Homework should be written neatly and clearly explained.

**Problem 1**
Let $X_1$, $X_2$, $X_3$, ... be independent random variables uniformly distributed on the interval $[0, 2]$.

1. What are $\mu$, the mean of $X_1$, and $\sigma^2$, the variance of $X_1$?

2. Simulate $\frac{1}{100} \sum_{i=1}^{100} \frac{X_i - \mu}{\sigma}$, 10000 times and give a histogram of the results.

3. Simulate $Y_n = \frac{1}{n} \sum_{i=1}^{n} \frac{X_i - \mu}{\sigma}$ for various $n$ and plot the histograms. What happens with the histogram of $Y_n$ when $n$ grows?

   If we want the convergence in distribution, then we need to scale $Y_n$, that is, we should multiply $Y_n$ by some $a_n$. What $a_n$ should be chosen so that $Z_n = a_n Y_n$ converge in distribution to a standard normal random variable? Plot the histogram of $Z_n$ for $n = 1000$.

*Solution:*