

Gruppe  
1

Velislav Slavov, 2385786

ucsmm@student.kit.edu

A1

$$A = \begin{pmatrix} 1 & 0 & 1 \\ 1 & 1 & 3 \\ 3 & 2 & 2 \end{pmatrix}$$

Σ 26

a)

1	0	1		1	0	0	. $(-1)$ ), . $(-3)$
1	1	3		0	1	0	
3	2	2		0	0	1	
<hr/>							
1	0	1		1	0	0	
0	1	2		-1	1	0	. $(-2)$
0	2	-1		-3	0	1	
<hr/>							
1	0	1		1	0	0	
0	1	2		-1	1	0	
0	0	-5		-1	-2	1	. $(-\frac{1}{5})$
<hr/>							
1	0	1		1	0	0	
0	1	2		-1	1	0	
0	0	1		$\frac{1}{5} + \frac{2}{5}$	$-\frac{1}{5}$		. $(-2)$ ), . $(-1)$
<hr/>							
1	0	0		$+\frac{4}{5} - \frac{2}{5}$	$\frac{1}{5}$		
0	1	0		$-\frac{7}{5}$	$\frac{2}{5}$		-1
<hr/>							
0	0	1		$\frac{1}{5}$	$-\frac{2}{5}$	$-\frac{1}{5}$	

$$A^{-1} = \begin{pmatrix} -\frac{1}{5} & \frac{2}{5} & \frac{1}{5} \\ -\frac{7}{5} & \frac{2}{5} & \frac{2}{5} \\ \frac{1}{5} & -\frac{2}{5} & -\frac{1}{5} \end{pmatrix} = \frac{1}{5} \begin{pmatrix} 4 & 2 & 1 \\ -7 & 2 & 2 \\ 1 & -2 & -1 \end{pmatrix}$$

b)  $\mathbb{F}_5$

+	0	1	2	3	4
0	0	1	2	3	4
1	1	2	3	4	0
2	2	3	4	0	1
3	3	4	0	1	2
4	4	0	1	2	3

•	0	1	2	3	4
0	0	0	0	0	0
1	0	1	2	3	4
2	0	2	4	1	3
3	0	3	1	4	2
4	0	4	3	2	1

1	0	1		1	0	0	1.4), .2) ← ←
1	1	3		0	1	0	
3	2	2		0	0	1	
<hr/>				<hr/>			
1	0	1		1	0	0	
0	1	2		4	1	0	1.3) ←
0	2	4		2	0	1	
<hr/>				<hr/>			
1	0	1		1	0	0	
0	1	2		4	1	0	
0	0	0		4	3	1	

Beim berechnen von  $A^{-1}$  kommt es zu eine Nullzeile  
somit ist  $A$  NICHT invertierbar. ✓

c)  $\mathbb{F}_3$

+	0	1	2
0	0	1	2
1	1	2	0
2	2	0	1

•	0	1	2
0	0	0	0
1	0	1	2
2	0	2	1

$$3 = 0 \mathbb{F}_3 \Rightarrow A = \begin{pmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 2 & 2 \end{pmatrix}$$

$$\begin{array}{cc|cc} 1 & 0 & 1 & 1 & 0 & 0 & \text{1.2} \\ 1 & 1 & 0 & 0 & 1 & 0 & \swarrow \\ 0 & 2 & 2 & 0 & 0 & 1 & \\ \hline 1 & 0 & 1 & 1 & 0 & 0 & \\ 0 & 1 & 2 & 2 & 1 & 0 & \text{1.1} \\ 0 & 2 & 2 & 0 & 0 & 1 & \swarrow \\ \hline 1 & 0 & 1 & 1 & 0 & 0 & \\ 0 & 1 & 2 & 2 & 1 & 0 & \\ 0 & 0 & 1 & 2 & 1 & 1 & \text{1.1}, \text{.2} \\ \hline 1 & 0 & 0 & 2 & 2 & 2 & \\ 0 & 1 & 0 & 1 & 2 & 1 & \\ 0 & 0 & 1 & 2 & 1 & 1 & \end{array}$$

$$A^{-1} = \begin{pmatrix} 2 & 2 & 2 \\ 1 & 2 & 1 \\ 2 & 1 & 1 \end{pmatrix}$$

9/10

$$A2 \quad B := \begin{pmatrix} a & x \\ b & y \end{pmatrix}^2 = \mathbb{1}_2$$

$$\begin{pmatrix} a & x \\ b & y \end{pmatrix}^2 = \begin{pmatrix} a^2 + bx & ax + xy \\ ab + by & bx + y^2 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

