34 DigitFactorialGraph

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1 The Problem

145 is a curious number, as $1! + 4! + 5! = 1 + 24 + 120 = 145$. Find the sum of all numbers which are equal to the sum of the factorial of their digits. Note: as $1! = 1$ and $2! = 2$ are not sums they are not included.

2 Finding an Upper Bound

Let's say we are interested in the digit factorial sum of a number n. Then we have that $n = `n_0 n_1 ... n_k$' with $k = \lfloor log_{10} n \rfloor$. The sum of the factorials of the digits of n, $DF(n) = \sum_{i=0}^{k} n_i!$, is bounded above by $(k+1)9! = \lceil log_{10} n \rceil 9!$, so the n for which $DF(n) = n$ is bounded above by the n for which $\lceil log_{10} n \rceil 9! = n$. Using the grapher tool an estimate for this upper bound is around 2.5402×10^6. Since n increases much faster than $DF(n)$ or $\lceil log_{10} n \rceil 9!$ we only need to look at $n < 2.5402 \times 10^6$.