

Reasoning with Formalized Statutes: The Case of Capital Gains and Losses

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Abstract

Reasoning with formalized statutes leads to insights into law that may be otherwise obscured by law’s complexity. Using Internal Revenue Code provisions related to capital gains and losses as an example, this Article shows that reasoning on formalized law can reveal previously overlooked errors in the law; demonstrate equivalence between the actual law and facially dissimilar administrative implementations of the law; and uncover technical corrections in these administrative implementations that have not been openly acknowledged. The Article presents both algebraic and graphical reasoning on formalization and computationally implements the analysis of the formalizations.

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I. INTRODUCTION

Formalizing complex law has many advantages.¹ Formalizations such as tax forms can help taxpayers comply with the law.² Formalizations help taxing authorities administer the law.³ Formalizing law at the drafting stage can draw drafters' attention to unintentional structural ambiguities or errors.⁴ This article provides an example of another benefit of formalization: formalized statutes become susceptible to a particular kind of formalized reasoning. This formalized reasoning can lead to insights about and highlight connections implicit in the statute but obscured by the statute's complexity.

The article focuses, as an example, on reasoning with formalizations of Internal Revenue Code sections related to capital gains and losses. Formalizing these sections reveals an error in the statute. Leading treatises and casebooks do not identify this error; they simply assume the statute works as intended. But formalizing the statute brings the error to light—and reasoning with formalizations makes it straightforward to show that IRS has corrected this error in its forms and worksheets implementing this law. Moreover, reasoning with formalizations shows that while the IRS presents facially different algorithms than those dictated by the statute, the algorithms the IRS presents are generally faithful to the statute—and actually improve upon the statute by making technical corrections that are inconsistent with the statute's (incorrect) language and consistent with the statute's intent. The article also uses formalizations to motivate a graphical presentation of the capital gain and loss statutes that unites the rules

¹ See, e.g., Sarah B. Lawsky, *Formalizing the Code*, 70 TAX L. REV. 377 (2017).

² Sarah B. Lawsky, *Form as Formalization*, 16 OHIO ST. TECH. L.J. 114 (2020).

³ E.g., INTERNAL REVENUE SERV., PUBL'N 5336: IRS INTEGRATED MODERNIZATION BUSINESS PLAN (2019).

⁴ E.g., Sarah B. Lawsky, *Formalizing the Code*, 70 TAX L. REV. 377 (2017) (arguing that “[t]he process of legislative drafting should include formalizing the proposed language of the statute” to avoid unintentional ambiguity).

regarding net capital loss, capital loss limitations, and capital loss carryforwards.

While the formalization does reveal what appears to be a mistake in the law, formalization was not strictly necessary to reveal that mistake—though the attention to detail required to formalize the statute and the precise reasoning necessary for the formalized reasoning were what caused the mistake to come to light. Nonetheless, for the most part, this article does not show anything new about the law. The article shows that a common claim people make about the law is *true*; the worksheet that the IRS has provided *matches the statute* (except for correcting the apparent mistake in the statute). Rather, this article puts forth formalization, and reasoning with formalizations, as a *tool* that can be used to understand the tax law. Reasoning with formalization can be concise and persuasive. Reasoning with formalizations takes using examples to explain the law a step further, reasoning about the structure of the law instead of just specific instances and providing justifications for outcomes.

Moreover, while reasoning on formalizations can be usefully done by humans, the reasoning in this article can be replicated computationally if the operations to be implemented are sufficiently specified.⁵ This article provides an example of such computational implementation, using the formalizations of the law to arrive at the same conclusions using a computer program: the Z3 prover in Python. All code for this article is available in a Github repository as well as reproduced as an appendix to the article.⁶

The article proceeds as follows. Part II provides specifications and operational rules for an operator pervasive in the tax code: “excess of.” This Part describes the law that is to be formalized, formalizes that law using the excess of operator, and through this formalization identifies what seems to be a previously unnoticed error in the statute. Part III provides examples of algebraic reasoning with the formalizations of the capital gains statute. It shows that a common claim about the capital gains statute is correct if the error in the statute is corrected, and

⁵ For a way to thus formalize tax law, together with a proof that the specification can be computationally reasoned upon, see Denis Merigoux, Nicolas Chataing, & Jonathan Protzenko. *Catala: A Programming Language for the Law*, 5 PROCEEDINGS OF THE ACM ON PROGRAMMING LANGUAGES 1 (2021). For a further description of formal methods as applied to tax law, see Liane Huttner & Denis Merigoux, *Catala: Moving Towards the Future of Legal Expert Systems*, ARTIFICIAL INTELLIGENCE AND LAW (2022).

⁶ That repository is <https://github.com/slask/tax-formalization/> and the appendix is Appendix C.

substantively justifies the IRS's implementation of the statute, including showing that the IRS itself corrects the statutory error.⁷ This Part also considers the policy issues raised by algebraic reasoning on formalizations, including showing the possibility and benefits of computational analysis of the formalization of statutes. Part IV uses formalizations to motivate graphical reasoning about the statute and discusses the distinct policy issues that graphical reasoning raises. Part V concludes.

II. FORMALIZING THE LAW

To lay the groundwork for reasoning on formalizations, this Part specifies certain terminology used in the formalization of the statutes in this article, and then provides formalizations of the capital gains and loss statutes that will, in subsequent Parts, be used as the basis for reasoning.

A. *Defining Operators: Excess Of*

To produce reliable results through reasoning with formalisms, the rules for that reasoning must be clearly defined.⁸ Reasoning with the Internal Revenue Code mostly involves reasoning using very basic rules from arithmetic, but the Code also includes some specialized terms. This Section focuses on one such term that pervades the Internal Revenue Code: the “excess of” one amount over another. This Section gives a precise definition of “excess of” and then establishes from that definition operational rules that make it easier to work with “excess of” when reasoning with “excess of” in formalizations.⁹

Throughout this article, “-*” signifies “excess of.”¹⁰ Representing “excess of” with a symbolic operator in this way offers several

⁷ As discussed *infra* Section III.C, while there is a substantive justification for the IRS's change, there is not a legal justification for that change.

⁸ Put another way, there must be a reliable meaning assigned to the formalization; the meaning of the formalization must be consistent, and the output of, for example, operators must be predictably the same every time.

⁹ Some sections refer to “the excess of [A] over [B].” E.g., IRC 1222(11) (“The term ‘net capital gain’ means the excess of the net long-term capital gain for the taxable year over the net short-term capital loss for such year”). Other sections refer to “the extent that [A] exceeds [B].” E.g., IRC 301(c)(1)(A) (“[T]he portion of the distribution which is not a dividend, to the extent that it exceeds the adjusted basis of the stock, shall be treated as gain from the sale or exchange of property.”). These two formulations are equivalent.

¹⁰ This is not, to my knowledge, a standard way to represent “excess of.” (I’m not aware of a standard way to represent excess of as an operator; note 12 discusses “excess of” as a shifted unary ramp function and mentions some standard ways to

benefits. First, it allows accurate and succinct formalization of algorithms involving “excess of.” Second, it emphasizes that excess of is “just” another function/operator, like addition or subtraction. Finally, it allows formalization of “excess of” without conflating it with subtraction, a common error.

“Excess of” is not another term for subtraction. While subtraction can yield a negative number, the “excess of” A over B is only ever positive or zero. If A is greater than B, then the excess of A over B is equivalent to subtraction, and the excess of A over B equals $A - B$. Otherwise, the excess of A over B is zero.¹¹ Another way to characterize the excess of A over B is as the maximum of $A - B$ and zero.¹²

“Excess of” is thus a piecewise function. With the $-*$ operator, another way to state the definition of “excess of” is:¹³

$$A -* B = \begin{cases} A - B, & \text{if } A > B \\ 0, & \text{if } A \leq B \end{cases}$$

This piecewise function is the “Excess Of Definition” or “definition of Excess Of” that will be referred to throughout the article.¹⁴

represent such a function.) There is nothing important about the particular symbol I’ve chosen, of course; it’s just easy to type and looks sort of but not exactly like a subtraction sign.

¹¹ One can see this throughout IRS Forms; see, for example, 2022 Instructions for Schedule D, Capital Loss Carryover Worksheet, D-11. “Excess of” has a long history in the Internal Revenue Code. Section 111(a) of the Internal Revenue Code of 1939, for example, states, “gain for the sale...of property shall be the excess of the amount realized therefrom over the adjusted basis...and loss shall be the excess of the adjusted basis...over the amount realized.”

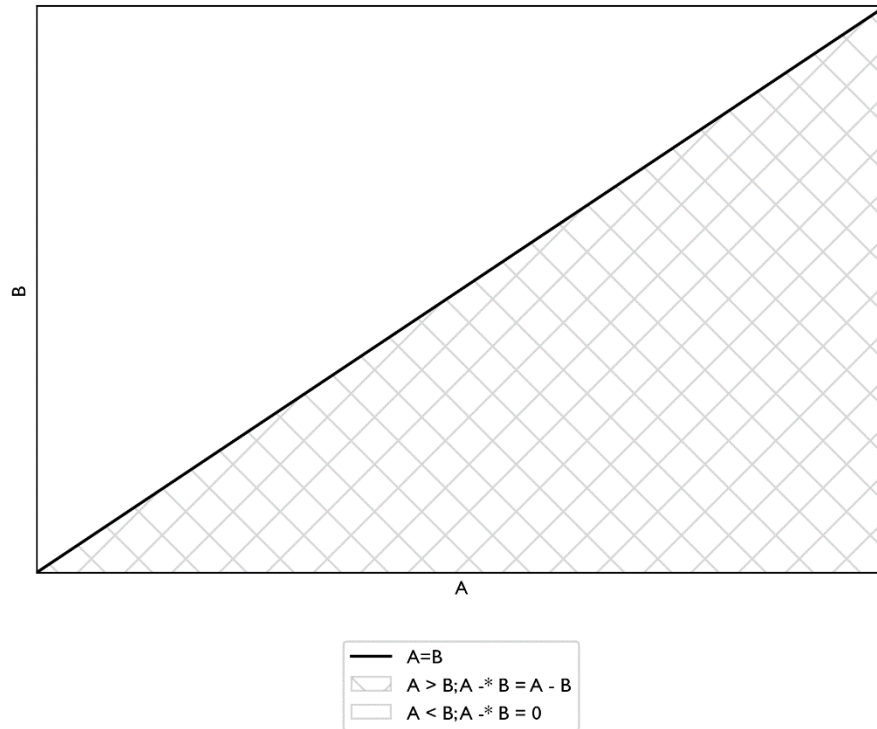
¹² Consider “maximum” to operate on a set of numbers, such that the maximum of a set is the largest number in that set. The maximum of the set $\{0,0\}$ is 0, because the set $\{0,0\}$ equals the set $\{0\}$. See Sarah B. Lawsky, *Teaching Algorithms and Algorithms for Teaching*, 24 FLA. TAX REV. 587, 611 n. 45 (2021) for an analogous discussion of this issue in the Internal Revenue Code in the context of “the lesser of.” More generally, there are lots of other ways to describe the “excess of” function, which is a shifted or delayed unary ramp function. It’s unary because it has a slope of 1; it’s shifted (or delayed) because it’s defined not as x when $x > 0$, but as $x - a$ when $x > a$. The “shift” is the presence of a which shifts the function over by a . Ramp functions come into play in electrical engineering, machine learning, finance (where, for example, a shifted ramp function is the payoff of a call option), and more.

¹³ This function is defined for these purposes over the rational numbers, because for law more generally and for tax calculations in particular, all numbers will be rationals.

¹⁴ Of course it’s true that when $A = B$, $A -* B = A - B = 0$, so it’s equivalent to say that $A -* B = A - B$ if $A \geq B$. I chose $>$ for two reasons. First, the Internal Revenue Service arguably divides things that way on their forms; they tend to say, “Subtract

For example, the extent to which 100 exceeds 40 is $100 - * 40 = 60$, and the extent to which 40 exceeds 100 is $40 - * 100 = 0$.

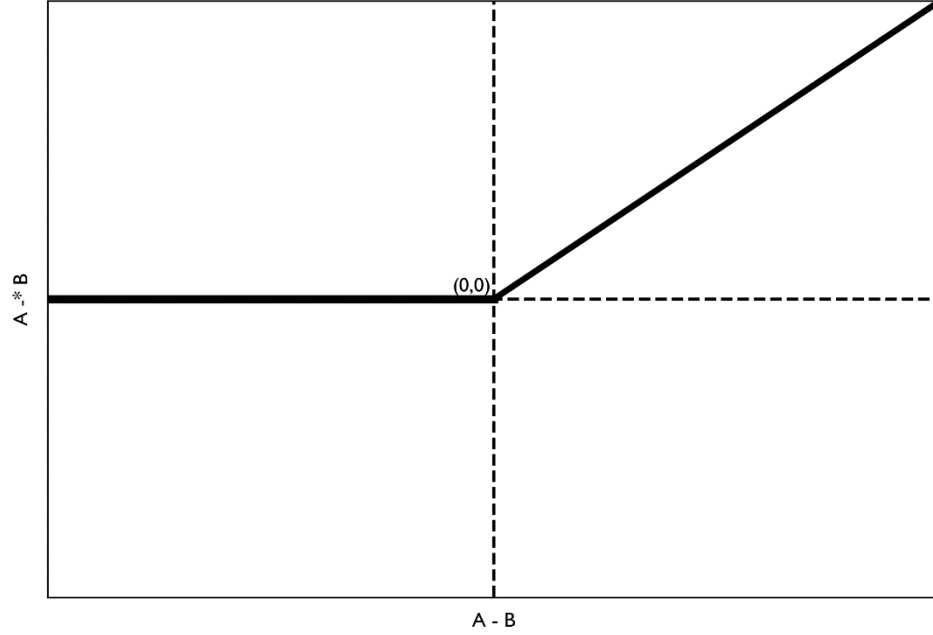
The excess of operator thus divides numerical space. Where A is on the x-axis and B is on the y-axis, in the space below the line $A = B$, where $A > B$, $A - * B = A - B$. In the space on and above that line, $A - * B = 0$.



Alternately, one can plot $A - B$ on the x-axis and $A - * B$ on the y-axis. Where $A - B$ is positive, $A - B = A - * B$; otherwise, $A - * B = 0$. (As noted above, it is also accurate to say that where $A - B$ is nonnegative,¹⁵ $A - B = A - * B$, because if $A - B = 0$, $A - B = A - * B = 0$.)

