EX1

Saturday, May 11, 2024 7:56 PM

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 49.05 & -200 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ -3.5 \\ 8 \end{bmatrix} \quad x = \begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \end{bmatrix}$$

a). C_o: $n=3$

$$[B \;\; AB \;\; A^2B] = \begin{bmatrix} 0 & -3.5 & 700 \\ -3.5 & 700 & -140171.675 \\ 8 & 0 & 0 \end{bmatrix}$$

$\det(C_o) = 9806.9 \neq 0 \rightarrow \text{Rang}_o = 3 = n$

$\therefore$ Si es controlable.

$T_{es} \leq 2$ seg

$M_p \leq 5\%$

Sist. de 3er orden $\rightarrow$ Se usa el de 2do: $\quad s_{1,2} = -\sigma_d \pm \omega_d j$

$\sigma_d = \dfrac{4}{T_{es}}$ $\quad \omega_d = \dfrac{-\pi\sigma_d}{\ln(M_p)}$

$\sigma_d = 2$ $\quad \omega_d = 2.097$

$s_1 = -2 + 2.097j$

$s_2 = -2 - 2.097j$

$s_3 = -10$

$$\alpha(s) = (s + 2 - 2.097j)(s + 2 + 2.097j)(s + 10)$$

$$s^2 + 2s + \cancel{2.097j}s + 2s + 4 + \cancel{4.194j} - \cancel{2.097}s - \cancel{4.194j} + 4.3974$$

$$(s^2 + 4s + 8.3974)(s+10) = s^3 + 4s^2 + 8.3974s + 10s^2 + 40s + 83.974$$

$$\alpha(s) = s^3 + 14s^2 + 48.3974s + 83.974$$

$$\phi(A) = A^3 + 14A^2 + 48.3974A + 83.974I$$

$$K = [0\;0\;1]\, C_o^{-1}\, \phi(A)$$

$$K = [-125.67 \;\; 52.654 \;\; -0.214]$$

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b). $y = Cx$

$$C_1 = [1\;0\;0] \vee \overset{C_2}{[0\;1\;0]} \vee \overset{C_3}{[0\;0\;1]}$$

O: $\begin{bmatrix} C \\ CA \\ CA^2 \end{bmatrix}$

C_1: $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 49.05 & -200 & 0 \end{bmatrix}$ $\quad \det(C_1) = 0 \;\therefore$ No

C_2: $\begin{bmatrix} 0 & 1 & 0 \\ 49.05 & -200 & 0 \\ -9810 & 40049.05 & 0 \end{bmatrix}$ $\quad \det(C_2) = 0$

2.

$$\ddot{h} = g - \frac{k}{m}\left(\frac{I}{h}\right)^2$$

$k = 0.001$

$g = 9.81$

$m = 0.2$

$$0 = g - \frac{k}{mh^2} \cdot I^2 - \ddot{h} = P(\ddot{h}, h, I)$$

$\bar{h} = 0.01$ m

$x_1 = h - \bar{h}$

$u = I - \bar{I}$

$$P = P(\bar{h}, \cancel{\bar{\ddot{h}}}^{\,0}, \bar{I}) + \frac{\partial P}{\partial \ddot{h}}\Delta\ddot{h} + \frac{\partial P}{\partial h}\cdot\Delta h + \frac{\partial P}{\partial I}\cdot\Delta I$$

$\bar{h} = 0.01 \rightarrow \bar{\ddot{h}} = 0$

$$0 = g - \frac{k}{m}\left(\frac{\bar{I}}{0.01}\right)^2 \Rightarrow \sqrt{\left(\frac{gm}{k}\right)(0.01)^2} = \bar{I} = 0.442945 \text{ A}$$

$$0 = -\Delta\ddot{h} + -\frac{k}{m}(-2)\frac{(\bar{I})^2}{(\bar{h})^3}\cdot\Delta h + \left(\frac{-2k}{m(\bar{h})^2}\cdot\bar{I}\right)\Delta I$$

$$0 = -\Delta\ddot{h} + 1961.6\,\underbrace{\Delta h}_{x_1} - 44.29\,\underbrace{\Delta I}_{u}$$

a). Sea $x_2 = \Delta\dot{h} = \dot{x}_1$

$\dot{x}_2 = \Delta\ddot{h}$

$$\dot{x}_2 = +1961.6\,x_1 - 44.29u$$

b). E.E.

