ECE 6604 Design Project:
Performance of STBC Schemes over Rayleigh Block-Fading Channel for 4G MIMO

Abhishek Obla Hema and Klaus Okkelberg

Abstract

Wireless mobile networks are a part of everyday life, but they have limited range and data rate to satisfy the demands of the exponentially increasing number of wireless mobile devices. In particular, high energy consumption and spectrum contraints mean that current 3.9G technology will not be able to satisfy the demands of mobile data traffic in the near future. To overcome these limitations, various diversity schemes are used, which fall into six categories, namely space, angle, polarization, frequency, and multipath. One implementation of a diversity scheme is to use MIMO links, which use multiple transmit and receive antennas to increase time and spatial diversity, the result of which is to increase range and data rate without requiring additional transmit power or bandwidth. To this end, this paper investigates the performance of space-time block coding (STBC) schemes for up to eight transmit antennas and up to two receive antennas, in line with current 4G standards for cellular phones. These schemes are analyzed for their spectral efficiency and bit error rates for transmitting data modulated using BPSK, QPSK, 16-QAM, and 64-QAM over a Rayleigh block-fading channel with additive white Gaussian noise. This is accomplished through software simulation in Matlab.

I. INTRODUCTION

The paper is organized as follows. Section II provides background on diversity-increasing schemes used in current 3.9G and 4G standards. Section III discusses current STBC schemes. Section IV explains the simulation methodology. The simulation results are presented and analyzed in section V. Finally, section VI summarizes the results and discusses how they can be applied to future 5G technologies. The simulation code used for this paper is included in the appendix.

II. BACKGROUND

Rayleigh fading of signals yields a very large performance loss by converting the exponential dependency of bit error probability P_b on the mean received bit energy-to-noise ratio E_b/N_0 into an inverse linear one. Diversity is one remedy that improves the reliability of communication by providing the receiver with multiple independently-faded copies of the same information. These copies are often referred to as diversity branches. Diversity methods can be categorized as those that exploit (1) spatial, (2) angle, (3) polarization, (4) frequency, (5) multipath, and (6) time diversity. At the receiver, the diversity branches are combined together, with the most effective combining method depending, among other things, on the type of additive impairment. For additive white Gaussian noise (AWGN) dominant channels, maximal ratio combining (MRC) is optimal in the maximum likelihood (ML) sense. For co-channel interference (CCI) dominant channels, optimum combining performs better.

The performance of a diversity scheme is measured by its diversity gain, defined as the number of independent receptions of the same signal. The independent copies of the signal increase the signal-to-interference ratio and allow a reduction in transmission power without a performance loss. A MIMO system with N_T transmit antennas and N_R receive antennas has a maximum diversity gain equal to $N_T N_R$.

III. Space-Time Block Coding

With the carrier frequencies used in current 3.9G and 4G standards, only up to two receive antennas can be used while maintaining spatial diversity. Thus, it is necessary to use transmitter diversity, which uses multiple transmit antennas to provide the receiver with multiple uncorrelated copies of the same signal. Simple forms of transmit diversity include selective transmit diversity for time division duplexed systems as well as time division transmit diversity, time-switched transmit diversity, and delay transmit diversity for frequency division duplexed systems. More complex forms of transmit diversity use space-time, space-frequency, or space-time-frequency coding.

For MIMO systems, knowledge of the channel state information (CSI) at the transmitter (CSIT) and at the receiver (CSIR) is necessary to choose a diversity technique [1,2]. In general, the channel can be estimated at the receiver using various techniques (see [3]–[5]), so the knowledge of the CSIR can be assumed. At the transmitter, if the CSIT is known, then beamforming techniques are used to assure both the diversity gain and the array gain. However, without CSIT, space-time (ST) codes can still be used to assure the diversity gain. ST codes are a general class of error correcting codes in which the control symbols are inserted in both spatial and temporal domains.

The multi-layered space-time architecture was introduced by Foschini in [6]. Later, Space-Time Trellis Codes (STTrC) [7] were proposed, which provide the optimal tradeoff between constellation size, data rate, diversity gain, and trellis complexity but have a greater decoding complexity. Regarding encoding and decoding complexity, Alamouti [8] introduced a simple repetition diversity scheme for two transmit antennas with maximum likelihood combining at the receiver. Alamouti's transmit diversity scheme provides a maximum diversity gain and no coding gain for a minimum decoding complexity. Later, Tarokh et al. generalized the Alamouti code for an arbitrary number of transmit antennas as the Space-Time Block Code (STBC) [9].