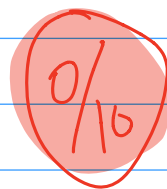
$$a^2 + bx = 1$$

$$ax + xy = 0 \Leftrightarrow x(a + y) = 0 \Rightarrow x = 0 \text{ oder } a + y = 0$$

$$ab + by = 0 \Leftrightarrow b(a + y) = 0 \Rightarrow b = 0 \text{ oder } a + y = 0$$

$$bx + y^2 = 1$$

$$(a, b) \in \{a, b \in K \mid \dots$$



A3

a) Seien:

$$B_{\mathbb{R}^{3 \times 3}} := \left( \overbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}}^{e_1}, \overbrace{\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}}^{e_2}, \overbrace{\begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}}^{e_3}, \overbrace{\begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}}^{e_4}, \overbrace{\begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}}^{e_5}, \right.$$

$$\left. \overbrace{\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}}^{e_6}, \overbrace{\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}}^{e_7}, \overbrace{\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}}^{e_8}, \overbrace{\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}}^{e_9} \right)$$

$$B_{S_3} := (e_9, e_8, e_7, e_6, e_5, e_4, e_3, e_2, e_1) \quad \text{! z.B. } e_8 \notin S_3$$

b) Seien  $r_1, r_2 \in \mathbb{R}^{3 \times 3}$ ,  $\lambda \in \mathbb{R}$

$$\begin{aligned} \text{(i) } \text{sym}(r_1 + r_2) &= \frac{1}{2} \left( (r_1 + r_2) + (r_1 + r_2)^T \right) \\ &= \frac{1}{2} (r_1 + r_2 + r_1^T + r_2^T) = \frac{1}{2} ((r_1 + r_1^T) + (r_2 + r_2^T)) \\ &= \frac{1}{2} (r_1 + r_1^T) + \frac{1}{2} (r_2 + r_2^T) = \text{sym}(r_1) + \text{sym}(r_2) \end{aligned}$$

$$\begin{aligned} \text{(ii) } \text{sym}(\lambda r_1) &= \frac{1}{2} (\lambda r_1 + (\lambda r_1)^T) \\ &= \frac{1}{2} (\lambda r_1 + \lambda r_1^T) = \frac{1}{2} (\lambda (r_1 + r_1^T)) = \lambda \left( \frac{1}{2} (r_1 + r_1^T) \right) \\ &= \lambda \text{sym}(r_1) \end{aligned}$$

Aus (i) und (ii)  $\Rightarrow$  sym ist linear

$$(\text{sym}(e_1))_{B_{S_3}} = \left( \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \right)_{B_{S_3}} = (0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 1)^T$$

Da die Basis in (\*) falsch war  
kann ich das richtig bewerten!

$$(\text{sym}(e_2))_{B_{S_3}} = \left( \begin{pmatrix} 0 & \frac{1}{2} & 0 \\ \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \right)_{B_{S_3}} = (0 \ 0 \ 0 \ 0 \ \frac{1}{2} \ 0 \ \frac{1}{2} \ 0)^T$$

$$(\text{sym}(e_3))_{B_{S_3}} = \left( \begin{pmatrix} 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 \\ \frac{1}{2} & 0 & 0 \end{pmatrix} \right)_{B_{S_3}} = (0 \ 0 \ \frac{1}{2} \ 0 \ 0 \ 0 \ \frac{1}{2} \ 0)^T$$

$$(\text{sym}(e_4))_{B_{S_3}} = \left( \begin{pmatrix} 0 & \frac{1}{2} & 0 \\ \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \right)_{B_{S_3}} = (0 \ 0 \ 0 \ 0 \ \frac{1}{2} \ 0 \ \frac{1}{2} \ 0)^T$$

$$(\text{sym}(e_5))_{B_{S_3}} = \left( \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \right)_{B_{S_3}} = (0 \ 0 \ 0 \ 0 \ 1 \ 0 \ 0 \ 0)^T$$

$$(\text{sym}(e_6))_{B_{S_3}} = \left( \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2} \\ 0 & \frac{1}{2} & 0 \end{pmatrix} \right)_{B_{S_3}} = (0 \ \frac{1}{2} \ 0 \ \frac{1}{2} \ 0 \ 0 \ 0 \ 0)^T$$

$$(\text{sym}(e_7))_{B_{S_3}} = \left( \begin{pmatrix} 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 \\ \frac{1}{2} & 0 & 0 \end{pmatrix} \right)_{B_{S_3}} = (0 \ 0 \ \frac{1}{2} \ 0 \ 0 \ 0 \ \frac{1}{2} \ 0)^T$$

$$(\text{sym}(e_8))_{B_{S_3}} = \left( \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2} \\ 0 & \frac{1}{2} & 0 \end{pmatrix} \right)_{B_{S_3}} = \left( 0 \frac{1}{2} 0 \quad \frac{1}{2} 0 0 \quad 0 0 0 \right)^T$$

$$(\text{sym}(e_9))_{B_{S_3}} = \left( \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \right)_{B_{S_3}} = (1 0 0 \quad 0 0 0 \quad 0 0 0)^T$$

$$\Rightarrow M_{B_{S_3}, B_{\mathbb{R}^{3 \times 3}}}(\text{sym}) = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1/2 & 0 & 1/2 & 0 \\ 0 & 0 & 1/2 & 0 & 0 & 0 & 1/2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1/2 & 0 & 1/2 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1/2 & 0 & 1/2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1/2 & 0 & 0 & 0 & 1/2 & 0 & 0 \\ 0 & 1/2 & 0 & 1/2 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

f. weil Basis in (a)

fürh. vrs.

