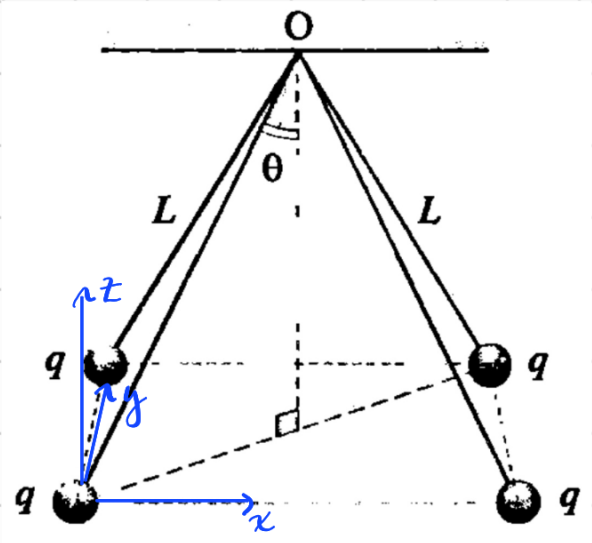
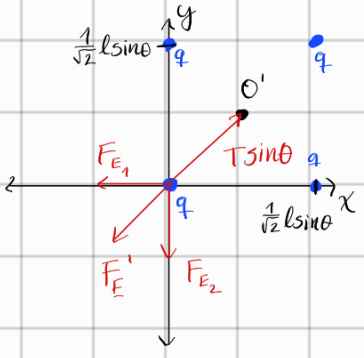
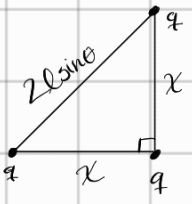
Preparcial 3



Como es un sistema en equilibrio y las cargas y longitudes de los hilos son iguales. Entonces, la $\sum \vec{F}_i = 0$ para cualquiera de las cargas es la misma para todas.

Plano xy





$4\ell^2\sin^2\theta = 2x^2$

$x = \sqrt{2}\ell\sin\theta$

$$F_{E_1} = F_{E_2} = \frac{kq^2}{2\ell^2\sin^2\theta} = \frac{kq^2}{2\ell^2\sin^2\theta}$$

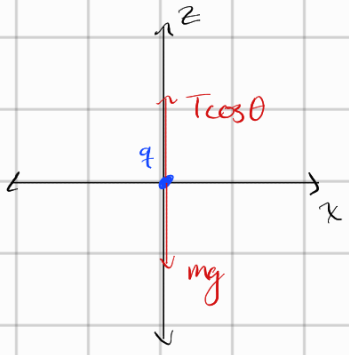
$$F_E' = \frac{kq^2}{4\ell^2\sin^2\theta} = \frac{1}{2}F_e$$

$$\left|F_{E_1}(-\hat{\imath}) + F_{E_1}(-\hat{\jmath})\right| = \sqrt{2F_{E_1}^2} = \sqrt{2}F_{E_1}$$

Como $\vec{F}_{E_1} + \vec{F}_{E_2}$ queda en la misma dirección que $\vec{F}_E'$, entonces sumamos sus magnitudes.

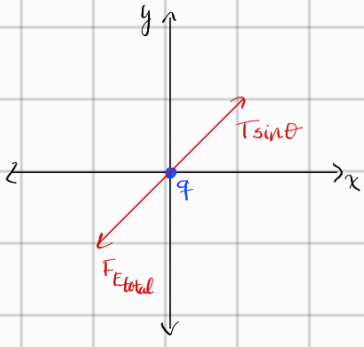
$$F_{E_{total}} = \sqrt{2}F_{E_1} + \frac{1}{2}F_{E_1} = \left(\frac{2\sqrt{2}+1}{2}\right)F_{E_1}$$

Plano xz



$T\cos\theta = mg$

$T = \frac{mg}{\cos\theta}$ (2)



$\sum F_i = 0$: $T\sin\theta = \frac{2\sqrt{2}+1}{2}F_{E_1}$

$$T = \frac{2\sqrt{2}+1}{2\sin\theta}F_{E_1} = \frac{2\sqrt{2}+1}{2\sin\theta}\left(\frac{kq^2}{2\ell^2\sin^2\theta}\right) = \frac{(2\sqrt{2}+1)kq^2}{4\ell^2\sin^3\theta}$$ (1)

