Gruppe 1

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A1

$$A = \begin{pmatrix} 1 & 0 & 1 \\ 1 & 1 & 3 \\ 3 & 2 & 2 \end{pmatrix}$$

a)

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 3 & 0 & 1 & 0 \\ 3 & 2 & 2 & 0 & 0 & 1 \end{array}\right) \quad | \cdot (-1), \ \cdot(-3)$$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 2 & -1 & 1 & 0 \\ 0 & 2 & -1 & -3 & 0 & 1 \end{array}\right) \quad | \cdot (-2)$$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 2 & -1 & 1 & 0 \\ 0 & 0 & -5 & -1 & -2 & 1 \end{array}\right) \quad | \cdot \left(-\frac{1}{5}\right)$$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 2 & -1 & 1 & 0 \\ 0 & 0 & 1 & \frac{1}{5} & -\frac{2}{5} & -\frac{1}{5} \end{array}\right) \quad | \cdot (-2), \ \cdot(-1)$$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & -\frac{1}{5} & \frac{2}{5} & \frac{1}{5} \\ 0 & 1 & 0 & -\frac{7}{5} & \frac{9}{5} & \frac{2}{5} \\ 0 & 0 & 1 & \frac{1}{5} & \frac{-2}{5} & -\frac{1}{5} \end{array}\right)$$

$$A^{-1} = \begin{pmatrix} -\frac{1}{5} & \frac{2}{5} & \frac{1}{5} \\ -\frac{7}{5} & \frac{9}{5} & \frac{2}{5} \\ \frac{1}{5} & \frac{-2}{5} & -\frac{1}{5} \end{pmatrix} = \frac{1}{5} \begin{pmatrix} -1 & 2 & 1 \\ -7 & 9 & 2 \\ 1 & -2 & -1 \end{pmatrix}$$

b) $\mathbb{F}_5$

+	0	1	2	3	4
0	0	1	2	3	4
1	1	2	3	4	0
2	2	3	4	0	1
3	3	4	0	1	2
4	4	0	1	2	3

·	0	1	2	3	4
0	0	0	0	0	0
1	0	1	2	3	4
2	0	2	4	1	3
3	0	3	1	4	2
4	0	4	3	2	1

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 3 & 0 & 1 & 0 \\ 3 & 2 & 2 & 0 & 0 & 1 \end{array}\right) \quad \text{I}\cdot 4,\ \cdot 2$$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 2 & 4 & 1 & 0 \\ 0 & 2 & 4 & 2 & 0 & 1 \end{array}\right) \quad \text{I}\cdot 3$$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 2 & 4 & 1 & 0 \\ 0 & 0 & 0 & 4 & 3 & 1 \end{array}\right)$$

Beim berechnen von A^{-1} kommt es zu eine Nullzeile somit ist A NICHT invertierbar.

c) $\mathbb{F}_3$

$+$	0	1	2
0	0	1	2
1	1	2	0
2	2	0	1

$\cdot$	0	1	2
0	0	0	0
1	0	1	2
2	0	2	1

$$3 = 0 \; \mathbb{F}_3 \implies A = \begin{pmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 2 & 2 \end{pmatrix}$$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 2 & 2 & 0 & 0 & 1 \end{array}\right) \quad 1.2$$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 2 & 2 & 1 & 0 \\ 0 & 2 & 2 & 0 & 0 & 1 \end{array}\right) \quad 1.1$$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 2 & 2 & 1 & 0 \\ 0 & 0 & 1 & 2 & 1 & 1 \end{array}\right) \quad 1.1, \; .2$$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 2 & 2 & 2 \\ 0 & 1 & 0 & 1 & 2 & 1 \\ 0 & 0 & 1 & 2 & 1 & 1 \end{array}\right)$$

$$A^{-1} = \begin{pmatrix} 2 & 2 & 2 \\ 1 & 2 & 1 \\ 2 & 1 & 1 \end{pmatrix}$$

A 2 $B := \begin{pmatrix} a & x \\ b & y \end{pmatrix}^2 = \mathbb{1}_2$

$$\begin{pmatrix} a & x \\ b & y \end{pmatrix}^2 = \begin{pmatrix} (a^2+bx) & (ax+xy) \\ (ab+by) & (bx+y^2) \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$a^2+bx = 1$

