Problemas parcial 2.

29-09-23

1. $$I=\int_a^b f(x)dx \approx \int_a^b P_1(x)dx = \int_a^b \frac{x-b}{a-b}f(a)+\frac{x-a}{b-a}f(b)\,dx = \int_a^b \frac{(-x+b)f(a)+(x-a)f(b)}{b-a}dx = \frac{1}{b-a}\left[f(a)\left(-\int_a^b x\,dx+\int_a^b b\,dx\right)+f(b)\left(\int_a^b x\,dx-\int_a^b a\,dx\right)\right]$$

$$=\frac{1}{b-a}\left[f(a)\left(-\left(\frac{x^2}{2}\Big|_a^b\right)+bx\Big|_a^b\right)+f(b)\left(\frac{x^2}{2}\Big|_a^b-ax\Big|_a^b\right)\right]=\frac{1}{b-a}\left[f(a)\left(\frac{(b-a)^2}{2}\right)+f(b)\left(\frac{(b-a)^2}{2}\right)\right]=\frac{(b-a)^{\cancel{2}}}{2\cancel{(b-a)}}(f(a)+f(b))$$

$$I=\int_a^b f(x)\,dx \approx \int_a^b P_1(x)dx = \frac{b-a}{2}\left(f(a)+f(b)\right)$$ //

3. $$x_m=\frac{b+a}{2},\quad h=\frac{b-a}{2}$$

$$-\int_a^b \frac{(x-b)(x-x_m)}{(a-b)(a-x_m)}f(a)dx=\frac{f(a)}{(a-b)(a-x_m)}\int_a^b x^2-\frac{3bx}{2}-\frac{ax}{2}+\frac{b^2}{2}+\frac{ba}{2}\,dx=\frac{f(a)}{(a-b)(a-x_m)}\left(\frac{x^3}{3}-\frac{3bx^2}{4}-\frac{ax^2}{4}+\frac{b^2x}{2}+\frac{bax}{2}\right)\Big|_a^b$$

$$=\frac{f(a)}{(-2h)(-h)}\left(\frac{b^3}{3}-\frac{3b^3}{4}-\frac{ab^2}{4}+\frac{b^3}{2}+\cancel{\frac{b^2a}{2}}-\frac{a^3}{3}+\frac{3a^2b}{4}+\frac{a^3}{4}-\cancel{\frac{ab^2}{2}}-\frac{ba^2}{2}\right)$$

$$=\frac{f(a)}{2h^2}\left(\frac{-a^3}{12}+\frac{3a^2b}{12}-\frac{3ab^2}{12}+\frac{b^3}{12}\right)=\frac{f(a)}{2h^2}\left(\frac{(b-a)^3}{12}\right)=\frac{f(a)}{2h^2}\left(\frac{(2h)^3}{12}\right)=\frac{f(a)}{2h^{\cancel{2}}}\left(\frac{8h^{\cancel{3}}}{12}\right)$$

$$=\frac{h}{3}f(a)$$

$$-\int_a^b \frac{(x-a)(x-b)}{(x_m-a)(x_m-b)}f(x_m)\,dx=\frac{f(x_m)}{(x_m-a)(x_m-b)}\int_a^b (x-a)(x-b)\,dx=\frac{f(x_m)}{\left(\frac{b}{2}-\frac{3a}{2}\right)\left(-\frac{b}{2}-\frac{a}{2}\right)}\int_a^b x^2-xb-ax+ab\,dx$$

$$=\frac{f(x_m)}{\left(\frac{b}{2}-\frac{a}{2}\right)\left(-\frac{b}{2}+\frac{a}{2}\right)}\left(\frac{x^3}{3}-\frac{bx^2}{2}-\frac{ax^2}{2}+abx\right)\Big|_a^b=\frac{f(x_m)}{(h)(-h)}\left(\frac{b^3}{3}-\frac{b^3}{2}-\frac{ab^2}{2}+ab^2-\frac{a^3}{3}+\frac{a^2b}{2}+\frac{a^3}{2}-a^2b\right)$$

$$=\frac{f(x_m)}{-h^2}\left(\frac{-b^3}{6}+\frac{3ab^2}{6}-\frac{3a^2b}{6}+\frac{a^3}{6}\right)=\frac{f(x_m)}{-h^2}\left(\frac{-(2h)^3}{6}\right)=\frac{f(x_m)}{-h^2}\left(\frac{-8h^3}{6}\right)$$

$$=\frac{h}{3}\left(4f(x_m)\right)$$

$$-\int_a^b \frac{(x-a)(x-x_m)}{(b-a)(b-x_m)} f(b)\,dx = \frac{f(b)}{(b-a)(b-x_m)}\int_a^b (x-a)(x-x_m)\,dx = \frac{f(b)}{(2h)(h)}\int_a^b x^2 - \frac{bx}{2} - \frac{ax}{2} - ax + \frac{ab}{2} + \frac{a^2}{2}\,dx$$

$$= \frac{f(b)}{2h^2}\left(\frac{x^3}{3} - \frac{bx^2}{4} - \frac{3ax^2}{4} + \frac{abx}{2} + \frac{a^2x}{2}\right)\Bigg|_a^b = \frac{f(b)}{2h^2}\left(\frac{b^3}{3} - \frac{b^3}{4} - \frac{3ab^2}{4} + \frac{ab^2}{2} + \cancel{\frac{a^2b}{2}} - \frac{a^3}{3} + \frac{a^2b}{4} + \frac{3a^3}{4} - \cancel{\frac{a^2b}{2}} - \frac{a^3}{2}\right)$$

$$= \frac{f(b)}{2h^2}\left(\frac{b^3}{12} - \frac{3ab^2}{12} + \frac{3a^2b}{12} - \frac{a^3}{12}\right) = \frac{f(b)}{2h^2}\left(\frac{(2h)^3}{12}\right) = \frac{f(b)}{2\cancel{h^2}}\left(\frac{8h^3}{12}\right)$$

$$= \frac{h}{3} f(b)$$

$$I = \int_a^b f(x)\,dx \cong \int_a^b P_2(x)\,dx = \int_a^b \frac{(x-b)(x-x_m)}{(a-b)(a-x_m)} f(a)\,dx + \int_a^b \frac{(x-a)(x-b)}{(x_m-a)(x_m-b)} f(x_m)\,dx + \int_a^b \frac{(x-a)(x-x_m)}{(b-a)(b-x_m)} f(b)\,dx$$

$$= \frac{h}{3} f(a) + \frac{h}{3}(4f(x_m)) + \frac{h}{3} f(b)$$

$$I = \int_a^b f(x)\,dx \cong \int_a^b P_2(x)\,dx = \frac{h}{3}\left(f(a) + 4f(x_m) + f(b)\right) \; //$$