A from B. If zero or less, enter 0.” See, e.g., 2022 Instructions for Schedule D, Capital Loss Carryover Worksheet, D-11. Second, $>$ better captures the idea that “excess of” is really meant to show when one thing is bigger than the other. As discussed further below, formalizations (algorithms, definitions) and drafting choices can communicate different things to humans even if the outputs are always identical for identical inputs.

¹⁵ “Positive” means “greater than zero.” “Nonnegative” means “not less than zero.” Thus while positive numbers do not include zero, nonnegative numbers do include zero.



Because “excess of” divides up numerical space in this way, one approach to the excess of operator is to operate by cases: if $A - B > 0$, then... and otherwise.... This can be unwieldy. But operational rules make the excess of operator easier to handle.

The article now introduces three equivalences, or operational rules, that hold involving the excess of operator if A , B , and C are all nonnegative (that is, greater than or equal to zero).¹⁶ Rule 1, “Extract the Second Sum,” states that $A - * (B + C) = (A - B) - * C$. Rule 2, “First Excess Of Equivalent to Subtraction,” states that $(A - B) - * C = (A - * B) - * C$. Rule 3, “Push Through the First Sum,” states that $(A + B) - * C = A - * (C - B)$. This Section demonstrates that each of these equivalences holds; the code verifying these equivalences is also available in Appendix C as well as on the Github repository.

1. Rule 1: Extract the Second Sum

“Extract the Second Sum” states that the excess of A over the sum of B and C equals the excess of $A - B$ over C . That is:

$$A - * (B + C) = (A - B) - * C$$

¹⁶ In fact, Rules 1 and 3 work even if some or all of A , B , and C are negative. The nonnegative assumption is relevant only in Rule 2. These rules and the names of these rules are not drawn from another sources; they are new with this article.

Intuitively, this makes sense. The excess of A over the sum of B and C equals the amount by which A is greater than the sum of B and C. If A is greater than B, then $A - B$ is greater than zero, and the excess of $A - B$ over C is positive when A is larger than B by more than the value of C. If A is not greater than B, then A is certainly not greater than the sum of B and C. Therefore both the lefthand and righthand sides of the equation will be zero.

Slightly more formally, proceed by cases (as so often with “excess of.”) For any three numbers A, B, and C, either $A > (B + C)$ (“Case 1”), or $A \leq (B + C)$ (“Case 2”).

Case 1: $A > (B + C)$

Given $A > (B + C)$, the “Case 1 Assumption,” show that $A - * (B + C) = (A - B) - * C$. Because $A > (B + C)$, by the definition of Excess Of, $A - * (B + C) = A - (B + C)$, which is the same as $(A - B) - C$.

From the Case 1 Assumption, $A - B > C$, so again by the definition of Excess of, $(A - B) - C$ equals the excess of $A - B$ over C, $(A - B) - * C$. Therefore, $A - * (B + C) = (A - B) - * C$.

Case 2: $A \leq B + C$

Given $A \leq (B + C)$, the “Case 2 Assumption,” show that $A - * (B + C) = (A - B) - * C$. Because $A \leq (B + C)$, by the definition of Excess Of, $A - * (B + C) = 0$. Because, by the Case 2 Assumption, $A - B \leq C$, it’s also true that $(A - B) - * C = 0$, by the definition of Excess of. Therefore, because both sides of the equation equal 0, $A - * (B + C) = (A - B) - * C$.

2. Rule 2: First Excess Of Equivalent to Subtraction

The definition of “excess of” also allows us to rewrite formulae in which there is (1) either a subtraction sign or an excess of operator followed by (2) an excess of operator. Specifically, *where A, B, and C are nonnegative*:

$$(A - B) - * C = (A - * B) - * C$$

Conceptually, this makes sense, because the second “excess of” operator can do what’s necessary to throw a zero. The first “excess of” operator means that $A - B$ will be either $A - B$ or zero; if it’s zero, then the whole statement will be zero. But if this first excess of operator is a subtraction sign, the whole statement will come out the same way, because $A - B$ will be $A - B$, zero, or a negative number, and if $A - B$

is zero or a negative number, the second excess of operator will return a zero (because C is assumed to be nonnegative).

Again slightly more formally, take two cases, $A > B$ and $A \leq B$.

Case 1: $A > B$

Given $A > B$, the “Case 1 Assumption,” show that $(A - B) - * C = (A - * B) - * C$. By the Case 1 Assumption and the definition of Excess Of, $A - * B = A - B$. It immediately follows that $(A - * B) - * C = (A - B) - * C$.

Case 2: $A \leq B$

Given $A \leq B$, the “Case 2 Assumption,” show that $(A - B) - * C = (A - * B) - * C$. By the Case 2 Assumption and the definition of Excess Of, $A - * B = 0$. Because C is, by assumption, nonnegative, $0 - * C = 0$, by the definition of Excess Of, so $(A - * B) - * C = 0$.

Again by the Case 2 Assumption, $A - B \leq 0$. Thus again by the assumption that C is nonnegative and the definition of Excess Of, $(A - B) - * C = 0$.

Because both sides of the equation equal 0, $(A - B) - * C = (A - * B) - * C$.

3. Rule 3: Push Through the First Sum

“Push Through the First Sum” states that the excess of the sum of A and B over C equals the excess of A over C minus B . That is:

$$(A + B) - * C = A - * (C - B)$$

Slightly more formally, proceed by cases. For any three numbers A , B , and C , either $A + B > C$ (“Case 1”), or $A + B \leq C$ (“Case 2”).

Case 1: $A + B > C$

Given $A + B > C$, the “Case 1 Assumption,” show that $(A + B) - * C = A - * (C - B)$. By the Case 1 Assumption and the definition of Excess Of, $(A + B) - * C = (A + B) - C$. $(A + B) - C = A - (C - B)$.

Also by the Case 1 Assumption, $A > C - B$, and therefore by the definition of Excess of, $A - * (C - B) = A - (C - B)$.

Therefore, because both sides of the equation equal $A - (C - B)$, $(A + B) - * C = A - * (C - B)$,

Case 2: $A + B \leq C$

Given $A + B \leq C$, the “Case 2 Assumption,” show that $(A + B) - * C = A - * (C - B)$. Here, both sides of the equation equal zero. Specifically, $(A + B) - * C = 0$ by the Case 2 Assumption and the definition of Excess Of.

Also by the Case 2 Assumption, $A \leq C - B$, so by the definition of Excess of, $A - * (C - B) = 0$.

Therefore, because both sides of the equation are zero, $(A + B) - * C = A - * (C - B)$.

With this understanding of the “excess of” operator, we can turn now to understanding code sections that rely heavily upon this operator: the law relating to capital gains and losses.

B. The Law of Capital Gains and Losses

This Section reviews and formalizes current law of capital gains and losses for individuals. Gains and losses from the sale or exchange of capital assets are treated differently from other gains and losses in two ways for individuals. Some capital gain is subject to tax at a lower rate than is ordinary income. Additionally, use of capital losses is limited. However, any losses that are not permitted to be used in the current year can be carried forward indefinitely.

Two caveats. First, this article does not incorporate into any of its analysis “qualified dividend income,” which under current law is treated as “net capital gain.”¹⁷ Second, as already mentioned, this article looks at capital gains and losses for individuals only. Different rules apply to corporate taxpayers.

1. Net Capital Gain

“Net capital gain” is taxed at lower rates than ordinary income.¹⁸ For example, in 2023, the highest earners would pay a tax at 37% of their next dollar of ordinary income, but only 20% of their next dollar of capital gain.¹⁹ Very roughly speaking, the idea of net capital gain is that long-term capital gain is subject to this favorable rate, but short-

¹⁷ IRC 1(h)(11).

¹⁸ IRC 1(h)(1). For a history of the origin of the capital gains statute, see Ajay K. Mehrotra, *The Curious Beginnings of the Capital Gains Tax Preference*, 84 *FORDHAM L. REV.* 2517 (2016).

¹⁹ IRC 1(h), (j). An additional 3.8% tax is imposed on certain income of high-earning individuals. IRC 1411.

term capital gain is not. (As will be discussed further, short-term capital gain still matters; it can affect the amount of gain subject to the favorable rate, and it can also affect the extent to which capital losses can be used in a given year.)

“Net capital gain” is the excess of “net long-term capital gain” over “net short-term capital loss.”²⁰ That is:

$$\text{NCG} = \text{NLTCG} - * \text{NSTCL}$$

“Net long-term capital gain” is the excess of long-term capital gain over long-term capital loss.²¹ Long-term capital gain (loss) is gain (loss) from the sale or exchange of a capital asset held for more than one year.²² Both long-term capital gain and long-term capital loss are nonnegative numbers in this definition. That is:

$$\text{NLTCG} = \text{LTCG} - * \text{LTCL}$$

“Net short-term capital loss” is the excess of short-term capital loss over short-term capital gain.²³ Short-term capital loss (gain) is the loss (gain) from the sale or exchange of a capital asset held for not more than one year.²⁴ Both short-term capital gain and short-term capital loss are nonnegative numbers in this definition. That is:

$$\text{NSTCL} = \text{STCL} - * \text{STCG}$$

“Net capital gain” is thus defined by two layers of “excess of,” where LTCG, LTCL, STCL, and STCG are all nonnegative numbers.

$$\text{NCG} = (\text{LTCG} - * \text{LTCL}) - * (\text{STCL} - * \text{STCG})$$

Some intuitions about net capital gain arise from inspecting the definition of net capital gain. Net capital gain is always nonnegative, because it is defined in terms of the excess of one number over another, and the output of excess of is always nonnegative. Both net long-term capital gain and net short-term capital loss are nonnegative numbers, because both are defined in terms of the “excess of” operator. If there is no long-term capital gain, then there is no net capital gain, because

²⁰ IRC 1222(11).

²¹ IRC 1222(7).

²² IRC 1222(3) (gain), (4) (loss).

²³ IRC 1222(6).

²⁴ IRC 1222(2) (loss), (1) (gain).

if LTCG = 0, then net long-term capital gain is 0, and the excess of 0 over a nonnegative number is 0.

If there is no short-term capital loss, then short-term capital gain has no effect on the amount of net capital gain, because short-term capital gain comes into play only to the extent that it reduces short-term capital losses that in turn go to reduce the amount that is passed on to net capital gain. For example, imagine a taxpayer with \$5000 of long-term capital gain, no long-term capital loss, \$2000 of short-term capital loss, and \$500 of short-term capital gain. Net long-term capital gain is the excess of long-term capital gain, \$5000, over \$0. $\$5000 - \$0 = \$5000$. Net short-term capital loss is the excess of short-term capital loss, \$2000, over short-term capital gain, \$500. $\$2000 - \$500 = \$1500$. Net capital gain is therefore the excess of \$5000 over \$1500. $\$5000 - \$1500 = \$3500$.

In the same scenario but with no short-term capital loss—that is, \$5000 of long-term capital gain, no long-term capital loss, no short-term capital loss, and \$500 of short-term capital gain—then net short-term capital loss is \$0, because the excess of \$0 over \$500 is \$0. In this scenario, there is $\$5000 - \$0 = \$5000$ of net capital gain.

Take the same scenario and reduce the short-term capital gain. For example, say there is \$5000 of long-term capital gain, \$500 of short-term capital gain, but now \$2000 of short-term capital loss and no short-term capital gain—then there is $\$2000 - \$0 = \$2000$ of net short-term capital loss, and $\$5000 - \$2000 = \$3000$ of net capital gain. There is *less* short-term capital gain, and thus *more* net short-term capital loss, which leads to *less* net capital gain.

Short-term capital gain in excess of short-term capital losses isn't taxed at a favorable rate. But if there are short-term capital losses, the more short-term capital gain there is (up to the amount of short-term capital losses), the less net short-term capital loss there is to offset net long-term capital gain, and thus the more net long-term capital gain is taxed at a favorable rate. In effect, the short-term capital gains are indirectly taxed as a favorable rate, up to the amount of short-term capital losses, because short-term capital gains can increase the income that is taxed at a favorable rate.

2. Limitation on Losses

The use of capital losses is limited.²⁵ Section 1211(b) permits an individual taxpayer to use capital losses only to extent of capital gains plus the lower of (1) \$3000 and (2) the excess of capital losses over capital gains.²⁶

The general idea of allowing the additional \$3000 is to let the taxpayer use some excess capital losses to offset ordinary income. The amount allowed shouldn't, though, be more capital losses than the taxpayer actually has—which is why the amount permitted is the lower of \$3000 and the excess of capital losses over capital gains.

For example, if the taxpayer has \$10,000 of capital gains and \$15,000 of capital losses, the taxpayer can use capital losses equal to capital gains, \$10,000, plus the lower of (1) \$3000, and (2) the excess of \$15,000 over \$10,000, which is \$5000. The lower of those two is \$3000, so the taxpayer is permitted to use capital losses equal to $\$10,000 + \$3000 = \$13,000$.

If the taxpayer has \$10,000 of capital gains and \$12,000 of capital losses, the taxpayer can use capital losses equal to capital gains, \$10,000, plus the lower of (1) \$3000, and (2) the excess of \$12,000 over \$10,000, which is \$2000. The taxpayer is therefore allowed to use all \$12,000 of capital losses—but not \$13,000 of capital losses, which would be an extremely strange result, since the taxpayer has only \$12,000 of capital losses!

Notice that this loss limitation rules does not mention long-term or short-term capital losses. For example, a taxpayer with \$100,000 of short-term capital gain and \$100,000 of long-term capital loss is allowed to use all \$100,000 of the long-term capital loss, because the

²⁵ The classic article Alvin Warren, *Deductibility by Individuals of Capital Losses Under the Federal Income Tax*, 40 U. CHI. L. REV. 291, 300-314 (1973), discusses (and finds wanting) three possible justifications for limiting the use of capital losses. The article was written when the law regarding capital gains was somewhat different, but the reasons outlined still may apply (or may still be problematic). First, if unlimited capital losses were permitted, tax revenue might drop when prices dropped. Essentially, the argument here is that limiting losses protects tax revenue. A second argument, perhaps more persuasive, is that capital losses are limited because capital gains are taxed favorably. If capital losses could be used against all kinds of income, then the law would have created a heads-taxpayer-wins-tails-government-losses situation: if a capital asset increases in value, that increase is taxed at a low tax rate; if a capital asset decreases in value, that decrease offsets income taxed at a high tax rate. A third argument is that “unlimited deductibility of capital losses would invite tax avoidance.”

²⁶ IRC 1211(b).

amount of loss permitted equals all capital gains, not limited to capital gains of the same length as the losses (plus of course the 1211(b)(1) or (2) amount).