The remainder of this section presents a mathematical model for a general MIMO system using STBC and details ML decoding methods for both linear and widely linear coding schemes. Additionally, some examples of STBCs are reviewed with emphasis on the code transmission matrix and the code parameters.

A. Space-Time Block Coded MIMO System

This paper considers digital wireless transmissions through a general MIMO system with N_T transmit antennas and N_R receive antennas, consisting of a ST encoder, a Rayleigh block-fading channel with AWGN, and a ST decoder with MRC and ML decoding. The channel is modeled as a discrete-time baseband-equivalent channel. This study assumes single-carrier modulated block transmission with zero padding is used to suppress intersymbol interference, as in [10,11] and uncorrelated path gains for the receive antennas.

1) Space-Time Encoding: Assume that the input to the ST encoder is a k-dimensional vector of complex symbols $\boldsymbol{s} = [s_1 \, s_2 \, \cdots \, s_k]^{\mathrm{T}}$. The encoding method for the STBC is defined by its transmission matrix in the form

$$\boldsymbol{G} = \begin{bmatrix} g_{11} & g_{12} & \cdots & g_{1N_T} \\ g_{21} & g_{22} & \cdots & g_{2N_T} \\ \vdots & \vdots & & \vdots \\ g_{T1} & g_{T2} & \cdots & g_{TN_T} \end{bmatrix},$$

where g_{ij} is the modulated symbol transmitted in the ith time slot from the jth antenna. Note that the g_{ij} are combinations of the input complex symbols and their conjugates and the encoding is independent of the number of receive antennas N_R.

During each time slot $i = 1, \ldots, T$, the symbols are sent simultaneously from the N_T transmit antennas. Because k symbols are transmitted during T time slots, the code rate is $R = k/T$, where the maximum possible rate is unity [9]. For simplicity, it is useful to refer to a code by its parameters as $C(N_T, k, T)$. With this notation, an uncoded transmission is $C(N_T, 1, 1)$ and Alamouti code with two transmit antennas and full rate is $C(2, 2, 2)$.

In practice, the class of useful codes is restricted to Orthogonal STBCs (OSTBCs), for which the transmission matrix $\boldsymbol{G}$ satisfies [8]:

- *linearity*: its elements g_{ij} are linear combinations of the input symbols and their conjugates
- *orthogonality*:

$$\boldsymbol{G}^{\mathrm{H}}\boldsymbol{G} = \|\boldsymbol{s}\|_2^2\mathbf{I}_{N_T} = \left(\sum_{i=1}^{k}|s_i|^2\right)\mathbf{I}_{N_T},$$

where $\|\cdot\|_2$ is the Euclidean norm and $\mathbf{I}_{N_T}$ is the $N_T \times N_T$ identity matrix. OSTBCs have the advantage of providing full diversity equal to $N_T N_R$ as well as making MRC of the received signals equivalent to the linear minimum mean square error (MMSE) estimate, as will be detailed in the decoding section. Note that these codes have very little or no coding gain in contrast to space-time trellis codes that spread a conventional trellis code over space and time.

Orthogonal matrices can be designed using Hurwitz-Radon (HR) theory [12,13], which shows that a HR family with $N-1$ matrices of size $N \times N$ exists only if $N \in \{2, 4, 8\}$. STBC transmission matrices can be constructed from the HR family of matrices. For real input constellations, square STBC matrices with full rate of unity exist only for $N_T \in \{2, 4, 8\}$. However, non-square full rate STBC matrices can be designed for $N_T \in \{3, 5, 6, 7\}$ by removing columns from the square full rate matrices.

For complex input constellations, a full rate OSTBC only exists for $N_T = 2$. However, for $N_T > 2$, $R = 1/2$ codes can be constructed for complex constellations from full rate codes for real constellations [9], in essence sacrificing code rate to gain an orthogonal design. In addition, $R = 3/4$ codes exist for $N_T = 4$ but the authors do not know of any orthogonal codes with $R > 1/2$ for $N_T > 4$. Also of interest are Quasi-Orthogonal STBCs (QOSTBCs), in which only adjacent columns of the transmission matrix are orthogonal. QOSTBCs can achieve full rate communication at the expense of decoding complexity and diversity gain due to the interference [14].

2) Space-Time Coded MIMO Channel: The transmission of the ST coded symbols over the MIMO channel considered by this paper is subject to fading and additive noise. Let h_{ijk} denote the path gain between in the ith time slot between the jth transmit antenna and the kth receive antenna. Then, the received symbol y_{ik} in the ith time slot by the kth receive antenna is

$$y_{ik} = [h_{i,1,k}\,h_{i,2,k}\,\cdots\,h_{i,N_T,k}]\begin{bmatrix}x_{i,1}\\x_{i,2}\\\vdots\\x_{i,N_T}\end{bmatrix} + n_{ik},$$

where x_{ij} is the symbol transmitted by the jth transmit antenna in the ith time slot and n_{ik} is the additive noise for the kth receive antenna at the ith time slot. Note that the x_{ij} are combinations, not necessarily linear, of the input symbols s_i and their conjugates, so this system is difficult to solve in general.

