

# On the CCN (de)activation nonlinearities

Sylwester Arabas and Shin-ichiro Shima

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  - ❖ 2017–2018: AETHON, Athens (H2020 “Innovation Associate”)

# introduction

## Shin-ichiro Shima

≡ Google Scholar



Shin-ichiro Shima

Associate Professor of Numerical Simulation, [University of Hyogo](#)

Verified email at sim.u-hyogo.ac.jp - [Homepage](#)

Nonlinear Science Complex Systems Computational Science

TITLE	CITED BY	YEAR
<a href="#">Rotating spiral waves with phase-randomized core in nonlocally coupled oscillators</a> S Shima, Y Kuramoto Physical Review E 69 (3), 036213	201	2004
<a href="#">The super-droplet method for the numerical simulation of clouds and precipitation: a particle-based and probabilistic microphysics model coupled with a non-hydro...</a> S Shima, K Kusano, A Kawano, T Sugiyama, S Kawahara Quarterly Journal of the Royal Meteorological Society 135 (642), 1307-1320	81	2009

## Arabas & Shima 2017

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Nonlinear Processes  
in Geophysics



Open Access

## On the CCN (de)activation nonlinearities

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<sup>2</sup>Chatham Financial Corporation Europe, Cracow, Poland

<sup>3</sup>Graduate School of Simulation Studies, University of Hyogo, Kobe, Japan

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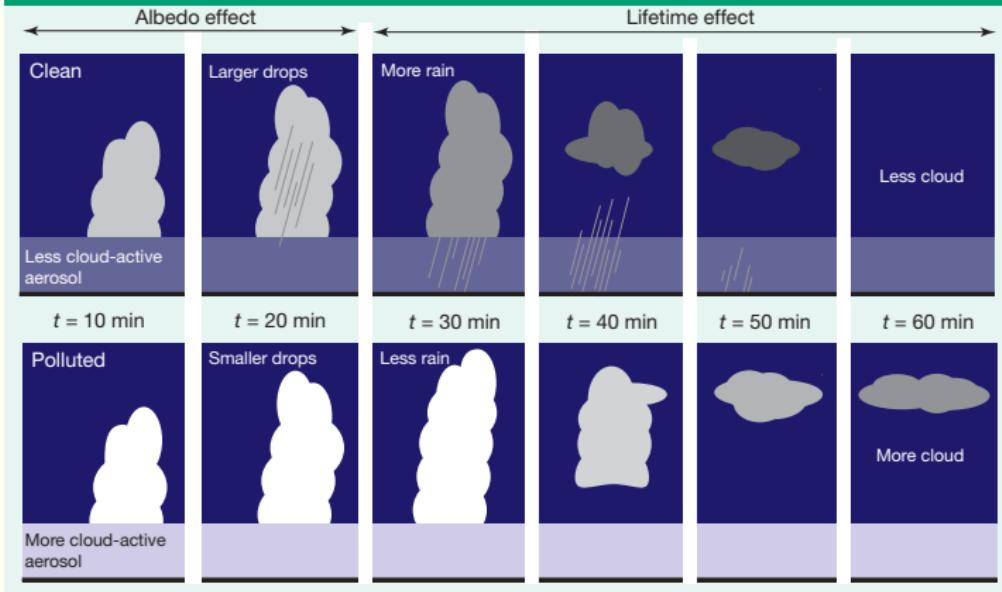
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# one-slide aerosol-cloud (micro-macro) interaction primer

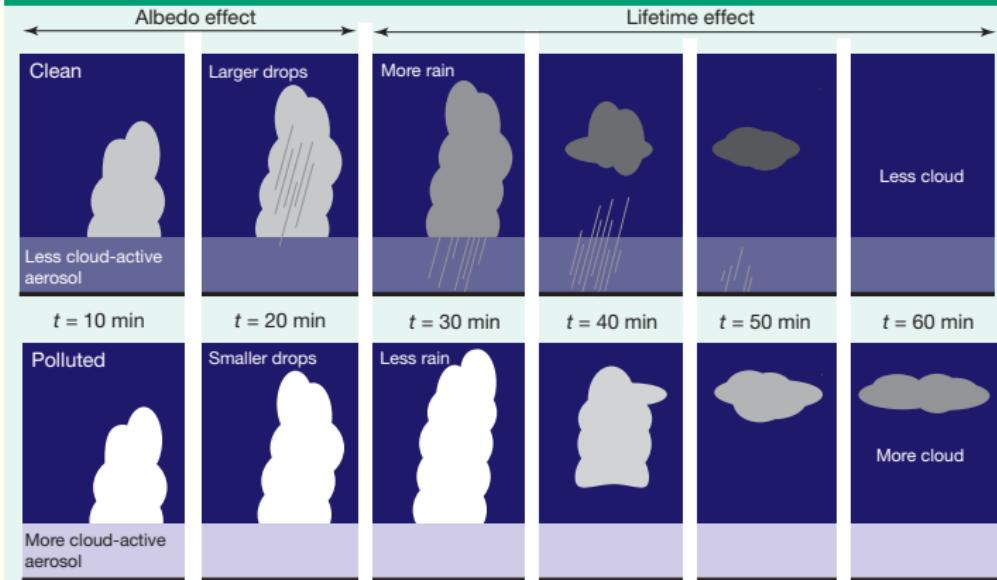
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Stevens and Feingold, 2009 (Nature)



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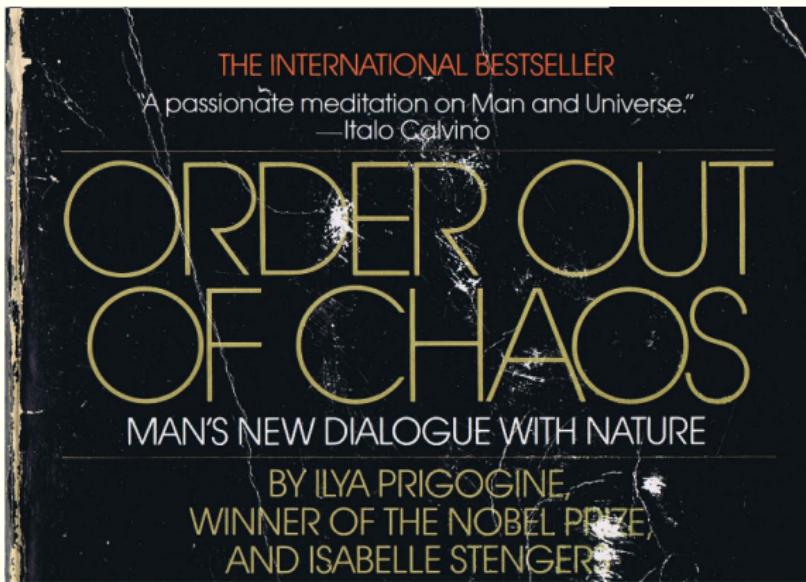


Stevens and Boucher, 2012 (Nature)

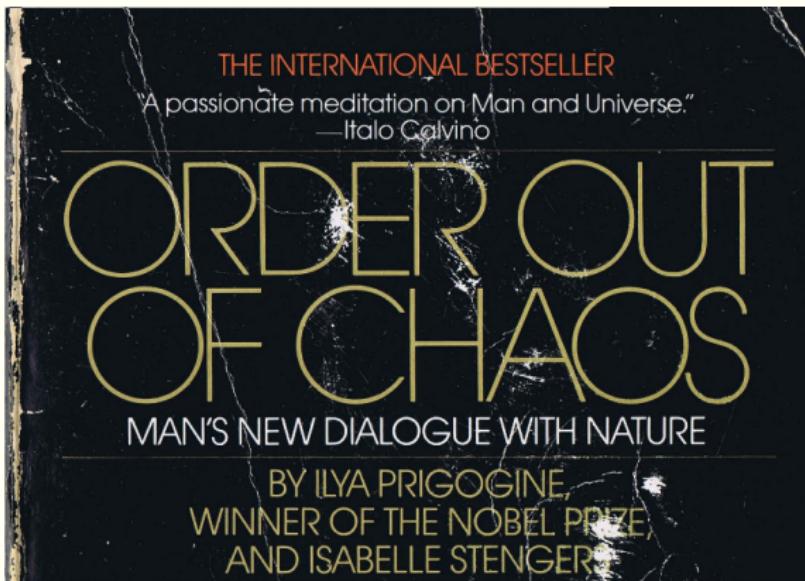
*"there is something captivating about the idea that fine particulate matter, suspended almost invisibly in the atmosphere, holds the key to some of the greatest mysteries of climate science"*

... others captivated by micro-macro interactions

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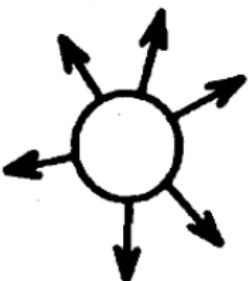
Prigogine and Stengers 1984

*"Much of this book has centered around the relation between the microscopic and the macroscopic. One of the most important problems in evolutionary theory is the eventual feedback between macroscopic structures and microscopic events: macroscopic structures emerging from microscopic events would in turn lead to a modification of the microscopic mechanisms."*

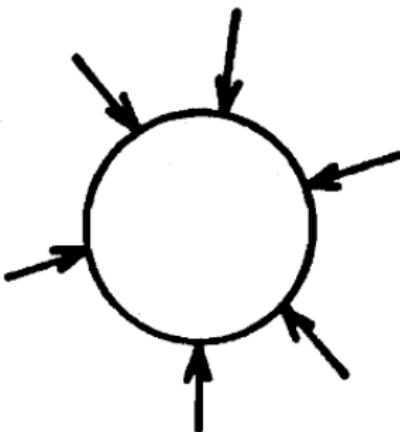
# regime-transition (bifurcation) example from P&S 1984

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(a)



(b)

Figure 19. Nucleation of a liquid droplet in a supersaturated vapor. (a) droplet smaller than the critical size; (b) droplet larger than the critical size. The existence of the threshold has been experimentally verified for dissipative structures.

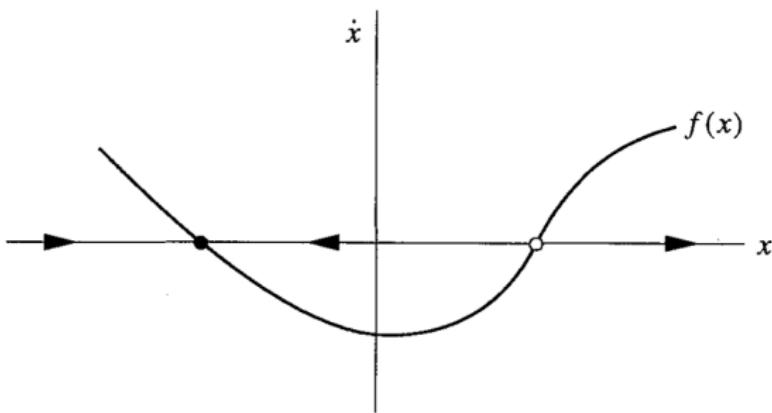
## two-slide bifurcation analysis primer (1/2)

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Strogatz 2014 (sect. 2.2): fixed points and stability

graphical (qualitative) analysis  
of a non-linear one-dimensional dynamical system:

$$\dot{x} = f(x)$$



NONLINEAR  
With Applications to Physics,  
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AND CHAOS



Steven H. Strogatz

SECOND EDITION

## two-slide bifurcation analysis primer (2/2)

Strogatz 2014 (sect. 3.1): saddle-node bifurcation

prototypical example of saddle-node bifurcation:

$$\dot{x} = r + x^2$$

$r$ : parameter (distinct regimes if positive, negative or zero)