3. *Loss Carryforwards*

It's commonly said that individuals can carry forward unused capital losses indefinitely.²⁷ This is true, as we will see,²⁸ but the statute does not actually state that in so many words. Rather, the statute says, in Section 1212(b)(1), that if a taxpayer other than a corporation has a net capital loss for any taxable year (more on that “net capital loss” requirement in a moment), then (A) the excess of net short-term capital loss over net long-term capital gain is a short-term capital loss in the next year, and (B) the excess of net long-term capital loss over net short-term capital gain is a long-term capital loss in the next year.²⁹

The term “net short-term capital loss” and “net short-term capital gain” do not have the standard meaning given in Section 1222, however, because under Section 1212(b)(2), to make the calculations in 1212(b)(1), “an amount equal to the lesser of—(i) the amount allowed for the taxable year under paragraph (1) or (2) of section 1211(b), or (ii) the adjusted taxable income for such taxable year” shall be treated as short-term capital gain. That is, added onto the amount that is *actually* short-term capital gain would be an amount equal to the lesser of (i) or (ii). Section 1212(b)(2), in other words, may require the

²⁷ E.g., JOSEPH BANKMAN, DANIEL SHAVIRO, ERIC SCHARFF, & KIRK STARK, *FEDERAL INCOME TAXATION* 550 (19th ed. 2023) (“[A]ny net loss above [this] \$3000 figure is defined as a ‘net capital loss’ and may be carried forward to subsequent years[.]”); BORIS BITTKER & LAWRENCE LOKKEN, *FEDERAL TAXATION OF INCOME, ESTATES AND GIFTS*, para. 46.2 (2023). (“If a noncorporate taxpayer has capital losses exceeding those that can be used to reduce capital gains or ordinary income in the taxable year in which sustained, the unused losses carry forward indefinitely under § 1212(b).”); MICHAEL J. GRAETZ & ANNE L. ALSTOTT, *FEDERAL INCOME TAXATION, PRINCIPLES AND POLITICS* (9th ed. 2022) (“Any excess [of capital losses over capital gains] not allowed in one taxable year is carried forward indefinitely until it is completely utilized.”); LIND ET AL., *supra* note 31, at 696 (“[C]apital losses not utilized in the year incurred are carried over into subsequent taxable years”); DANIEL L. SIMMONS, MARTIN J. MCMAHON, JR., BRADLEY T. BORDEN, & BRET WELLS, *FEDERAL INCOME TAXATION* (8th ed. 2020) (“Section 1212 provides non-corporate taxpayers an unlimited carryover of unused capital losses[.]”); RICHARD SCHMALBECK, LAWRENCE ZELENAK & SARAH B. LAWSKY, *FEDERAL INCOME TAXATION* 878 (5th ed. 2018) (“[L]osses that are nondeductible because of the [Section] 1211 limitations may be carried over to future years indefinitely[.]”).

²⁸ *Infra* Section III.A.

²⁹ IRC 1212(b)(1).

amount treated as short-term capital gain to be increased beyond actual short-term capital gain.

To develop an intuition around the purpose of Section 1212(b)(2), consider that an amount treated as short-term capital gain for these purposes *reduces* the loss carryforward. 1212(b)(1)(A) permits as a loss carryforward the excess of net short-term capital loss over net long-term capital gain. Net short-term capital loss equals the excess of short-term capital loss over short-term capital gain. The more the short-term capital gain, the less the loss carryforward. Similarly, 1212(b)(1)(B) permits as a loss carryforward the excess of net long-term capital loss over net short-term capital gain. Net short-term capital gain equals the excess of short-term capital gain over short-term capital loss. The larger this number, the more the loss carryforward is reduced.

But this still isn't quite right, because sometimes a taxpayer is unable to use all permitted capital losses because of insufficient other taxable income. Thus Section 1212(b)(2) treats as short-term capital gain the *lesser* of the amount allowed, or the “adjusted taxable income” for the year, where “adjusted taxable income” equals taxable income, increased by the amount allowed under 1211(b)(1) or (2), and the deduction allowed under Section 151 (\$0 currently³⁰). For these purposes, the excess of deductions allowed over gross income for the year is negative taxable income.

By limiting the amount treated as short-term capital gain to the lesser of the amount allowed under 1211(b)(1) or (2), or the adjusted taxable income (i.e., taxable income with the 1211(b)(1) or (2) amount added back), the statute lets permitted but unused capital losses to be carried forward.

For example, consider the situation where all other income except \$1000 is completely wiped out by all other losses, and in addition there is \$6000 in long-term capital losses and \$3500 in long-term capital gains. The excess of capital losses over capital gains is \$2500, which is less than \$3000. Under Section 1211(b), losses equal to the lesser of the excess of capital losses over capital gains and \$3000 are allowed (in addition the losses up to the amount of capital gains). Therefore, the amount of permitted capital losses is therefore \$3500 (the amount of capital gains) plus the excess of capital losses over capital gains (\$2500)—in other words, all the capital losses are allowed.

³⁰ IRC 151(d)(5)(A).

This all makes good sense, but this example also shows why the net capital loss requirement in Section 1212(b)(1) no longer makes sense. Net capital loss is the excess of losses from sales or exchanges of capital assets over the sum allowed under Section 1211—but that is $\$6000 - \$6000 = \$0$. Although there is no net capital loss, Section 1212(b)(2) operates to permit \$1500 to be carried forward by treating only \$1000, not the full \$2500, as short-term capital gain, and therefore reducing the carryforward by only \$1000, not the full \$1500. Specifically, Section 1212(b) says to treat as a short-term capital gain the amount equal to the lesser of (i) the amount allowed under 1211(b)(1) or (2), and (ii) adjusted taxable income. Adjusted taxable income in this scenario is taxable income increased by the amount allowed under 1211(b)(1) or (2). Adjusted taxable income is thus $\$3500 + \1000 (capital gain plus other income) $- \$6000$ (total capital losses permitted) $+ \$2500$ (amount allowed under 1211(b)(1) or (2)) $= \$1000$. So \$1000 of the excess losses are considered short-term capital gain under Section 1212(b)(2)(A), and \$1500, the amount that went unused because there was insufficient taxable income to absorb it, are not considered short-term capital gain and do not reduce the carryforward.

Net short-term capital loss is \$0; there are no short-term capital losses.

Net long-term capital loss is the excess of \$6000 over \$3500, or \$2500. Net short-term capital gain is \$1000 (there is no short-term capital loss). Therefore the carryforward should be the excess of \$2500 over \$1000, or \$1500, even though there is no net capital loss.

The conditional “if a taxpayer...has a net capital loss” thus seems to be a drafting error—or more precisely, the failure to remove it seems to be a drafting error.³¹ This language was added before Section 1212(b)(2). The purpose of 1212(b)(2) is to allow the taxpayer to

³¹ *Contra, e.g.*, STEPHEN A. LIND, DANIEL J. LATHROPE, & HEATHER M. FIELD, FUNDAMENTALS OF FEDERAL INCOME TAXATION 695-696 (20th ed. 2022) (“The ‘if’ clause at the beginning of Section 1212(b)(1) makes the carryover provisions dependent upon the taxpayer having a ‘net capital loss,’ which is defined in Section 1222(10) as ‘the excess of the losses from the sales or exchanges of capital assets over the sum allowed under section 1211.’ If the sum allowed under Section 1211(b) does not exceed \$3,000 of excess losses over gain (the maximum Section 1211(b) \$3,000 deduction against ordinary income), there is no ‘net capital loss,’ no carryover, and no need or permission to use Section 1212(b).”). *Cf. also* JOSEPH BANKMAN, DANIEL SHAVIRO, ERIC SCHARFF, & KIRK STARK, FEDERAL INCOME TAXATION 550 (19th ed. 2023) (“[T]axpayers sometimes have capital losses that exceed their capital gains for the year, if this is the case, the result loss may be used to offset \$3,000 of ordinary income. Any net loss above this \$3000 figure is defined as a ‘net capital loss’ and may be carried forward to subsequent years[.]”).

carryover the “extra” amount permitted in 1211(b)(1) or (2). Prior to amendment in 1986, Section 1211(b) allowed taxpayers to use capital losses to the extent of the least of (1) taxable income reduced by the zero-bracket amount, (2) the “applicable amount,” which was \$3000 for taxable years starting after 1977, and (3) effectively, the excess of capital losses over capital gains, though modified to reflect that fewer long-term capital losses were permitted to be carried over due to interactions with other permitted exclusions for capital gains (this aspect of the capital gains statutes is outside the scope of this paper).

There had been a deduction for net capital gains, and when this deduction was repealed in 1986, Section 1211(b) was amended to take into account this change but continue to limit the use of capital losses. Specifically, Section 1211(b) was amended to allow the use of capital losses only to the extent of capital gains plus up to \$3000.³² Section 1212(b)(2) was also amended at that time to treat amounts allowed under paragraph (1) or (2) of Section 1211(b) as short-term capital gain for the year when determining loss carryforwards—that is, while Section 1212(b)(2) had previously said that any of the three amounts in Section 1211(b) (taxable income, applicable amount, modified capital losses over capital gains) would be taken into account as short-term capital gain, the statute was amended to treat only the *new* amounts in Section 1211(b) (that is, the (1) \$3000 or (2) the excess of capital losses over capital gains) as short-term capital gain.

This 1986 initial amendment overlooked the situation in which taxpayers were unable to use the “extra” capital losses due to having insufficient taxable income, so Section 1212(b)(2) was subsequently further amended, as part of the 1988 technical corrections act, to treat as a short-term capital gain the *lesser* of the amount allowed, or the “adjusted taxable income” for the year.³³ When 1212(b)(2)(A) was

³² Committee Report, 1986 TRA, PL 99-514; *see also* W. Kirk Baker, *Capital Loss Deduction Limits After the Tax Reform Act of 1986*, 66 TEX. L. REV. 159 (1987).

³³ “[T]he conference agreement provides that the amount that the capital loss carryforwards to the subsequent taxable year are reduced by reason of section 1212(b)(2) cannot exceed the taxpayer’s taxable income for the year increased by (1) the amount of capital losses allowed for the year under paragraph (1) or (2) of section 1211(b) and (2) the deduction for personal exemptions. Where the deductions exceed gross income for the year, the computation is made starting with a negative number.” Committee Report 1988 TAMRA, PL 100-647 (Nov. 10, 1988), p. 7. There is not discussion of this provision in any of the Joint Committee discussions of this bill, nor was the provision in the House bill or the Senate bill. The provision appears to have been added in during the conference. *See also* Office of Chief Counsel Memorandum FILEN-120035-06 (May 3, 2006) (“[Section] 1212(b)(2) generally preserves the carryover to the extent the deduction did not produce an actual tax

amended to take adjusted taxable income into account, 1212(b)(1) should have been amended to remove the language stating that loss carryforwards were permitted “[i]f a taxpayer...*has a net capital loss*” (emphasis added).³⁴ Alternately, the definition of net capital loss in 1222(10) could have been amended to include adjusted taxable income—which it did by cross-reference before adjusted taxable income was removed from Section 1211 in 1986.

The intention of the 1988 amendment was clearly to permit a loss carryover of permitted but unused losses when there is insufficient taxable income fully to use the loss. When capital losses exceed capital gains by not more than \$3000, there is no net capital loss, because Section 1211(b) allows all of the capital losses. But if there is also insufficient taxable income to actually absorb the amount permitted in 1211(b)(1) or (2), then there should be a loss carryforward, equal to the amount that was permitted but could not actually be used. And Section 1212(b)(2) clearly envisions that there should be a loss carryforward in this situation. As we shall see, the net capital loss language is unnecessary, because even without that language, the statute permits carryforwards only when there are unused capital losses.³⁵ And the IRS has no requirement for a net capital loss in its implementation of the statute in the form and worksheet.³⁶

Requiring a net capital loss made sense with the statutory scheme when Section 1211(b) limited the use of capital loss to the amount of capital gains plus the least of three things, one of which was taxable income reduced by the zero-bracket amount, because 1222 defines net capital loss as the excess of capital losses over “the sum allowed under Section 1211.” But when the taxable income limitation was no longer listed explicitly in Section 1211, Sections 1211, 1212, and 1222 no longer fit together in a way that makes requiring a net capital loss work for permitting a loss carryforward.³⁷

benefit because there was insufficient ordinary income available for offset and taxable income was actually negative.”)

³⁴ It’s true that this is an “if” statement and not an “if and only if” statement. But loss carryforwards are permitted only if explicitly allowed by the statute, and capital loss carryforwards are not allowed by any other part of the statute.

³⁵ See discussion *infra* Section III.A, using reasoning on formalizations to show that this is the case even without an explicit requirement of a net capital gain.

³⁶ See *infra* Section III.B.

³⁷ The regulations have not been updated since the 1986 amendments, and according to the regulations, the permitted 1211(b) “additional allowance” is still the least of taxable income reduced by the zero bracket amount; \$3000; and the sum of the excess of net short-term capital loss over net long-term capital gain, plus half the excess of

III. ALGEBRAIC REASONING ON FORMALIZATIONS

Once statutes are formalized, they are susceptible to formalized reasoning. This Part provides two examples of what might be called algebraic reasoning on the formalized capital gain and loss statutes (as opposed to graphical reasoning, which is presented in Part IV), and then considers policy issues related to this type of reasoning.

A. *Carrying Forward “Unused Losses”: Justifying a Common Description of the Law*

It’s a common informal claim that unused capital losses can be carried forward.³⁸ The statute does not, of course, say that in so many words.³⁹ This Part establishes that the carryforward permitted by the statute equals the carryforward of unused capital losses. That the amount of losses that can be carried forward equals the unused losses may be obvious to people who are very familiar with the statute. Beyond showing that the requirement of “net capital loss” within the statute is likely an error, the argument below does not establish anything new; rather, it provides a concise and exhaustive argument to justify a familiar claim.

Capital losses can go unused in a given year for two reasons; both reasons can come into play at once. First, there might be capital losses in excess of capital gains plus \$3000—that is, there could be a net

net long-term capital loss over net short-term capital gain. Treas. Reg. 1.1211-1(b)(2). Because this produces such a radically different permitted carryforward from the current statute, it doesn’t seem a solution to rely on the regulation to “solve” the definition of net capital loss/1212(b)(1) “net capital loss” disconnection.

