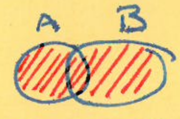
A set is an unordered collection of distinct objects/elements.

A list is a ordered collection of possibly not distinct elements

Let A, B be sets

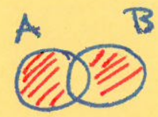
- UNION: $A \cup B := \{x \mid x \in A \text{ or } x \in B\}$ "is defined to be"



- INTERSECTION: $A \cap B := \{x \mid x \in A \text{ and } x \in B\}$

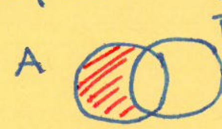
- symmetric DIFFERENCE "XOR"

  $A \oplus B := A \setminus B \cup B \setminus A$



- DIFFERENCE: $\underbrace{A - B}_{} := \underbrace{A \setminus B}_{} = \{x \in A \mid x \notin B\}$

  two ways to write it

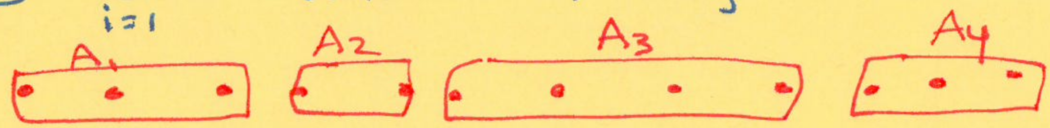


e.g., $A = \{1, 2, 3\}$, $B = \{2, 3, 5\}$

$A \setminus B = \{1, 2, 3\} - \{2, 3, 5\} = \{1\}$

- SIZE or CARDINALITY $|A| :=$ the number of elements in A.

- PARTITION $A_1, \ldots, A_n$ is a partition of B if $B = \bigcup_{i=1}^{n} A_i$ and $A_i \cap A_j = \emptyset \ \forall i \neq j$



Well-defined = $\exists$ exactly one meaning

sum / multiplication PRINCIPLES

1. $|A \cup B| \leq |A| + |B|$

   ex of wwc in cs: estimating storage

2. Let $A_1, \ldots, A_n$ be sets

   Then: $\left|\bigcup_{i=1}^{n} A_i\right| \leq \sum_{i=1}^{n} |A_i|$

3. If $A_1, \ldots, A_n$ is a partition of B,

   and if ~~$|A_i| = \frac{|B|}{n} \forall i$~~. $|A_i| = |B|/n \ \forall i$,

   then $|B| = |A_i| \cdot n$.

   [more generally stated:
   If $\exists c \geq 0$ such that $B = \bigcup_{i=1}^{n} A_i$ and $|A_i| = c$
   and ~~$A_i \cap A_j$~~ $A_i \cap A_j = \emptyset \ \forall i \neq j$, then $|B| = c \cdot n$.

4. $S :=$ the set of lists of length m
   with l_j possibilities for object j

   Then, $|S| = l_1 \cdot l_2 \cdot \ldots \, l_m = \prod_{i=1}^{m} l_i$

   the product of

   e.g.: PASSWORDS

   → 3 numbers followed by 2 letters (lower case)

   $$\underset{\#}{\underline{10}} \cdot \underset{\#}{\underline{10}} \cdot \underset{\#}{\underline{10}} \cdot \underset{\text{let.}}{\underline{26}} \cdot \underset{\text{let.}}{\underline{26}} = 10^3 \cdot 26^2$$

INTERVIEW QUESTION: How many piano movers are in Chicago?

TODAY'S QUESTION: How many license plates can Montana issue (w/ current standards)?

Answers: $\underbrace{56}_{\text{counties}}\cdot 36^6$

$56\cdot 36^6 + \underbrace{26^3 10^3}_{\text{OLDIES BUT GOODIES}}$

$56\cdot 36^6 + 26^3 10^3 + \sum_{i=1}^{7} 36^i$

Same but $36 \to 35$ b/c $\emptyset$ + O conf.

go back to 36 b/c % allowed

MINUS ones w/ cursewords ~ 50-100

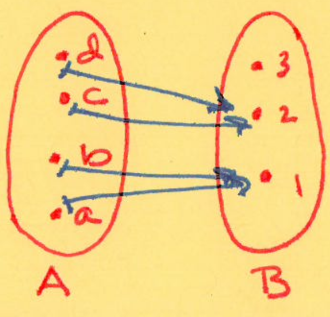
MINUS those over counted.

e.g. P = NP

FUNCTIONS

A function is a map from one set to another.

$f: A \to B$ sets a value $f(a) \in B$ $\forall a \in A$.



- $f: A \to B$ is injective if $\forall a_1 \neq a_2 \in A$, $f(a_1) \neq f(a_2)$. (1-1)
  i.e., each elt. of A goes to a distinct elt of B

- $f: A \to B$ is surjective if $\forall b \in B$, $\exists a \in A$ such that $f(a) = b$. (ONTO)
  i.e., no man in B is left out.

FOR ALL

element of

$f: A \to B$

DOMAIN CO-DOMAIN

- $f(A) := \{ b \in B \mid \exists a \in A \text{ s.t. } f(a) = b \}$
  is the <u>range</u> of ~~A~~ f.

NOTATION: • $\forall b \in B$ reads
"for all elements b in B"

- Let $b \in B$
  "Let b be an element of B"

- $$\bigcup_{i=1}^{n} A_i = A_1 \cup A_2 \cup \ldots \cup A_n$$
  $$= A_1 \cup (A_2 \cup (\ldots \cup A_n)) \ldots)$$