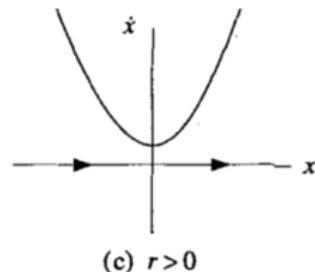
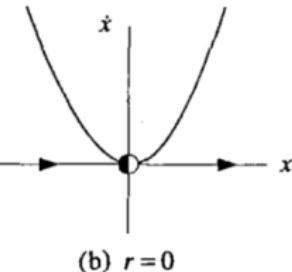
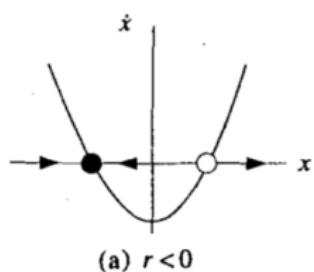
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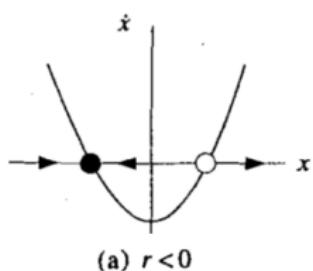
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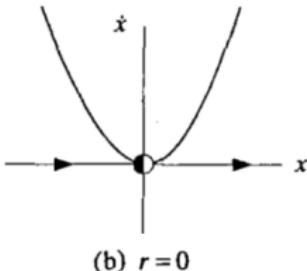
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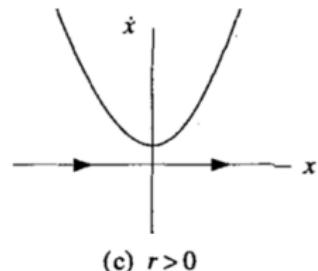
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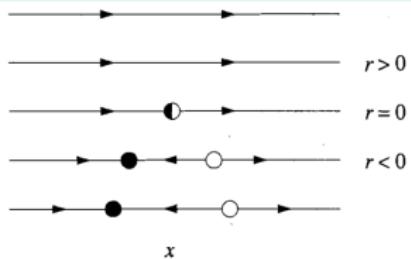
(a)  $r < 0$



(b)  $r = 0$



(c)  $r > 0$



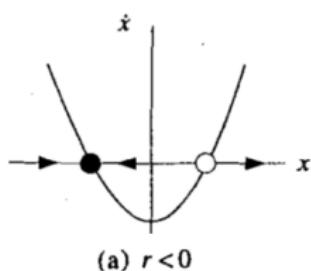
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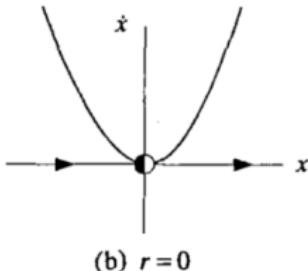
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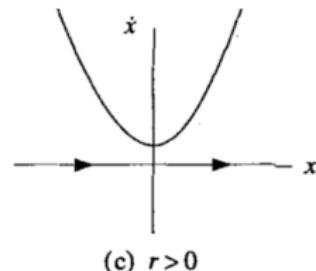
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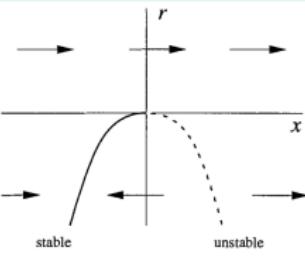
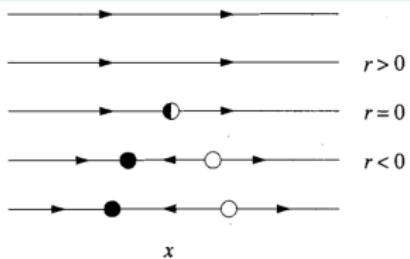
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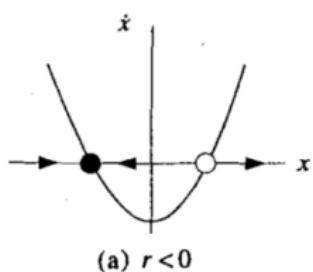
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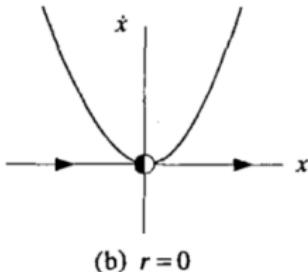
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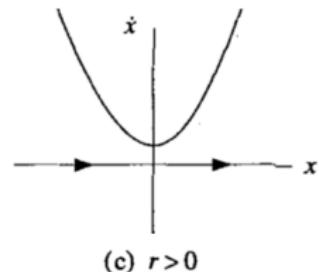
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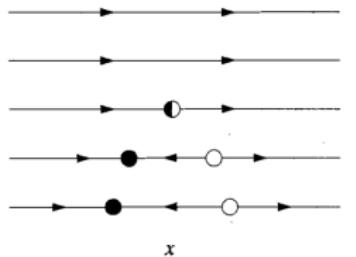
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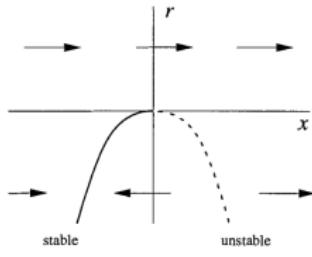


$r > 0$

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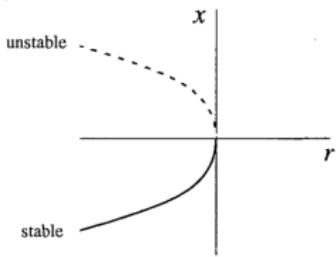
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stable

unstable

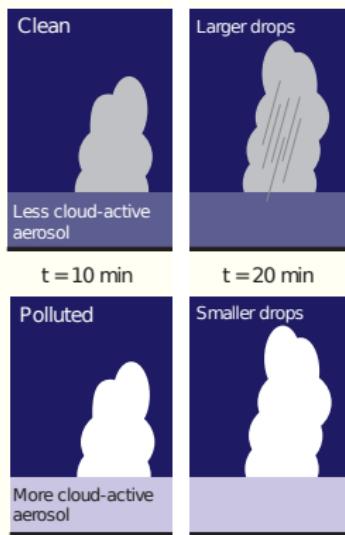


stable

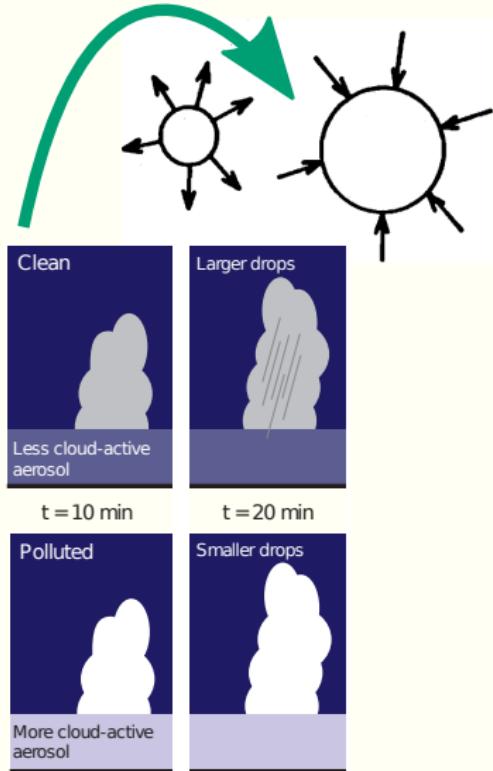
$r$

connecting the dots ...

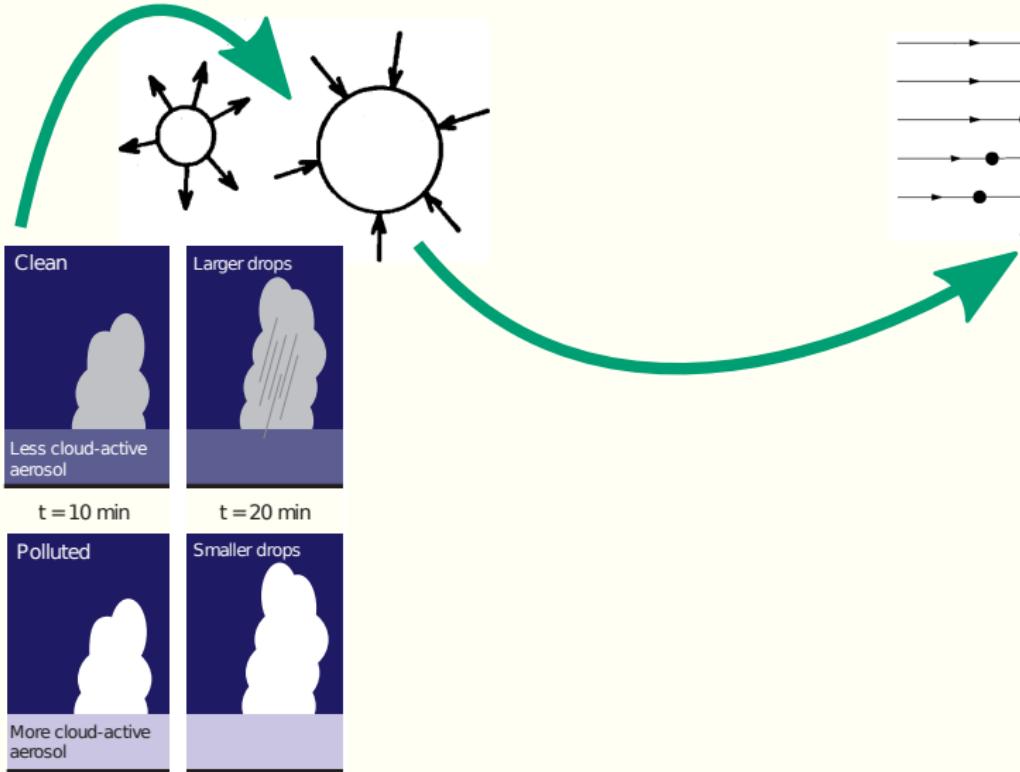
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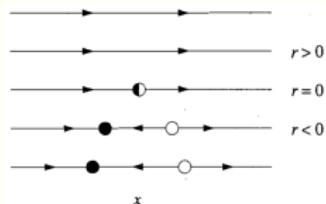
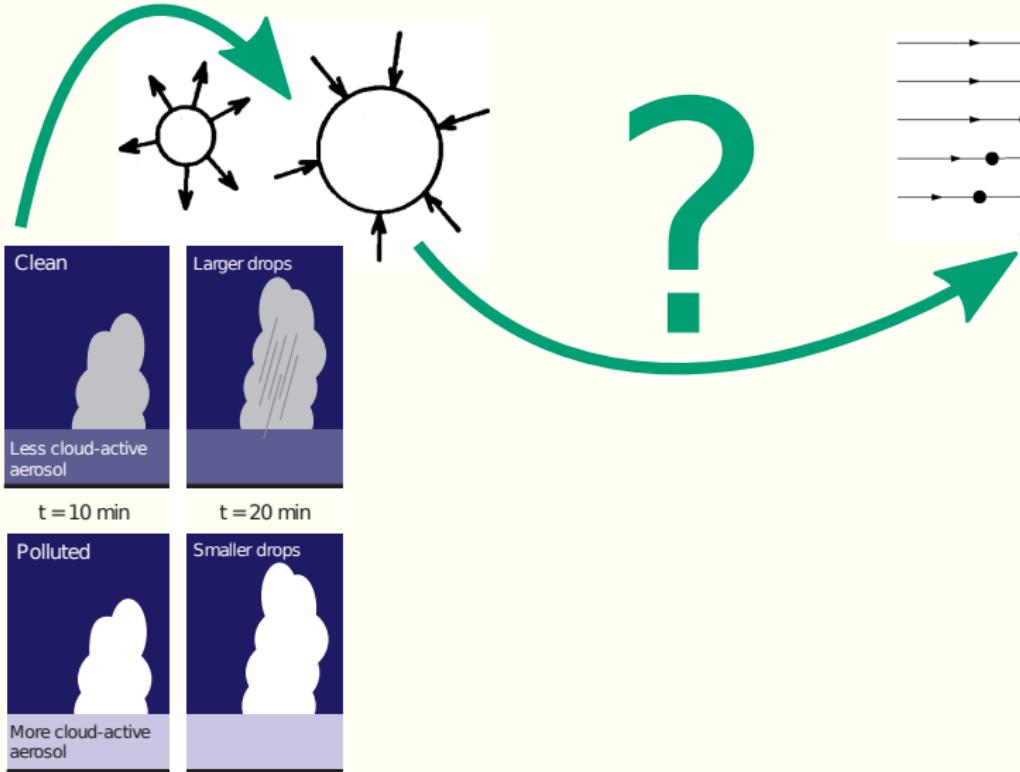
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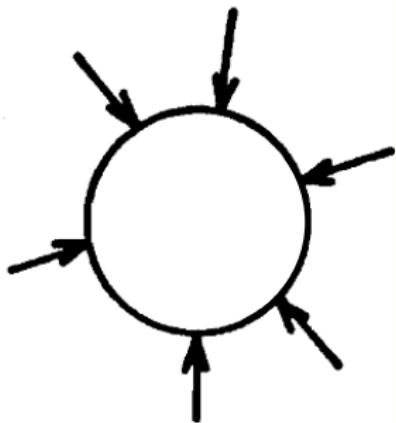
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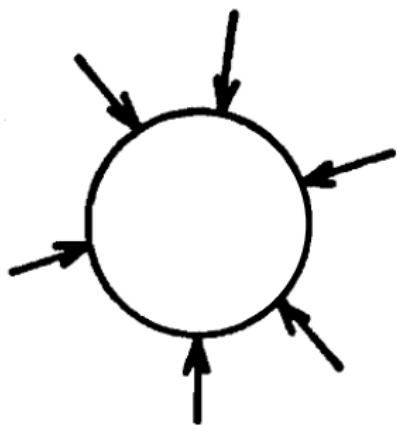