³⁸ E.g., JOSEPH BANKMAN, DANIEL SHAVIRO, ERIC SCHARFF, & KIRK STARK, *FEDERAL INCOME TAXATION* 550 (19th ed. 2023) (“[A]ny net loss above [this] \$3000 figure is defined as a ‘net capital loss’ and may be carried forward to subsequent years[.]”); BORIS BITTKER & LAWRENCE LOKKEN, *FEDERAL TAXATION OF INCOME, ESTATES AND GIFTS*, para. 46.2 (2023). (“If a noncorporate taxpayer has capital losses exceeding those that can be used to reduce capital gains or ordinary income in the taxable year in which sustained, the unused losses carry forward indefinitely under § 1212(b).”); MICHAEL J. GRAETZ & ANNE L. ALSTOTT, *FEDERAL INCOME TAXATION, PRINCIPLES AND POLITICS* (9th ed. 2022) (“Any excess [of capital losses over capital gains] not allowed in one taxable year is carried forward indefinitely until it is completely utilized.”); LIND ET AL., *supra* note 31, at 696 (“[C]apital losses not utilized in the year incurred are carried over into subsequent taxable years”); DANIEL L. SIMMONS, MARTIN J. MCMAHON, JR., BRADLEY T. BORDEN, & BRET WELLS, *FEDERAL INCOME TAXATION* (8th ed. 2020) (“Section 1212 provides noncorporate taxpayers an unlimited carryover of unused capital losses[.]”); RICHARD SCHMALBECK, LAWRENCE ZELENAK & SARAH B. LAWSKY, *FEDERAL INCOME TAXATION* 878 (5th ed. 2018) (“[L]osses that are nondeductible because of the [Section] 1211 limitations may be carried over to future years indefinitely[.]”).

³⁹ *Supra* Section II.B.3.

capital loss. These losses are disallowed under Section 1211. Second, there might be not enough ordinary income to use capital losses that are permitted under Section 1211(b). The claim is that whatever combination of these two situations is true, the amount of capital losses that can be carried forward equals the unused capital losses.

To tackle this through somewhat formalized reasoning, first formalize the total losses that the statute permits to be carried forward, and then formalize “unused losses.” Show that these two are always equal.

First, formalize what the statute permits. Section 1212(b)(1) states that the amount permitted to be carried forward is the sum of (A) the excess of net short-term capital loss over net long-term capital gain, and (B) the excess of net long-term capital loss over net short-term capital gain. For these purposes, short-term capital gain is increased by the lesser of (i) the amount allowed under 1211(b)(1) or 1211(b)(2) (call this the “1211(b) Amount”), and (ii) adjusted taxable income. Call the lesser of the 1211(b) Amount and the adjusted taxable income, the “1212(b)(2) Amount.” The losses permitted to be carried forward by the statute are thus, by the definitions of net short-term capital gain and loss and net long-term capital gain and loss in Section 1222:

$$(\text{STCL} - * (\text{STCG} + 1212(\text{b})(2) \text{ Amount})) - * (\text{LTCG} - * \text{LTCL}) +$$

$$(\text{LTCL} - * \text{LTCG}) - * ((\text{STCG} + 1212(\text{b})(2) \text{ Amount}) - * \text{STCL})$$

Now formalize “unused capital losses”—not a formal term from the Code, but rather the term this article will use for capital losses that were not able to be used in a taxable year. Unused capital losses will, thus, equal the excess of the total capital losses over the losses able to be used. (Use “excess of” here and not just subtraction because if there are not more losses than there are losses able to be used, then the correct amount to carry forward is 0.)

The total capital losses in a given year:

$$\text{LTCL} + \text{STCL}$$

Section 1211 says that the amount of capital losses that can be used in a given year equals sum of the amount of capital gains plus the 1211(b) Amount. So this might be a first attempt at formalizing the amount of losses that can be used:

$$\text{LTCG} + \text{STCG} + 1211(\text{b}) \text{ Amount}$$

But in fact, the full 1211(b) Amount cannot be used unless there is sufficient ordinary income to use it all. The extra amount that can be used is the lesser of the 1211(b) Amount and taxable income not counting the reduction by the 1211(b) Amount. This is, by no coincidence at all, the 1212(b)(2) Amount. The amount of capital losses that can be used in a given year is thus

$$\text{LTCG} + \text{STCG} + 1212(\text{b})(2) \text{ Amount}$$

Therefore, the unused capital losses will equal the excess of the total capital losses over the losses able to be used:

$$(\text{LTCL} + \text{STCL}) - * (\text{LTCG} + \text{STCG} + 1212(\text{b})(2) \text{ Amount})$$

I have made a judgment call here that the unused capital losses include not the excess of capital losses over capital gains plus the 1211(b) amount (i.e., the lesser of two things: \$3000 or the excess of capital losses over capital gains), but rather the excess capital losses over capital gains plus the 1212(b)(2) amount—essentially, the least of three things: \$3000, the excess of capital losses over capital gains, *or adjusted taxable income*. I’ve described above why I think this is the right way to characterize unused losses,⁴⁰ but I will go a step further here—if this is not the right way to characterize unused losses, then the below reasoning shows that *carryforwards do not always equal unused losses*. Carryforwards equal unused losses only when the 1212(b)(2) Amount equals the 1211(b) Amount; when the 1212(b)(2) Amount equals adjusted taxable income, then unused losses would not equal what the statute prescribes to be carried forward.

With the formalization of unused losses as above, then, the goal is to show that the statutory prescription of capital losses that can be used equals the unused capital losses:

$$\begin{aligned} & (\text{STCL} - * (\text{STCG} + 1212(\text{b})(2) \text{ Amount})) - * (\text{LTCG} - * \text{LTCL}) + \\ & (\text{LTCL} - * \text{LTCG}) - * ((\text{STCG} + 1212(\text{b})(2) \text{ Amount}) - * \text{STCL}) = \\ & (\text{LTCL} + \text{STCL}) - * (\text{LTCG} + \text{STCG} + 1212(\text{b})(2) \text{ Amount}) \end{aligned}$$

For ease of presentation, use the notation of STCG# to represent STCG + 1212(b)(2) Amount. The claim to demonstrate is:

$$((\text{STCL} - * \text{STCG\#}) - * (\text{LTCG} - * \text{LTCL})) +$$

⁴⁰ See *supra* Section II.B.3.

$$\begin{aligned} & ((LTCL - * LTCG) - * (STCG\# - * STCL)) = \\ & (LTCL + STCL) - * (LTCG + STCG\#) \end{aligned}$$

Call the amount described in 1212(b)(1)(A):

$$A = (STCL - * STCG\#) - * (LTCG - * LTCL)$$

Call the amount described in 1212(b)(1)(B):

$$B = (LTCL - * LTCG) - * (STCG\# - * STCL)$$

First show that if there are no unused losses, the statute does not permit any loss carryforward. That is, if

$$(LTCL + STCL) - * (LTCG + STCG\#) = 0$$

then $A + B = 0$.

If

$$(LTCL + STCL) - * (LTCG + STCG\#) = 0$$

then

$$(LTCL + STCL) \leq (LTCG + STCG\#).$$

Therefore

$$LTCL - LTCG \leq STCG\# - STCL$$

$$B = (LTCL - * LTCG) - * (STCG\# - * STCL).$$

$$(LTCL - * LTCG) - * (STCG\# - * STCL) =$$

$$(LTCL - LTCG) - * (STCG\# - * STCL)$$

by Rule 2.

Either $(STCG\# - * STCL) = 0$ or $(STCG\# - * STCL) > 0$, by the definition of Excess Of.

If $(STCG\# - * STCL) > 0$, then $(STCG\# - * STCL) = (STCG\# - STCL)$, and because $LTCL - LTCG \leq STCG\# - STCL$, $B = 0$.

If $(STCG\# - * STCL) = 0$, then $STCL > STCG\#$. But we also know that $LTCL - LTCG \leq STCG\# - STCL$, so we also know that $LTCL -$

LTCG is negative. The excess of a negative number over 0 is 0. So again, $B = 0$.

The argument to show that $A = 0$ is identical, but work from

$$STCL - STCG\# \leq LTCG - LTCL.$$

Notice that this demonstrates that there is no need for the explicit requirement that there be a net capital loss (or that there must be unused losses) for there to be carryforwards; following the terms of the rest of the statute, if there are no unused losses, then there will be no carryforward.

Now show that the claim holds when there are unused losses, i.e., when

$$LTCL + STCL > LTCG + STCG\#$$

Call this the Net Loss Assumption.⁴¹ Because the “excess of” operator has different results depending on which of the two terms is larger, there are four cases:

	$LTCL > LTCG$	$LTCL \leq LTCG$
$STCL \leq STCG\#$	Case 2	Case 1
$STCL > STCG\#$	Case 3	Case 4

For each of these cases, given the Net Loss Assumption, either the claim holds, or the case cannot occur.

Case 1: $STCL \leq STCG\#$ and $LTCL \leq LTCG$

This never occurs. If $STCL \leq STCG\#$ and $LTCL \leq LTCG$, then $LTCL + STCL \leq LTCG + STCG\#$, which violates the Net Loss Assumption.

Case 2: $STCL \leq STCG\#$ and $LTCL > LTCG$

$$A = (STCL - * STCG\#) - * (LTCG - * LTCL)$$

Because $STCL \leq STCG\#$ and $LTCL > LTCG$, $A, = 0 - * 0 = 0$.

⁴¹ This is *not* an assumption that there is a net capital loss as defined in 1222(10); net capital loss is the excess of capital losses over the amount allowed by 1211(b), not 1212(b).

For B, by assumption, $STCL \leq STCG\#$ and $LTCL > LTCG$. By the definition of Excess Of, therefore

$$\begin{aligned} B &= (LTCL - LTCG) - (STCG\# - STCL) = \\ &= (LTCL - LTCG) - (STCG\# - STCL) \end{aligned}$$

If $LTCL - LTCG \leq STCG\# - STCL$, then $LTCL + STCL \leq LTCG + STCG\#$. This violates the Net Loss Assumption.

Therefore $LTCL - LTCG > STCG\# - STCL$, and

$$\begin{aligned} B &= (LTCL - LTCG) - (STCG\# - STCL) \\ &= (LTCL + STCL) - (LTCG + STCG\#). \\ A + B &= 0 + (LTCL + STCL) - (LTCG + STCG\#) = \\ &= (LTCL + STCL) - (LTCG + STCG\#) \end{aligned}$$

Because of the Net Loss Assumption and the definition of Excess Of,

$$\begin{aligned} &= (LTCL + STCL) - (LTCG + STCG\#) = \\ &= (LTCL + STCL) - (LTCG + STCG\#) \end{aligned}$$

This is what was to be shown.

Case 3: $STCL > STCG\#$ and $LTCL > LTCG$

By the definition of Excess Of and the Case 3 Assumption:

$$\begin{aligned} A &= (STCL - STCG\#) - 0 = (STCL - STCG\#) \\ B &= (LTCL - LTCG) - 0 = (LTCL - LTCG) \\ A+B &= \\ &= (STCL - STCG\#) + (LTCL - LTCG) = \\ &= (LTCL + STCL) - (LTCG + STCG\#) \end{aligned}$$

Because of the Net Loss Assumption and the definition of Excess Of,

$$\begin{aligned} &= (LTCL + STCL) - (LTCG + STCG\#) = \\ &= (LTCL + STCL) - (LTCG + STCG\#) \end{aligned}$$

This is what was to be shown.

Case 4: $STCL > STCG\#$ and $LTCL \leq LTCG$

By the definition of Excess of and the Case 4 Assumption,

$$A = (STCL - STCG\#) - (LTCG - LTCL)$$

If

$$(STCL - STCG\#) \leq (LTCG - LTCL),$$

then

$$(STCL + LTCL) \leq (LTCG + STCG\#)$$

which violates the Net Loss Assumption. Therefore

$$(STCL - STCG\#) > (LTCG - LTCL)$$

$$A = (STCL - STCG\#) - (LTCG - LTCL)$$

by the definition of Excess Of.

$$(STCL - STCG\#) - (LTCG - LTCL) =$$

$$(STCL + LTCL) - (STCG\# + LTCG).$$

Therefore

$$A = (STCL + LTCL) - (STCG\# + LTCG).$$

$B = 0$, because $LTCL \leq LTCG$, so $(LTCL - LTCG) = 0$.

$$A + B =$$

$$(STCL + LTCL) - (STCG\# + LTCG) + 0 =$$

$$(LTCL + STCL) - (LTCG + STCG\#)$$

Because of the Net Loss Assumption and the definition of Excess Of,

$$(LTCL + STCL) - (LTCG + STCG\#) =$$

$$(LTCL + STCL) - (LTCG + STCG\#)$$

This is what was to be shown.

Loss carryforwards can occur under the statutory prescription only when there are unused losses. If there are unused losses, the statutory prescription for loss carryforwards equals unused losses in all possible combinations of long-term capital loss, long-term capital gain, short-term capital loss, and short-term capital gain. Therefore, *if and only if* the net capital loss requirement is dropped out, the common claim regarding the carrying forward of unused losses is accurate.

This can also be verified, including the fact that the common claim is true only if the net capital loss requirement is dropped out, using the Z3 prover. Code with this verification is available at the Github repository for this article.

B. Justifying the IRS Implementation of the Law

The IRS creates forms and worksheets to allow taxpayers to comply with the law more easily. These forms and worksheets do not have legal authority, but as a practical matter, they are how most taxpayers interact with the law.⁴² The IRS implements loss carryforwards in the worksheet in the instructions for the Form 1040 Schedule D. This worksheet is of interest for two reasons. First, as this Section shows, the worksheet implements what is essentially a technical correction, because it does not require taxpayers to have a net capital loss as defined in Section 1222(10) in order to carry forward unused capital losses. Second, the algorithm prescribed by the worksheet does not on its face track the statute. This Section shows that the apparently different approach on the worksheet is a more efficient computation that results in exactly the output the statute prescribes, except for the technical correction already described. You may find it easier to read this Section while consulting the worksheet in Appendix A, which is labeled to match to the following discussion.

To see how the worksheet executes the correction and the efficient algorithm, it's first necessary to look at Schedule D itself, where the taxpayer provides information about capital gains and losses.

1. The Groundwork: Netting Capital Gains and Losses

On the Schedule D to the Form 1040, the taxpayer calculates and then sums all gains and losses from transactions involving short-term

⁴² For further discussion of the legal significance, or lack thereof, of forms, see Sarah B. Lawsky, *Form as Formalization*, 16 OHIO ST. TECH. L.J. 114, 120-121 and accompanying notes (2020).

capital assets. For each transaction, the taxpayer subtract proceeds from basis, the result may be, as the form says, “[g]ain (or loss).” The taxpayer then combines all these numbers, positive and negative, to obtain “[n]et short-term capital gain (or loss).” The taxpayer then does the same for long-term capital assets.

The taxpayer combines these two numbers.

