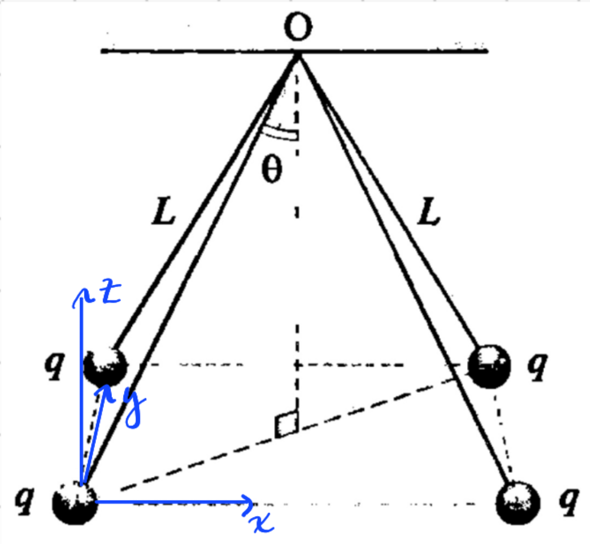
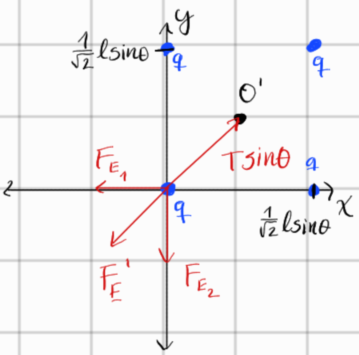
Preparcial 3



Como es un sistema en equilibrio y las cargas y longitudes de los hilos son iguales. Entonces, la $\Sigma \vec{F}_i = 0$ para cualquiera de las cargas es la misma para todas.

Plano xy



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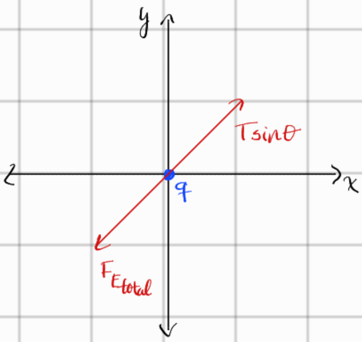
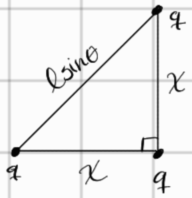




$$l^2\sin^2\theta = 2x^2$$

$$x = \frac{1}{\sqrt{2}} l\sin\theta$$

$$F_{E_1} = F_{E_2} = \frac{kq^2}{\frac{1}{2} l^2\sin^2\theta} = \frac{2kq^2}{l^2\sin^2\theta}$$

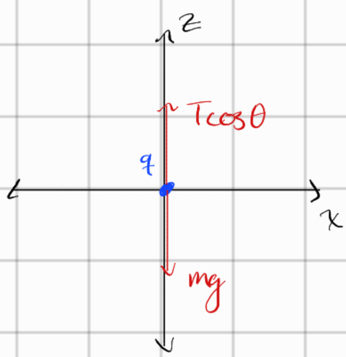
$$F_E' = \frac{kq^2}{l^2\sin^2\theta} = \frac{1}{2} F_{E_1}$$

$$\left| F_{E_1}(-\hat{\imath}) + F_{E_1}(-\hat{\jmath}) \right| = \sqrt{2F_{E_1}^2} = \sqrt{2} F_{E_1}$$

Como $\vec{F}_{E_1} + \vec{F}_{E_2}$ queda en la misma dirección que $\vec{F}_E'$, entonces sumamos sus magnitudes.

$$F_{E_{total}} = \sqrt{2} F_{E_1} + \frac{1}{2} F_{E_1} = \frac{2\sqrt{2}+1}{2} F_{E_1}$$

Plano xz



$$T\cos\theta = mg$$

$$T = \frac{mg}{\cos\theta} \quad ②$$

$$\Sigma F_i = 0: \quad T\sin\theta = \frac{2\sqrt{2}+1}{2} F_{E_1}$$

$$T = \frac{2\sqrt{2}+1}{2\sin\theta} F_{E_1} = \frac{2\sqrt{2}+1}{\cancel{2}\sin\theta}\left(\frac{\cancel{2}kq^2}{l^2\sin^2\theta}\right) = \frac{(2\sqrt{2}+1)\,kq^2}{l^2\sin^3\theta} \quad ①$$

