

A Modularised Java library for 3D Euclidean geometry

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Software

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Summary

A three-dimensional (3D) [Euclidean geometry](#) Java library for modelling 3D changes in geometries over time. The library is modularised, based on [openJDK 17](#), and depends on [cgc-math](#) which in turn depends on both [big-math](#) and [cgc-io](#).

Point locations in space are defined using 3D [cartesian](#) coordinates with [orthogonal](#) X, Y and Z axes that meet at the origin - a point $\langle x, y, z \rangle$ where $x=y=z=0$. The coordinate system is “right handed”: so if X increases to the right of this page, and Y increases towards the top of this page, then Z increases out from the page, (see [orientation](#) for details of handedness, essentially there are two choices - a “left handed” system would have the Z direction of increase reversed). Right handed is thought to be more commonly used in both physics and geography.

Coordinates are either stored as [Math_BigRational](#) numbers - a subset of [rational numbers](#) with java.math.BigDecimal numerators and denominators, or [Math_BigRationalSqrt](#) - numbers that also represent square roots (calculated to a given [Order of Magnitude](#) precision as required).

More light weight geometries are in the [uk.ac.leeds.ccg.v3d.geometry.light](#) package. These use [V3D_V](#) which have three [Math_BigRational](#) components (nominally x, y and z) and extend [V3D_VGeometry](#) - an abstract class that holds a [V3D_V](#) “offset” which can be thought of as the translation of the geometry from the origin. - [V3D_VPoint](#) - is for representing points. A [V3D_V](#) instance gives the location of the point relative to the offset. - [V3D_VLine](#) - is for representing finite lines. [V3D_V](#) instances p and q give the location of the ends of the line relative to the offset (and can be the same). - [V3D_VTriangle](#) - is for representing triangles. [V3D_V](#) instances p, q and r give the location of each corner of the triangle relative to the offset (two or more of them can be the same). - [V3D_VTetrahedron](#) - is for representing [tetrahedron](#). [V3D_V](#) instances p, q, r and s give the location of each corner of the tetrahedron relative to the offset (two or more can be the same).

Heavier geometries use offsets and relative locations stored as [Math_BigRationalSqrt](#) numbers. This allows for example a point to be located at $x=y=z=\sqrt{2}$. For these geometries there are methods which test for intersection, that return the geometry of the intersection, and that calculate the minimum distance between any two geometries. The methods that test if two geometries intersect involve no rounding. Calculating geometries of intersection may involve rounding (as coordinates are only a subset of [algebraic numbers](#)).

For any finite [cgc-v3d](#) geometry, a [V3D_Envelope](#) - a [rectangular cuboid](#) aligned with the axes can be computed. Such envelopes (also commonly referred to as Axis Aligned Bounding Boxes - AABB) assist in the computation of intersections or tests for intersection by quickly ruling out intersection between geometries that are in different X, Y, Z domains.

[V3D_Geometry](#) is an abstract class with a [V3D_Vector](#) offset, and an [Order of Magnitude](#). Similar to [V3D_V](#), this allows geometries to be readily translated and rotated. [V3D_Geometry](#) is extended to define the following finite and infinite geometries: - [V3D_Point](#) - a point geometry where a [V3D_Vector](#) gives the position relative to the offset. - [V3D_Line](#) - an infinite line geometry that passes through two defined points. - [V3D_Ray](#) - an infinite line

43 geometry that extends from a point in one direction through another point. - [V3D_Plane](#)
44 - an infinite [plane geometry](#). The plane is defined either by 3 points (p, q and r) that are
45 not [collinear](#) or coincident, or by a normal vector and a point (where all points orthogonal to
46 the normal vector at the point are on the plane). - [V3D_LineSegment](#) - represents a single
47 continuous finite part of a line with non-zero length. Line segments are considered equal
48 irrespective of the order of the end points. The centroid of a line segment can be computed
49 and stored. - [V3D_Triangle](#) - a triangular part of a plane with a non-zero area. The three
50 coplanar points {p, q and r} are the corners of the triangle. Triangle sides can be stored as
51 line segments. The centroid of a triangle can be computed and stored. - [V3D_Tetrahedron](#) -
52 represents a [tetrahedron](#). Any selection of three points are not coplanar with the fourth, so
53 this always defines a volume. A tetrahedron surface can be thought of as comprising of four
54 different triangles.

55 The following geometry collections are supported: - [V3D_Points](#) - for collections of points.
56 - [V3D_LineSegments](#) - for collections of collinear line segments. - [V3D_TriangleCoplanar](#) -
57 for collections of coplanar triangles. These do not have the constraint of all the component
58 triangles having to intersect. - [V3D_Tetrahedrons](#) - for collections of tetrahedrons. These do
59 not have the constraint of all the component tetrahedra being interconnecting.

60 Statement of need

61 Whilst open source 3D spatial tools exist, those which are known about rely on [floating point](#)
62 arithmetic for which precision varies and results in uncertainties. Utilising more heavyweight
63 numerical representations for coordinates allows for greater precision. Shapes can be approxi-
64 mated using points and straight line segments which connect these. Real spatial objects can
65 be represented in this way, particularly around human scales. For generating models of what is
66 happening on or near the surface and within the [Earth](#), such representation offers a way to link
67 data together for a Digital Twin Earth.

68 Everything is moving and changing at rates that vary with scale. Exploring patterns of
69 change spatially is common to much scientific activity. 3D models and maps are important
70 in telecommunications, transportation, architecture and engineering. This library could be
71 applied to represent objects in 3D and model things physically involving motion, energy transfer
72 and the dissolution and formation of objects. An important next steps is to build some basic
73 models and visualise them. Some preliminary work has been done to develop 2D pictures of
74 3D spatial objects. My hope is to develop a basic model of the solar system which inputs data
75 that is known about the masses, orientation and location of the [Sun](#), the [Moon](#) and the rest
76 of the solar system. I also plan to create a basic static model of Earth using available data,
77 and on a smaller scale, I would like to develop a model of a glaciated region.

78 Future development

79 Ideally, a collective effort is wanted to develop the library sustainably. Issue reporting and
80 tracking, feature requests and a more detailed roadmap for developing the functionality is
81 wanted. Perhaps rather than creating a new community, what might best happen is to migrate
82 the functionality to [Apache Commons Geometry](#) (see also: [Apache Commons Geometry GitHub](#)
83 [Repository](#)). This parallel development appears to be developing similar arbitrary precision
84 geometrical functionality.

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⁸⁸ ([Obermühlner, 2020](#)) library, in particular [BigRational](#) upon which Math_BigRational is almost
⁸⁹ entirely based.

⁹⁰ References

⁹¹ Obermühlner, E. (2020). Big-math: Advanced java BigDecimal math functions (pow, sqrt,
⁹² log, sin, ...) Using arbitrary precision. In *GitHub repository*. GitHub. [https://github.com/](https://github.com/eobermuhlner/big-math)
⁹³ [eobermuhlner/big-math](https://github.com/eobermuhlner/big-math)

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