## droplet growth laws in a nutshell: mass and heat diffusion



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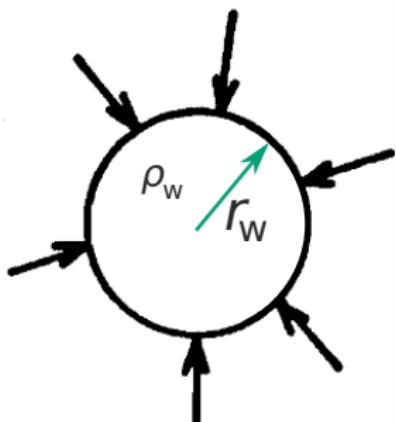
Fick's and Fourier's laws combined  
spherical geometry



$$\dot{r}_w = \frac{1}{r_w} \frac{D_{\text{eff}}}{\rho_w} (\rho_v - \rho_o)$$

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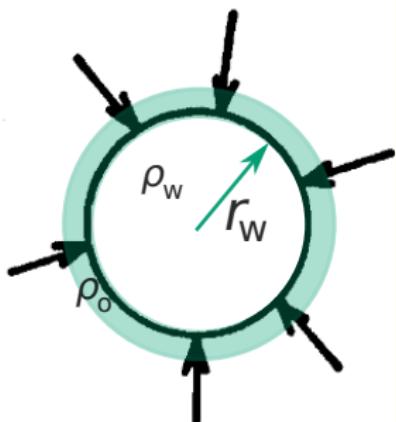
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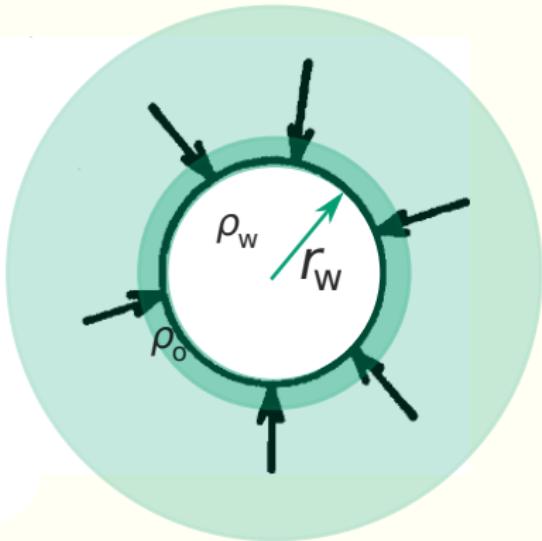


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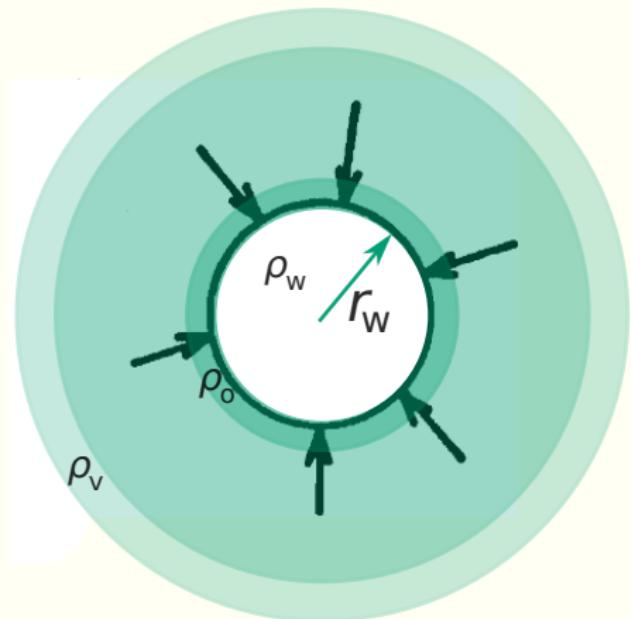
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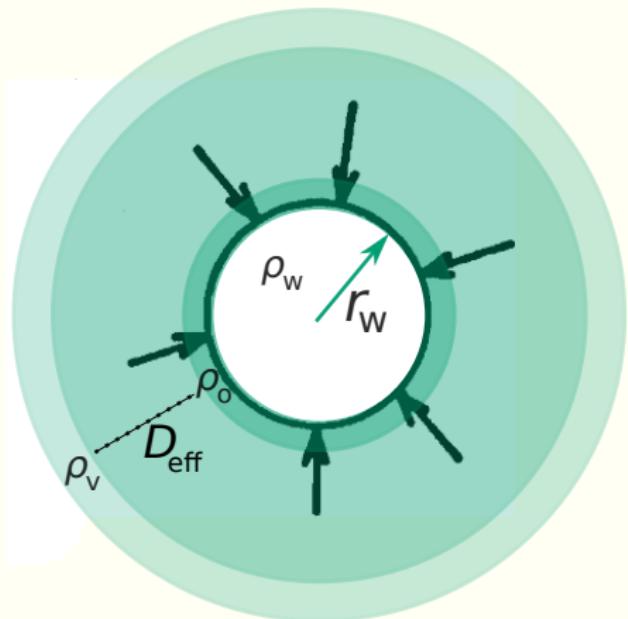
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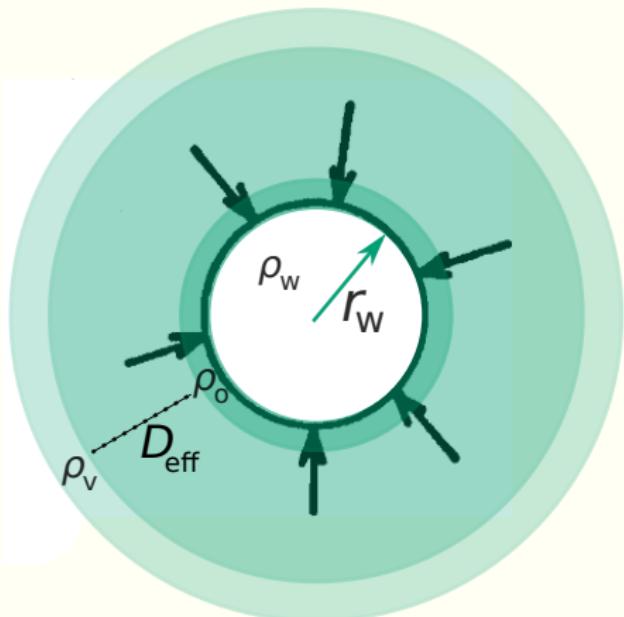
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non-dimensional numbers:

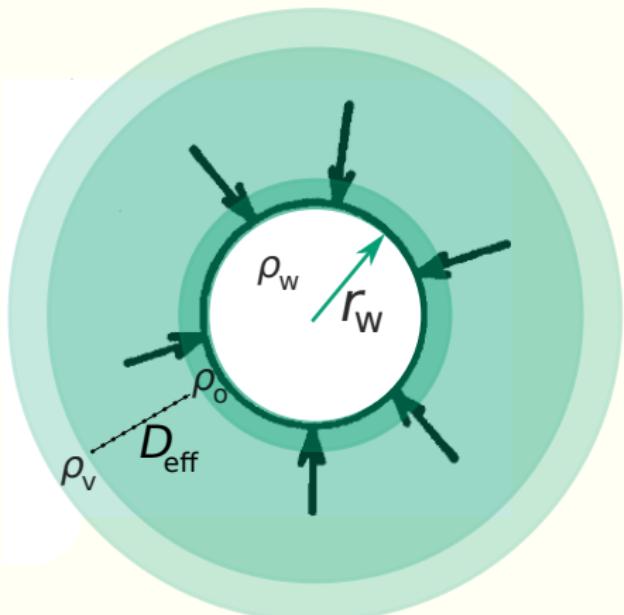
$$\text{RH} = \rho_v / \rho_{vs}$$

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## droplet growth laws in a nutshell: Köhler curve

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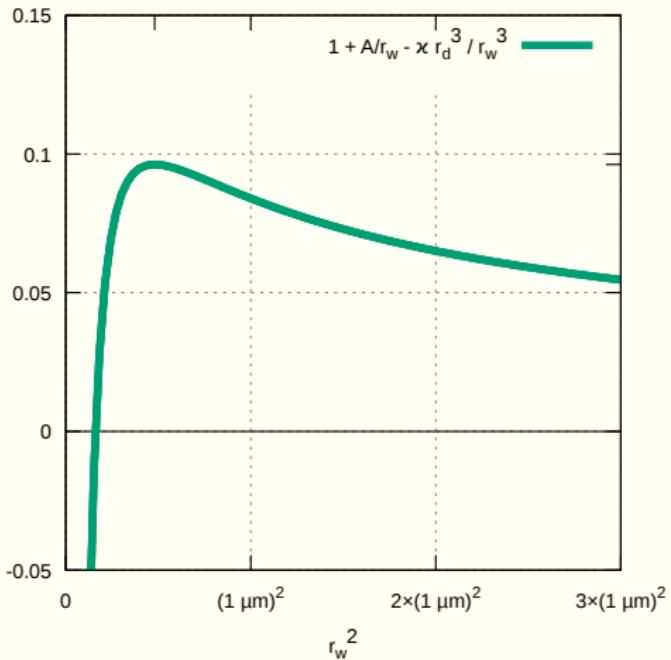
$$\dot{r}_w = \frac{1}{r_w} D_{\text{eff}} \frac{\rho_{vs}}{\rho_w} (\text{RH} - \text{RH}_{\text{eq}}) \quad \text{RH}_{\text{eq}} = \frac{r_w^3 - r_d^3}{r_w^3 - r_d^3(1 - \kappa)} \exp\left(\frac{A}{r_w}\right)$$
$$\approx 1 + \frac{A}{r_w} - \frac{\kappa r_d^3}{r_w^3}$$

