A 48 i) a) $z_1 := \frac{3+5i}{2-i}$

$$z_1 = \frac{3+5i}{2-i} \cdot \frac{2+i}{2+i} = \frac{6+3i+10i-5}{2^2+1^2} = \frac{1+13i}{5} = \frac{1}{5} + \frac{13}{5}i$$

Realteil $= \frac{1}{5}$ Imaginärteil $= \frac{13}{5}$

$$|z_1| = \sqrt{\frac{1}{5} + \frac{169}{5}} = \sqrt{\frac{170}{5}} = \sqrt{34}$$

b) $z_2 := \frac{1}{(1+i)^2} = \frac{1}{1+2i-1} = \frac{1}{2i} = \frac{2i}{-4} = -\frac{1}{2}i$

Realteil $= 0$ Imaginärteil $= -\frac{1}{2}$

$$|z| = \sqrt{0+(-\tfrac{1}{2})^2} = |-\tfrac{1}{2}| = \frac{1}{2}$$

c) $z_3 := (1-i\sqrt{3})^{3n} \quad (n \in \mathbb{N})$

$$(1-i\sqrt{3})^{3n} = \sum_{k=0}^{3n} \binom{3n}{k} 1^{3n-k} (-i\sqrt{3})^k = \sum_{k=0}^{3n} \binom{3n}{k} (-i\sqrt{3})^k$$

d) $z_4 := \sum_{k=0}^{26} (2i)^k$

$$z_4 = \frac{1-(2i)^{27}}{1-2i} = \frac{(1-2i)\sum_{k=0}^{26}\binom{26}{k}1^{26-k}(2i)^k}{1-2i} = \sum_{k=0}^{26}\binom{26}{k}(2i)^k$$

A 48 ii) a) $z^2 - 2\bar{z} + 1 = 0$

Definiere $z = x + iy \Rightarrow \bar{z} = x - iy$

$\Rightarrow z^2 - 2\bar{z} + 1 = z^2 - 2(x - iy) + 1$
$= z^2 - 2x + 2iy + 1 = z^2 - 2x + 4x - 4x + 2iy + 1$

$= z^2 + 2(x+iy) + (1-4x) = z^2 + \underbrace{2}_{p}z + \underbrace{(1-4x)}_{q} = 0$

$\Rightarrow z = -\frac{2}{2} \pm \sqrt{\frac{2^2}{4} - (1-4x)}$

$= -1 \pm \sqrt{1-1+4x} = -1 \pm 2\sqrt{x}$

$z \in \{-1 \pm 2\sqrt{x} : x \in \mathbb{R}\}$

b) $e^z = \sqrt{12} + 2i$

Definiere $w := \sqrt{12} + 2i$

Es gilt: $w = |w|(\cos(\varphi) + i\sin(\varphi))$

$|w| = \sqrt{12+4} = \sqrt{16} = 4$
$\Rightarrow \sqrt{12} + 2i = 4(\cos(\varphi) + i\sin(\varphi))$
$\Leftrightarrow \frac{2\sqrt{3}}{4} + \frac{2i}{4} = \cos(\varphi) + i\sin(\varphi)$

$\Leftrightarrow \cos(\varphi) + i\sin(\varphi) = \frac{\sqrt{3}}{2} + \frac{1}{2}i$

$\Rightarrow \cos(\varphi) = \frac{\sqrt{3}}{2}$ und $\sin(\varphi) = \frac{1}{2}$

$\Rightarrow \varphi = \frac{\pi}{6}$ (Argument von $w$)

Weiter gilt:

$w = |w| \cdot e^{i \cdot \pi/6} \Rightarrow w = 4 \cdot e^{i \pi/6}$

$\Rightarrow e^{z} = 4 \cdot e^{i \pi/6}$

$\Rightarrow z = \log(4 \cdot e^{i \pi/6})$

$\Rightarrow z \in \{ \log(|w|) + i\varphi + 2k\pi i \mid k \in \mathbb{Z} \}$

$\Rightarrow z \in \{ \log(4) + \frac{\pi}{6} i + 2k\pi i \mid k \in \mathbb{Z} \}$