

UMKC SUMMER OF 2021 RESEARCH SUMMARY

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# Analysis of Residual Galaxy Images

## Technical Report

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# Contents

<b>1 Abstract</b>	<b>3</b>
<b>2 Introduction</b>	<b>4</b>
2.1 Motivation . . . . .	4
2.2 Application to Larger Research Project . . . . .	4
2.3 Research Goals and Description . . . . .	4
<b>3 Sample</b>	<b>5</b>
3.1 Sample Description . . . . .	5
3.2 Graphic User Interface (GUI) . . . . .	5
3.3 Creating of the Classification Schemes . . . . .	6
<b>4 Analysis</b>	<b>7</b>
4.1 Justifying the Classification Schemes . . . . .	7
4.2 Tidal Feature (TF) Finder . . . . .	9
4.3 TF Finder Analysis . . . . .	11
<b>5 Results</b>	<b>18</b>
5.1 Key Takeaways . . . . .	18
<b>6 Conclusion</b>	<b>18</b>

# List of Figures

1 Graphic User Interface (GUI) . . . . .	6
2 Name and descriptions of the 6 classification schemes . . . . .	7
3 For loops to produce random images . . . . .	8
4 Negligible examples . . . . .	8
5 Outer Layer examples . . . . .	9
6 Center/behind examples . . . . .	9
7 Shedding examples . . . . .	10
8 Splotches examples . . . . .	10
9 Two Dots examples . . . . .	10
10 TF finder output: residual image of Negligible example . . . . .	12
11 TF finder output: highlighted residual image of Negligible example . . . . .	12
12 TF finder output: residual image of Outer layer example . . . . .	13
13 TF finder output: highlighted residual image of Outer layer example . . . . .	13
14 TF finder output: residual image of Center/behind example . . . . .	14
15 TF finder output: highlighted residual image of Center/behind example . . . . .	14
16 TF finder output: residual image of Shedding example . . . . .	15
17 TF finder output: highlighted residual image of Shedding example . . . . .	15
18 TF finder output: residual image of Splotches example . . . . .	16
19 TF finder output: highlighted residual image of Splotches example . . . . .	16

20	TF finder output: residual image of Two dots example . . . . .	17
21	TF finder output: highlighted residual image of Two dots example . . . . .	17

# 1 Abstract

In this analysis, we analyze the residual substructures of a representative sample of 500 galaxies obtained from a larger sample of 10,000 galaxies, all with stellar masses  $> 10^{9.5} M_\odot$  and redshift between  $0.5 < z < 3$ . I develop 6 classification schemes of residual substructures that appear to be universal to a multitude of tidal features. Through running a tidal feature (TF) finder python code on a sample of these galaxies, I found that at least 3 of the 6 classification schemes are strongly supported by the results of the TF finder code and are applicable as ground truths for a residual classification Machine Learning (ML) software. Whereas, the other 3 classification schemes are not strongly supported by this analysis and should be analyzed further into whether or not they can be considered ground truths for a residual ML software.

## 2 Introduction

### 2.1 Motivation

The morphological development of galaxies is rather elusive. There have been a multitude of astronomers, many years spent and thousands of papers written to try and understand the morphological development of galaxies (Jackson et al. 2019 [3], Mancillas et al. 2019 [4], Baugh et al. 1996 [1], Governato et al. 1997 [2], Nipoti et al. 2009 [7], Mantha et al. 2017 [5]). To understand galaxy evolution and growth, there are some crucial questions to be answered to formulate the processes that galaxies undergo. Two prominent unanswered questions relevant to this analysis are: How can we empirically distinguish galaxies undergoing rapid spheroid formation by major merging versus predicted non-merging processes? And, How do we better constrain the rates of spheroid formation (e.g. rate of merging)? These, and questions alike, are at the forefront of our knowledge and understanding of the development of galaxies in our universe. Answering these questions are the motivation behind characterizing residual images of galaxies.

### 2.2 Application to Larger Research Project

Characterizing residual images of galaxies is a crucial aspect of the larger research project of the research lead Kameswara Mantha. The overarching conceptual goal of Mantha's current research project is to devise an automated Machine Learning (ML) software to identify galaxies hosting interesting residual substructures that could be due to physical processes such as galaxy merging. This ML based approach should characterize and learn these residual shapes of galaxies and be able to bin them in similar classifications schemes. In this project there are two approaches that Mantha wants to test on his ML software: supervised (guided) and unsupervised (unguided). Supervised entails giving the ML software some ground truth (human input) where the software tries to reproduce these target values. While unsupervised equates to allowing the software to produce its own outputs based on what it recognizes and learns from the residual images. Both approaches will be useful in characterizing residual substructures and ultimately help us learn what these substructures physically represent.

### 2.3 Research Goals and Description

The characterization of residual images of galaxies are a small, but central, aspect of understanding galaxy evolution and growth. Mapping out universal classification schemes that are applicable to a plethora of residual images can better help us fathom the physical processes that these galaxies are enduring, distinguish major merging versus non-merging processes and better pin point the rate at which spheroid formation occurs. Moreover, while these schemes themselves will not answer these questions, as mentioned previously the overarching goal of binning residual images of galaxies into universal classification schemes is to train Mantha's automated ML software to recognize these schemes and be able to label these schemes on these residual images by itself.