$\dot{x}_1 = x_2$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ +1961.6 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ -44.29 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix} u$$

$T_{es} = 1\,s$

$M_p = 1\%$

$\Rightarrow \quad \sigma_d = \frac{4}{1} = 4$

$w_d = 2.729$

$s_{1,2} = -4 \pm 2.729j$

$s_3 = -20$

$$((s+4)-2.729j)((s+4)+2.729j)$$

$$(s+4)^2 - (2.729j)^2 = s^2 + 8s + 16 + 7.447 = s^2 + 8s + 23.45 \rightarrow \alpha(s) = (s^2+8s+23.45)(s+20)$$

$$s^3 + 8s^2 + 23.45s + 20s^2 + 160s + 469$$

$$\alpha(s) = s^3 + 28s^2 + 183.45s + 469$$

Considere m=0.2Kg, g=9.81m/s², k=0.001. Se desea controlar la posición de la bola.

a) Linealizar el modelo dinámico alrededor de un punto de equilibrio $\bar{h} = 0.01$ m. Para ello deberá calcular $\bar{I}$. Considerar $x_1 = h - \bar{h}$, $x_2 = I - \bar{I}$. (2 puntos)

b) Diseñar un servosistema agregando integrador para controlar x_1 con las especificaciones de diseño $T_{es} = 1$; $M_p = 1\%$. (4 puntos)

c) Diseñar un observador de estados. (2 puntos)

d) Graficar el diagrama de simulación del servosistema con observador con la planta lineal. (2 puntos)

e) Graficar el diagrama de simulación del servosistema con la planta no lineal. Considerar un bloque de división y un bloque de elevación al cuadrado. (2 puntos)

Servosistema agregando integrador:

$$\begin{bmatrix} \dot{x}(t) \\ \dot{\xi}(t) \end{bmatrix} = \underbrace{\begin{bmatrix} A & 0 \\ -C & 0 \end{bmatrix}}_{AN}\begin{bmatrix} x(t) \\ \xi(t) \end{bmatrix} + \underbrace{\begin{bmatrix} B \\ 0 \end{bmatrix}}_{BN} u(t) + \underbrace{\begin{bmatrix} 0 \\ 1 \end{bmatrix}}_{B_r} r(t) \qquad y = Cx + D$$

$$AN = \begin{bmatrix} 0 & 1 & 0 \\ 1961.6 & 0 & 0 \\ -1 & 0 & 0 \end{bmatrix} \qquad BN = \begin{bmatrix} 0 \\ -44.29 \\ 0 \end{bmatrix} \qquad B_r = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

Usando Ackermann:

$$C_o = \begin{bmatrix} B_N & A_N B_N & A_N^2 B_N \end{bmatrix} = \begin{bmatrix} 0 & -44.29 & 0 \\ -44.29 & 0 & -86879.264 \\ 0 & 0 & 44.29 \end{bmatrix}$$

$\det(C_o) = -86879.44559 \neq 0 \rightarrow$ si es controlable

$$\alpha(s) = s^3 + 28s^2 + 183.45s + 469$$

$$\phi(A_N) = A^3 + 28A^2 + 183.45A + 469I$$

$$\hat{K} = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} C_o^{-1}\phi(A_N) = [\underbrace{-48.43 \quad -0.6322}_{K} \quad \underbrace{10.389}_{K_I}]$$

c). Observabilidad:

$s_{1,2} = -4 \pm 2.729j$

$s_3 = -20$

$$O = \begin{bmatrix} C \\ CA \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \rightarrow \det(O) = 1 \neq 0 \quad \therefore \text{ si es}$$

$$s_4 = 2s_1 = -8 + 5.458j \qquad s_5 = 2s_2 = -8 - 5.458j$$

$$\Rightarrow \alpha_e(s) = ((s+8) - 5.458j)((s+8) + 5.458j)$$

$$(s+8)^2 - (5.458j)^2 = s^2 + 16s + 64 + 29.79 = s^2 + 16s + 93.79$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ +1961.6 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ -44.29 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix} u$$

$$\alpha_e(A) = A^2 + 16A + 93.79I$$

Ackermann:

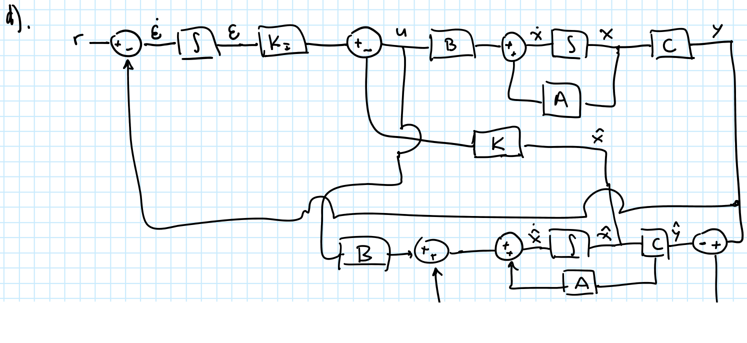
$Ax + Bu$

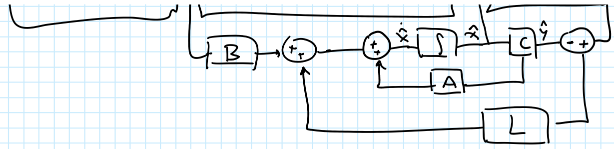
$$L = \alpha_e(A) \cdot Ob^{-1}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 16 \\ 2055.39 \end{bmatrix}$$

