Homework 2

ECE6553 Optimal Control

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Due: February 16, 2017 (and Feb. 23 for DL students – sections Q and QSZ)

1

Consider the system

$$\dot{x} = f(x), \quad x(0) = x_0$$

and assume that the state undergoes a discrete jump at a particular time τ, such that

$$x(\tau_+) = x(\tau_-) + \rho,$$

i.e., at time τ the state jumps from its current value $x(\tau_-)$ to its new value $x(\tau_+) = x(\tau_-) + \rho$, where $\rho \in \mathbb{R}^n$ is the jump. (This is a vector so ρ is both the magnitude and the direction of the jump.)

a

Find the first order necessary conditions that give us the best jump ρ by solving

$$\min_{\rho} J(\rho) = \int_0^T L(x(t))dt,$$

subject to the dynamics given above. (Note, τ and x_0 are fixed and given...)

b

Now assume that the direction of the jump is given and we only have control of the magnitude, i.e., $\rho = \alpha w$, where $w \in \mathbb{R}^n$ is fixed and given and our job is to find the best $\alpha \in \mathbb{R}$ by minimizing

$$\min_{\alpha} J(\alpha) = \int_0^T L(x(t))dt,$$

subject to the same dynamics as before. How do the first order necessary conditions change?

2

Now, assume that in Question **1a**, instead of just selecting ρ, we are free to select both ρ and τ, i.e., the time of the jump as well as the jump direction and magnitude are under our control.

a

Draw the variation in x as a result of the variations $\tau \mapsto \tau + \epsilon\theta$ and $\rho \mapsto \rho + \epsilon\nu$. In the figure, use x to denote the original trajectory and $\hat{x}$ to denote the new (perturbed) trajectory.

b

Do you foresee any trouble if you were to tackle this problem using the direct application of Calculus of Variations as we have done in class? Motive your answer to this question!

3

Consider the switch-time minimization problem

$$\min_{\tau} J(\tau) = \int_0^1 \frac{1}{2}(x(t) - \alpha)^2 dt$$

such that

$$\dot{x}(t) = \begin{cases} -\alpha & \text{if } t \in [0, \tau) \\ \alpha & \text{if } t \in [\tau, 1] \end{cases}$$
$$x(0) = 1,$$

where $0 < \alpha < 2$.

a

Find an explicit expression for the optimal switch time τ, where you can assume that $0 < \tau < 1$.

b

Simulate the optimal solution in matlab. Note, you should not solve the optimal timing control problem numerically. Just simulate the system over the interval $[0, 1]$ as it switches at your optimal switching time from Question **b**. For $\alpha = 0..25, \alpha = 1$, and $\alpha = 1.75$ compute the costs, plot you solutions (x vs t), and include those plots and costs together with the code in your homework submission.

4

Let

$$\dot{x} = Ax$$

and let the cost to be minimized with respect to the initial condition x_0 be

$$\int_0^T \frac{1}{2} x(t)^T Q x(t) dt,$$

where $Q = Q^T \succ 0$.

a.

What is the costate equation (given explicitly in terms of the A and Q matrices)?

b.

If A is Hurwitz (the system $\dot{x} = Ax$ is asymptotically stable), what are the stability properties of the costate equation?

c.

What does your answer to **1b** mean for the efficiency/robustness of the numerical solution to this problem presented in class and made available in Tsquare under Resources/m-files as `optinit.m`?

5

A student in ECE6553 working on Homework 2 has decided to maximize the following utility function

$$\max_{g,T,c} F(g,T,c) = g - \frac{1}{2}\alpha T^2 - \frac{1}{2}\beta c^2,$$

where g is the grade on the homework, T is the time spent working on the homework, and c is how hard the student concentrates while working (α and β are positive constants). Moreover, g, T and c are coupled through the constraint

$$g = \gamma(T + c),$$

where $\gamma > 0$.

What grade does the student receive on the homework if he/she studies optimally?

6

If we wish to solve the following problem

$$\min_u J(u) = \int_0^T L(x(t), u(t), t)dt$$

where instead of a dynamical system, we have an algebraic coupling between x and u through the constraint

$$\Phi(x(t), u(t), t) = 0, \quad \forall t \in [0, T].$$

What do the associated optimality conditions look like?