For a block-fading channel with coherence time T_c, it is assumed that the fading coefficients h_{ijk} remain constant within a frame of length T_c time slots. With Rayleigh fading, these coefficients are generated as independent complex Gaussian random variables with zero mean and variance $\Omega_p/2$ in both the real and imaginary dimensions, where Ω_p is the average energy of the transmitted discrete-time symbol. In addition, the additive noise is assumed to be complex Gaussian random variables with zero mean and variance $1/(2E_b/N_0)$, where the E_b/N_0 is the bit energy-to-noise ratio of the channel.

3) Space-Time Decoding: ST decoding for a STBC consists of MRC followed by ML decoding of the combined symbols. For an OSTBC, MRC is equivalent to solving a linear system, which is advantageous for decoding performance. As the combined symbols are independent and assuming no ISI, the ML decoding can be performed independently of the MRC. A similar approach works for QOSTBC, but the performance is reduced to due interference between symbols.

For a general OSTBC or QOSTBC, the effect of the channel can be written in matrix form as

$$\bar{\boldsymbol{y}} = [\boldsymbol{H}_1\,\boldsymbol{H}_2]\begin{bmatrix}\boldsymbol{x}\\\boldsymbol{x}^*\end{bmatrix} + \bar{\boldsymbol{n}},$$

where x is the input to the STBC encoder, $\bar{y}$ is the output of the MIMO channel, $\bar{n}$ is the AWGN, and H_1 and H_2 are matrices representing the equivalent channel formed by the ST encoder and the MIMO channel. For a linearly decodable ST code where x and its complex conjugate x^* are uncorrelated, it is possible to rewrite the equation as

$$\begin{bmatrix} \bar{y}_1 \\ \bar{y}_2^* \end{bmatrix} = \begin{bmatrix} H_1 \\ H_2^* \end{bmatrix} x + \begin{bmatrix} n_1 \\ n_2^* \end{bmatrix},$$

where $\bar{y} = [\bar{y}_1^T \, \bar{y}_2^T]^T$ and $\bar{n} = [\bar{n}_1^T \, \bar{n}_2^T]^T$. Then, let $y = [\bar{y}_1^T \, \bar{y}_2^T]^T$, $H_{ef} = [H_1^T \, H_2^H]^T$, and $n = [\bar{n}_1^T \, \bar{n}_2^T]^T$ to obtain:

$$y = H_{ef}x + n, \tag{1}$$

where H_{ef} is the matrix of the equivalent channel formed by the ST encoder and the MIMO channel. Note that for an OSTBC, H_{ef} is an orthogonal matrix over all channel realizations because

$$H_{ef}^H H_{ef} = \|H\|_F^2 I_k,$$

where H is the channel matrix, $\| \cdot \|_F$ is the Frobenius norm, and k is the dimension of the input.

Assuming perfect CSIR, the MRC coefficients equal the complex conjugate of the equivalent channel matrix [14,15]:

$$\tilde{x} = H_{ef}^H y, \tag{2}$$

where $\tilde{x}$ is the vector of the combined symbols.

Using (1) and assuming an orthogonal ST code, (2) can be written as

$$\tilde{x} = H_{ef}^H H_{ef} x + H_{ef}^H n = \|H\|_F^2 I_k x + \tilde{n},$$

where $\tilde{n}$ has zero-mean and autocorrelation $\mathrm{E}[\tilde{n}\tilde{n}^H] = \sigma_n^2 \|H\|_F^2 I_k$. Thus, the combined symbols are decoupled into

$$\tilde{x}_i = \|H\|_F^2 x_i + \tilde{n}_i, \quad i = 1, \ldots, k, \tag{3}$$

where k is the dimension of the x. With this decoupling, ML decoding is trivial for the case of AWGN.

To see why MRC for the case of linearly decodable orthogonal ST codes is equivalent to the linear MMSE solution, consider solving for the input x in (1) using the pseudo-inverse:

$$\hat{x} = H_{ef}^\dagger y = (H_{ef}^H H_{ef})^{-1} H_{ef}^H y = \frac{1}{\|H\|_F^2} I_k \tilde{x} = x + \frac{1}{\|H\|_F^2} \tilde{n},$$

which is equivalent to the solution for (3).

