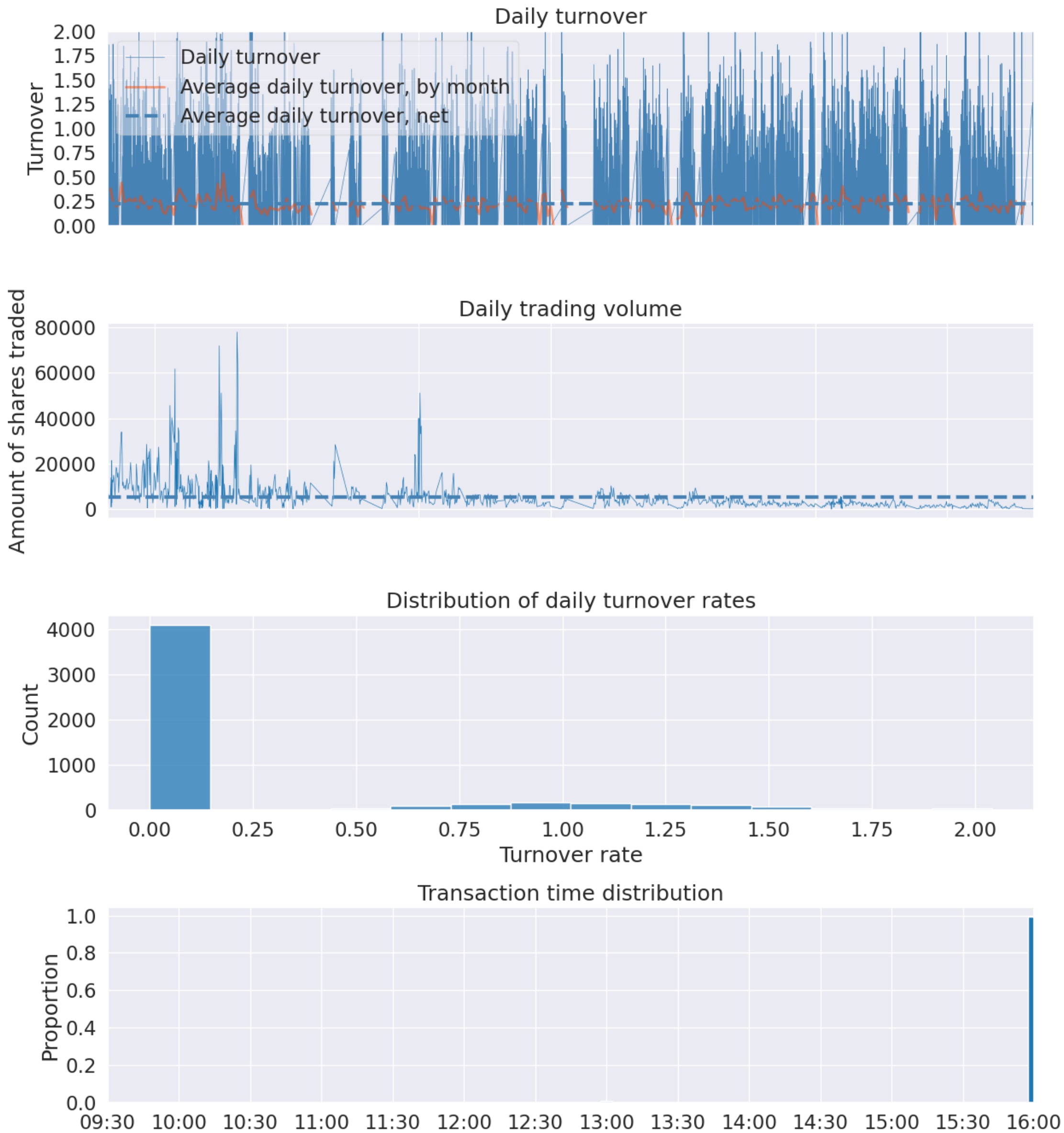
ii.


Dotcom
Lehman
9/11
US downgrade/European Debt Crisis
Fukushima
US Housing
EZB IR Event
Aug07
Mar08
Sept08
2009Q1
2009Q2
Flash Crash
Apr14
Oct14
Fall2015
Low Volatility Bull Market
GFC Crash
Recovery
New Normal
Returns
Algo

iii.


Exposure
Net
Long
Short
Portfolio allocation over time, only top 10 holdings
Exposure by holding
AMCR
KDP
SNA
NOC
SEE
Long/short max and median position concentration
max_long
median_long
median_short
max_short
Total holdings
Holdings
Daily holdings
Average daily holdings, by month
Average daily holdings, overall
Long and short holdings
Long (max: 28, min: 0)
Short (max: 2, min: 0)
Overlap
Gross leverage
1996
2000
2004
2008
2012
2016
2020

$iv.$


Daily turnover
Turnover
2.00
1.75
1.50
1.25
1.00
0.75
0.50
0.25
0.00
Daily turnover
Average daily turnover, by month
Average daily turnover, net
Daily trading volume
Amount of shares traded
80000
60000
40000
20000
0
Distribution of daily turnover rates
Count
4000
3000
2000
1000
0
0.00
0.25
0.50
0.75
1.00
1.25
1.50
1.75
2.00
Turnover rate
Transaction time distribution
Proportion
1.0
0.8
0.6
0.4
0.2
0.0
09:30
10:00
10:30
11:00
11:30
12:00
12:30
13:00
13:30
14:00
14:30
15:00
15:30
16:00