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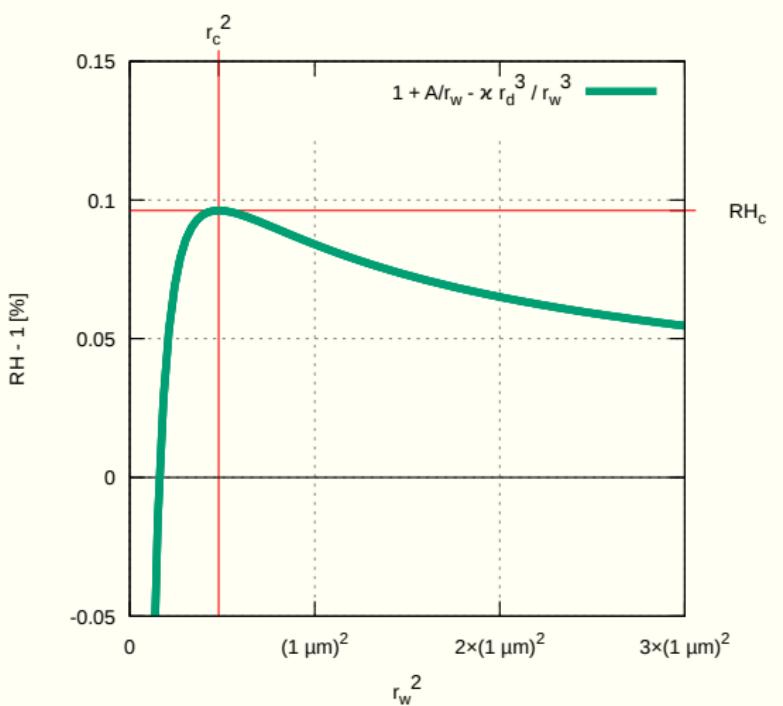


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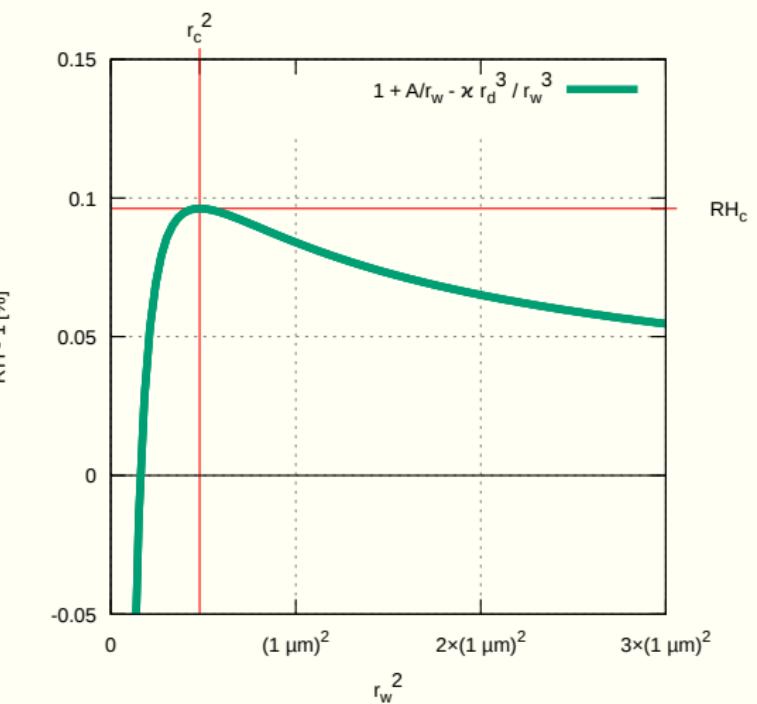


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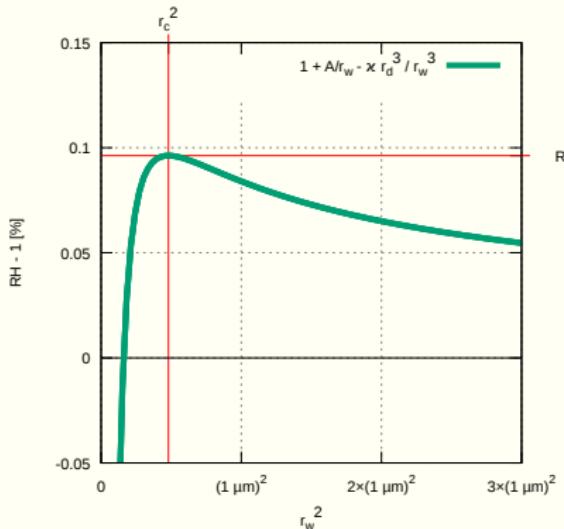


maximum at  $(r_c, \text{RH}_c)$ :

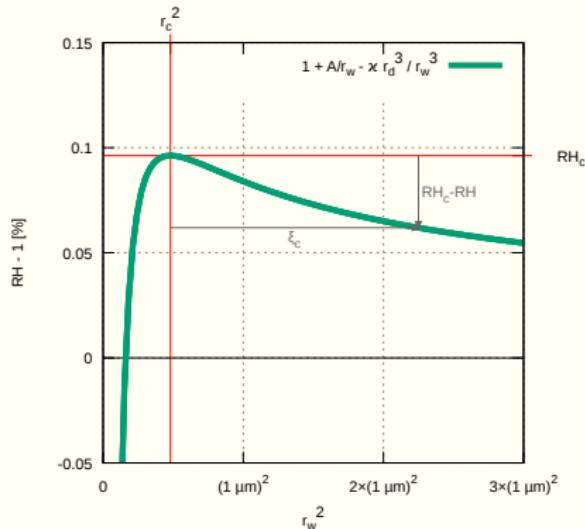
$$r_c = \sqrt{3\kappa r_d^3 / A}$$

$$\text{RH}_c = 1 + \frac{2A}{3r_c}$$

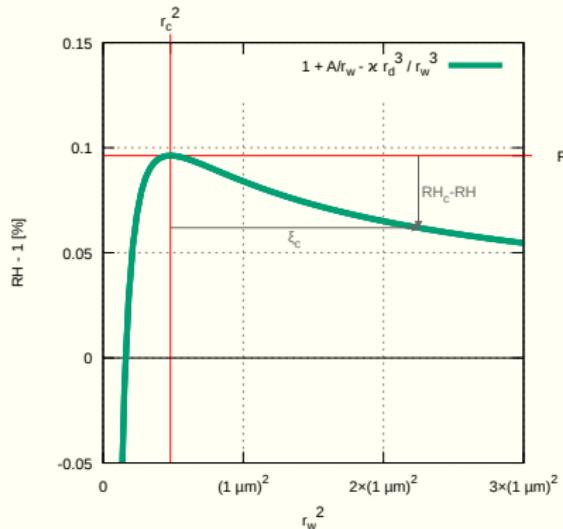
# phase portrait of the system: flipped Köhler curve



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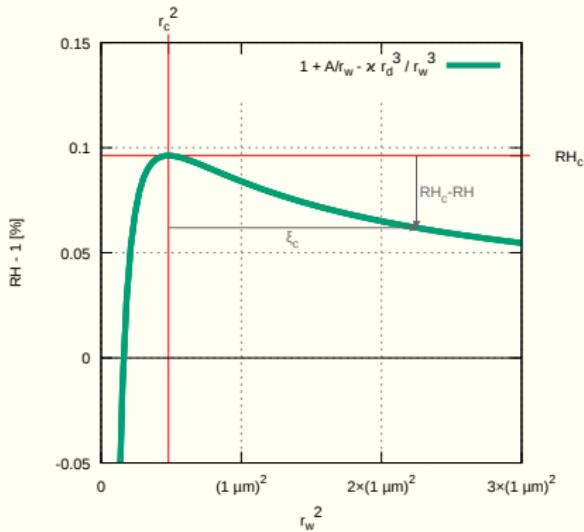
# phase portrait of the system: flipped Köhler curve



$$\xi = r_w^2 + C$$

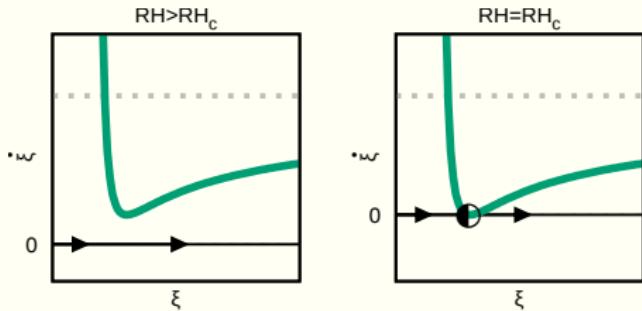
$$\dot{\xi} = 2D_{\text{eff}} \frac{\rho_{\text{vs}}}{\rho_w} (\text{RH} - \text{RH}_{\text{eq}}(\xi))$$

# phase portrait of the system: flipped Köhler curve



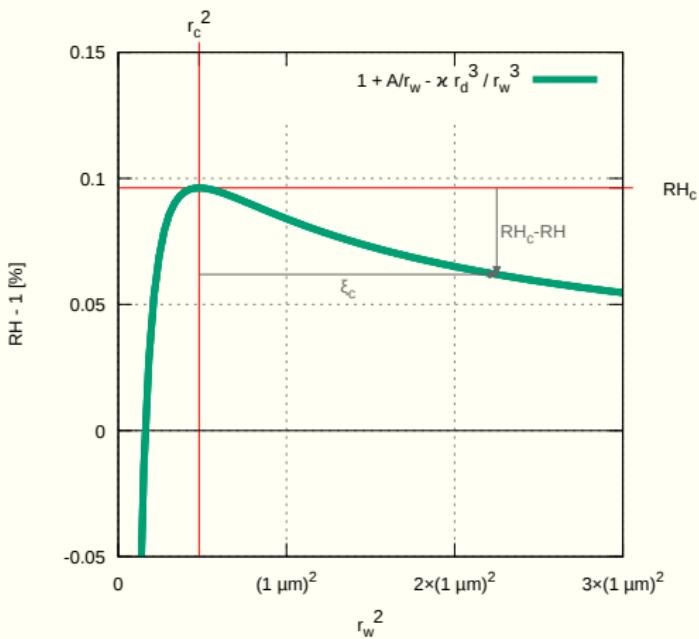
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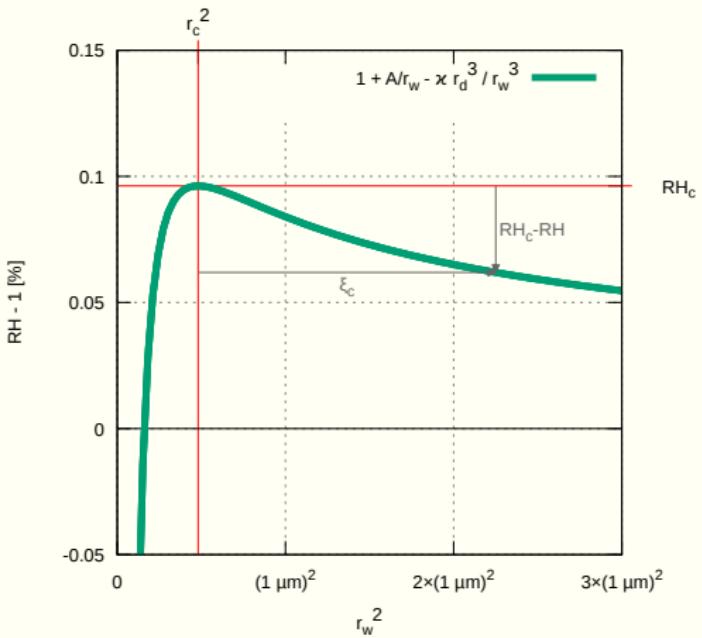
# saddle-node bifurcation at Köhler curve maximum

