25 1000digitFibonacciNumber

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1 Problem

What is the index of the first term in the Fibonacci sequence to contain 1000 digits?

2 Solution

Using linear algebra one can deduce that the generic formula for a Fibonacci number is as follows:

$$F(n) = [\frac{\phi^n}{\sqrt{5}}]$$

where the square brackets denotes the closest integer. You can actually use $F(n) = \frac{\phi^n}{\sqrt{5}}$ to a good approximation for large n. So we set about solving $\frac{\phi^n}{\sqrt{5}} = 10^{1000}$ to yield $n = \lceil \log_\phi \sqrt{5} + 999 log_\phi 10 \rceil = 4782$