Now the form branches: if the combined amount is a gain, the taxpayer follows one path; if the combined amount is a loss, the taxpayer follows another path.

Combined Amount Is Gain

If the combined amount is a gain, the taxpayer is asked whether “lines 15 and 16 [are] **both** gains” (emphasis in original). That is, the form is asking whether the taxpayer has net long-term capital gain (line 15), and whether net long-term capital gain exceeds net short-term capital loss, if any—line 16 combines the long-term and short-term numbers, and if it is positive, either there is net short-term capital gain (two positives make a positive) or the net long-term capital gain exceeds the net short-term capital loss (such that the combined number is still positive).

If both lines are *not* gains, then the taxpayer does not have net capital gain (disregarding qualified dividends) and can proceed with the Form 1040 without including net capital gain.

If both lines *are* gains, then the taxpayer, after taking into account other possible rates, does have capital gain, and they go on to complete the relevant worksheet and enter the correct amount of capital gain into the 1040.

Combined Amount Is Loss

If the “combined amount” is a loss,

$$\text{LTCG} + \text{STCG} - \text{LTCL} - \text{STCL}$$

is a loss. That is,

$$(\text{LTCG} - \text{LTCL}) + (\text{STCG} - \text{STCL}) < 0$$

In this scenario, it can’t be that there is both net long-term capital gain (i.e., that $\text{LTCG} - \text{LTCL} > 0$) and net short-term capital gain (i.e., that $\text{STCG} - \text{STCL} > 0$), because then this amount would not be a loss.

There could be net long-term capital gain and net short-term capital loss, if $NSTCL > NLTCG$. There could be net long-term capital loss and net short-term capital gain, if $NLTCL > NSTCG$. Or there could be both net long-term capital loss and net short-term capital loss.

If the combined amount is a loss, then the taxpayer is told to enter the smaller of the (1) combined amount or (2) \$3000 on Form 1040, Line 16. This applies Section 1211(b)(2). All of the losses that offset gains are permitted to be used, because none of the capital gains are included on the 1040. The amount that is being included on the 1040 if the combined number is a loss is the 1211(b)(1) or (2) number.

Now consider what happens if there is not enough taxable income to use this loss in full. This loss is combined to obtain adjusted gross income, which is then reduced further. Line 15 of the Form 1040 says that if, when all these numbers are combined, the result is zero or less, enter taxable income as 0, not as a negative number. Where does the capital loss carryforward occur? And are they ever permitted to be taken without there being net capital loss?

2. Transforming Net Capital Gains and Losses into Loss Carryforwards

The capital loss carryforwards occur on a worksheet. Each year's worksheet calculates how much capital loss can be carried from the prior year.⁴³ As this Section shows, the IRS worksheet provides a facially different algorithm than that provided by the statute, but with the same outcome, with one important difference: the IRS worksheet does not require the taxpayer to have a net capital loss in order to carry forward unused losses.⁴⁴

The worksheet first has the taxpayer calculate the 1212(b)(2) Amount. Start with the amount on Line 15, but here if the number could have been negative, include a negative amount, not just 0. This is taxable income with the adjustment as described in the flush language to 1212(b)(2)(B) ("any excess of the deductions allowed for the taxable year over the gross income for such year shall be taken into account as

⁴³ The worksheet with the various lines linked to the statute is available in Appendix B.

⁴⁴ See also IRS Pub. 550 INVESTMENT INCOME AND EXPENSES (INCLUDING CAPITAL GAINS AND LOSSES) 66 (2023) ("If you have a total net loss on line 16 of Schedule D (Form 1040) that is more than the yearly limit on capital loss deductions, you can carry over the unused part to the next year....The amount of your capital loss carryover is the amount of your total net loss that is more than the lesser of (1) Your allowable capital loss deduction for the year, or (2) Your taxable income increased by your allowable capital loss deduction for the year.").

negative taxable income”). Then take the total loss from Line 21 on Schedule D, as a positive number. This is the amount permitted in 1211(b)(1) or (2): the smaller of the excess of capital losses or capital gains, or \$3000. Combine these two numbers to get adjusted taxable income as defined in 1212(b)(2)(B): taxable income (which can, according to the terms of Section 1212(b)(2)(B), go negative for this purpose), increased by the amount allowed under 1211(b)(1) or (2). There is one slight discrepancy between the statute and the worksheet, which is that the statute apparently would allow adjusted taxable income to be negative, but the worksheet states that if the two numbers combined is less than 0, the taxpayer should enter 0.

This could be considered another technical correction. The statute says that the *lesser* of the amount allowed under 1211(b)(1) or (2) and the adjusted taxable income is treated as short-term capital gain. The goal here is, as stated above, to make sure that if the taxpayer didn’t actually benefit from the 1211(b)(1) or (2) allowance, the taxpayer’s loss carryforward isn’t reduced by that allowance. The purpose is not to provide additional capital loss carryforward where there should not be any. Consider the scenario where adjusted taxable income is negative—for example, there is \$10,000 of deduction allowed over gross income, and say that \$3000 of that \$10,000 deduction is what’s allowed under 1211(b)(2). Adjusted taxable income is then $(\$10,000) + \$3000 = (\$7000)$. The taxpayer effectively got to use none of the \$3000 that was allowed under 1211(b)(2), so the right amount by which to treat as a short-term capital gain should be the lesser of \$3000 (the 1211(b)(2) amount) and \$0, which is \$0. According to the strict terms of the statute, though, $(\$7000)$ is less than \$0, suggesting that $(\$7000)$ should be treated as a short-term capital gain. This cannot be right.

Then take the smaller of the combined number and the loss from the Schedule D as a positive amount—this is the Section 1212(b)(2) Amount as prescribed by the statute.

If the amount on Line 15 is a gain, then the smaller of combined numbers and the loss from Schedule D as a positive number will be the loss from Schedule D as a positive number. If the amount on Line 15 is a loss, then the smaller of the two will be the combined amount. If the loss is less than the 1211(b) Amount the combined amounts will be a positive number less than the 1211(b) Amount, and if it is a loss greater than the 1211(b) amount, then the combined number will be 0, and the smaller of the two numbers will be 0.

On Line 5, the taxpayer enters the net short-term capital loss, if any (line 7 of the Schedule D, but only if it is negative; it is entered as a positive number). On line 6, the taxpayer enters the long-term capital gain, if any (line 15 of the Schedule D, but only if it is positive). The taxpayer then adds the 1212(b)(2) Amount to the net long-term capital gain, and subtracts this sum from net short-term capital loss. If the amount is 0, then enter zero. This is an excess of calculation:

$$\text{NSTCL} - * (\text{NLTCG} + 1212(b)(2)).$$

(For ease of presentation, in the formalizations for this next portion I will use “1212(b)(2)” for “1212(b)(2) Amount.”)

Why is the 1212(b)(2) Amount combined with net long-term capital gain, when the statute tells us that the 1212(b)(2) Amount is treated as short-term capital gain? The calculation on the form is identical to that treatment.

The statute, formalized, is $(\text{STCL} - * (\text{STCG} + 1212(b)(2))) - * \text{NLTCG}$. By Extract the Second Sum (Rule 1), this becomes

$$((\text{STCL} - \text{STCG}) - * 1212(b)(2))) - * \text{NLTCG}$$

Because STCL, STCG, and the 1212(b)(2) Amount are all positive, by First Excess Of Equivalent to Subtraction (Rule 2), this can be written as

$$((\text{STCL} - * \text{STCG}) - * 1212(b)(2))) - * \text{NLTCG}$$

By the definition of Net Short Term Capital loss, this is simply

$$(\text{NSTCL} - * 1212(b)(2)) - * \text{NLTCG}$$

These numbers are all positive, so again by First Excess Of Equivalent to Subtraction, rewrite this as

$$(\text{NSTCL} - 1212(b)(2)) - * \text{NLTCG}$$

Extract the Second Sum results in

$$\text{NSTCL} - * (\text{NLTCG} + 1212(b)(2))$$

And this is what is on the worksheet as short-term capital loss to be carried forward. Code verifying this equivalence using the Z3 prover is available at the Github repository for this article.

To determine the long-term capital loss carryforward, the worksheet says to start by writing down Schedule D, Line 15, if it is a negative amount—that is, write down net long-term capital loss if it exists. Then write down net short-term capital gain, if it exists (Schedule D, line 7, if it is a gain).

The form then says to find the excess of line 4 over line 5—that is, it says to subtract line 5 from line 4 and if zero or less, enter 0. Line 5 is net short-term capital loss. Line 4 is the 1212(b) Amount. Thus this line is saying:

$$1212(b)(2) - * NSTCL$$

And then the next step is line 10 plus line 11,

$$NSTCG + (1212(b)(2) - * NSTCL)$$

Finally, take the excess of NLTCL over this full amount.

$$NLTCL - * (NSTCG + (1212(b)(2) - * NSTCL))$$

The statute says that the long-term capital loss carryforward is

$NLTCL - * NSTCG$, with the 1212(b)(2) Amount included in short-term capital gain, so:

$$NLTCL - * ((STCG + 1212(b)(2)) - * STCL)$$

Show that $NSTCG + (1212(b)(2) - * NSTCL)$ is equivalent to $(STCG + 1212(b)(2)) - * STCL$, and this will show that the statement on the form is equivalent to the statute.

Consider $NSTCG + (1212(b)(2) - * NSTCL)$. Either $NSTCG = 0$ or $NSTCL = 0$ (or both).

Case 1: NSTCG = 0

If $NSTCG = 0$,

$NSTCG + (1212(b)(2) - * NSTCL) = 1212(b)(2) - * NSTCL$. Either $1212(b)(2) > NSTCL$ or not.

If $1212(b)(2) > NSTCL$, then

$$1212(b)(2) > STCL - STCG$$

$$1212(b)(2) - * (STCL - STCG) =$$

$$1212(b)(2) - (STCL - STCG) =$$

$$1212(b)(2) + STCG - STCL =$$

$(STCG + 1212(b)(2)) - STCL$, because by assumption, $1212(b)(2) + STCG > STCL$.

If $1212(b)(2) \leq NSTCL$, then $1212(b)(2) - NSTCL = 0$.

But if $1212(b)(2) \leq NSTCL$, then $(STCG + 1212(b)(2)) - STCL$ is also 0, because if $1212(b)(2) \leq NSTCL$, $1212(b)(2) \leq (STCL - STCG)$, and thus $STCG + 1212(b)(2) \leq STCL$.

Therefore in Case 1, $NSTCG + (1212(b)(2) - NSTCL) = (STCG + 1212(b)(2)) - STCL$.

Case 2: $NSTCL = 0$

By the Case 2 Assumption, $NSTCG + (1212(b)(2) - NSTCL) = NSTCG + 1212(b)(2)$, which by the definition of Net Short Term Capital Gain can be written as

$$(STCG - STCL) + 1212(b)(2)$$

Again by the Case 2 Assumption, this can be written as

$$(STCG - STCL) + 1212(b)(2)$$

Thus

$$(STCG - STCL) + 1212(b)(2) = (STCG - STCL) + 1212(b)(2)$$

These terms can be associated and rewritten as $(STCG + 1212(b)(2)) - STCL$. But by the Case 2 Assumption, we know that $STCG \geq STCL$. Because the $1212(b)(2)$ Amount is positive, we can therefore conclude that $STCG + 1212(b)(2) \geq STCL$. Therefore, by the definition of Excess Of,

$$(STCG + 1212(b)(2)) - STCL = (STCG + 1212(b)(2)) - STCL$$

Thus in Case 2 as well, $NSTCG + (1212(b)(2) - NSTCL) = (STCG + 1212(b)(2)) - STCL$.

Code verifying this equivalence using the Z3 prover is available at the Github repository for this article.

Thus while the IRS worksheet presents an algorithm to follow that seems quite distinct from that prescribed by the statute, reasoning on the formalization shows that the IRS has prescribe a method that is consistent with the statute, with one important difference: the IRS implementation does not require that the taxpayer have a net capital loss in order to carry forward unused losses.

C. Implications of Algebraic Reasoning and Computational Implementation

Algebraic reasoning, finding equivalences between formalization, allowed justification of the IRS form and worksheet. Equivalences between formalizations also makes the form and worksheet possible. Consider two aspects of the worksheet: first, it fixes errors in the statute; and, second, the calculations that the worksheet prescribes produce the same outcomes as the calculations that the statute prescribes (with the errors fixed), but the algorithm that the worksheet provides is not the same as the algorithm the statute provides. Fixing clear errors and creating efficient, easy to implement algorithms both may be desirable actions by administrative agencies, but both raise issues related to transparency and legitimacy.

The worksheet corrects what appears to be at least one and perhaps two drafting errors in the statute. As explained above, the statute requires that a taxpayer have net capital loss in order to carry forward unused capital losses, and the worksheet, in a technical correction, does not require a net capital loss. This is an example of what Leigh Osofsky calls an “agency legislative fix.”⁴⁵ As Osofsky explains, legislative drafting is rife with errors, and agencies routinely fix these errors, whether through informal guidance or otherwise. Osofsky notes the significant normative problems with agency legislative fixes, and proposes that in addition to certain legislative procedural fixes that might allow easier correction of errors, agencies should make their fixes more “transparent and systematic,”⁴⁶ including publicizing and justifying these fixes.

This algorithm is *not transparent*, because it does not track the statute, and someone inspecting it could not easily back out the motivation behind each of the parts. The IRS has not provided a justification of

⁴⁵ Leigh Osofsky, *Agency Legislative Fixes*, 105 IOWA L. REV. 2107 (2020).

⁴⁶ Leigh Osofsky, *Agency Legislative Fixes*, 105 IOWA L. REV. 2107, 2163-2165 (2020).

the worksheet algorithm to the public. Taxpayers simply have to trust that the IRS is correct.

But the algorithm is also very *efficient*, because it is short and all the inputs are precalculated; nothing needs to be recalculated to use this algorithm. All of the inputs for this algorithm are also generated for the net capital gain algorithm. The worksheet asks the taxpayer to pull information from just a few lines of the Schedule D. The way the statute puts forward the algorithm, the taxpayer would have to recalculate net short term capital gain, to take into account the 1212(b)(2) amount. But the algorithm on the worksheet pulls out the 1212(b)(2) amount, so the taxpayer can take just the two non-zero numbers from the Form 1040, and, together with the 1212(b)(2) amount, determine the carryforwards. In other words, while the worksheet doesn't provide a formula that obviously tracks the statute, what it does provide is easy to implement. An algorithm that tracked the statute closely would be less efficient, and even that algorithm would be opaque to almost all taxpayers.