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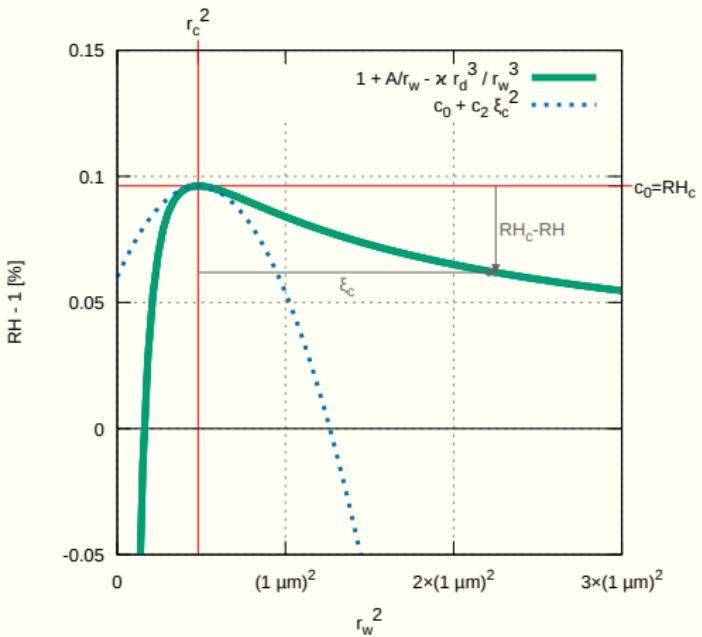
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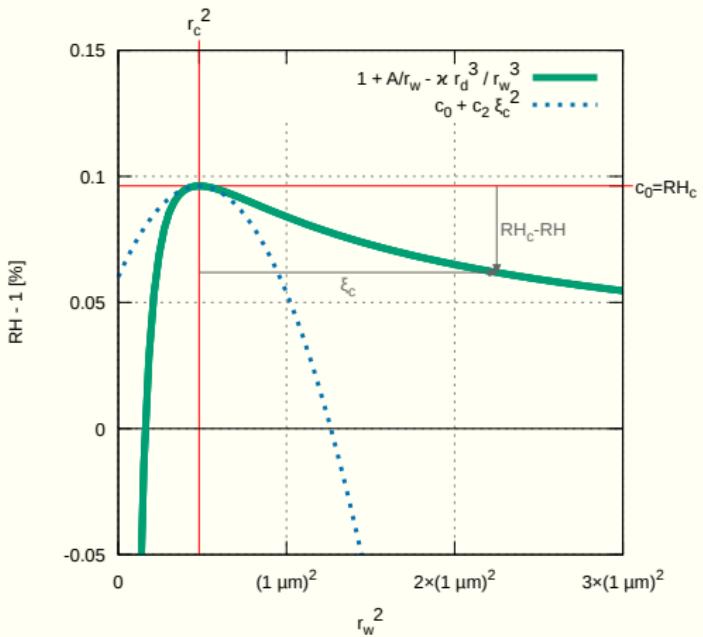
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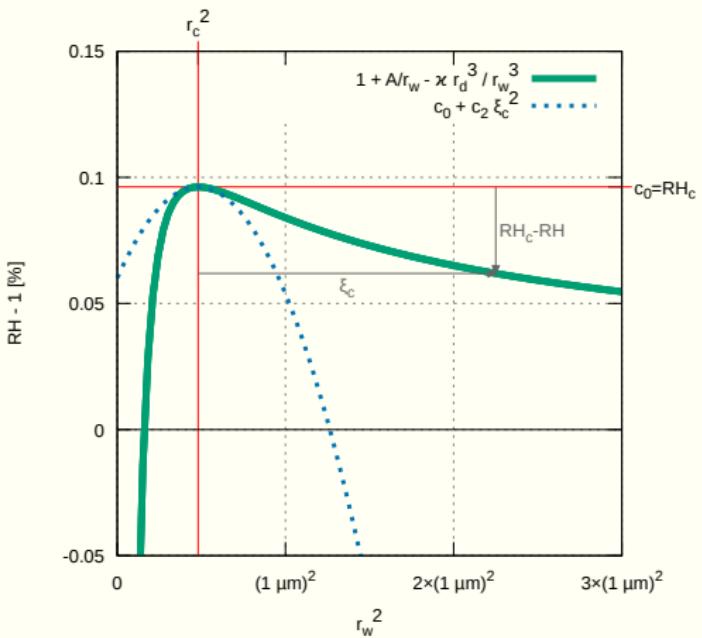


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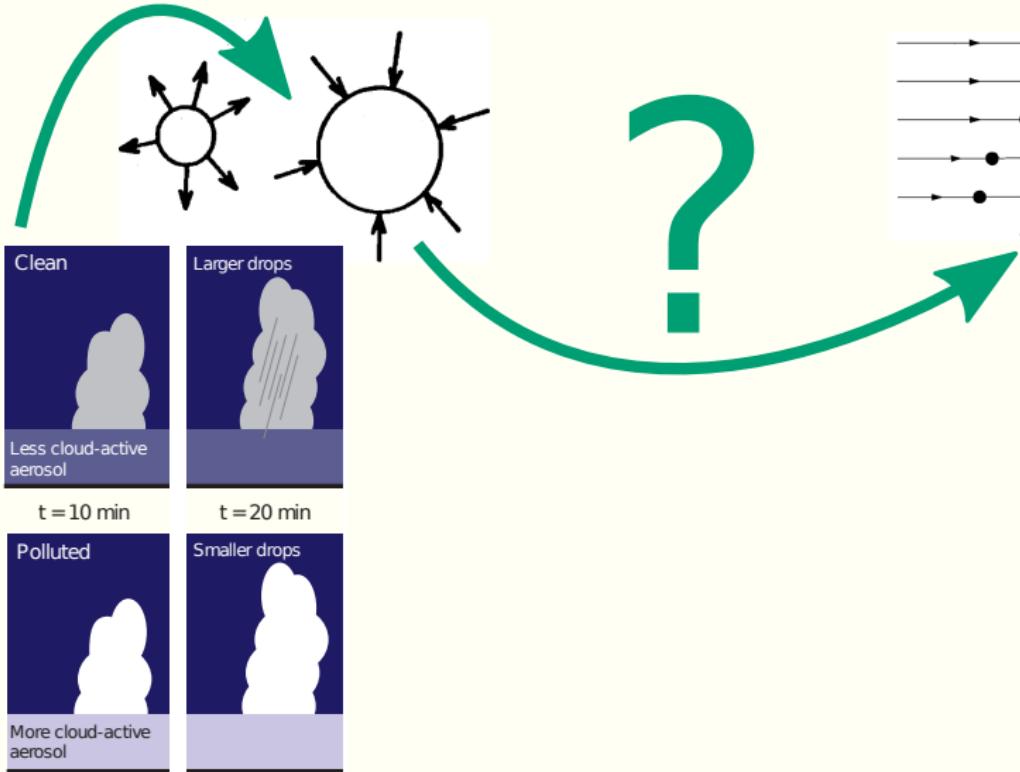
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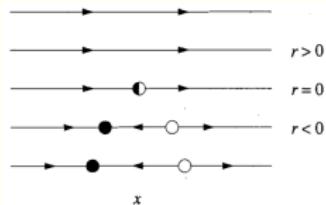
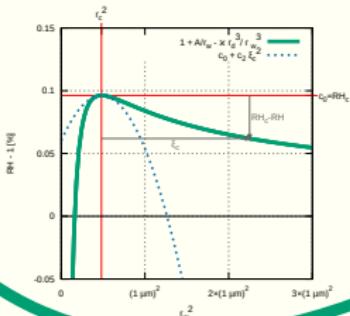
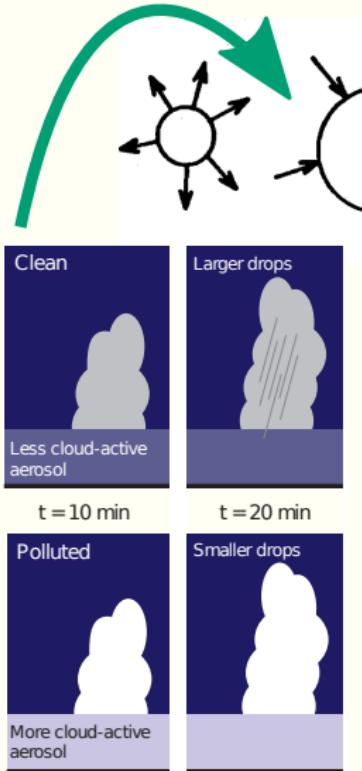
$$\dot{x} = r + x^2$$



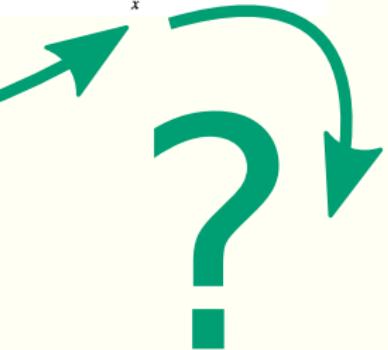
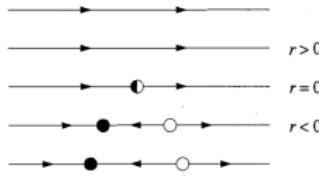
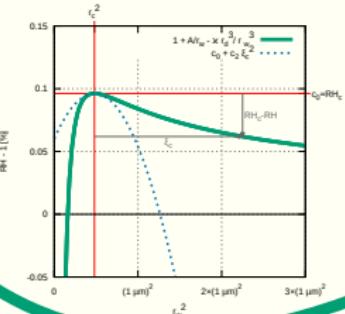
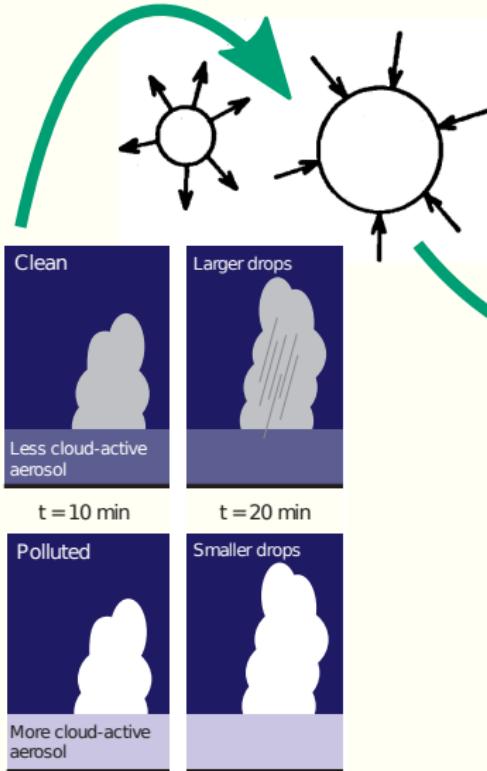
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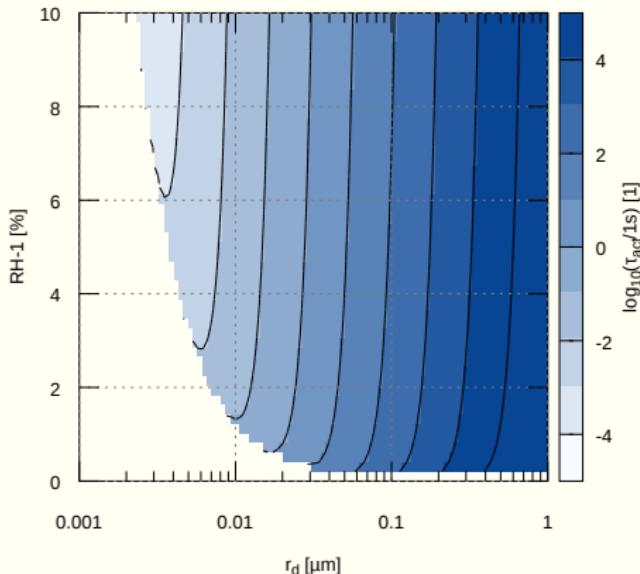
$$\begin{aligned}\tau_{act} &\approx \int_{-\infty}^{+\infty} \frac{d\xi_c}{\dot{\xi}_c} \\ &= \frac{r_c^{5/2}}{\sqrt{A}} \frac{\rho_w/\rho_{vs}}{D_{\text{eff}}} \frac{\pi}{\sqrt{RH - RH_c}}\end{aligned}$$