My role in this research project is to create the supervised ground truth for the ML software by establishing approximately 5 residual classification schemes. This was done

by observing 500 residual images of galaxies through a Graphic User Interface (GUI) and creating bins based on patterns I recognize. Also, I need to be able to justify these schemes on how they are applicable to these residual substructures. This includes running a tidal feature (TF) finders python code on the galaxy images and their corresponding residuals to help justify and interpret my classification schemes. The idealized goal is that my classification schemes can be justified by the TF finder code. Ultimately, my function in the larger research effort is to serve as the "test run" for the ML software, in order for the software in the supervised approach to get a blueprint of what it needs to be looking for in order to bin these residual substructures effectively.

## 3 Sample

### 3.1 Sample Description

In this analysis of residual galaxy images, I use a sample of 500 galaxies derived from a larger sample of 10,000 galaxies. These 10,000 galaxies all have a stellar mass  $> 10^{9.5} M_{\odot}$ , a redshift between  $0.5 < z < 3$  and an H-band brightness  $< 24.5$ . Of this population of galaxies, the 500 galaxies in this sample was chosen to be a representative subset spanning the diversity of residual features.

### 3.2 Graphic User Interface (GUI)

The first steps I took in contributing to this research effort was simply looking at the sample of 500 images of galaxies to get an initial understanding of what the residual images looked like. I did this through the GUI, which is a python code written by the research lead Kameswara Mantha. When running the GUI it contained three images with the simple commands; comments, yes and no boxes and save and exit. The 3 images were the original image of the galaxy, the modeled Sérsic profile image, and the residual image. Figure 1 below is an example of what the GUI looks like when you execute the code.

The modeled Sérsic profile <sup>1</sup> of the galaxy is a mathematical function that describes the intensity ( $I$ ) of a galaxy as a function of radius from the center of the galaxy. Mathematically, the Sérsic profile is given by:

$$I(r) = I_e \times \exp(-\kappa [(\frac{r}{r_e})^{1/n} - 1]). \quad (1)$$

The equation conceptually illustrates how the light of a galaxy at a given radius from its center falls off as a function of radius towards the outskirts. This is extremely important in creating the residual image of galaxies. The residual is created by subtracting the original image of the galaxy by the modeled Sérsic profile image. What is left behind is any substructures not contained in the modeled Sérsic profile.

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<sup>1</sup>For more information on Sérsic profiles of galaxies and applications visit: Vitral 2020 [8]

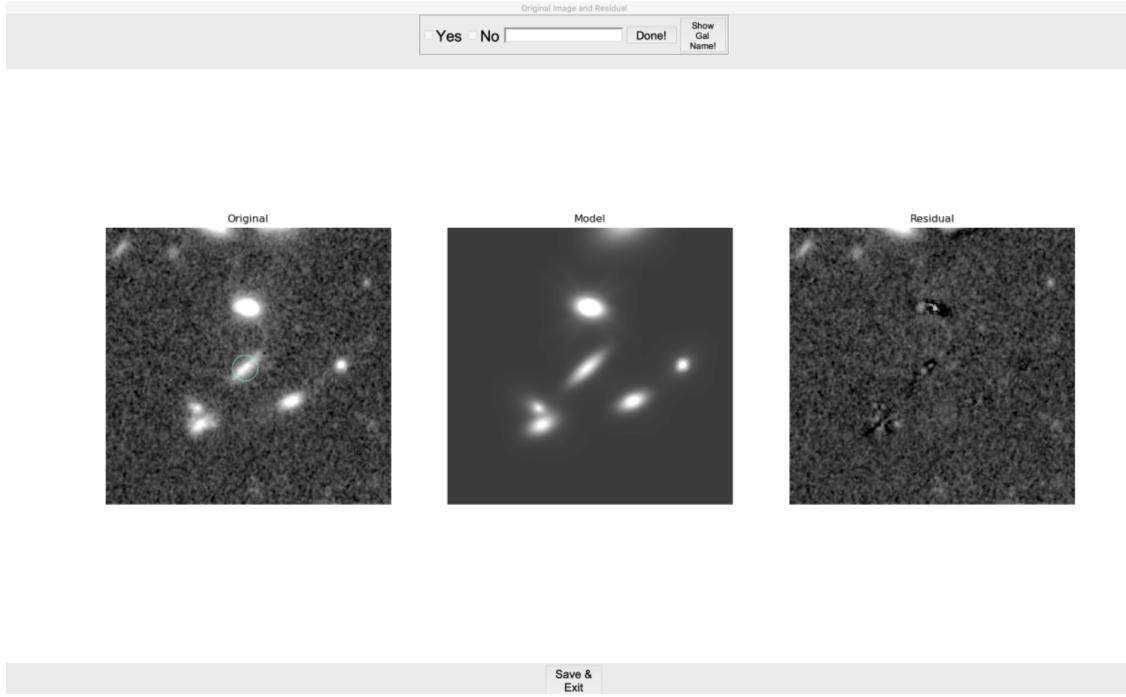


Figure 1: Graphic User Interface (GUI)

### 3.3 Creating of the Classification Schemes

After running through approximately 50-100 of these images in the sample, I decided to began characterizing some residual substructures. After looking through these images I noticed some patterns in the residuals. The first pattern I noticed was an insignificant residual. Therefore, the first classification scheme I created was called Negligible. The second pattern I noticed was diffuse spread out light along, but not always, the outskirts of the galaxies. The second classification scheme I created was called Outer layer. The third noticeable pattern was concentrated light at or near the center of the galaxy. Thus, this third scheme I called Center/behind. Finally, the fourth pattern I noticed initially was a combination of diffuse spread out light and concentrated light. From which I called this scheme Shedding.

