Homework 5

ECE6553 Optimal Control

Magnus Egerstedt

Due: April 13, 2017 (and April 20 for DL students – sections Q and QSZ)

1

Consider the LQ problem

$$\min_u \int_0^T \Big(x^T Q x + u^T R u\Big) dt + x^T(T) S x(T),$$

such that

$$\dot{x} = Ax + Bu,$$

and where Q, R, S are symmetric matrices.

a

What conditions do we need on the Q, R, S, A, B matrices in order for the optimal control problem to be well-posed. (Don't just state the conditions, explain why we need them.)

b

If instead of the finite time problem, we have the inifinite horizon problem, i.e., $T \to \infty$. In that case, we no longer have a terminal cost (i.e., no S-term). What conditions do we now need on the Q, R, A, B matrices in order for the optimal control problem to be well-posed. (Again, don't just state the conditions, explain why we need them.)

c

Let $P(t)$ solve the Riccati equation

$$\dot{P} = -A^T P - PA - Q + PBR^{-1}B^T P, \quad P(T) = S.$$

Show that $P(t) = P^T(t),\ \forall t \in [0, T]$.

2

Consider the tracking problem, where we want the scalar state of a system $x(t)$ to track a reference signal $\sigma(t)$, i.e.,

$$\min_u \frac{1}{2} \int_0^T \Big(\rho(x(t) - \sigma(t))^2 + u(t)^2\Big) dt,$$

where $\rho > 0$ and where $\dot{x} = ax + bu,\ x(0) = x_0$.

Now, assume that someone claimed that the solution is given by

$$u(t) = -b(p(t)x(t) - w(t)),$$

and the optimal cost-to-go is

$$J^\star(x,t) = \frac{1}{2}x^2 p(t) + wx(t) + v(t)$$

where p, w, v satisfy

$$\begin{aligned} \dot{p} &= -2ap + p^2b^2 - \rho,\ p(T) = 0 \\ \dot{w} &= -(a - b^2p)w + \rho\sigma,\ w(T) = 0 \\ \dot{v} &= \tfrac{1}{2}(w^2b^2 - \sigma^2\rho),\ v(T) = 0. \end{aligned}$$

Prove or disprove that this is indeed the optimal solution.

3

Go to T-square under Resources/m-files and download the m-file `dist.m`. This is a model of a distillation process with five states and two inputs.

a

Is the system completely controllable? Is the uncontrolled system (i.e., when $u = 0$) stable?

b

Use infinite-horizon LQ to control this system. Your job is to pick weights that makes the system behave "well". (This is vague - just like life as a control designer is...) Explain why you are happy with your control design.

The weights will have to be inserted where it says:

```
%% Put your weights here.
Q=????;
R=????;
```

This is the only place where the code should be changed.

4

Consider the problem

$$\min_u \int_0^\infty ((x_1 + x_2)^2 + u^2)dt$$

such that

$$\begin{aligned} \dot{x}_1 &= x_1 + x_2 \\ \dot{x}_2 &= -x_1 + u. \end{aligned}$$

a

Is this problem well-defined?

b

If the problem is indeed well-defined, what is the optimal controller?

5

If a linear system does not have an input ($B = 0$) then the Algebraic Riccati Equation becomes the so-called Lyapunov Equation instead

$$0 = A^T P + PA + Q,$$

which has a solution $P = P^T \succ 0$ if and only if $\dot{x} = Ax$ is asymptotically stable.

Show that the solution to the Lyapunov Equation is given by

$$P = \int_0^\infty e^{A^T t} Q e^{At} dt.$$

6.

Go to

`gatech.smartevals.com`

and check out the course survey for ECE6553 Sections A, Q, or QSZ (depending on what section you are enrolled in). (Supposedly, the Course Instructor Opinion Survey will open at 6:00 AM on Apr. 10.)

How many questions were there?