An IRS-provided write-up of the form and worksheet describing reasoning like the reasoning in this paper would address concerns about the agency's change in the law and the lack of transparency of the worksheet. A write-up could address concerns about the IRS's functional change in the law, in line with Osofsky's proposal for increasing transparency around legislative fixes. It would also make the algorithm more transparent, connecting the algorithm of the worksheet with the language of the statute. And it could do this while allowing continued efficient implementations of algorithms as presented on the current form and worksheet. (The IRS has not in the past provided similar write-ups showing the connections between forms and worksheets, on the one hand, and statutes.)

Such write-ups of justifications would be relatively easy to provide because reasoning such as the reasoning done in this Part can be done computationally. The formalizations in this paper were rewritten in code that allowed the conclusions to be checked by the Z3 prover, a computer program that can, among other things, check whether a set of logical formulas is satisfied under all theories, and if the set is not satisfied, can provide a counterexample.⁴⁷ As noted above, this check confirmed that the three "excess of" rules were correct. The prover also

⁴⁷ An introduction to Z3 is available at <https://microsoft.github.io/z3guide/docs/logic/intro/>. The code and the verifications for this article are available at the Github repository <https://github.com/slawsk/tax-formalization/> and also attached as Appendix C to this article.

confirmed as accurate the common statement that unused capital losses can be carried forward—but only if the net capital loss requirement is read out of the statute. If the net capital loss requirement is kept in the formalization, the common statement is not true—and the program was able to provide a counterexample (this is only one counterexample among many, of course):⁴⁸

```
Carryforward unused losses, require Net Capital Loss does not
work
[LTCL = 1600,
 STCG = 100,
 STCL = 100,
 LTCG = 0,
 Other_Income = 700]
```

Similarly, the program confirmed that the loss carryforward worksheet matches the statute—but again, only reading the net capital loss requirement out of the statute, and again, the program provided counterexamples if the net capital loss requirement is not read out of the statute. For example, the worksheet prescription for the amount of long-term gain to be carried forward does not match the statute if the net capital loss requirement is not read out of the statute, and the program proposed the following counterexample:⁴⁹

```
Statute v. Worksheet Long, require Net Capital Loss does not work
[LTCL = 554100,
 STCG = 1978100,
```

⁴⁸ This is indeed a counterexample. When there is \$1600 of long-term capital loss, \$100 of short-term capital gain, \$100 of short-term capital loss, no long-term capital gains, and \$700 of other income, the statute would allow no loss carryforward, because there would be no net capital loss. The capital losses permitted under Section 1211 would equal the capital gains, \$100, plus the lesser of \$3000 or the excess of capital losses over capital gains. The excess of capital losses over capital gains is \$1600, and therefore Section 1211 would allow all $\$100 + \$1600 = \$1700$ of the capital losses. The excess of capital losses over capital losses permitted under Section 1211 is therefore $\$1700 - \$1700 = \$0$, and there would be no net capital loss, and therefore no loss carryforward permitted. But there is also no other income against which to use the capital losses. Reading out the net capital loss requirement, under Section 1212(b)(2), adjusted taxable income for the taxable year is taxable income with the 1211(b) Amount added back. Taxable income is $\$100 + \$700 - \$1700 = (\$900)$; adding back the 1211(b) amount of \$1600 comes to \$700. This is the amount that should be treated as short-term capital gain when computing the carryforward. This makes sense, because this is the amount of “extra” capital loss that was able to be used. The total unused losses would equal $(\$1600 + \$100) - (\$100 + \$700) = \$900$.

⁴⁹ In the proposed set of facts, there is no net capital loss; the total capital losses equal $\$1,424,100 + \$554,100 = \$1,978,200$, and the total capital gains equal \$1,978,100. All of the capital losses are permitted to be used, because there are only \$100 of capital losses in excess of capital gains. Therefore there should be, if the statute is taken literally, no loss carryforward. Yet of course there should be \$100 of losses carried forward, because there is no excess income against which to use the losses.

```
STCL = 1424100,  
LTCG = 0,  
Other_Income = 0]
```

Reasoning on formalizations, including computational analysis, thus can help verify that drafted statutes were consistent with desired outcomes; could identify problems within the statute that agencies needed to fix; and could help verify that agency implementations were consistent with current law (if the law needed no fixing) or with the desired outcome (if the law did need to be fixed).

IV. GRAPHICAL REASONING ON FORMALIZATIONS

Formalizations also open the way for a different kind of reasoning: reasoning through graphical representations. This Part uses the formalizations of the statute to present graphs that bring together the various capital gain rules and help readers of the statute develop intuitions about the statute. It also demonstrates graphically some of what was demonstrated algebraically above.

A. The Net Capital Loss Carryforward Requirement

Section 1212(b)(1) permits capital loss carryforwards “[i]f a taxpayer other than a corporation has a net capital loss for any taxable year.” However, as argued above, in certain factual situations, a taxpayer with no net capital loss should still be able to carry capital losses forward. This Section provides a graphical representation of when that situation occurs.

Section 1222(10) defines “net capital loss” as the excess of the losses from sales or exchanges of capital assets over the sum allowed in Section 1211:

Net Capital Loss =

Capital Losses - * Sum Allowed in Section 1211

The sum allowed in Section 1211 is the sum of the gains from the sale of capital losses and the lesser of \$3000 and the excess of capital losses over capital gains.

Net Capital Loss =

Capital Losses - * (Capital Gains + Lesser(\$3000, Capital Losses - * Capital Gains))

The lesser of these is the excess of capital losses over capital gains, then

Net Capital Loss =

Capital Losses -* (Capital Gains + (Capital Losses -* Capital Gains))

Assuming that there are more capital losses than capital gains,

Net Capital Loss =

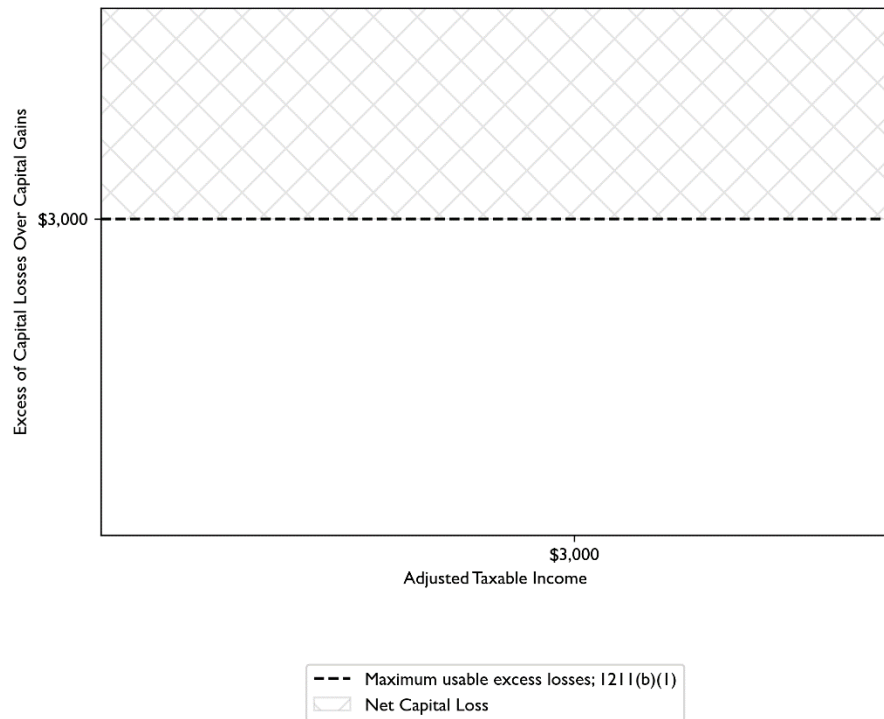
Capital Losses -* (Capital Gains + (Capital Losses - Capital Gains)) =

Capital Losses -* Capital Losses = 0

Therefore, there is net capital loss (or, more precisely, net capital loss exceeds zero) exactly when the relevant number from Section 1211 is \$3000, that is, when capital losses exceed capital gains by more than \$3000.

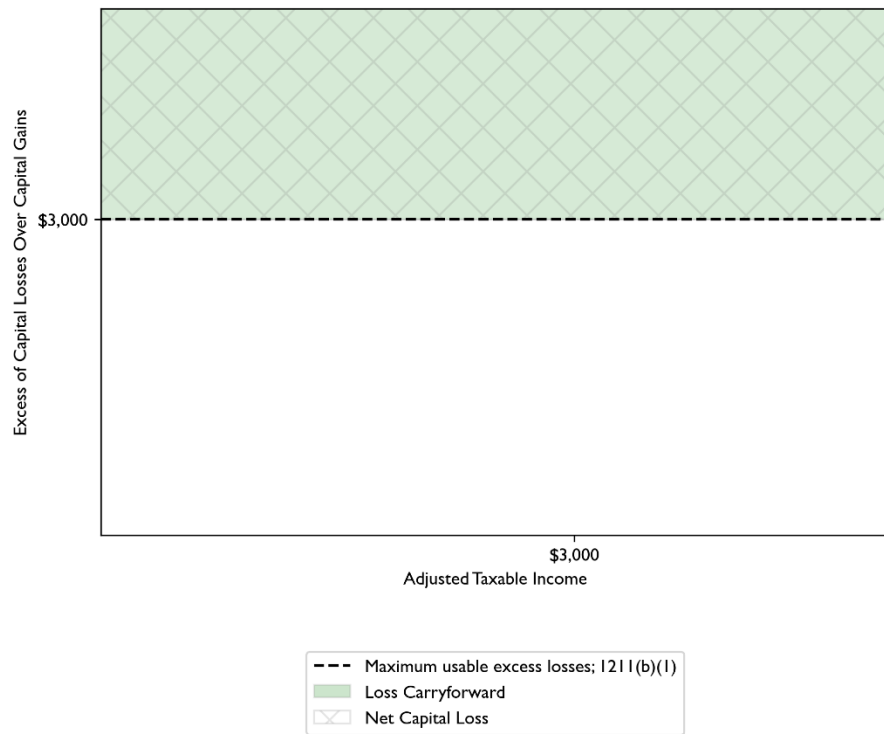
Now graph the excess of capital losses over capital gains on the y-axis, and adjusted taxable income, as defined in Section 1212(b)(2), on the x-axis. The area above \$3000 on the y-axis, marked with cross-hatching, is where there is a net capital loss.

Reasoning with Formalized Statutes

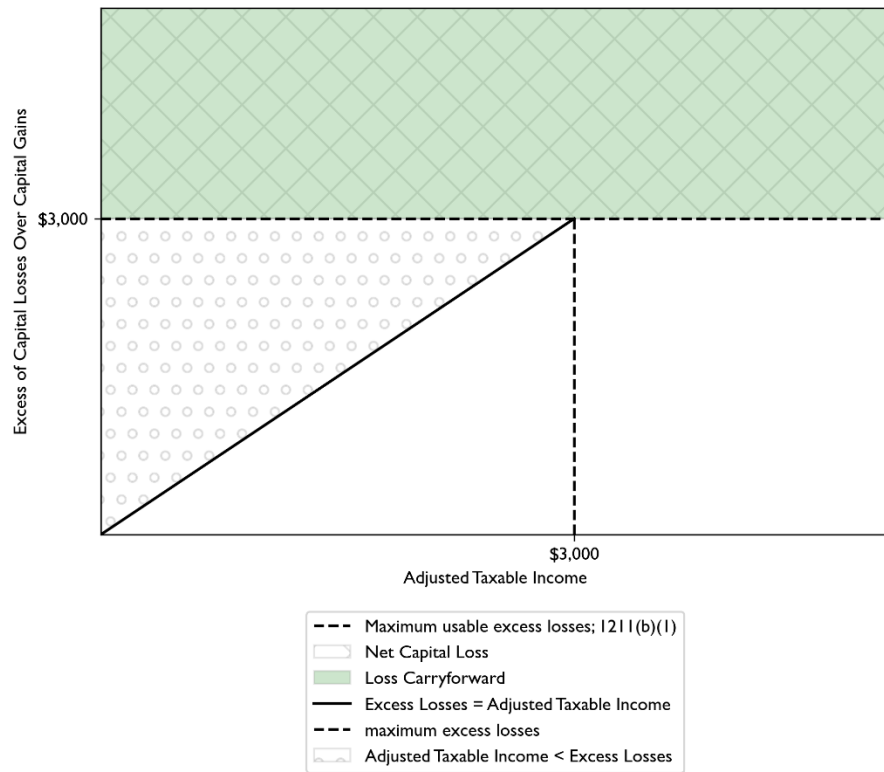


According to the statute, if the taxpayer has a net capital loss, then the taxpayer can carry forward certain capital losses. Shade the area with net capital loss to show that this is the area with loss carryforward.

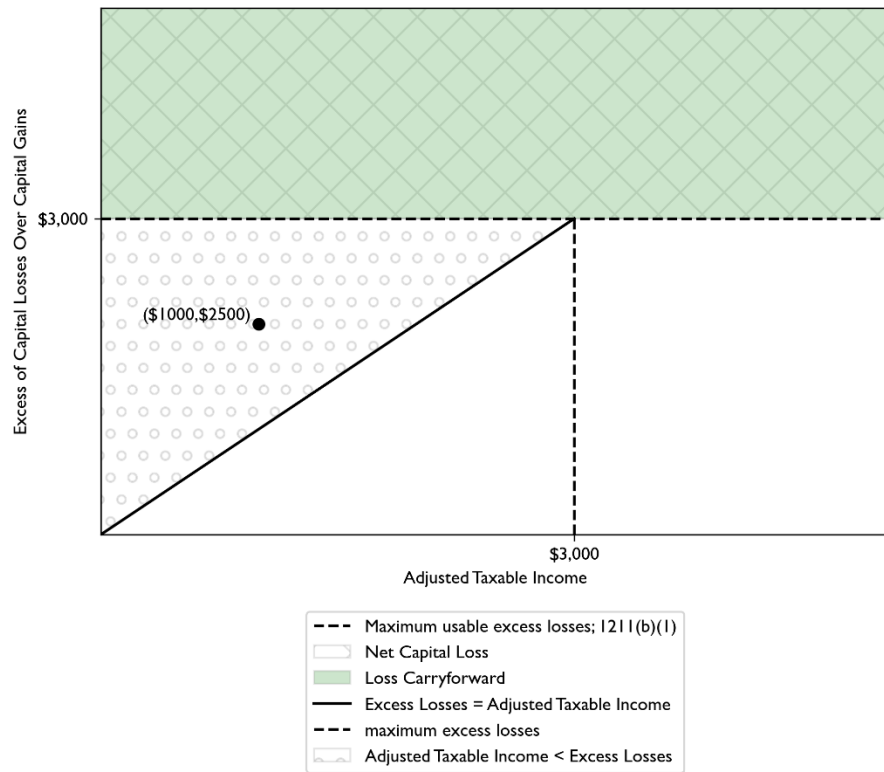
Reasoning with Formalized Statutes



Sometimes a taxpayer will have insufficient adjusted taxable income to be able to use all permitted capital losses. This could occur when the taxpayer has more permitted excess losses (up to \$3000) than adjusted taxable income. The diagonal line represents when adjusted taxable income equals excess losses, up to \$3000. Above that diagonal line, and below \$3000, the taxpayer has more permitted excess losses than adjusted taxable income (the area marked filled with “o” below).