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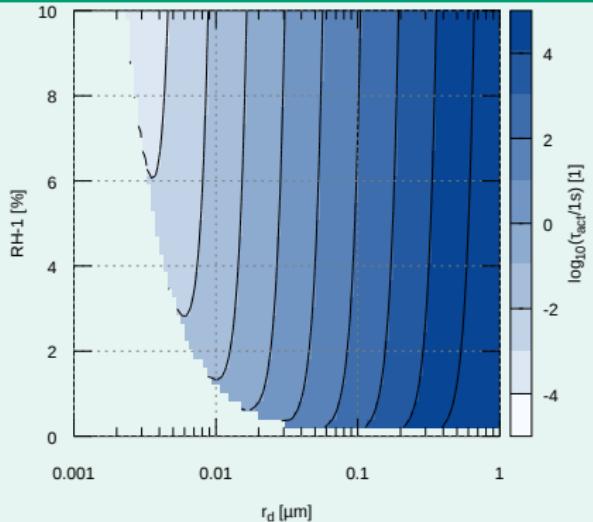
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# activation timescale: analytic vs. numerical

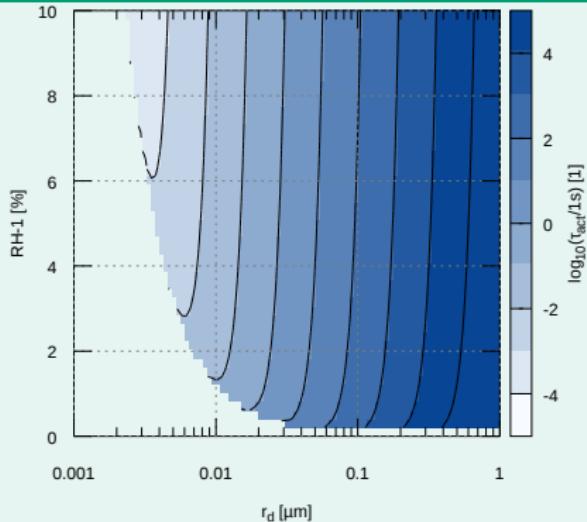
Arabas & Shima 2017



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Hoffmann, 2016 (MWR)

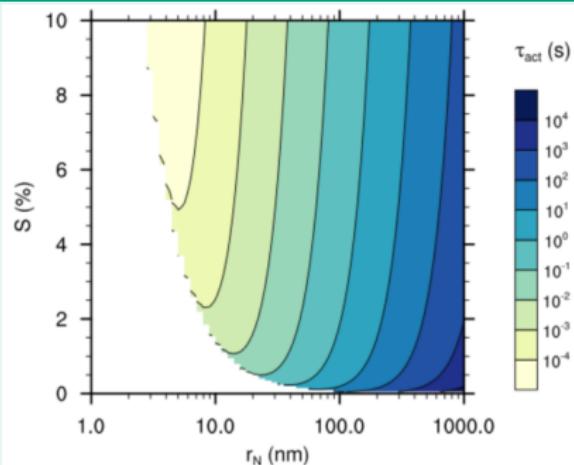
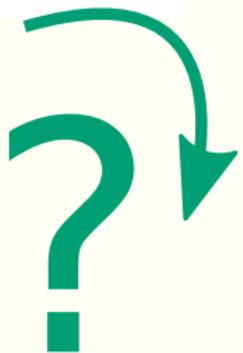
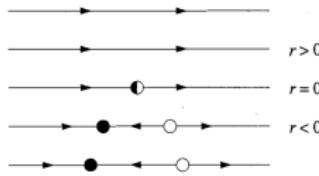
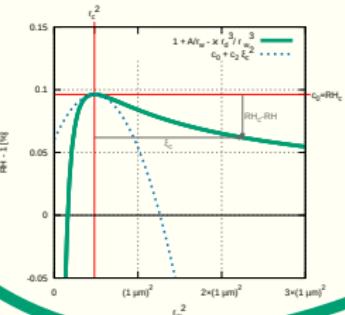
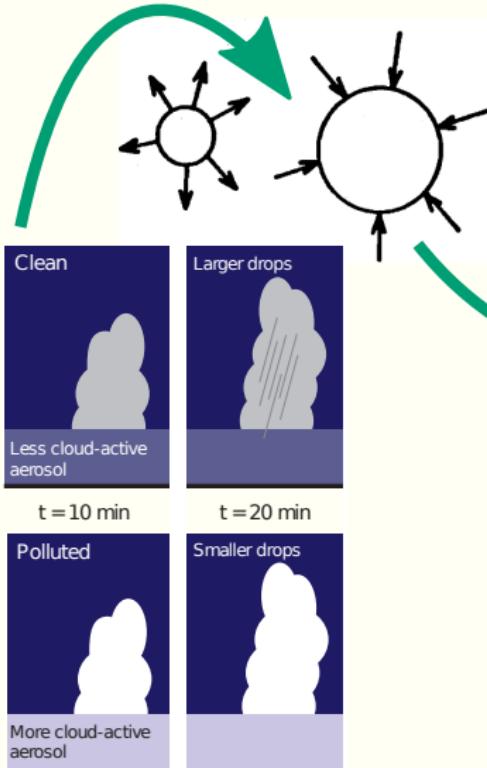


FIG. 2. The activation time scale  $\tau_{\text{act}}$  as a function of dry aerosol radius  $r_N$  and supersaturation  $S$ . For values of  $S < S_{\text{crit}}$  (white areas),  $\tau_{\text{act}}$  does not exist.

$$r \frac{dr}{dt} = \left( S - \frac{A}{r} + \frac{Br_N^3}{r^3} \right) / (F_k + F_D), \quad (10)$$

The second time scale is associated with the activation of particles, for which Köhler theory is essential. This makes an analytic solution for (10) impossible. Numerically calculated values of  $\tau_{\text{act}}$  measuring the time needed for a wetted aerosol to grow beyond its critical radius  $r_{\text{crit}} = \sqrt{3Br_N^3/A}$  are given in Fig. 2 as a function of

# connecting the dots ...



# RH-coupled system & particle concentration as parameter

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simple moisture budget (const T,p):

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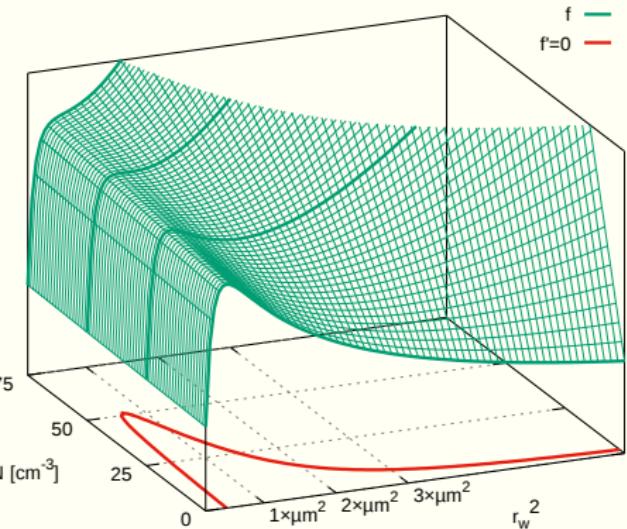
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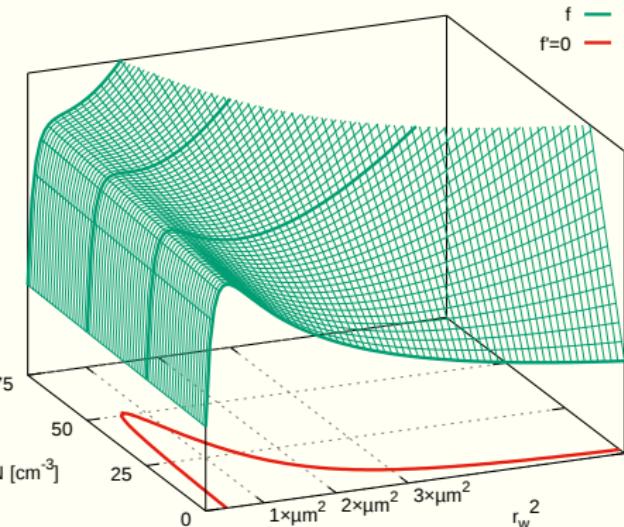
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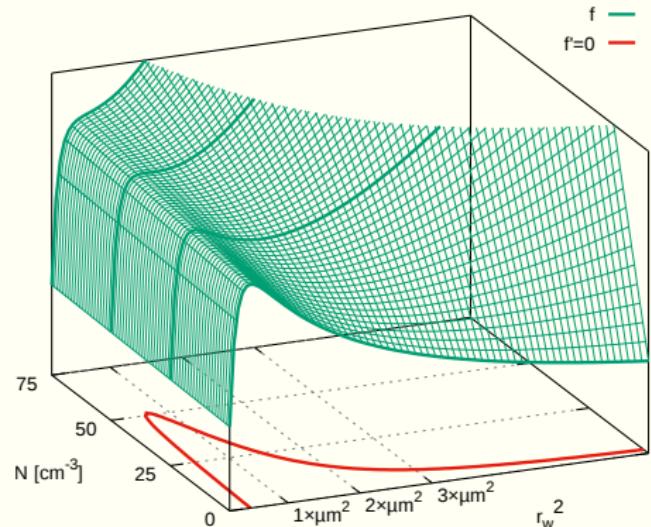
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$$\text{sgn}(f') = \text{sgn}\left(\kappa r_d^3 - \frac{A}{3} r_w + \alpha N r_w^3\right)$$

# bifurcations (and catastrophe) in the RH-coupled system

Prigogine & Stengers 1984

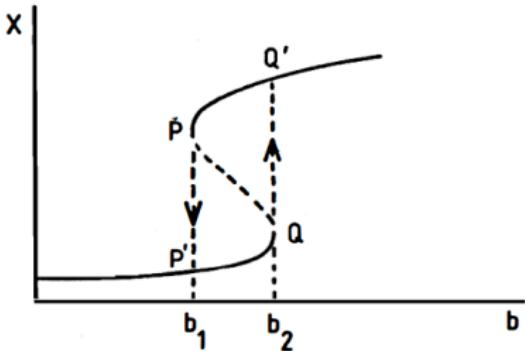


Figure 15. This figure shows how a "hysteresis" phenomenon occurs if we have the value of the bifurcation parameter  $b$  first growing and then diminishing. If the system is initially in a stationary state belonging to the lower branch, it will stay there while  $b$  grows. But at  $b=b_2$ , there will be a discontinuity: The system jumps from  $Q$  to  $Q'$ , on the higher branch. Inversely, starting from a state on the higher branch, the system will remain there till  $b=b_1$ , when it will jump down to  $P'$ . Such types of bistable behavior are observed in many fields, such as lasers, chemical reactions or biological membranes.

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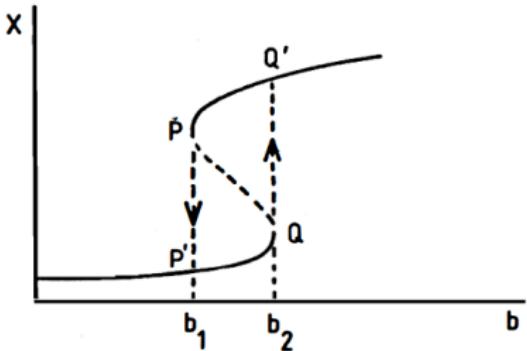
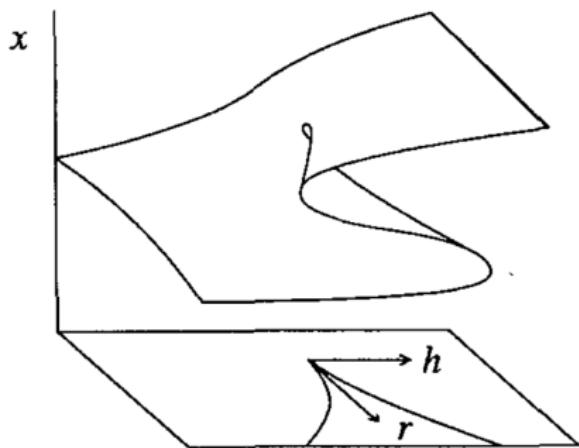


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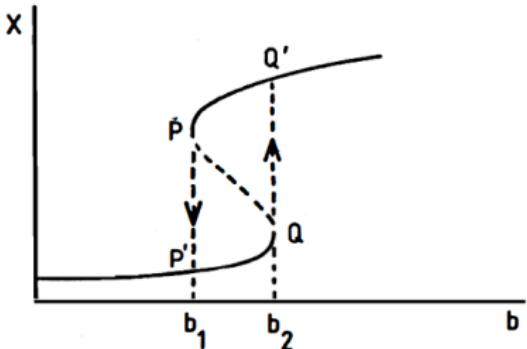
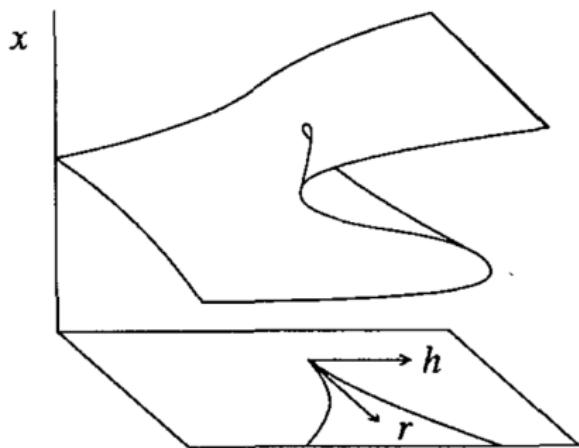


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↔ "jumps", hysteretic behaviour ( $r_w$ , RH) for small enough  $N$ , close to equilibrium (slow process)

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  - spectral broadening (mixing, parcel history, ...)