① − ② $$0 = \frac{(2\sqrt{2}+1)\,kq^2}{4\ell^2 \sin^3\theta} - \frac{mg}{\cos\theta}$$

$$\frac{\sin^3\theta}{\cos\theta} = \frac{(2\sqrt{2}+1)\,kq^2}{4\ell^2 mg}$$

$$\frac{\sin^6\theta}{\cos^2\theta} = \left(\frac{(2\sqrt{2}+1)\,kq^2}{4\ell^2 mg}\right)^2$$

$$\frac{\sin^6\theta}{(1-\sin^2\theta)} = \left(\frac{(2\sqrt{2}+1)\,kq^2}{4\ell^2 mg}\right)^2$$

$$\sin^6\theta = \left(\frac{(2\sqrt{2}+1)\,kq^2}{4\ell^2 mg}\right)^2 (1-\sin^2\theta)$$

$$\sin^6\theta = \left(\frac{(2\sqrt{2}+1)\,kq^2}{4\ell^2 mg}\right)^2 - \left(\frac{(2\sqrt{2}+1)\,kq^2}{4\ell^2 mg}\right)^2 \sin^2\theta$$

$$\sin^6\theta + \left(\frac{(2\sqrt{2}+1)\,kq^2}{4\ell^2 mg}\right)^2 \sin^2\theta - \left(\frac{(2\sqrt{2}+1)\,kq^2}{4\ell^2 mg}\right)^2 = 0$$

$u = \sin\theta \rightarrow$ $$u^6 + \left(\frac{(2\sqrt{2}+1)\,kq^2}{4\ell^2 mg}\right)^2 u^2 - \left(\frac{(2\sqrt{2}+1)\,kq^2}{4\ell^2 mg}\right)^2 = 0, \qquad \boxed{\theta = \sin^{-1}(u)}$$

21. *Calculo de raíces en física:* Cuatro esferas de pesos iguales $w = 114.6\ N$ y cargas iguales $q = 3\times10^{-4}\ C$ se encuentran en los extremos de hilos inelásticos y aislantes de longitudes $L = 5\ m$. Los que a su vez se encuentran unidos en $\mathcal{O}$. Para la aplicación numérica use $g = 10\ m/s^2$ (Tomado de [5]).

$$u^6 + \left(\frac{(2\sqrt{2}+1)\,kq^2}{4\ell^2 mg}\right)^2 u^2 - \left(\frac{(2\sqrt{2}+1)\,kq^2}{4\ell^2 mg}\right)^2 = 0$$

$$u^6 + \left(\frac{(2\sqrt{2}+1)\,9\times10^9\,\cancel{N}\,\cancel{m^2}\,\cancel{C^{-2}}\,(3\times10^{-4}\,\cancel{C})^2}{4\cdot(5\,\cancel{m})^2\,114{,}6\,\cancel{N}}\right)^2 u^2 - \left(\frac{(2\sqrt{2}+1)\,9\times10^9\,\cancel{N}\,\cancel{m^2}\,\cancel{C^{-2}}\,(3\times10^{-4}\,\cancel{C})^2}{4\cdot(5\,\cancel{m})^2\,114{,}6\,\cancel{N}}\right)^2 = 0$$

$$u^6 + \left(\frac{(2\sqrt{2}+1)\,27}{382}\right)^2 u^2 - \left(\frac{(2\sqrt{2}+1)\,27}{382}\right)^2 = 0$$

$$u^6 + \frac{(9+4\sqrt{2})\,729}{145\,924}\,u^2 - \frac{(9+4\sqrt{2})\,729}{145\,924} = 0, \qquad \boxed{u = \sin\theta,\ \theta = \sin^{-1}(u)}$$

Raices del polinomio

```
u= [(-0.6003712, -0.0),
 (-0.3677575, -0.5616657),
 (-0.3677575, 0.5616657),
 (0.3677575, -0.5616657),
 (0.3677575, 0.5616657),
 (0.6003712, -0.0)]
```

Como $0 \leq \theta \leq \pi/2$, $\theta \in \mathbb{R}$. Entonces escogemos el valor real positivo de u.

$\theta = \sin^{-1}(0,6)$

$\theta \approx 37°$ //