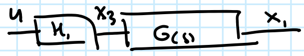
PC2-2017-1

Sunday, May 12, 2024 9:29 PM

$$G_1(s) = \frac{-0.125}{s^2+0.226\,s+0.02535}$$

$$G_2(s) = \frac{s+0.435}{s+1.23}$$

$$G(s) = \frac{-3.228}{10.5\,s^2+13.91\,s+1.23} \qquad H_1(s) = \frac{2}{s+2}$$



$$Y(s)\left[10.5\,s^2+13.91\,s+1.23\right] = -3.228\,X_3(s)$$

$$10.5\,\ddot{y} + 13.91\,\dot{y} + 1.23\,y = -3.228\,x_3$$

$$\frac{X_1}{X_3} = \frac{-3.228}{\ldots}$$

$$\left.\begin{aligned} y &= \theta_y(t) \\ x_1 &= \theta_y(t) = y \\ x_2 &= \dot{\theta}_y = \dot{y} = \dot{x}_1 \\ \ddot{y} &= \dot{x}_2 \end{aligned}\right\} \quad \begin{aligned} &10.5\,\dot{x}_2 + 13.91\,x_2 + 1.23\,x_1 = -3.228\,x_3 \\ &\dot{x}_2 = \frac{-1.23}{10.5}x_1 - \frac{13.91}{10.5}x_2 - 3.228\,x_3 \\ &\dot{x}_2 = -0.11714\,x_1 - 1.32476\,x_2 - 3.228\,x_3 \\ &\dot{x}_1 = x_2 \end{aligned}$$

$$\frac{x_3}{u} = \frac{2}{s+2} \rightarrow X_3(s+2) = 2U$$

$$\dot{x}_3 + 2x_3 = 2u$$

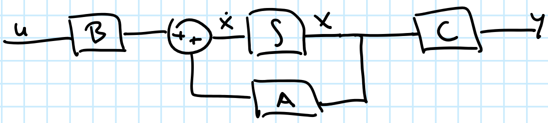
$$\dot{x}_3 = -2x_3 + 2u$$

a).

$$\begin{pmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\end{pmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ -0.11714 & -1.32476 & -3.228 \\ 0 & 0 & -2 \end{bmatrix}\begin{pmatrix}x_1\\ x_2\\ x_3\end{pmatrix} + \begin{bmatrix}0\\0\\2\end{bmatrix}u$$

$$y = \begin{bmatrix}1 & 0 & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix}$$

b).



c).

$$M_p = 10\% \qquad T_{ss} = 5\,s$$

$$\sigma_d \approx \frac{4}{T_{ss}} = \frac{4}{5} \qquad s_1 = -\frac{4}{5} + 1.0915j\,, \quad s_2 = -\frac{4}{5} - 1.0915j$$

$$w_d = \frac{-\pi\sigma_d}{\ln(M_p)} = \frac{-\pi\left(\frac{4}{5}\right)}{\ln(0.1)} = 1.0915$$

Tercer polo, no dominante:

$$s_3 = -4$$

$$\alpha(s) = \left(\left(s+\frac{4}{5}\right)-1.0915j\right)\left(\left(s+\frac{4}{5}\right)+1.0915j\right)(s+4)$$

$$\alpha(s) = \left[\left(s+\frac{4}{5}\right)^2 - (1.0915j)^2\right](s+4)$$

$$\alpha(s) = \left(s^2+\frac{8}{5}s+1.8314\right)(s+4) = s^3+\frac{8}{5}s^2+1.8314s+4s^2+\frac{32}{5}s+7.3256$$

$$\alpha(s) = s^3 + 5.6\,s^2 + 8.2314\,s + 7.3256 \quad \text{(deseado)}$$

$$A = \begin{bmatrix} 0 & 1 & 0 \\ -0.11714 & -1.32476 & -3.228 \\ 0 & 0 & -2 \end{bmatrix}$$

$$B = \begin{bmatrix}0\\0\\2\end{bmatrix}$$

$$\phi(A) = A^3 + 5.6\,A^2 + 8.2314\,A + 7.3256\,I$$

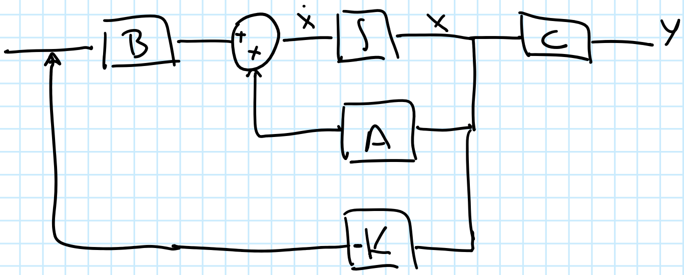
Matriz de control.