Continuing through the images, I noticed more and more intricate patterns and shapes. I began to get more descriptive in my classification naming; attributing certain patterns and shapes to possible physical processes that could be occurring. Along this line I came up with 3 schemes that represented physical processes. These were: Active Galactic Nuclei (AGN) activity, Merger, and Star Formation (SF) activity. In discussing my progress with Kameswara he advised me to stray away from naming/labeling possible physical processes in the residual images and focus more on broad unspecific naming of schemes. Understanding his line of thought, I scratched these 3 physical process schemes and tried to adhere to creating bins based solely on the patterns and shapes of the residual substructures and not on the physical processes these residual substructures could represent.

Progress was made when I noticed two more patterns that were relatively universal to many other residual images. The fifth pattern I noticed was a radial splotch pattern at or

near the center of the galaxy. I called this classification scheme Splotches. The sixth and last pattern I noticed of these residual images were two concentrated light sources. I called this last classification scheme Two dots. Below is a table consisting of the names and descriptions of the 6 classification schemes I created for these residual substructures.

<b>Classification name</b>	<b>Description</b>
1- Negligible residual	This residual is characterized as having little to none of noticeable features.
2- Outer layer residual	Characterized as having light spread out and diffuse, covering at least a quarter or more of the perimeter of the galaxy. This can be in the outer edges or more central.
3- Center/behind residual	Distinguished by having a bright concentrated source of light at or near the center of the galaxy.
4- Shedding residual	This classification scheme is symptomatic by having both diffuse and concentrated light sources throughout the residual image.
5- Splotches residual	Distinctive by having a radial splotch pattern at or near the center of the residual image. This radial splotch pattern has a bright center and slowly becomes more dim towards the outskirts.
6- Two dots residual	Identified by having two bright concentrated dots at or near the center of the residual image

Figure 2: Name and descriptions of the 6 classification schemes

It should be noted that the names of these classification schemes are not what is important, but rather it is what the names represent about the residual substructures that is important. Each description of these bins hold good ground as being distinct and universal among a plethora of tidal features. The names of these 6 bins can easily be changed to be a better representation of their description. For example, the fourth bin, Shedding, can easily be called Concentrated and Diffuse, as this is what this bins description entails. The main point I want to make here is that the description of each classification scheme is more important and holds more truth than the name of the schemes.

## 4 Analysis

### 4.1 Justifying the Classification Schemes

To first justify my 6 classification schemes, Kameswara advised me to obtain some example images of each scheme. To do this, I wrote, with significant mentoring help from Kameswara, a python code to extract 5 random images from each classification scheme and save them in a new folder. Below is a picture of the bulk of the code that Kameswara and I wrote to achieve this.

```

18     for each_flag, each_folder in zip(flags, folders):
19         truth_array = [each_flag in each_gal['Comments'] for each_gal in sample]
20         subset = np.random.sample(truth_array)
21         random_objects = np.random.choice(subset, 5)
22         iteration+=1
23         for each_random_object in random_objects:
24             image_cube = fits.open('/Users/slaterjonesoden/Desktop/Summer_Research_Set/Images/%s'
25                                   %each_random_object[0], memmap=True)
26             original = image_cube[1].data
27             residual = image_cube[3].data
28             fig,axs = plt.subplots(nrows=1,ncols=2)
29             ax1,ax2 = axs
30             normalization = ImageNormalize(original, interval=PercentileInterval(99.),
31                                             stretch=AsinhStretch())
32             ax1.imshow(original, origin='lower', cmap='gray', norm=normalization)
33             ax2.imshow(residual, origin='lower', cmap='gray', norm=normalization)
34             ax1.xaxis.set_visible(False)
35             ax1.yaxis.set_visible(False)
36             ax2.xaxis.set_visible(False)
37             ax2.yaxis.set_visible(False)
38             plt.tight_layout()
39             plt.savefig('/Users/slaterjonesoden/Desktop/Summer_Research_Set/Examples/%s/%s'
40                         %(each_folder,each_random_object[0].strip('.fits')+'.png'))
41             plt.show()
42             iteration += 1
43             if iteration > 30:
44                 break

```

Figure 3: For loops to produce random images

The code essentially loops through each classification name (labeled as 'each flag'), produces 5 random pictures from each name, and saves these images under the galaxies ID. Writing this code was of tremendous learning experience because it taught me how to iterate over multiple objects (or galaxies) sequentially and save them in different folders, all in one line of code.