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vertically displaced (velocity  $w$ , hydrostatic background) adiabatic parcel:  
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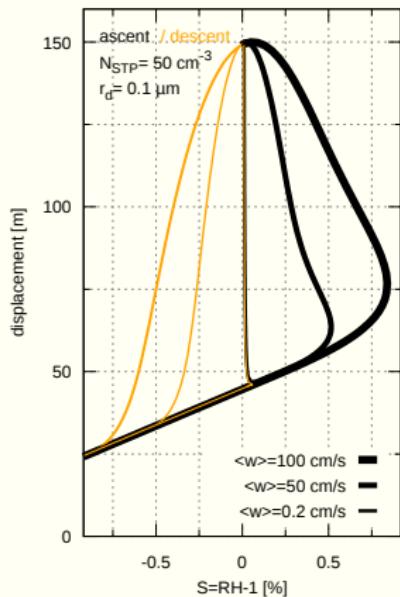
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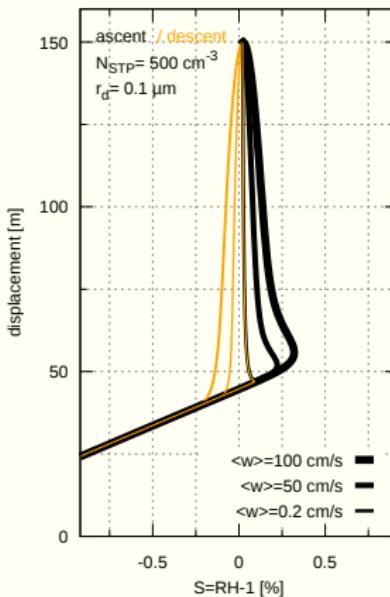
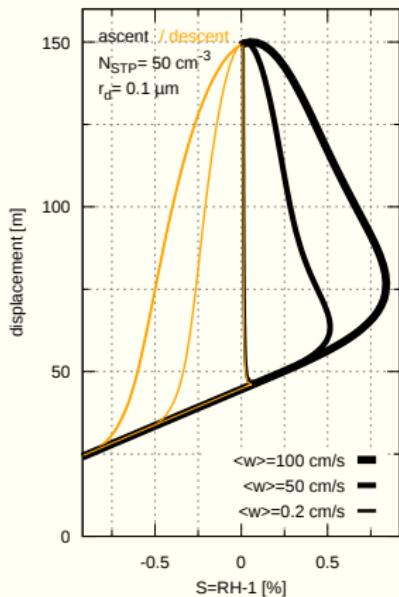
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- $N \rightarrow 0$  (and hence  $\dot{q} \approx 0$ ) i.e., weak coupling between particle size evolution and ambient thermodynamics (pertinent to the case of low particle concentration).

# parcel model: numerical integration (sinusoidal $w$ )



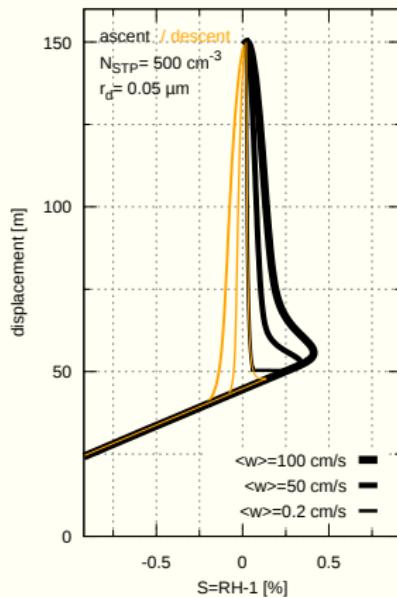
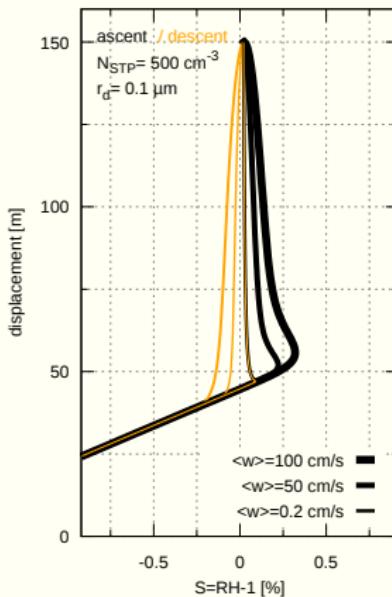
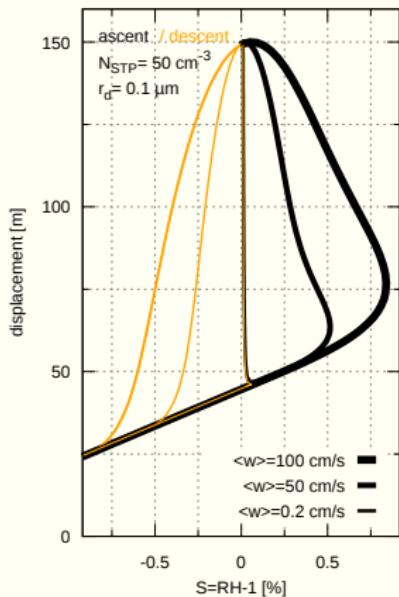
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open source code (based on libcloudph++) as electronic paper supplement

