13 i) $a_n := \left(3 + (-1)^n\right) \sqrt[2]{(-1)^{n(n+1)}}$

Falls n gerade: $(3+1)\ \sqrt[2]{1} = 4$
$\Rightarrow (a_{n_k}) \to 4 \ (n = 2k)$

Falls n ungerade: $(3-1)\ \sqrt[2]{1} = 2$
$\Rightarrow (a_{n_k}) \to 2 \ (n = 2k-1)$

also $H(a_n) = \{4, 2\}$

$\limsup\limits_{n\to\infty} a_n = 4$

$\liminf\limits_{n\to\infty} a_n = 2$

13 iv) $\left(8+\frac{12}{n}+\frac{6}{n^2}+\frac{1}{n^3}\right)^n = \left(1+7+\frac{12}{n}+\frac{6}{n^2}+\frac{1}{n^3}\right)^n \quad 8^n \cdot \left(\frac{1}{\left(8+\frac{12}{n}+\frac{6}{n^2}+\frac{1}{n^3}\right)^n}\right)$

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$\left(1+7+\frac{12}{n}+\frac{6}{n^2}+\frac{1}{n^3}\right)^n \geq 1+n\left(7+\frac{12}{n}+\frac{6}{n^2}+\frac{1}{n^3}\right) = 1+7n+12+\frac{6}{n}+\frac{1}{n^2}$

$\Rightarrow \frac{1}{\left(8+\frac{12}{n}+\frac{6}{n^2}+\frac{1}{n^3}\right)^n} \leq \frac{1}{\underbrace{\frac{1}{n^2}}_{\to 0}+\underbrace{\frac{6}{n}}_{\to 0}+7n+13} \Rightarrow a_n \longrightarrow 8^n \frac{1}{7n+13} \to 0$

$\frac{8n^3+12n^2+6n+1}{n^3} = \frac{(2n+1)^3}{n^3} = \left(\frac{2n+1}{n}\right)^3$

$\Rightarrow \frac{(2^3)^n}{\left(\left(\frac{2n+1}{n}\right)^3\right)^n} = \left(\frac{2}{\frac{2n+1}{n}}\right)^{3n} = \left(\frac{2n}{2n+1}\right)^{3n} = \left(1-\frac{1}{2n+1}\right)^{3n}$

$\frac{1}{2n} < \frac{2n}{2n+1} \quad \forall n \in \mathbb{N} \Rightarrow \underbrace{\left(\frac{1}{2n}\right)^{3n}}_{\to 0} < \left(\frac{2n}{2n+1}\right)^{3n} \overset{?}{\Rightarrow} a_n \to 0 \Rightarrow \dots$

I.A. $n=1$

$\frac{1}{2} < \frac{2}{3}$

I.V. n: $\quad \frac{1}{2n} < \frac{2n}{2n+1} \Rightarrow 2n+1 < 4n^2$

I.S. $n+1$

$\frac{1}{2n+2} \quad \frac{2n+2}{2n+3}$

$2n+3 \quad (2n+2)^2 = 4n^2+8n+4$

$2n+1+2 < 4n^2+2 < 4n^2+2+8n+2$ ✓

15 i) $a_n := \frac{1}{\sqrt{n}+\sqrt{n-1}} = \frac{1}{\sqrt{n}+\sqrt{n-1}} \cdot \frac{\sqrt{n}-\sqrt{n-1}}{\sqrt{n}-\sqrt{n-1}} = \frac{\sqrt{n}-\sqrt{n-1}}{n-n+1} = \sqrt{n} - \sqrt{n-1}$

$a_1 = 1$

$a_2 = \sqrt{2} - 1$ $\qquad S_2 = 1 + \sqrt{2} - 1 = \sqrt{2}$

$a_3 = \sqrt{3} - \sqrt{2}$ $\qquad S_3 = \sqrt{2} + \sqrt{3} - \sqrt{2} = \sqrt{3}$ ...

$\Rightarrow s_n = \sqrt{n}$

Da $\sqrt{n}$ divergent $\Rightarrow$ s_n auch divergent

15 ii) $s_n := \sum_{n=0}^{\infty} \frac{2}{n^2+3n+2}$

$n^2+3n+2 = n^2+n+2n+2 = n(n+1)+2(n+1) = (n+1)(n+2)$

$\frac{2}{n^2+3n+2} = \frac{2}{(n+1)(n+2)} = \frac{2}{n+1} - \frac{2}{n+2}$

$s_n = 1 - \frac{2}{3} + \frac{2}{3} - \frac{2}{4} + \frac{2}{4} - \frac{2}{5} \dots + \frac{2}{n} - \frac{2}{n+1} + \frac{2}{n+1} - \frac{2}{n+2}$

$= 1 - \frac{2}{n+2} \rightarrow 1 \Rightarrow s_n \rightarrow 1$