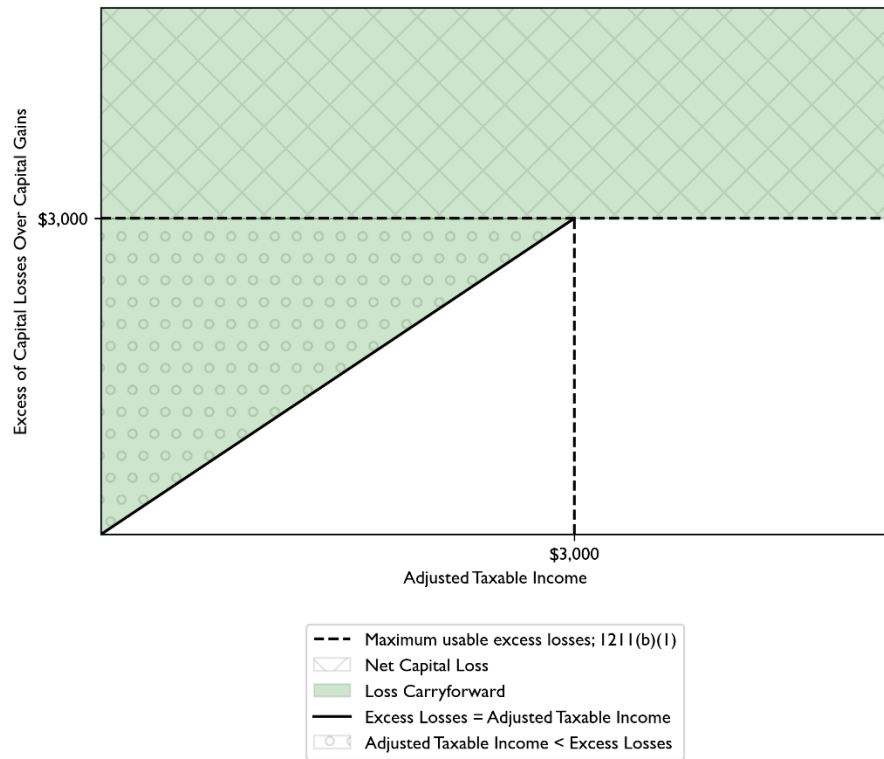


The area filled with circles represents situations in which a taxpayer has less than \$3000 of excess capital losses (because $y < \$3000$), and therefore has no net capital loss. Yet the taxpayer also cannot use all of those excess capital losses, because adjusted taxable income is less than the excess capital losses. For example, as in the example above, adjusted taxable income of \$1000 and excess of capital losses over capital gains of \$2500:



As described above, the taxpayer is permitted under Section 1211(b) to use the lesser of (1) \$3000 or (2) the excess of capital losses over capital gains, here, \$2500. Net capital loss equals the excess of capital losses over the sum permitted by Section 1211. Thus net capital loss equals 0. Yet according to Section 1212(b)(2), the taxpayer should calculate loss carryforwards taking into account as short-term capital gain the lesser of the 1211(b) amount—here, \$2500—or adjusted taxable income—here, by assumption, \$1000. Therefore the taxpayer should reduce the carryforward by only \$1000, not \$2500, and should, under 1212, be able to carry forward \$1500 of losses.

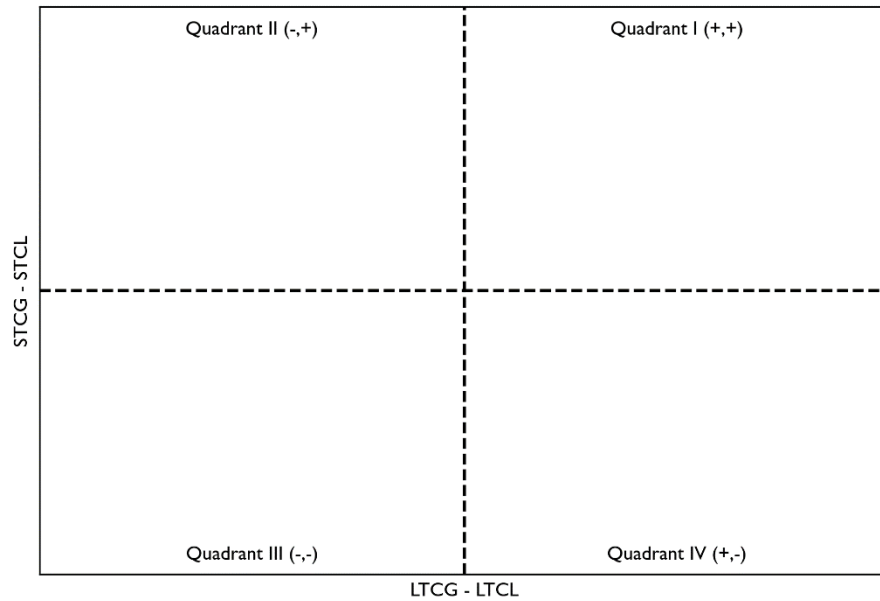
Thus, notwithstanding that Section 1212(b)(1), given the reference to adjusted taxable income in 1212(b)(2), the correct reading of the statute would seem to provide this representation, where the area with excess losses greater than adjusted taxable income but less than \$3000 do not have net capital loss, but still permit a loss carryforward.



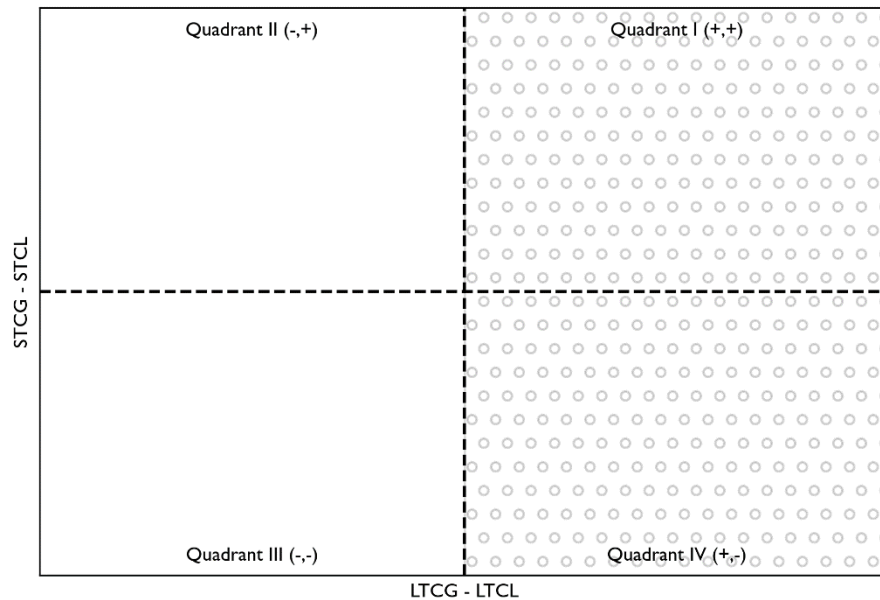
This graphical representation does not, of course, prove that the statute is wrong, but it provides a way to understand why the statute may be wrong, and to precisely identify the area of concern, where there seems to be a disconnect between two parts of the statute.


B. *Picturing Capital Gains and Losses*

Net capital gain and the limitations on the use of capital losses are in some ways distinct calculations, but they can be united in a single picture. Short-term capital gains, short-term capital losses, long-term capital gains, and long-term capital losses are all independent from each other. They simply represent facts about the world—what assets were sold, how long each of those assets had been held for, and the basis and amount realized for each of those assets. Take the difference between long-term capital gains and long-term capital losses as the x-axis. The x-axis is *not* equal only to the “excess of” long-term capital gains over long-term capital losses, though it does include that. The “excess of” is always positive, whereas the x-axis is simply long-term capital gains minus long-term capital losses; it can be negative or positive. The y-axis is, similarly, the difference between short-term capital gains and short-term capital losses. It too can be positive or negative.

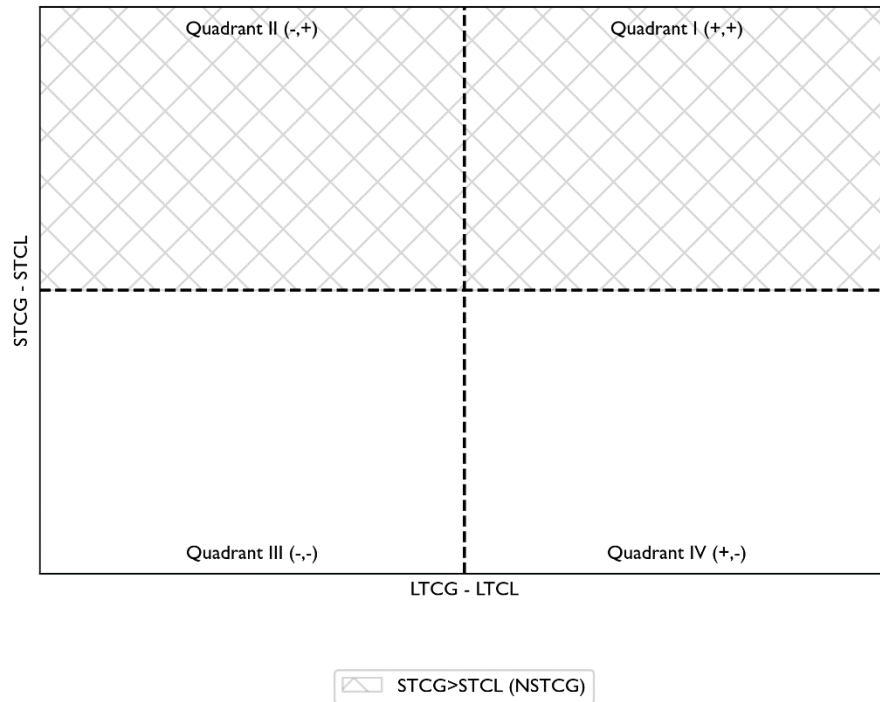


On the right-hand side of the x-axis, $LTCG - LTCL$ is positive; long-term capital gain is greater than long-term capital loss. Fill in with circles the area where long-term capital gain is greater than long-term capital loss; that is, where there is an excess of long-term capital gain over long-term capital loss—put another way, where there is net long-term capital gain.

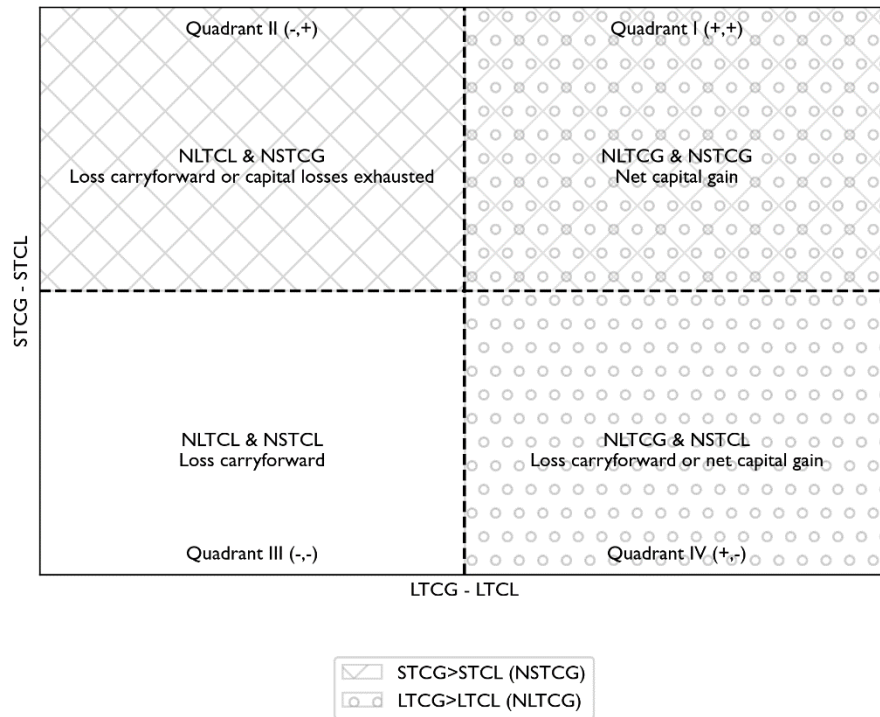


 $LTCG > LTCL$ (NLTCG)

Similarly, where the y-axis is positive, short-term capital gain minus short-term capital loss is positive; short-term capital gain exceeds short term capital loss; and thus there is net short-term capital gain. Fill this area in with cross-hatches.



Combine these two shadings to shade the four quadrants.