$-kx$

$\dot{x} = Ax + Bu$

d).

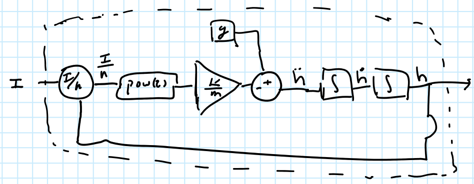




e).

Planta:

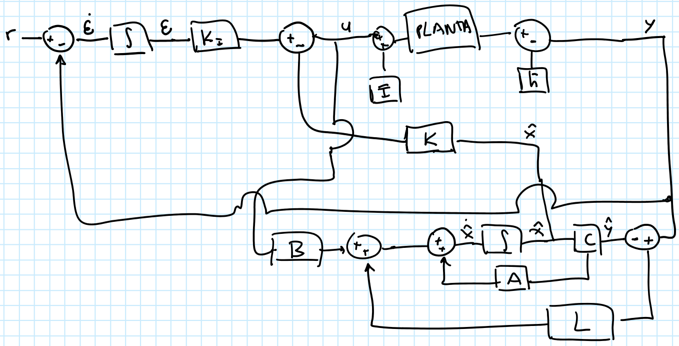
$\ddot{h} = g - \frac{k}{m}\left(\frac{I}{h}\right)^2$ I es la entrada



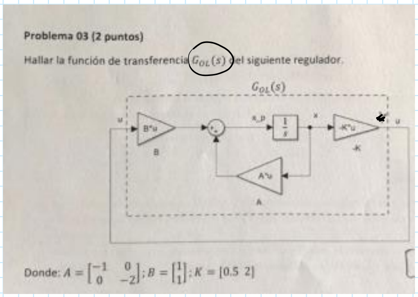
$u = I - \bar{I}$

$I = u + \bar{I}$

$y = h - \bar{h}$



3.



Problema 03 (2 puntos)

Hallar la función de transferencia $G_{OL}(s)$ del siguiente regulador.

Donde: $A = \begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix}; B = \begin{bmatrix} 1 \\ 1 \end{bmatrix}; K = [0.5\ \ 2]$

$$G_{OL}(s) = \frac{Y(s)}{U(s)} = 1$$

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} 1 \\ 1 \end{pmatrix} u$$

$$y = u = -KX = -[0.5\ \ 2]\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$\dot{X} = \begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix} X + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u$$

$$sX = AX + BU$$

$$sX - AX = BU$$

$$(sI - A)X = BU \rightarrow X = (sI-A)^{-1} \cdot BU$$

$$Y = -KX = -K(sI-A)^{-1}BU$$

$$\frac{Y}{U} = -K\underbrace{(sI-A)^{-1}}_{\begin{bmatrix} s+1 & 0 \\ 0 & s+2 \end{bmatrix}^{-1}} \cdot B = \underbrace{[-0.5\ \ -2]}_{1\times 2} \underbrace{\begin{bmatrix} \frac{1}{s+1} & 0 \\ 0 & \frac{1}{s+2} \end{bmatrix}}_{2\times 2} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \left[\frac{-0.5}{s+1}\ \ \frac{-2}{s+2}\right]\begin{bmatrix} 1 \\ 1 \end{bmatrix} = -\left(\frac{0.5}{s+1} + \frac{2}{s+2}\right) = -\left(\frac{0.5s + 1 + 2s + 2}{(s+1)(s+2)}\right) = \frac{-(2.5s + 3)}{s^2+3s+2} = \frac{-5s-6}{2s^2+6s+4}$$

$$(sI-A)^{-1} = \frac{adj(sI-A)}{|sI-A|} = \frac{\begin{bmatrix} s+2 & 0 \\ 0 & s+1 \end{bmatrix}}{(s+1)(s+2)} = \begin{bmatrix} \frac{1}{s+1} & 0 \\ 0 & \frac{1}{s+2} \end{bmatrix}$$