The output of this code was then organized in a PowerPoint of example images to get a better visualization of my classification schemes. Below are the 6 classification schemes visualized with 4 example images for each scheme

### 1- Negligible examples

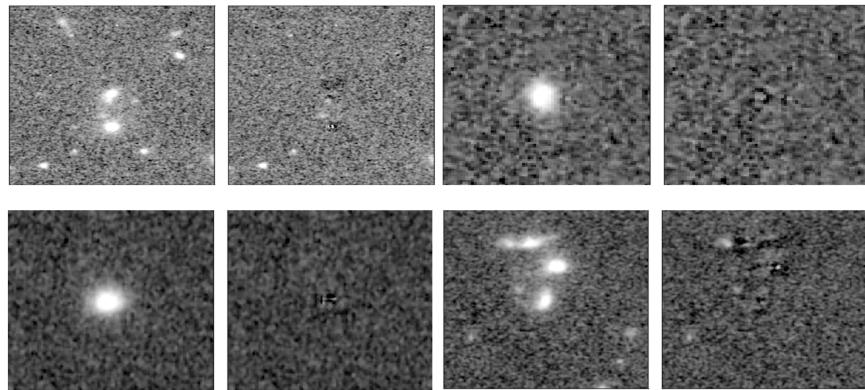


Figure 4: Negligible examples

## 2- Outer Layer examples

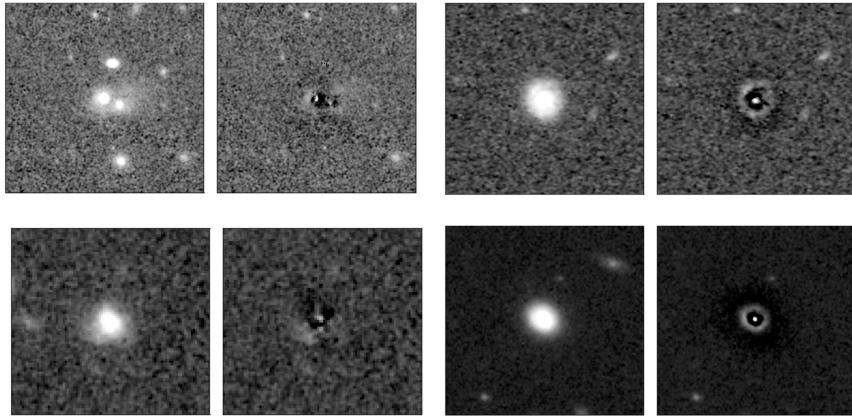


Figure 5: Outer Layer examples

## 3- Center/behind examples

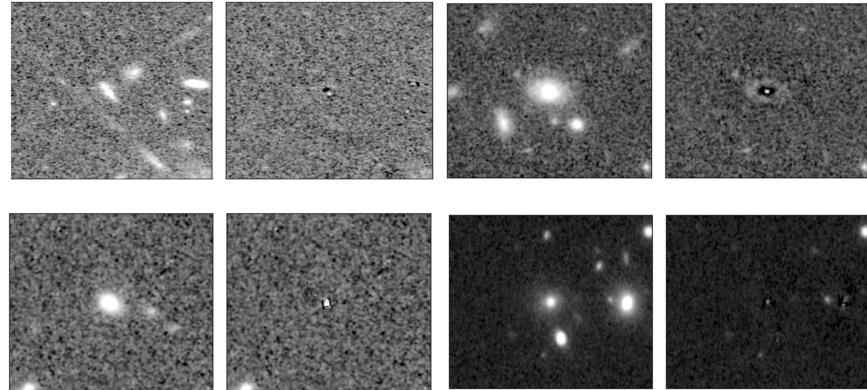


Figure 6: Center/behind examples

As you can see from a few of these example images (e.g., the two right sided Outer layer examples and the top right Center/behind example) there are some residuals that can easily fit two classification schemes. While this is true, my justification behind these examples, and ultimately behind creating 6 distinct bins, includes picking the most prominent residual feature. While this can be rather selective in nature, I tried to classify residuals that could fit two schemes, by what was the most "eye-catching" residual substructure. Or in more technical language, the residual that appeared to have a brighter overall flux.

## 4.2 Tidal Feature (TF) Finder

The next step to analyze and justify my 6 classification schemes is running a tidal feature (TF) finder code on each galaxy image and its corresponding residual. The TF finder code is outlined extensively in the paper Mantha et al. 2019 [6]. Essentially, the code extracts tidal features (i.e. residual substructures) and highlights/masks these residual features in

