

## BONUS ASSIGNMENT

The sigmoid function is defined as

$$\sigma(x) = \frac{1}{1+e^x}$$

$$\sigma(x) = (1+e^{-x})^{-1}$$

Applying Chain Rule

$$\frac{d}{dx} \sigma(x) = \frac{d}{dx} (1+e^{-x})^{-1}$$

$$u = 1+e^{-x}$$

$$\frac{d}{dx} \sigma(x) = \frac{d}{du} (u^{-1})$$

$$\frac{d}{dx} \sigma(x) = -u^{-2} \frac{du}{dx}$$

$$\frac{du}{dx} = \frac{d}{dx} (1+e^{-x}) = 0 + \frac{d}{dx} (e^{-x}) = -e^{-x}$$

Substitute  $\frac{du}{dx}$

$$\frac{d}{dx} \sigma(x) = -u^{-2} (-e^{-x})$$

$$\frac{d}{dx} \sigma(x) = -(1+e^{-x})^{-2} (-e^{-x}) = \frac{e^{-x}}{(1+e^{-x})^2}$$

In terms of  $\sigma(x)$

$$\sigma(x) = \frac{1}{1+e^{-x}}, \quad 1-\sigma(x) = \frac{e^{-x}}{1+e^{-x}}$$

$$\sigma(x) \cdot (1-\sigma(x)) = \frac{1}{1+e^{-x}} \cdot \frac{e^{-x}}{1+e^{-x}} = \frac{e^{-x}}{(1+e^{-x})^2}$$

$$\frac{d}{dx} \sigma(x) = \sigma(x)(1-\sigma(x))$$