$$C_o = \begin{bmatrix} B & AB & A^2B \end{bmatrix} = \begin{bmatrix} 0 & 0 & -6.456 \\ 0 & -6.456 & 21.465 \\ 2 & -4 & 8 \end{bmatrix} \rightarrow \det(C_o) = -83.36 \neq 0$$

$\therefore$ Rango$(C_o) = 3 = n \rightarrow$ Controlable!

$$K = [0 \ 0 \ 1] C_o^{-1} \phi(A) = [-1.057 \quad -0.3796 \quad 1.13762]$$

d).



P2).

$n=3$

a). $C_o = [B \quad AB \quad A^2B]$

$$A = \begin{bmatrix} -0.9 & 0.6 & 0 \\ 1 & -2.1 & 0 \\ 1 & 1.5 & -1.1 \end{bmatrix}_{3\times3} \qquad B = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \\ 0 & 0 \end{bmatrix}_{3\times2}$$

$$C_o: \begin{bmatrix} 0.5 & 0 & -0.45 & 0.3 & 0.705 & -0.9 \\ 0 & 0.5 & 0.5 & -1.05 & -1.5 & 2.505 \\ 0 & 0 & 0.5 & 0.75 & -0.25 & -2.1 \end{bmatrix}$$

Se toma una sub matriz 3x3:

$$\det\left(\begin{bmatrix} 0.5 & 0 & -0.45 \\ 0 & 0.5 & 0.5 \\ 0 & 0 & 0.5 \end{bmatrix}\right) = 0.125 \neq 0 \rightarrow \text{Rango}(C_o) = 3$$

b). Sin q_2:

$$B = \begin{bmatrix} 0.5 \\ 0 \\ 0 \end{bmatrix} \Rightarrow C_o = \begin{bmatrix} 0.5 & -0.45 & 0.705 \\ 0 & 0.5 & -1.5 \\ 0 & 0.5 & -0.25 \end{bmatrix} \rightarrow \det(C) = 0.3125 \neq 0$$

Si es.

c). Sin q_1:

$$B = \begin{bmatrix} 0 \\ 0.5 \\ 0 \end{bmatrix} \Rightarrow C_o = \begin{bmatrix} 0 & 0.3 & -0.9 \\ 0.5 & -1.05 & 2.505 \\ 0 & 0.75 & -2.1 \end{bmatrix} = -0.0225 \neq 0$$

$$C = \begin{bmatrix} -0.9 & 0.6 & 0 \\ 1 & -2.1 & 0 \\ 1 & 1.5 & -1.1 \end{bmatrix}$$

∴ Controlable

d). Obs:

$$O = \begin{bmatrix} C \\ CA \\ CA^2 \end{bmatrix} = \begin{bmatrix} -0.9 & 0.6 & 0 \\ 1 & -2.1 & 0 \\ 1 & 1.5 & -1.1 \\ & & \\ & & \end{bmatrix}$$

$\det(O) \neq 0 \rightarrow$ Obs

e). Solo midiendo q_1' y q_3':

$$C = \begin{bmatrix} 1.5 & 0 & 0 \\ 0 & 0 & 1.4 \end{bmatrix} \qquad Obs: \begin{bmatrix} 1.5 & 0 & 0 \\ 0 & 0 & 1.4 \\ -1.35 & 0.9 & 0 \\ 1.4 & 2.1 & -1.54 \end{bmatrix} \rightarrow \det(-1.85) \neq 0 \checkmark$$

$$C = \begin{bmatrix} 1.5 & 0 & 0 \\ 0 & 0 & 1.4 \end{bmatrix}$$

$$\begin{bmatrix} q_{x_1} \\ q_{x_3} \end{bmatrix} = C \begin{bmatrix} h_1 \\ h_3 \end{bmatrix}$$

Obs:

$$\begin{bmatrix} 1.5 & 0 & 0 \\ 0 & 0 & 1.4 \\ -1.35 & 0.9 & 0 \\ 1.4 & 2.1 & -1.54 \end{bmatrix}$$

$\rightarrow \det(-1.85) \neq 0$ ✓