#### 4- Sheding examples

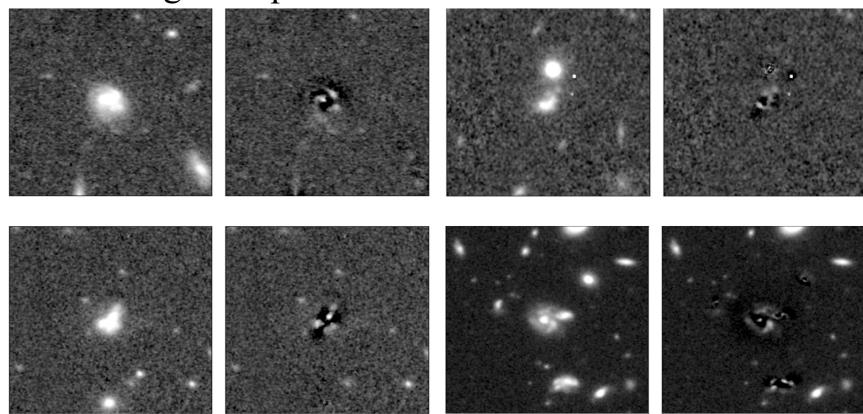


Figure 7: Sheding examples

#### 5-Splotches examples

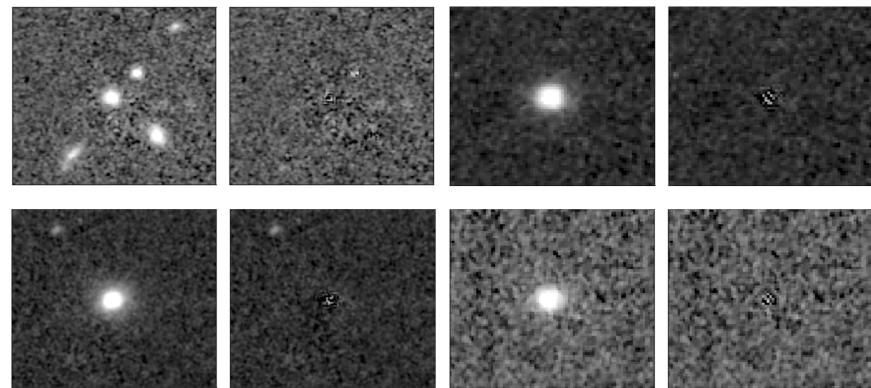


Figure 8: Splotches examples

#### 6- Two dots examples

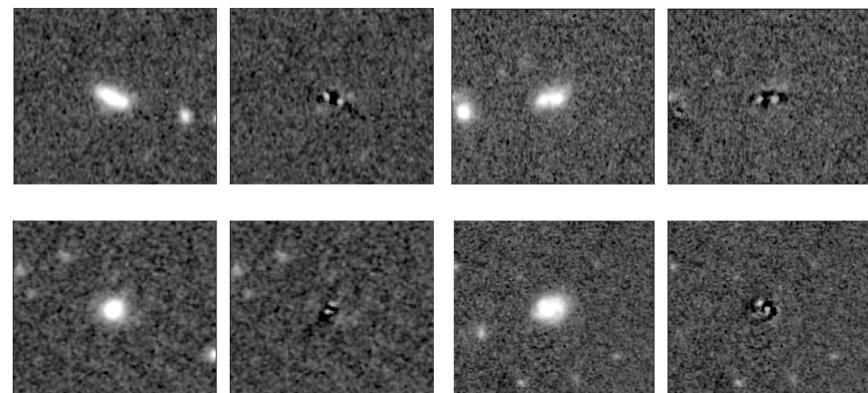


Figure 9: Two Dots examples

the original image. The highlights/masks are in orange and overlays on top of the residual features in the image. The orange highlight is also sensitive to the flux or magnitude of the residual features. Where part of a residual feature is very luminous, the orange highlight will appear lighter in color (e.g. light orange to almost white in color). Whereas if the residual feature is faint, the orange highlight will appear darker in color (e.g. dark orange). There is also a minimum flux that is required in the TF finder code in order to be differentiated from the background noise. This is extremely important in creating effective residual highlights because without this minimum flux the TF finder code would highlight sources of light in the image that is not a tidal feature of the galaxy. Finally, in this model of the TF finder code there is a red ellipse to ignore the center regions of the residual images. Ultimately, there are 3 main reasons in executing the TF finder code and producing these orange highlights on top of the residual substructures. The first reason is to obtain a clearer outline of where exactly these residual substructures are. The second is to legitimize and justify the pattern recognition, of which forms the bases of the 6 classification schemes. The last reason is to see if new features are highlighted which might have not been identified previously.

In order for me to successfully execute the TF finder code, there were some crucial steps that needed to be taken which were advised from the research lead, Kameswara. These steps included creating new folders for which the output(s) of the code will be saved, correcting the directories in the original TF finder code as to be applicable on my machine and to write a new code in conjunction with the TF finder code to create parameter files for each galaxy in the sample and to be saved as the galaxies ID. This last step is crucial because each galaxies ID in the sample is needed every time you run the TF finder code in order for the code to know which galaxy to analyze.

### 4.3 TF Finder Analysis

In this section we will discuss the results and analysis of the TF finder code on the galaxies in the sample. While time did not permit to be able to run the TF finder code on all 500 galaxies, I was however able to run and analyze the results of the TF finder code on roughly 100 of these galaxies. While the TF finder code should be ran on the whole sample of 500 galaxies to get a better representation of the highlighted residual substructures, there nevertheless was enough information to justify at least 3 of the 6 classifications schemes I created. Below is an example of the TF finder output for the first classification scheme: Negligible.