For a nonlinearly decodable ST code where x and its complex conjugate x^* are correlated, it is still possible to perform MRC and ML decoding by solving a linear system of augmented symbols given by

$$\begin{bmatrix} \bar{y} \\ \bar{y}^* \end{bmatrix} = \begin{bmatrix} H_1 & H_2 \\ H_2^* & H_1^* \end{bmatrix} \begin{bmatrix} x \\ x^* \end{bmatrix} + \begin{bmatrix} \bar{n} \\ \bar{n}^* \end{bmatrix}.$$

This type of system is refered to as a widely linear system [16,17]. Then, the MMSE solution to this system is given by the pseudo-inverse as before and the ML decoding of the input symbols is given by the first half of the augmented input vector.

For QOSTBC, while the decoding is more complex than the diagonal decoding presented in (3), the system is still linear so a linear MMSE solution still exists. This increased decoding complexity is offset by its increased data rate over OSTBC, particularly for low E_b/N_0.

B. Examples of STBCs

1) Alamouti's STBC: The Alamouti STBC uses two transmit antennas and any number of receive antennas.

IV. Methodology

V. Results

VI. Summary

References

[1] G. Caire and S. Shamai, "On the achievable throughput of a multi-antenna Gaussian broadcast channel," *IEEE Trans. Inf. Theory*, July 2003.

[2] H. Weingarten, Y. Steinberg, and S. Shamai, "The capacity region of the Gaussian multiple-input multiple-output broadcast channel," *IEEE Trans. Inf. Theory*, vol. 52, pp. 3936–3964, Sept 2006.

[3] M. Nasseri and H. Bakhshi, "Iterative channel estimation algorithm in multiple input multiple output orthogonal frequency division multiplexing systems," *J. of Comput. Sci.*, vol. 6, no. 2, pp. 224–228, 2010.

[4] E. Björnson and B. Ottersten, "A framework for training-based estimation in arbitrary correlated Rician MIMO channels with Rician disturbance," *IEEE Trans. Signal Process.*, vol. 58, pp. 1807–1820, Mar 2010.

[5] M. Biguesh and A. Gershman, "Training-based MIMO channel estimation: a study of estimator tradeoffs and optimal training signals," *IEEE Trans. Signal Process.*, vol. 54, pp. 884–893, Mar 2006.

[6] G. J. Foschini, "Layered space-time architecture for wireless communications in a fading environment when using multi-element antennas," *Bell Labs Technical Journal*, vol. 1, no. 2, pp. 41–59, 1996.

[7] V. Tarokh, N. Seshadri, and A. R. Calderbank, "Space-time codes for high data rate wireless communication: performance analysis and code construction," *IEEE Trans. Inf. Theory*, vol. 44, pp. 744–765, Mar 1998.

[8] S. M. Alamouti, "A simple transmit diversity technique for wireless communications," *IEEE J. Sel. Areas Commun.*, vol. 16, no. 8, pp. 1451–1458, 1998.

[9] V. Tarokh, H. Jafarkhani, and A. R. Calderbank, "Space-time block codes from orthogonal designs," *IEEE Trans. Inf. Theory*, vol. 45, pp. 1456–1458, July 1999.

[10] Z. Wang and G. B. Giannakis, "Wireless multicarrier communications," *IEEE Signal Processing Mag.*, vol. 17, no. 3, pp. 29–48, 2000.

[11] Z. Wang, X. Ma, and G. B. Giannakis, "OFDM or single-carrier block transmissions?" *IEEE Trans. Commun.*, vol. 52, pp. 380–394, Mar 2004.

[12] J. Seberry, S. A. Spence, and T. A. Wysocki, "A construction technique for generalized complex orthogonal designs and applications to wireless communications," *Linear algebra and its applications*, vol. 405, pp. 163–176, 2005.

[13] W. Wolfe, "Amicable orthogonal designs—existence," *Canad. J. Math.*, vol. 28, no. 5, pp. 1006–1020, 1976.

[14] H. Jafarkhani, *Space-time coding: theory and practice*. New York: Cambridge University Press, 2005.

[15] E. G. Larsson and P. Stoica, *Space-time block coding for wireless communications*. New York: Cambridge University Press, 2003.

[16] Y. A. S. Dama, R. A. Abd-alhameed, T. S. Ghazaany, and S. Zhu, "A new approach for OSTBC and QOSTBC," *Int. J. of Comput. Applicat.*, vol. 67, no. 6, pp. 45–48, 2013.

[17] B. Picinbono and P. Chevalier, "Widely linear estimation with complex data," *IEEE Trans. Signal Process.*, vol. 43, pp. 2030–2033, Aug 1995.

Appendix