

When and why should you logarithm your values?

Log-scale informs on relative changes (multiplicative), while linear-scale informs on absolute changes (additive). When do you use each? When you care about relative changes, use the log-scale; when you care about absolute changes, use linear-scale. This is true for distributions, but also for any quantity or changes in quantities.

Note, I use the word "care" here very specifically and intentionally. Without a model or a goal, your question cannot be answered; the model or goal defines which scale is important. If you're trying to model something, and the mechanism acts via a relative change, log-scale is critical to capturing the behavior seen in your data. But if the underlying model's mechanism is additive, you'll want to use linear-scale.

Example. Stock market.

Stock A on day 1: \$100.

On day 2, \$101.

Every stock tracking service in the world reports this change in two ways! (1) +\$1. (2) +1%. The first is a measure of absolute, additive change; the second a measure of relative change.

Illustration of relative change vs absolute: Relative change is the same, absolute change is different

Stock A goes from \$1 to \$1.10.

Stock B goes from \$100 to \$110.

Stock A gained 10%, stock B gained 10% (relative scale, equal)

...but stock A gained 10 cents, while stock B gained \$10 (B gained more absolute dollar amount)

If we convert to log space, relative changes appear as absolute changes.

Stock A goes from $\log_{10}(\$1)$ to $\log_{10}(\$1.10)$ = 0 to .0413

Stock B goes from $\log_{10}(\$100)$ to $\log_{10}(\$110)$ = 2 to 2.0413

Now, taking the *absolute difference in log space*, we find that both changed by .0413.

Both of these measures of change are important, and which one is important to you depends solely on your model of investing. There are two models. (1) Investing a fixed amount of principal, or (2) investing in a fixed number of shares.

Model 1: Investing with a fixed amount of principal.

Say yesterday stock A cost \$1 per share, and stock B costs \$100 a share. Today they both went up by one dollar to \$2 and \$101 respectively. Their absolute change is identical (\$1), but their relative change is dramatically different (100% for A, 1% for B). Given that you have a fixed amount of principal to invest, say \$100, you can only afford 1 share of B or 100 shares of A. If you invested yesterday you'd have \$200 with A, or \$101 with B. So here you "care" about the *relative* gains, specifically because you have a finite amount of principal.

Model 2: fixed number of shares.

In a different scenario, suppose your bank only lets you buy in blocks of 100 shares, and you've decided to invest in 100 shares of A or B. In the previous case, whether you buy A or B your gains will be the same (\$100 - i.e. \$1 for each share).

Now suppose we think of a stock value as a random variable fluctuating over time, and we want

to come up with a model that reflects generally how stocks behave. And let's say we want to use this model to maximize profit. We compute a probability distribution whose x-values are in units of 'share price', and y-values in probability of observing a given share price. We do this for stock A, and stock B. If you subscribe to the first scenario, where you have a fixed amount of principal you want to invest, then taking the log of these distributions will be informative. Why? What you care about is the shape of the distribution in relative space. Whether a stock goes from 1 to 10, or 10 to 100 doesn't matter to you, right? Both cases are a 10-*fold* relative gain. This appears naturally in a log-scale distribution in that unit gains correspond to fold gains directly. For two stocks whose mean value is different but whose relative change is identically distributed (they have the same distribution of daily *percent* changes), their log distributions will be *identical* in shape just shifted. Conversely, their linear distributions will not be identical in shape, with the higher valued distribution having a higher variance.

If you were to look at these same distributions in linear, or absolute space, you would think that higher-valued share prices correspond to greater fluctuations. For your investing purposes though, where only relative gains matter, this is not necessarily true.

Źródło: <http://stats.stackexchange.com/questions/18844/when-and-why-should-you-take-the-log-of-a-distribution-of-numbers>
vector07