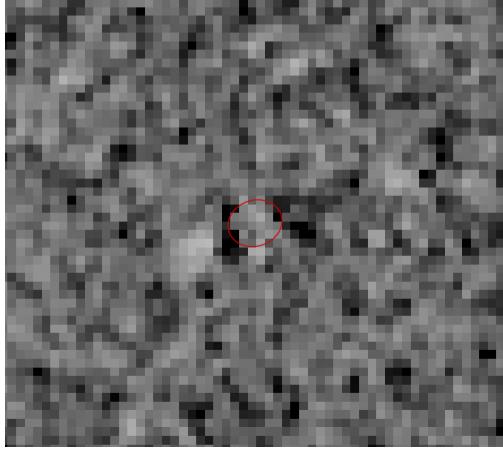


Figure 10: TF finder output: residual image of Negligible example

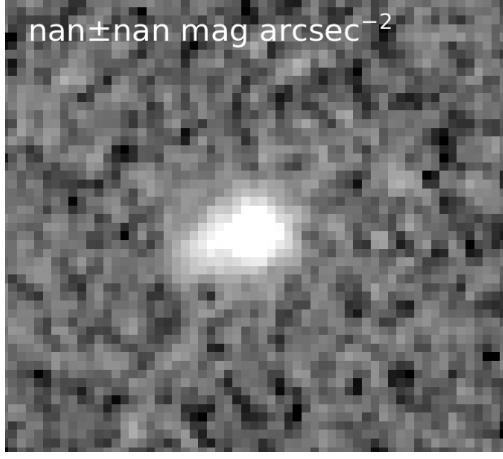


Figure 11: TF finder output: highlighted residual image of Negligible example

Figure 10 is the residual image of this galaxy, where Figure 11 is the highlighted image, or output of the TF finder code. As you can see from what is supposed to be the highlighted image (Figure 11), there is no orange highlighted area. This is exactly what we would expect in the output of the TF finder code for a negligible residual. These essentially nonexistent residual features of this galaxy are indistinguishable from the background sky noise and are below the minimum flux required to produce a highlighted tidal feature. The outputs of the TF finder producing images like these are strong evidence to support the classification scheme of a negligible residual. Galaxies with tidal features grouped in the negligible residual classification scheme most likely haven't had any recent (< 1 billion years) physically disturbing activities to change their morphology.

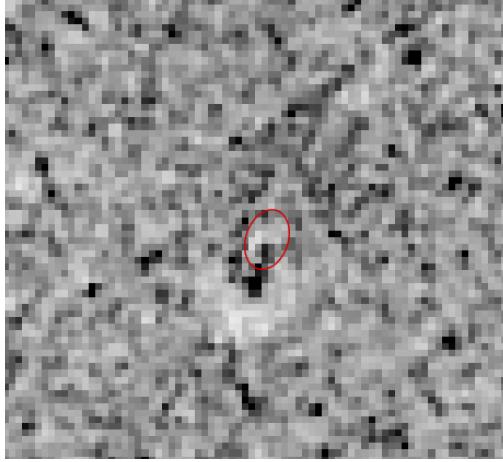


Figure 12: TF finder output: residual image of Outer layer example

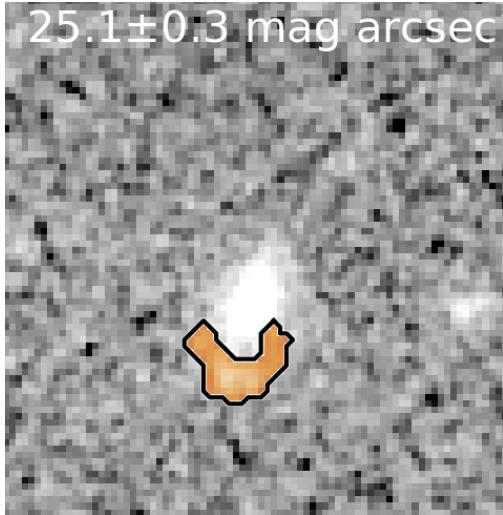


Figure 13: TF finder output: highlighted residual image of Outer layer example

This is an example of the TF finder output for the second classification scheme: Outer layer. In the top residual image, you can clearly see a diffuse residual substructure. This is confirmed in the output of the TF finder code, where the orange highlight is overlaid on top of this feature. We can also notice that this highlight is relatively the same shade of orange throughout. Finally, notice that the orange highlight subtends around roughly a half of the outer perimeter of the galaxy. In analyzing this highlighted residual feature we can clearly see that this TF finder output matches the description of the classification scheme Outer layer. In running the TF finder code on the other galaxies in the sample, we see similar highlighted features with the attributes listed above. Since this highlighted tidal feature matches the Outer layer classification scheme, as well as seeing many similar highlighted features, it is good evidence that this residual classification bin is universal throughout a multitude of residual features and could be used as one of the ground truths for the automated ML software.