17/10

A4

$$B := \begin{matrix} & b_1 & b_2 & b_3 \\ \begin{pmatrix} 0 \\ 2 \\ 0 \end{pmatrix}, & \begin{pmatrix} 0 \\ 0 \\ 2 \end{pmatrix}, & \begin{pmatrix} 2 \\ 0 \\ 2 \end{pmatrix} \end{matrix} \quad C := \begin{matrix} & c_1 & c_2 & c_3 \\ \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}, & \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}, & \begin{pmatrix} 2 \\ 2 \\ 2 \end{pmatrix} \end{matrix}$$

$$c_1 = 2 \begin{pmatrix} 0 \\ 2 \\ 0 \end{pmatrix} + 2 \begin{pmatrix} 0 \\ 0 \\ 2 \end{pmatrix} + 1 \begin{pmatrix} 2 \\ 0 \\ 2 \end{pmatrix}$$

$$\stackrel{(i)}{\Rightarrow} (c_1)_B = (2 \ 2 \ 1)^T$$

$$c_2 = 2 \begin{pmatrix} 0 \\ 2 \\ 0 \end{pmatrix} + 2 \begin{pmatrix} 0 \\ 0 \\ 2 \end{pmatrix} + 2 \begin{pmatrix} 2 \\ 0 \\ 2 \end{pmatrix}$$

$$\stackrel{(ii)}{\Rightarrow} (c_2)_B = (2 \ 2 \ 2)^T$$

$$c_3 = 1 \begin{pmatrix} 0 \\ 2 \\ 0 \end{pmatrix} + 0 \begin{pmatrix} 0 \\ 0 \\ 2 \end{pmatrix} + 1 \begin{pmatrix} 2 \\ 0 \\ 2 \end{pmatrix}$$

$$\stackrel{(iii)}{\Rightarrow} (c_3)_B = (1 \ 0 \ 1)^T$$

$$\text{Aus (i), (ii), (iii)} \Rightarrow M_{B,C}(\text{id}_{\mathbb{F}_3}) = \begin{pmatrix} 2 & 2 & 1 \\ 2 & 2 & 0 \\ 1 & 2 & 1 \end{pmatrix}$$





$$M_{C,B}(\text{id}_{\mathbb{F}_3^3}) = M_{B,C}(\text{id}_{\mathbb{F}_3^3})^{-1}$$

$$\begin{array}{ccc|ccc} 2 & 2 & 1 & 1 & 0 & 0 \\ 2 & 2 & 0 & 0 & 1 & 0 \\ 1 & 2 & 1 & 0 & 0 & 1 \end{array} \quad \begin{array}{l} | \cdot 2, \cdot 1 \\ \swarrow \\ \swarrow \end{array}$$

$$\begin{array}{ccc|ccc} 2 & 2 & 1 & 1 & 0 & 0 \\ 0 & 0 & 2 & 2 & 1 & 0 \\ 0 & 1 & 2 & 1 & 0 & 1 \end{array} \quad \begin{array}{l} | \cdot 2 \\ | \cdot 1, \swarrow \\ \swarrow \end{array}$$

$$\begin{array}{ccc|ccc} 1 & 1 & 2 & 2 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 1 \end{array}$$

$$\begin{array}{ccc|ccc} 0 & 0 & 2 & 2 & 1 & 0 \end{array} \quad | \cdot 2,$$

$$\begin{array}{ccc|ccc} 1 & 1 & 2 & 2 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 2 & 0 \end{array} \quad \begin{array}{l} \swarrow \\ | \cdot 2, \cdot 1 \\ \swarrow \end{array}$$

$$\begin{array}{ccc|ccc} 1 & 1 & 0 & 0 & 2 & 0 \\ 0 & 1 & 0 & 2 & 2 & 1 \\ 0 & 0 & 1 & 1 & 2 & 0 \end{array} \quad \begin{array}{l} \swarrow \\ | \cdot 2 \\ \swarrow \end{array}$$

$$\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 2 \\ 0 & 1 & 0 & 2 & 2 & 1 \\ 0 & 0 & 1 & 1 & 2 & 0 \end{array}$$

$$M_{C,B}(\text{id}_{\mathbb{F}_3^3}) = \begin{pmatrix} 1 & 0 & 2 \\ 2 & 2 & 1 \\ 1 & 2 & 0 \end{pmatrix} \checkmark$$

W/10