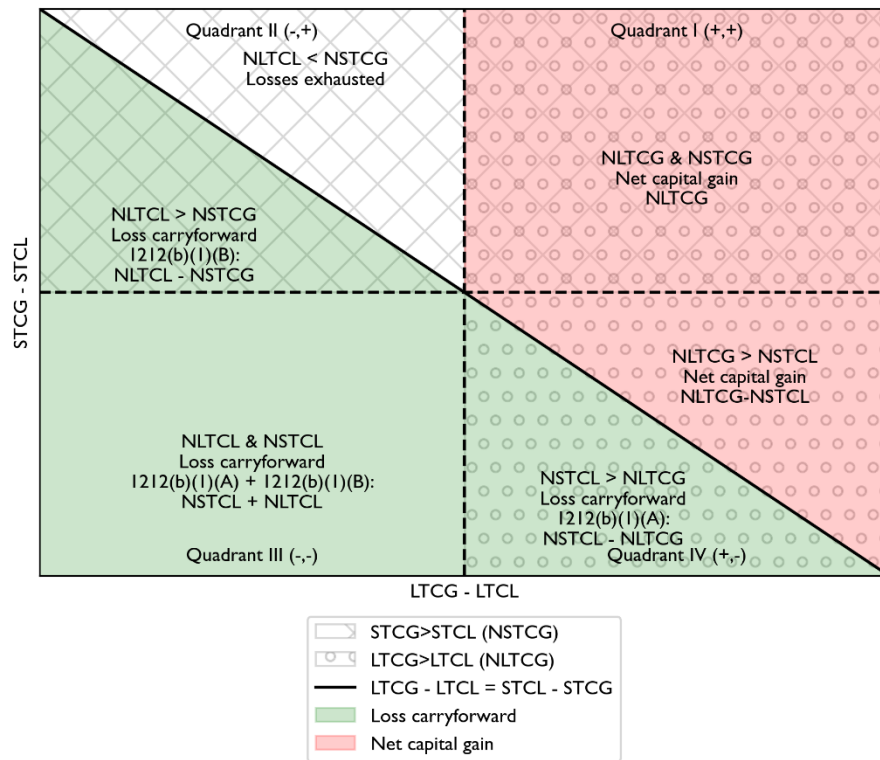
Quadrant I, the area with both net long-term capital gain and net short-term capital gain, must result in net capital gain taxed at a favorable rate, because there is no net short-term capital loss to reduce net long-term capital gain to zero.

Quadrant II, the area with net short-term capital gain and net long-term capital loss, has only x-hatching. Quadrant II can be subdivided into two regions. This area might have loss carryforward (if there is an excess of net long-term capital loss over net short-term capital gain) or losses might be exhausted. There cannot be any net capital gain in this region because there is no net long-term capital gain.

Quadrant III, the area that has no shading, is an area where there is neither net long-term capital gain nor net short-term capital gain. There must be loss carryforward here, because there are no capital gains against which to use the capital losses.

Quadrant IV, the area with net long-term capital gain and net short-term capital loss, has only circle-hatching. Quadrant IV can be subdivided into two regions. There is net capital gain if there is an excess of net long-term capital gain over net short-term capital loss—that is, definitionally, net capital gain. If there is an excess of net short-term capital loss over net long-term capital gain, that amount will be carried forward under 1212(b)(1)(A).

Representing this graphically:



There are other ways to see that this graph is correct.

The area to the left of the diagonal line is shaded green—that is the area where there is loss carryforward. The left of the line is

$$\text{LTCG} - \text{LTCL} < \text{STCL} - \text{STCG}$$

This can be rewritten as

$$\text{LTCG} + \text{STCG} < \text{STCL} + \text{LTCL}$$

Which of course must be right; that just says that there is loss carryforward when capital losses exceed capital gains. (This isn't exactly right; we will add in the 1212(b)(2) amount shortly.)

The right of the line is

$$\text{LTCG} - \text{LTCL} > \text{STCL} - \text{STCG}$$

Which can be rewritten as

$$\text{LTCG} + \text{STCG} > \text{LTCL} + \text{STCL}$$

But it isn't true that all of this area is shaded red, indicating that there is net capital gain in that area; there is a triangle that isn't shaded, the area where

$$\text{LTCG} - \text{LTCL} < 0$$

In that area, even though there are more capital gains than capital losses, the long-term capital gains are entirely absorbed by long-term capital losses; the remaining capital gains are short-term capital gains, and those are not subject to a favorable rate. Thus in that area, the losses are exhausted, but what remains is short-term capital gain, taxed at ordinary rates. The graph is asymmetric because short-term and long-term capital gains are treated differently from each other.

Now let's make the graph actually fit the law for individuals. It's not true that there is loss carryforward when losses exceed gains. Rather, up to \$3000 of losses can be used against ordinary income, to the extent there is ordinary income—that is, the graph should incorporate the 1212(b)(2) Amount. The relevant trigger for loss carryforwards isn't

$$\text{LTCG} + \text{STCG} < \text{STCL} + \text{LTCL}$$

It is

$$\text{LTCG} + \text{STCG} + 1212(b)(2) < \text{STCL} + \text{LTCL}$$

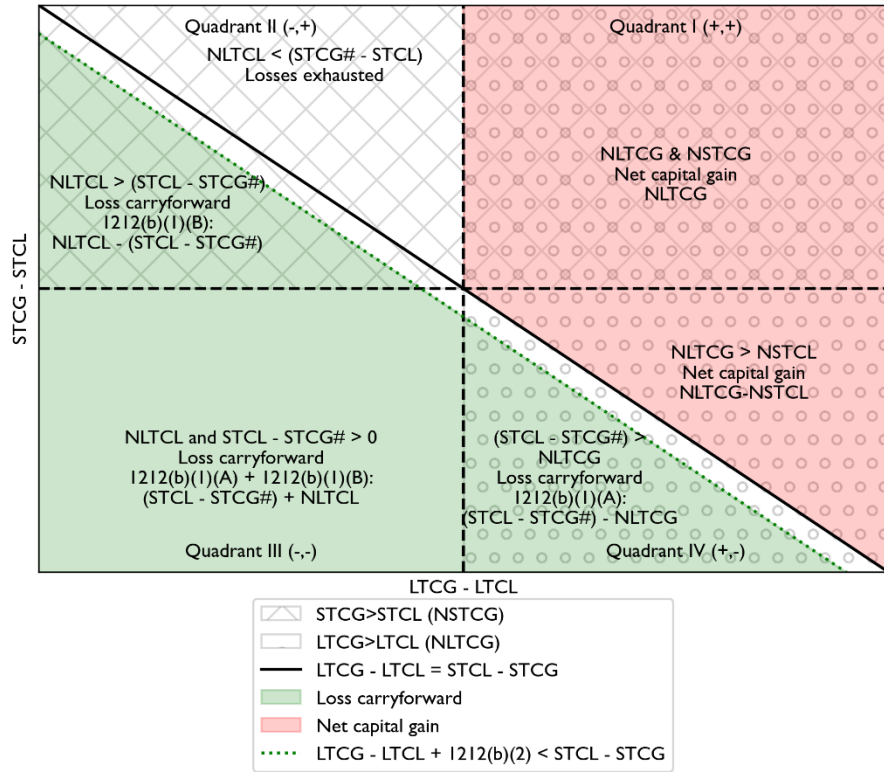
There won't be loss carryforwards until losses are greater than capital gains *plus the amount of capital losses that were permitted to offset ordinary income*. This can be rewritten as:

$$\text{LTCG} - \text{LTCL} + 1212(b)(2) < \text{STCL} - \text{STCG}$$

Graphing this line is the same as graphing

$$x + 1212(b)(2) < -y.$$

Notate $\text{STCG} + 1212(b)(2)$ as STCG\# , as above.



This comports with the statute. The statute says that for the purpose of determining loss carryforward, treat the lesser of (1) adjusted taxable income, or (2) the amount allowed under 1211(b)(1) or (2) as a short-term capital gain—that is, what we have called the 1212(b)(2) Amount.

$$LTCG - LTCL < STCL - STCG - 1212(b)(2) =$$

$$LTCG - LTCL < STCL - (STCG + 1212(b)(2))$$

This is what the statute says to do.⁵⁰

C. Implications of Graphical Reasoning

While the formalizations may initially seem more obscure, the formalizations permit a graphical representation of the statute that allows readers of the statute to develop intuitions about the meaning of

⁵⁰ This actually represents what the statute as fixed, with no net capital loss requirement, says to do. This graph does not include adjusted taxable income, but one could make a three-dimensional graph with a z-axis that represented adjusted taxable income and combine the graph from the prior section with the graph from this section.

the statute. Charts and diagrams are familiar tools to help clarify the structure of statutes and arguments⁵¹ or provide information about the structure of entities. Some legislation includes graphics such as charts, tables, decision trees, or flowcharts.⁵² The graphs proposed in this article are related to but slightly different from those tools. Instead of clarifying the structure of a statute or argument, these graphs represent the *substance* of the statute. The graphs in this article do not present proofs of anything, but rather, like graphical “proofs,” or “proofs without words,” within mathematics, are meant to help a person understand the ideas and relationships that the statute and thus allow stimulate insights and allow further, perhaps more rigorous, reasoning.⁵³

V. CONCLUSION

This article has shown another advantage of formalizing law: formalized law can be reasoned upon, either algebraically or graphically. As the article has shown, such reasoning can unearth errors within statutes and provide concise, reliable justifications of restatements or implementations of the law. This article has created the reasoning nonmechanically and has also shown that reasoning such as that presented in this article can be implemented computationally.

⁵¹ E.g., Adam L. Rosman, *Visualizing the Law: Using Charts, Diagrams, and Other Images to Improve Legal Briefs*, 63 J. LEGAL ED. 70 (2013)

⁵² E.g., Hilary Penfold, *When Words Aren't Enough: Graphics and Other Innovations in Legislative Drafting* (2001).

⁵³ See generally Marcus Giaquinto, *The Epistemology of Visual Thinking in Mathematics*, *The Stanford Encyclopedia of Philosophy* (2020) available at <https://plato.stanford.edu/archives/spr2020/entries/epistemology-visual-thinking/>.

APPENDIX A: RELEVANT CODE SECTIONS

A. *Section 1211(b)*

(b) OTHER TAXPAYERS. In the case of a taxpayer other than a corporation, losses from sales or exchanges of capital assets shall be allowed only to the extent of the gains from such sales or exchanges, plus (if such losses exceed such gains) the lower of—

- (1) \$3,000 (\$1,500 in the case of a married individual filing a separate return), or
- (2) the excess of such losses over such gains.

B. *Section 1212(b)*

(b) OTHER TAXPAYERS.

(1) In general. If a taxpayer other than a corporation has a net capital loss for any taxable year—

(A) the excess of the net short-term capital loss over the net long-term capital gain for such year shall be a short-term capital loss in the succeeding taxable year, and

(B) the excess of the net long-term capital loss over the net short-term capital gain for such year shall be a long-term capital loss in the succeeding taxable year.

(2) Treatment of amounts allowed under section 1211(b)(1) or (2)

(A) In general. For purposes of determining the excess referred to in subparagraph (A) or (B) of paragraph (1), there shall be treated as a short-term capital gain in the taxable year an amount equal to the lesser of—

- (i) the amount allowed for the taxable year under paragraph (1) or (2) of section 1211(b), or
- (ii) the adjusted taxable income for such taxable year.

(B) Adjusted taxable income. For purposes of subparagraph (A), the term “adjusted taxable income” means taxable income increased by the sum of—

- (i) the amount allowed for the taxable year under paragraph (1) or (2) of section 1211(b), and
- (ii) the deduction allowed for such year under section 151 or any deduction in lieu thereof.


For purposes of the preceding sentence, any excess of the deductions allowed for the taxable year over the gross income for such year shall be taken into account as negative taxable income.

C. Section 1222

(10) NET CAPITAL LOSS. The term “net capital loss” means the excess of the losses from sales or exchanges of capital assets over the sum allowed under Section 1211....

(11) NET CAPITAL GAIN. The term “net capital gain” means the excess of the net long-term capital gain for the taxable year over the net short-term capital loss for such year.

APPENDIX B: ANNOTATED CAPITAL LOSS CARRYOVER WORKSHEET

Capital Loss Carryover Worksheet—Lines 6 and 14		Keep for Your Records 
<p>Use this worksheet to figure your capital loss carryovers from 2021 to 2022 if your 2021 Schedule D, line 21, is a loss and (a) that loss is a smaller loss than the loss on your 2021 Schedule D, line 16; or (b) if the amount on your 2021 Form 1040 or 1040-SR, line 15, (or your 2021 Form 1040-NR, line 15, if applicable) would be less than zero if you could enter a negative amount on that line. Otherwise, you don't have any carryovers.</p> <p>If you and your spouse once filed a joint return and are filing separate returns for 2022, any capital loss carryover from the joint return can be deducted only on the return of the spouse who actually had the loss.</p> <p>If you excluded canceled debt from income in 2022, see Pub. 4681.</p>		
<p>1. Enter the amount from your 2021 Form 1040 or 1040-SR, line 15; or your 2021 Form 1040-NR, line 15. If the amount would have been a loss if you could enter a negative number on that line, enclose the amount in parentheses</p> <p>2. Enter the loss from your 2021 Schedule D, line 21, as a positive amount</p> <p>3. Combine lines 1 and 2. If zero or less, enter -0-</p> <p>4. Enter the smaller of line 2 or line 3</p> <p>If line 7 of your 2021 Schedule D is a loss, go to line 5; otherwise, enter -0- on line 5 and go to line 9.</p> <p>5. Enter the loss from your 2021 Schedule D, line 7, as a positive amount</p> <p>6. Enter any gain from your 2021 Schedule D, line 15. If a loss, enter -0-</p> <p>7. Add lines 4 and 6</p> <p>8. Short-term capital loss carryover for 2022. Subtract line 7 from line 5. If zero or less, enter -0-. If more than zero, also enter this amount on Schedule D, line 6</p> <p>If line 15 of your 2021 Schedule D is a loss, go to line 9; otherwise, skip lines 9 through 13.</p> <p>9. Enter the loss from your 2021 Schedule D, line 15, as a positive amount</p> <p>10. Enter any gain from your 2021 Schedule D, line 7. If a loss, enter -0-</p> <p>11. Subtract line 5 from line 4. If zero or less, enter -0-</p> <p>12. Add lines 10 and 11</p> <p>13. Long-term capital loss carryover for 2022. Subtract line 12 from line 9. If zero or less, enter -0-. If more than zero, also enter this amount on Schedule D, line 14</p>	<p>1. Taxable income, can go negative</p> <p>2. 1211(b)(1) or (2)</p> <p>3. Adjusted taxable income</p> <p>4. 1212(b)(2) Amount</p> <p>5. Net Short-Term Capital Loss</p> <p>6. Net Long-Term Capital Gain</p> <p>7. Net Long-Term Capital Gain + 1212(b)(2) Amount</p> <p>8. Net Short-Term Capital Loss * (Net Long-Term Capital Gain + 1212(b)(2) Amount)</p> <p>9. Net Long-Term Capital Loss</p> <p>10. Net Short-Term Capital Gain</p> <p>11. 1212(b)(2) Amount</p> <p>12. Net Short-Term Capital Gain + (1212(b)(2) Amount - * Net Short-Term Capital Loss)</p> <p>13. Net Long-Term Capital Loss - * (Net Short-Term Capital Gain + (1212(b)(2) Amount - * Net Short-Term Capital Loss))</p>	

APPENDIX C: CODE AND OUTPUT FOR Z3 FORMALIZATIONS

[See <https://github.com/slawsk/tax-formalization> for current code; this section will be added when the paper and code are closer to finalized.]