$ax+xy = 0 \iff x(a+y)=0 \implies x=0$ oder $a+y=0$

$ab+by = 0 \iff b(a+y)=0 \implies b=0$ oder $a+y=0$

$bx+y^2 = 1$

$(a,b) \in \{a, b \in K \mid$

A3

a) Seien:

$$B_{\mathbb{R}^{3\times3}} := \left(\underbrace{\begin{pmatrix} 1&0&0\\0&0&0\\0&0&0 \end{pmatrix}}_{e_1}, \underbrace{\begin{pmatrix} 0&1&0\\0&0&0\\0&0&0 \end{pmatrix}}_{e_2}, \underbrace{\begin{pmatrix} 0&0&1\\0&0&0\\0&0&0 \end{pmatrix}}_{e_3}, \underbrace{\begin{pmatrix} 0&0&0\\1&0&0\\0&0&0 \end{pmatrix}}_{e_4}, \underbrace{\begin{pmatrix} 0&0&0\\0&1&0\\0&0&0 \end{pmatrix}}_{e_5}, \underbrace{\begin{pmatrix} 0&0&0\\0&0&1\\0&0&0 \end{pmatrix}}_{e_6}, \underbrace{\begin{pmatrix} 0&0&0\\0&0&0\\1&0&0 \end{pmatrix}}_{e_7}, \underbrace{\begin{pmatrix} 0&0&0\\0&0&0\\0&1&0 \end{pmatrix}}_{e_8}, \underbrace{\begin{pmatrix} 0&0&0\\0&0&0\\0&0&1 \end{pmatrix}}_{e_9} \right)$$

$$B_{S_3} := (e_9, e_8, e_7, e_6, e_5, e_4, e_3, e_2, e_1)$$

b) Seien $r_1, r_2 \in \mathbb{R}^{3\times3}$, $\lambda \in \mathbb{R}$

(i) $sym(r_1+r_2) = \frac{1}{2}\left((r_1+r_2) + (r_1+r_2)^T\right)$

$= \frac{1}{2}\left(r_1+r_2+r_1^T+r_2^T\right) = \frac{1}{2}\left((r_1+r_1^T) + (r_2+r_2)^T\right)$

$= \frac{1}{2}(r_1+r_1^T) + \frac{1}{2}(r_2+r_2^T) = sym(r_1) + sym(r_2)$

(ii) $sym(\lambda r_1) = \frac{1}{2}\left(\lambda r_1 + (\lambda r_1)^T\right)$

$= \frac{1}{2}\left(\lambda r_1 + \lambda r_1^T\right) = \frac{1}{2}\left(\lambda(r_1+r_1^T)\right) = \lambda\left(\frac{1}{2}(r_1+r_1^T)\right)$

$= \lambda\, sym(r_1)$

Aus (i) und (ii) $\Rightarrow$ sym ist linear

$$\left(sym(e_1)\right)_{B_{s_3}} = \left(\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}\right)_{B_{s_3}} = (0\ 0\ 0\ \ 0\ 0\ 0\ 0\ 0\ 1)^T$$

$$\left(sym(e_2)\right)_{B_{s_3}} = \left(\begin{pmatrix} 0 & \frac{1}{2} & 0 \\ \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}\right)_{B_{s_3}} = (0\ 0\ 0\ \ 0\ 0\ \tfrac{1}{2}\ \ 0\ \tfrac{1}{2}\ 0)^T$$

$$\left(sym(e_3)\right)_{B_{s_3}} = \left(\begin{pmatrix} 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 \\ \frac{1}{2} & 0 & 0 \end{pmatrix}\right)_{B_{s_3}} = (0\ 0\ \tfrac{1}{2}\ \ 0\ 0\ 0\ \ \tfrac{1}{2}\ 0\ 0)^T$$

$$\left(sym(e_4)\right)_{B_{s_3}} = \left(\begin{pmatrix} 0 & \frac{1}{2} & 0 \\ \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}\right)_{B_{s_3}} = (0\ 0\ 0\ \ 0\ 0\ \tfrac{1}{2}\ \ 0\ \tfrac{1}{2}\ 0)^T$$

$$\left(sym(e_5)\right)_{B_{s_3}} = \left(\begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}\right)_{B_{s_3}} = (0\ 0\ 0\ \ 0\ 1\ 0\ \ 0\ 0\ 0)^T$$

$$\left(sym(e_6)\right)_{B_{s_3}} = \left(\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2} \\ 0 & \frac{1}{2} & 0 \end{pmatrix}\right)_{B_{s_3}} = (0\ \tfrac{1}{2}\ 0\ \ \tfrac{1}{2}\ 0\ 0\ \ 0\ 0\ 0)^T$$

$$\left(sym(e_7)\right)_{B_{s_3}} = \left(\begin{pmatrix} 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 \\ \frac{1}{2} & 0 & 0 \end{pmatrix}\right)_{B_{s_3}} = (0\ 0\ \tfrac{1}{2}\ \ 0\ 0\ 0\ \ \tfrac{1}{2}\ 0\ 0)^T$$