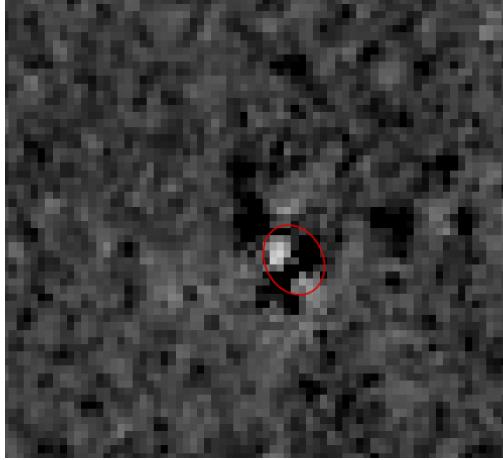


Figure 14: TF finder output: residual image of Center/behind example

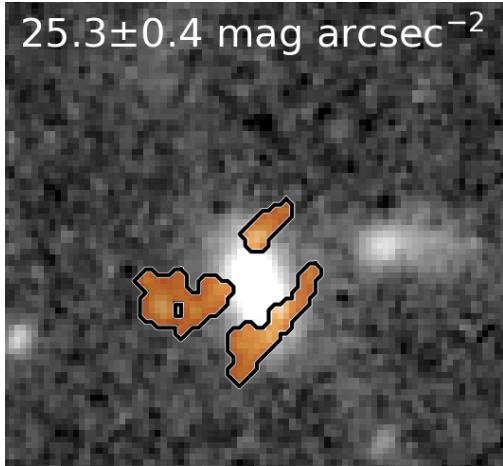


Figure 15: TF finder output: highlighted residual image of Center/behind example

This is an example of the TF finder output for the third classification scheme: Center/behind. In the residual image you can see a very concentrated bright source of light located in the middle of the where the galaxy was. You can also see some fainter diffuse tidal features scattered throughout. In the highlighted image you can see that the code picked up on these scattered tidal features, but not on the bright central residual. This is due to the fact that the TF code ignores the central areas in the residual image, indicated by the red ellipse. Because of this, there is no highlighted image from the TF finder code that supports the classification scheme Center/behind. This does not mean however that this classification scheme should be entirely ignored and thrown out. The TF finder code can be adjusted to include the center regions of the residual images, in which case we might have seen this classification scheme supported. Although, the first TF test run on these images does not support the classification scheme Center/behind.

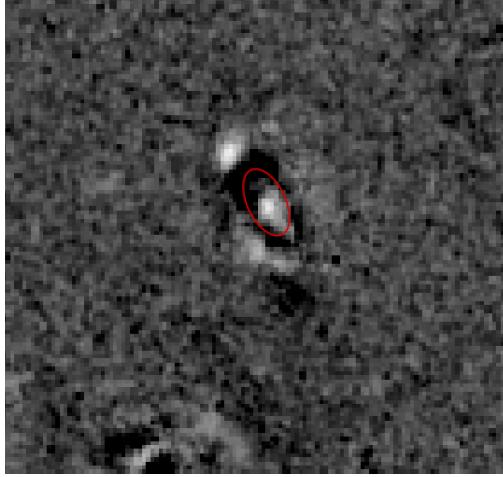


Figure 16: TF finder output: residual image of Shedding example

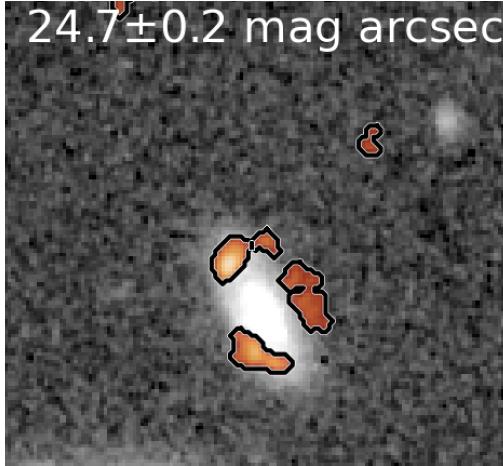


Figure 17: TF finder output: highlighted residual image of Shedding example

This is an example of the TF finder output for the fourth classification scheme: Shedding. In the above residual image there are two prominent features of the residual substructures. These prominent features are bright concentrated light and diffuse spread out light. Both of these are evident in the residual image. These features are also evident in the highlighted TF output image. Where we see very light, almost white, orange colored highlights in the top left part of the galaxy and very dark an faint orange colored highlights on the right side of the galaxy. The different shades of orange colors of this TF output image is very strong evidence of bright and diffuse residual substructures. All of which are the description of the Shedding classification scheme. Therefore, the TF output of this galaxy support the classification scheme Shedding. It is interesting to note that it is primarily the shade of the orange highlights that support this classification scheme and not the shape of the residual substructures.

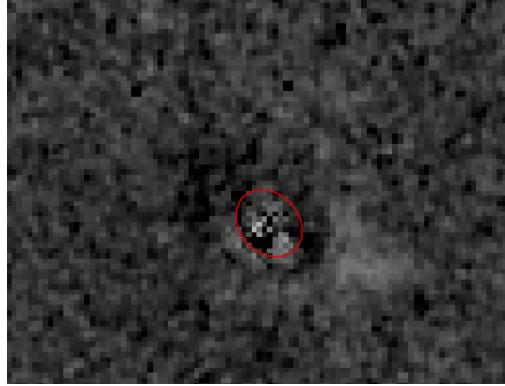


Figure 18: TF finder output: residual image of Splotches example

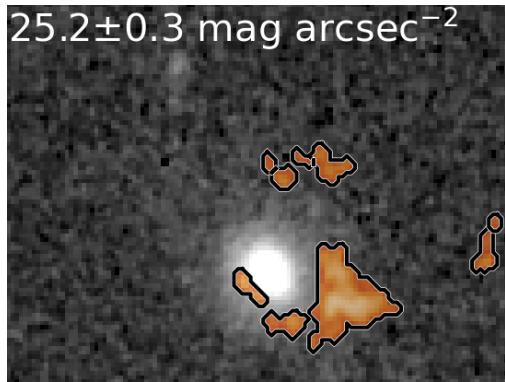


Figure 19: TF finder output: highlighted residual image of Splotches example

This is an example of the TF finder output for the fifth classification scheme: Splotches. As you can see the splotch pattern is located in the center of the red ellipse in figure 18. You may also notice some other fainter residual features throughout the image. In the highlighted TF image you can see that the fainter features were captured and highlighted. Where the splotch pattern in the center of the residual image was ignored. This is solely because it is located inside the red ellipse. If we were to run the TF finder code again on this galaxy without a limit on where the tidal features should be looked for, I believe the output of the TF code would include this center radial splotch pattern. At the present however, the outputs of the TF code do not support the fifth classification scheme Splotches. Just because the first run of this code does not support this classification scheme I don't think we should exclude it at the moment. There definitely is a recognizable pattern of this classification scheme that should be analyzed in future work.

