Ejercicio 1 Álgebra Lineal

$x_{n+1} = 4x_n - x_n^2$; $x_0 = 4\sin^2\theta$, $x_{n+1} = 4\sin^2(2^{n+1}\theta)$

$$\boxed{\begin{array}{l} * \ \sin(2\theta) = 2\sin\theta\cos\theta \\ \sin^2(2\theta) = 4\sin^2\theta\cos^2\theta \end{array}}$$

$x_1 = 4(4\sin^2\theta) - (4\sin^2\theta)^2$

$x_1 = 16\sin^2\theta - 16\sin^4\theta$

$\Rightarrow \quad 16\sin^2\theta(1-\sin^2\theta)$

$16\sin^2\theta\cos^2\theta$
$(4\cdot 4)$

$= 4\sin^2(2\theta) \longrightarrow$ Se obtiene lo que se esperaba.

Generalizando lo anterior

$x_{n+1} = 4(4\sin^2(2^n\theta)$

$= 16\sin^4(2^n\theta)$

$= 16\sin^2(2^n\theta)\cos^2(2^n\theta)$

$= 4\sin^2(2\,2^n\theta)$

$x_{n+1} = 4\sin^2(2^{n+1}\theta)$

Ejercicio 5 de Álgebra Lineal

$$x_i = \frac{b_i - \sum_{j=i+1}^{n} A_{ij}x_j}{A_{ii}} \qquad ; i = n, n-1, \ldots, 0$$

A es una matriz triangular superior

$$\begin{pmatrix} A_{11}x_1 + A_{12}x_2 + A_{13}x_3 + \cdots + A_{1n}x_n \\ 0 + A_{22}x_2 + \cdots\cdots + A_{2n}x_n \\ \vdots \qquad \vdots \qquad \vdots \qquad \vdots \\ 0 + \cdots \cdots \ddots \cdots + A_{nn}x_n \end{pmatrix} = \begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{pmatrix}$$

$$x_n = \frac{b_n}{A_{nn}} \qquad\qquad x_{n-2} = \frac{b_{n-2} - A_{n-1,n-1}x_{n-1} - A_{n-1,n}x_n}{A_{n-2,n-2}}$$

$$x_{n-1} = \frac{b_{n-1} - A_{n-1,n}x_n}{A_{n-1,n-1}} \qquad \Rightarrow x_i = \frac{b_i - \sum_{j=i+1}^{n} A_{ij}x_j}{A_{ii}}$$