A50 i) $b_n = \frac{1}{\pi} \cdot \int_0^{\pi} x \sin(nx)\, dx$

$= \frac{1}{\pi} \left(\left[-\frac{x}{n} \cos(nx) \right]_0^{\pi} - \int_0^{\pi} -\frac{1}{n} \cos(nx) \right)$

$= \frac{1}{\pi} \cdot \left(-\frac{\pi}{n} \cos(n\pi) + \frac{1}{n} \int_0^{\pi} \cos(nx) \right)$

$= -\frac{1}{n} (-1)^n + \frac{1}{n\pi} \left[\frac{1}{n} \sin(nx) \right]_0^{\pi}$

$= -\frac{1}{n\pi} (-1)^n$

Die Fourierreihe ist gegeben durch:

$\frac{\pi}{4} + \sum_{n=1}^{\infty} \left(-\frac{1}{\pi n^2} (1-(-1)^n) \cos(nx) + -\frac{1}{n\pi} (-1)^n \sin(nx) \right)$

$= \frac{\pi}{4} + \sum_{n=1}^{\infty} \left(-\frac{1}{n} \left(\frac{(1-(-1)^n) \cos(nx)}{\pi n} + \frac{(-1)^n \sin(nx)}{\pi} \right) \right)$

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ii) $a_0 = \pi$

$$a_n = -\frac{2}{\pi n^2}\left(1-(-1)^n\right)$$

$$f(x) = \frac{\pi}{2} + \sum_{n=1}^{\infty}\left(-\frac{2}{\pi n^2}\left(1-(-1)^n\right)\cos(nx)\right)$$

$$= \frac{\pi}{2} - \frac{2}{\pi}\sum_{n=1}^{\infty}\left(\frac{1}{n^2}\cos(nx)\left(1-(-1)^n\right)\right)$$

$$= \frac{\pi}{2} - \frac{2}{\pi}\sum_{n=1}^{\infty}\left(\frac{1}{n^2}\cos(nx)\left(1-(-1)^n\right)\right)$$

$$\| $$

$$2\cos(x) + \frac{2\cos(3x)}{9} + \frac{2\cos(5x)}{25} + \frac{2\cos(7x)}{49} \dots$$

$$= \frac{\pi}{2} - \frac{4}{\pi}\sum_{n=1}^{\infty}\frac{\cos(nx)}{n^2} \quad (n \text{ ungerade})$$

Für $x = \frac{\pi}{2n}$ gilt: $\cos(nx) = 0$ (n ungerade)

$$\Rightarrow \frac{\pi}{2} - \frac{4}{\pi}\sum_{n=1}^{\infty}\frac{\cos(nx)}{n^2} \longrightarrow \frac{\pi}{2}$$