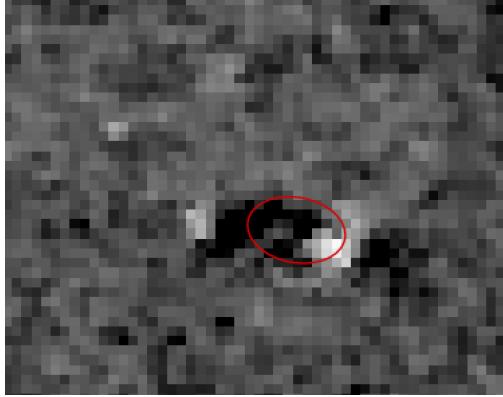


Figure 20: TF finder output: residual image of Two dots example

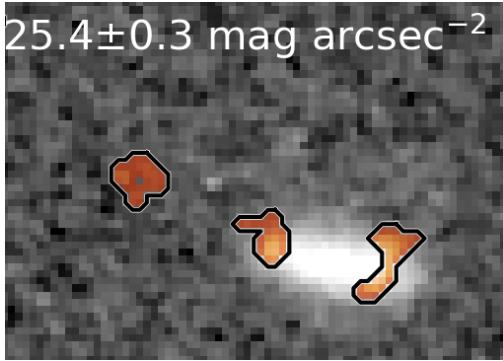


Figure 21: TF finder output: highlighted residual image of Two dots example

This is an example of the TF finder output for the sixth classification scheme: Two dots. In Figure 20 you can see two prominent bright sources of light. This is similarly captured in the highlighted TF image where you can see two orange highlights on opposite sides of the galaxy. The left highlight is a better representation of a dot or circle than the right highlight. This is due to the location of the right source on the red ellipse that tells the TF code to ignore this section. Just as in the Splotches example, if we were to rerun the TF code on this galaxy without the red ellipse to ignore certain sections, I believe the right highlight would be more representative of a dot or circle. While a multitude of good examples are needed to clearly justify a classification scheme from the TF finder code, this example is in moderate support of the classification scheme Two dots.

The highlighted residual features of the galaxies from running the TF finder code was successful in helping to justify the classification schemes and quite possibly throw out some the classification schemes. Of the 6 classification schemes devised in this analysis, at least 3 of them (1- Negligible, 2- Outer layer and 4- Shedding) were strongly supported by the results of the TF finder code. The classification scheme Two dots was moderately supported by the results of the TF finder code, however through more examples of the TF finder outputs, we could come to find that the classification scheme Two dots is strongly supported. The last two classification schemes, Center/behind and Splotches, were not supported by the results of the TF finder code. This was the first of many test runs on these examples to try and

justify the 6 classification schemes. More runs of this code along with tweaking some of the attributes (e.g. allowing the code to search for tidal features in the central regions) of the code should be done to get a better sense of the classifications schemes that should be thrown out. Nevertheless, 3 of the 6 classifications schemes were strongly justified.

## 5 Results

### 5.1 Key Takeaways

The key takeaways from this analysis of residual images of galaxies falls into two categories. The first one is very general. Of all of the residual shapes, substructures and varying fluxes that these tidal features can have, they can be binned by their similarities in a moderate sized number of groupings. Whether these similarities are due to their shape, location or relative brightness they can be categorized in a relatively small number of groups. Here, I categorized the sample of 500 galaxy residual images into 6 classification schemes. This is good news for the future of the supervised approach for the automated ML software in Mantha's larger research project. The second category of the key takeaways is the justification of these classification schemes. Through running the TF finder code on these residual images and producing orange highlighted features of the residual substructures I found that 3 of the 6 classification schemes devised in this analysis were supported. The classification schemes supported were: 1- Negligible, 2- Outer Layer and 4- Shedding. The other 3 bins not supported will need more validation to be considered a ground truth for the residual ML software. This justification process is extremely important in fine tuning these classification schemes because a human based approach in creating these bins can be selective or biased in nature. Having a second nonhuman validation of these classification schemes will better help create more robust schemes.

## 6 Conclusion

In conclusion of this work, lets recap the procedures taken in this analysis. We took a sample of 500 galaxies derived from a larger sample of 10,000 galaxies with stellar mass  $> 10^{9.5} M_{\odot}$ , redshift between  $0.5 < z < 3$  and an H-band brightness  $< 24.5$ . This sample contained original images of each galaxy, the modeled Sérsic profile and the residual image of any substructures left over. With this sample we looked at them through a python Graphic User Interface (GUI) and characterized them based on universal commonalities between residual features. Through this process I created 6 classification schemes: 1- Negligible, 2- Outer layer, 3- Center/behind, 4- Shedding, 5- Splotches and 6- Two dots. After creating these bins I took the same sample of galaxies and ran them through a tidal feature (TF) finder code. This code outputs an orange highlighted area over top of the residual substructures in the image of the galaxy. Applying this validation approach to justify my classification schemes strongly supported 1- Negligible, 2- Outer Layer and 4- Shedding. The other classification schemes were not strongly supported by this validation approach. The classification schemes that were supported by the TF finder code can be used as ground truths for Mantha's residual ML software.

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