Homework 4

ECE6553 Optimal Control

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Due: March 30, 2017 (and April 6 for DL students – sections Q and QSZ)

1

The standard approach to solving the min-time problem is to write the cost as

$$J(u,T)=\int_0^T dt+0,$$

i.e., with $L=1$ and $\Psi=0$, for the system

$$\dot{x}=f(x,u),\quad x(0)=x_0,\ x(T)=x_T.$$

But, we could just as well have written this as

$$\hat{J}(u,T)=\int_0^T 0dt+T,$$

i.e., with $L=0$ and $\Psi(x(T),T)=T$ (and the same dynamics and boundary conditions in the state).

Verify that these two formulations do indeed give the same optimality conditions (same u, same costates, and same transversality conditions).

You do not need to re-derive any optimality conditions. Just write them down for the two problems and verify that they are indeed the same.

2.

Consider a kinematic "car" driving to a stop sign:

$$\min_{u,T}\int_0^T dt$$

under the (scalar) dynamics

$$\dot{x}=u,$$

where $x(0)=0$ and $x(T)=x_T$ are given. Moreover, assume that we have to "consume" a fixed amount of energy, i.e., we have the additional constraint that

$$\int_0^T u^2(t)dt=E,$$

where $E>0$ is fixed and given.

What is the optimal solution (in terms of u and T) to this problem?

3.

Consider the undamped oscillator

$$\begin{aligned}\dot{x}_1 &= x_2\\ \dot{x}_2 &= -x_1 + u.\end{aligned}$$

The problem is to drive this system to $x(T) = 0$ as quickly as possible when u is constrained by

$$u(t) \in [-1, 1],\ \ \forall t \in [0, T].$$

a

Formulate this problem as a minimum-time optimal control problem and show that the costates satisfy

$$\begin{aligned}\lambda_1(t) &= \alpha\cos(T-t) + \beta\sin(T-t)\\ \lambda_2(t) &= \alpha\sin(T-t) - \beta\cos(T-t).\end{aligned}$$

(Note, don't try to solve the costate equations to find λ. Instead, verify that the proposed λ is indeed the solution.)

b

Use the transversality condition to find the possible values that α and β can have.

4.

How many times does the optimal controller u switch on the time interval $t \in (k\pi, (k+1)\pi]$, where k is an integer?

5.

Sketch (or plot) the switching curves in the state space (x_1x_2-space) corresponding to the problem in Question 3, i.e., the curves in the x_1x_2-space where the optimal controller switches from ± 1 to ∓ 1.

6

For a linear system

$$\dot{x} = Ax + Bu,\quad y = Cx,$$

the relative degree d is the smallest integer such that

$$CA^{d-1}B \neq 0.$$

We would like to solve the LQ-problem, with instantaneous cost $L = x^TQx + u^TRu$, dynamics, $\dot{x} = Ax + Bu,\ x(0) = x_0$, subject to the constraint that $y(t) = 0, \forall t$, where $y = Cx$. Assume that the system has relative degree d. What are the optimality conditions (Hamiltonian, optimal u (in terms of the state and costate), costate equation, and conditions on x_0), associated with this constrained optimal control problem? ($Q = Q^T \succ 0,\ R = R^T \succ 0$.)