27 QuadraticPrimes

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1 The Problem

Considering quadratics of the form: $n^2 + an + b$ where $|a| < 1000$ and $|b| < 1000$ find the product of the coefficients, a and b, for the quadratic expression that produces the maximum number of primes for consecutive values of n, starting with $n = 0$.

2 Reducing the search space

- $n = 0 \Rightarrow n^2 + an + b = b$, so we need that b is prime (and > 0)

- $n = 1 \Rightarrow n^2 + an + b = 1 + a + b$, so, given that b is prime, unless $b = 2$ we need a to be odd (because then that would make $1 + a + b$ odd, which we want because it's meant to be prime)

- $n^2 + an + b = n(n + a) + b$ which is a multiple of b if n is a multiple of b, so we have incentive to make b a large prime, such that as the n value ascends, it can do so for a while before it reaches b.

- Let $n = b - a$. Then $n^2 + an + b = b(b - a + 1)$, and is therefore not prime. So we have incentive to look for an a as small as possible.