code

January 11, 2018

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In [1]: using PyPlot
        =1
        =3
        iteration=10<sup>4</sup>
        N = 10
        M=N
Out[1]: 10
In [2]: # compute, to how big error step with omega size would lead
        function errorOmega(x, xavr, xopt, omega)
            return norm( ((1-omega)*x + omega*xavr) - xopt )
        end
Out[2]: errorOmega (generic function with 1 method)
In [3]: # compute size of the best for particular point x
        # starts wih interval, where to look for , splits it into 3 parts, throw away wrong part
        # suppose, that error is unimodal function
        function getBestOmega(x, xavr, xopt, lower, upper, accuracy)
            while upper-lower>accuracy
                middle1=(2*lower+upper)/3
                middle2=(lower+2*upper)/3
                if errorOmega(x, xavr, xopt, middle1) > errorOmega(x, xavr, xopt, middle2)
                    lower=middle1
                else
                    upper=middle2
                end
            end
            return (lower+upper)/2
        end
Out[3]: getBestOmega (generic function with 1 method)
In [4]: # get a random vector of size M+1 with ones
        function setS()
            S=zeros(M+1,1)
```

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for i=1:
                r=rand(1:M+1)
                while S[r]==1
                    r=rand(1:M+1)
                end
                S[r]=1
            end
            return S
        end
Out[4]: setS (generic function with 1 method)
In [5]: # compute average of projections
        function projection(A, b, x)
            S=setS()
            xsum=x*0
            for coord=1:M
                if S[coord] == 1
                    xsum += x - A[coord,:].*(A[coord,:]'*x - b[coord])/(A[coord,:]A[coord,:])
                end
            end
            if S[M+1] == 1
                # project on ball
                if norm(x) > 10
                    xsum += 10*x/norm(x)
                else
                    xsum += x
                end
            end
            return xsum/
        end
Out[5]: projection (generic function with 1 method)
In [6]: # randomly initialize variables
        A=randn(M,N)
        b=randn(M)
        x0=1000*rand(M)
        println("norm xopt: ", norm(A\b))
        println("norm x0: ", norm(x0))
norm xopt: 2.567477664141803
norm x0: 1784.3656339772504
In [7]: # find projection to measure distances (same algorithm runned 10x longer)
        xopt=x0
        for t=1:10*iteration
            xproj = projection(A,b,xopt)
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xopt = (1-)*xopt + *xproj
        end
        print("xopt:", norm(A*xopt-b))
xopt:2.9748540179341104e-15
In [8]: # run algorithm and save variables to be plotted
        dist=zeros(iteration)
        bestOmega=zeros(iteration)
        x=x0
        for t=1:iteration
            dist[t] = norm(x-xopt)
            xproj = projection(A,b,x)
            bestOmega[t] = getBestOmega(x, xproj, xopt, 0, 1000, 1/10^10)
            x = (1-)*x + *xproj # project on
            \# x = (1-best0mega[t])*x + best0mega[t]*xproj \# project on best omega
        end
In [9]: # plot variables
        semilogy(dist)
        v = eigvals(A'*A)
        min = minimum(v[v.>10.0^(-13)]) # for us it is not zero, but for comp it is
        rate = 1 - min/sum(v)
        semilogy(1:iteration,dist[1]*rate.^(1:iteration))
        scatter(1:iteration,bestOmega, s=0.1, color="grey", alpha=0.5)
        10^{3}
        10<sup>0</sup>
       10^{-3}
       10^{-6}
       10^{-9}
      10^{-12}
```

6000

8000

10000

4000

0

2000

Out[9]: PyObject <matplotlib.collections.PathCollection object at 0x1228dfd90>