$$\left(\mathrm{sym}(e_8)\right)_{B_{S_3}} = \left(\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2} \\ 0 & \frac{1}{2} & 0 \end{pmatrix}\right)_{B_{S_3}} = \left(0\ \tfrac{1}{2}\ 0\ \ \tfrac{1}{2}\ 0\ 0\ \ 0\ 0\ 0\right)^T$$

$$\left(\mathrm{sym}(e_9)\right)_{B_{S_3}} = \left(\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}\right)_{B_{S_3}} = \left(1\ 0\ 0\ \ 0\ 0\ 0\ \ 0\ 0\ 0\right)^T$$

$$\Rightarrow M_{B_{S_3}, B_{\mathbb{R}^{3\times 3}}}(\mathrm{sym}) = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1/2 & 0 & 1/2 & 0 \\ 0 & 0 & 1/2 & 0 & 0 & 0 & 1/2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1/2 & 0 & 1/2 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1/2 & 0 & 1/2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1/2 & 0 & 0 & 0 & 1/2 & 0 & 0 \\ 0 & 1/2 & 0 & 1/2 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

A 4

$$B := \left(\overset{b_1}{\begin{pmatrix}0\\2\\0\end{pmatrix}}, \overset{b_2}{\begin{pmatrix}0\\0\\2\end{pmatrix}}, \overset{b_3}{\begin{pmatrix}2\\0\\2\end{pmatrix}} \right) \qquad C := \left(\overset{c_1}{\begin{pmatrix}2\\1\\0\end{pmatrix}}, \overset{c_2}{\begin{pmatrix}1\\1\\2\end{pmatrix}}, \overset{c_3}{\begin{pmatrix}2\\2\\2\end{pmatrix}} \right)$$

$$c_1 = 2\begin{pmatrix}0\\2\\0\end{pmatrix} + 2\begin{pmatrix}0\\0\\2\end{pmatrix} + 1\begin{pmatrix}2\\0\\2\end{pmatrix}$$

$$\overset{(i)}{\Longrightarrow} (c_1)_B = (2 \quad 2 \quad 1)^T$$

$$c_2 = 2\begin{pmatrix}0\\2\\0\end{pmatrix} + 2\begin{pmatrix}0\\0\\2\end{pmatrix} + 2\begin{pmatrix}2\\0\\2\end{pmatrix}$$

$$\overset{(ii)}{\Longrightarrow} (c_2)_B = (2 \quad 2 \quad 2)^T$$

$$c_3 = 1\begin{pmatrix}0\\2\\0\end{pmatrix} + 0\begin{pmatrix}0\\0\\2\end{pmatrix} + 1\begin{pmatrix}2\\0\\2\end{pmatrix}$$

$$\overset{(iii)}{\Longrightarrow} (c_3)_B = (1 \quad 0 \quad 1)^T$$

Aus (i), (ii), (iii) $\Rightarrow M_{B,C}(id_{\mathbb{F}_3^3}) = \begin{pmatrix}2 & 2 & 1\\ 2 & 2 & 0\\ 1 & 2 & 1\end{pmatrix}$

$$M_{C,B}(id_{\mathbb{F}_3^3}) = M_{B,C}(id_{\mathbb{F}_3^3})^{-1}$$

$$
\begin{array}{ccc|ccc l}
2 & 2 & 1 & 1 & 0 & 0 & \mid \cdot 2, \ \cdot 1 \\
2 & 2 & 0 & 0 & 1 & 0 & \\
1 & 2 & 1 & 0 & 0 & 1 & \\
\hline
2 & 2 & 1 & 1 & 0 & 0 & \mid \cdot 2 \\
0 & 0 & 2 & 2 & 1 & 0 & \mid \cdot 1, \\
0 & 1 & 2 & 1 & 0 & 1 & \\
\hline
1 & 1 & 2 & 2 & 0 & 0 & \\
0 & 1 & 1 & 0 & 1 & 1 & \\
0 & 0 & 2 & 2 & 1 & 0 & \mid \cdot 2, \\
\hline
1 & 1 & 2 & 2 & 0 & 0 & \\
0 & 1 & 1 & 0 & 1 & 1 & \\
0 & 0 & 1 & 1 & 2 & 0 & \mid \cdot 2, \ \cdot 1 \\
\hline
1 & 1 & 0 & 0 & 2 & 0 & \\
0 & 1 & 0 & 2 & 2 & 1 & \mid \cdot 2 \\
0 & 0 & 1 & 1 & 2 & 0 & \\
\hline
1 & 0 & 0 & 1 & 0 & 2 & \\
0 & 1 & 0 & 2 & 2 & 1 & \\
0 & 0 & 1 & 1 & 2 & 0 &
\end{array}
$$

$$M_{C,B}(id_{\mathbb{F}_3^3}) = \begin{pmatrix} 1 & 0 & 2 \\ 2 & 2 & 1 \\ 1 & 2 & 0 \end{pmatrix}$$