①-② $$0=\frac{(2\sqrt{2}+1)kq^2}{\ell^2\sin^3\theta}-\frac{mg}{\cos\theta}$$

$$\frac{\sin^3\theta}{\cos\theta}=\frac{(2\sqrt{2}+1)kq^2}{\ell^2 mg}$$

$$\frac{\sin^6\theta}{\cos^2\theta}=\left(\frac{(2\sqrt{2}+1)kq^2}{\ell^2 mg}\right)^2$$

$$\frac{\sin^6\theta}{(1-\sin^2\theta)}=\left(\frac{(2\sqrt{2}+1)kq^2}{\ell^2 mg}\right)^2$$

$$\sin^6\theta=\left(\frac{(2\sqrt{2}+1)kq^2}{\ell^2 mg}\right)^2(1-\sin^2\theta)$$

$$\sin^6\theta=\left(\frac{(2\sqrt{2}+1)kq^2}{\ell^2 mg}\right)^2-\left(\frac{(2\sqrt{2}+1)kq^2}{\ell^2 mg}\right)^2\sin^2\theta$$

$$\sin^6\theta+\left(\frac{(2\sqrt{2}+1)kq^2}{\ell^2 mg}\right)^2\sin^2\theta-\left(\frac{(2\sqrt{2}+1)kq^2}{\ell^2 mg}\right)^2=0$$

$u=\sin\theta \rightarrow$ $$u^6+\left(\frac{(2\sqrt{2}+1)kq^2}{\ell^2 mg}\right)^2u^2-\left(\frac{(2\sqrt{2}+1)kq^2}{\ell^2 mg}\right)=0, \quad \boxed{\theta=\sin^{-1}(u)}$$

21. *Calculo de raíces en física:* Cuatro esferas de pesos iguales $w=114.6\ N$ y cargas iguales $q=3\times10^{-4}\ C$ se encuentran en los extremos de hilos inelásticos y aislantes de longitudes $L=5\ m$. Los que a su vez se encuentran unidos en $\mathcal{O}$. Para la aplicación numérica use $g=10\ m/s^2$ (Tomado de [5]).

$$u^6+\left(\frac{(2\sqrt{2}+1)kq^2}{\ell^2 mg}\right)^2u^2-\left(\frac{(2\sqrt{2}+1)kq^2}{\ell^2 mg}\right)^2=0$$

$$u^6+\left(\frac{(2\sqrt{2}+1)\,9\times10^9\cancel{N}\cancel{m^2}\cancel{C^{-2}}\,(3\times10^{-4}\cancel{C})^2}{(5\cancel{m})^2\ 114{,}6\cancel{N}}\right)^2u^2-\left(\frac{(2\sqrt{2}+1)\,9\times10^9\cancel{N}\cancel{m^2}\cancel{C^{-2}}\,(3\times10^{-4}\cancel{C})^2}{(5\cancel{m})^2\ 114{,}6\cancel{N}}\right)^2=0$$

$$u^6+\left(\frac{(2\sqrt{2}+1)270}{191}\right)^2u^2-\left(\frac{(2\sqrt{2}+1)270}{191}\right)^2=0$$

$$u^6+\frac{(9+4\sqrt{2})72900}{36481}u^2-\frac{(9+4\sqrt{2})72900}{36481}=0, \quad \boxed{u=\sin\theta,\ \theta=\sin^{-1}(u)}$$

Raices del polinomio

$u =$
```
[(-0.8323421, -0.0),
 (-0.6721389, -0.8934022),
 (-0.6721389, 0.8934022),
 (0.6721389, -0.8934022),
 (0.6721389, 0.8934022),
 (0.8323421, 0.0)]
```

Como $0 \leq \theta \leq \pi/2$, $\theta \in \mathbb{R}$. Entonces escogenos el valor real positivo de u.

$\theta = \sin^{-1}(0{,}8323)$

$\theta \approx 0{,}99 \text{ rad} \approx 56{,}34^{\circ}$