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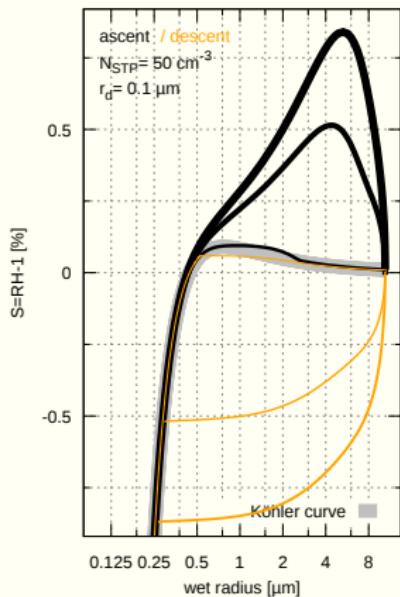
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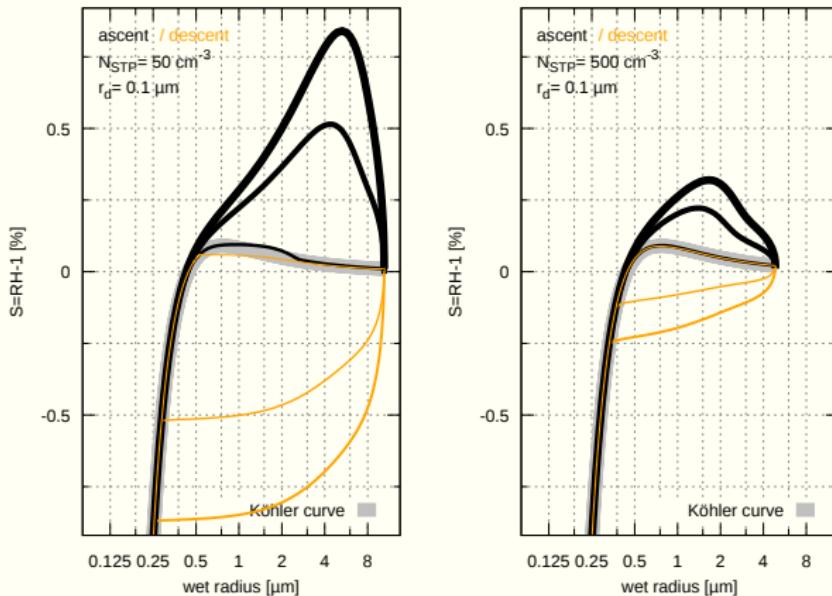
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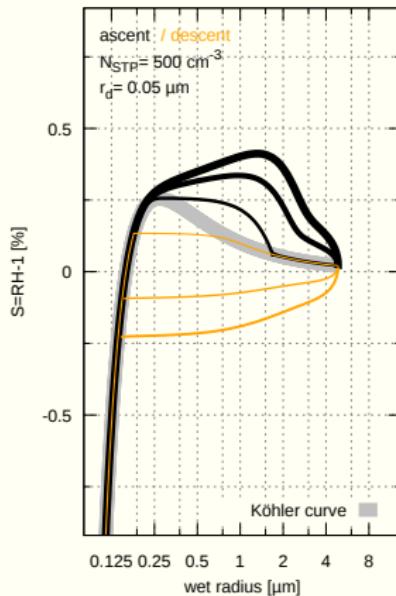
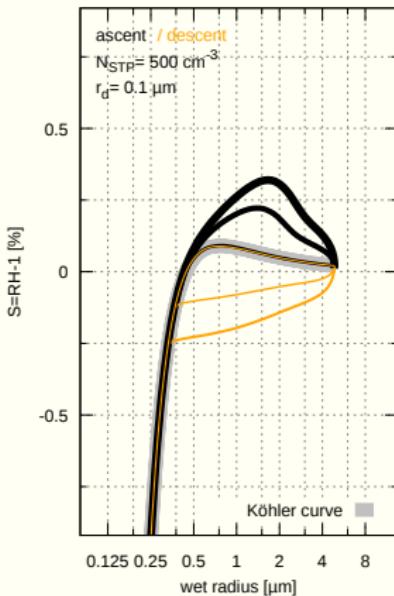
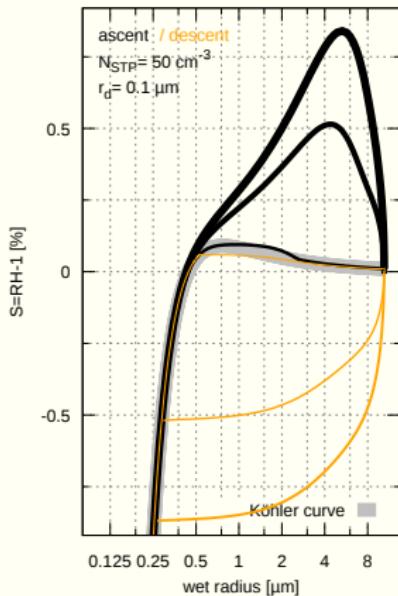
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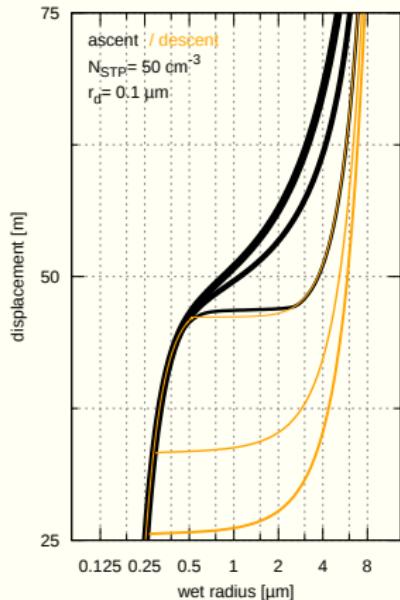
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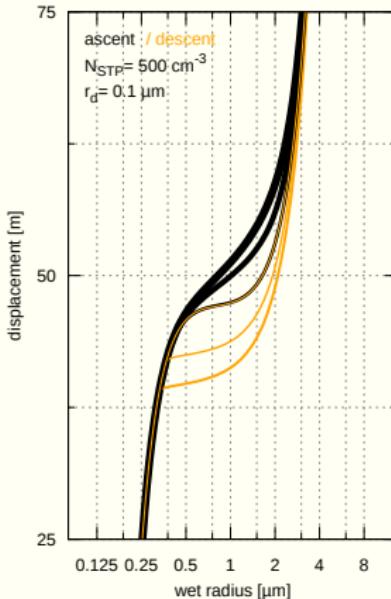
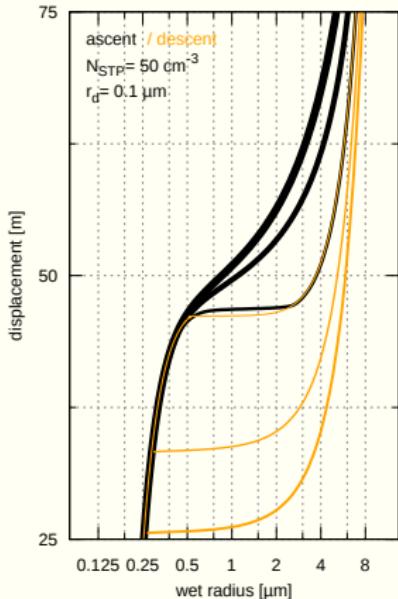
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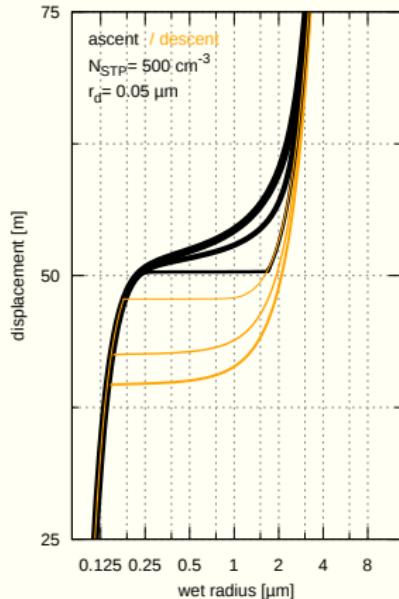
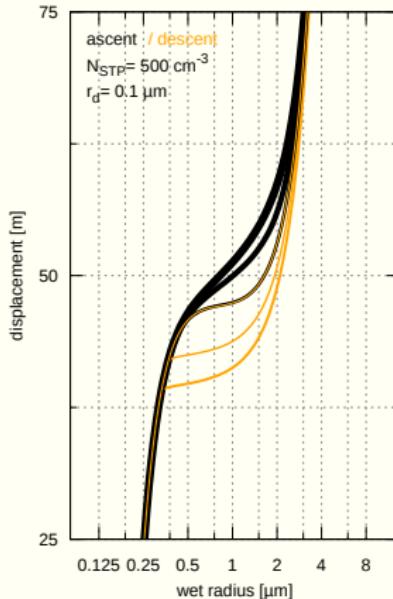
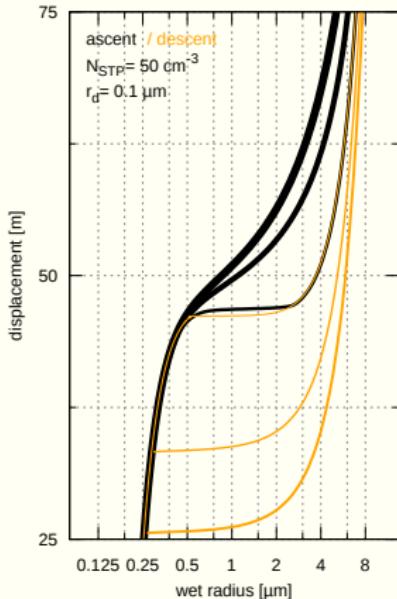
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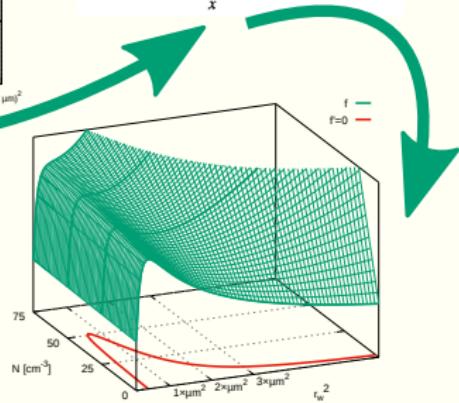
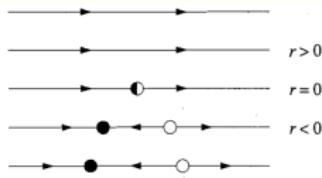
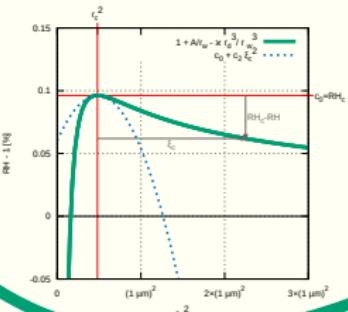
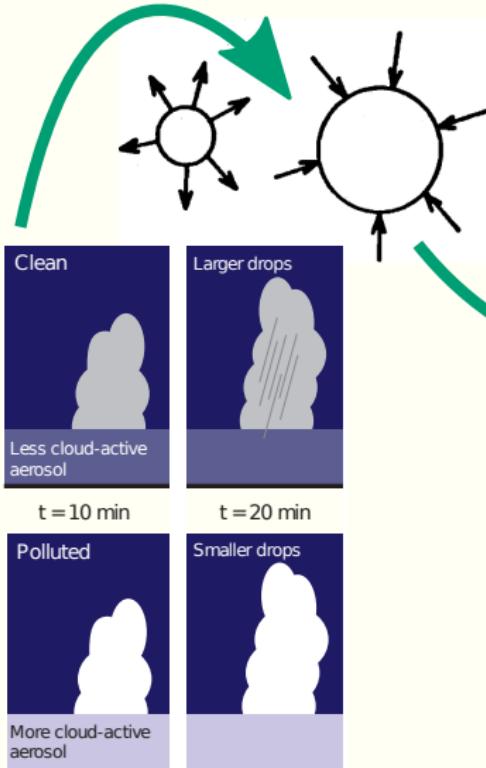
# parcel model: numerical integration (sinusoidal $w$ )



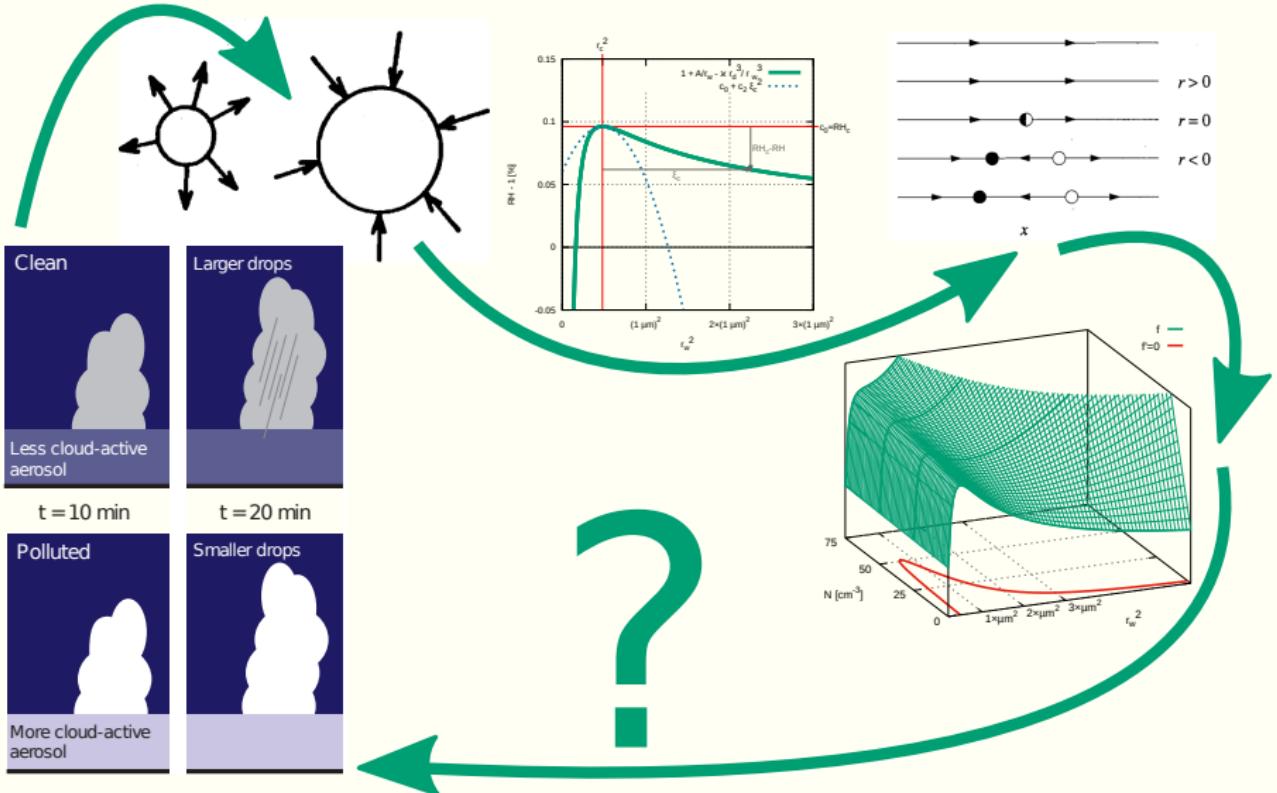
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connecting the dots ...

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monodisperse system: limitations and applicability

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(key for modelling precipitation onset)

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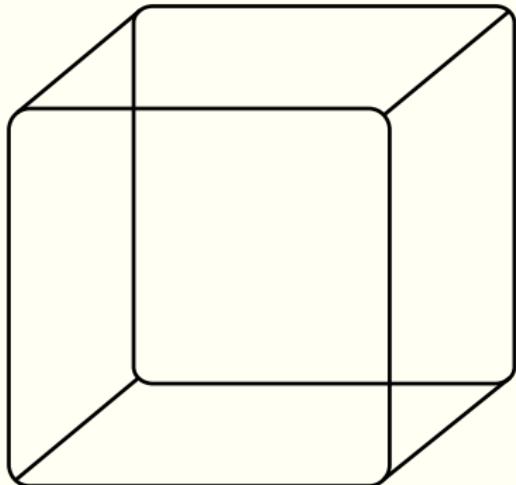
... applicability?

**particle-based  $\mu$ -physics schemes for LES!**  
(Lagrangian Cloud Models / Super-Droplet Models)

# particle-based $\mu$ -physics for LES

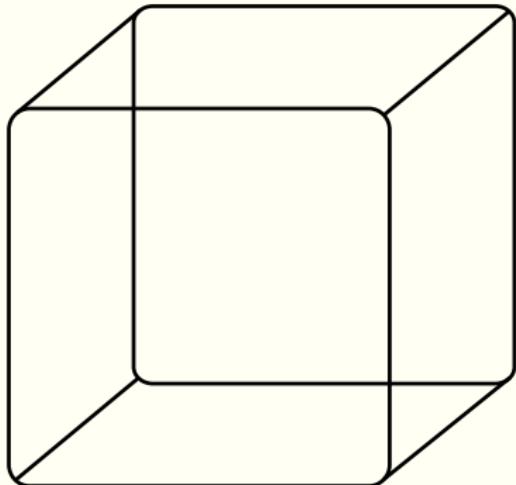
- “information carriers” in LES domain

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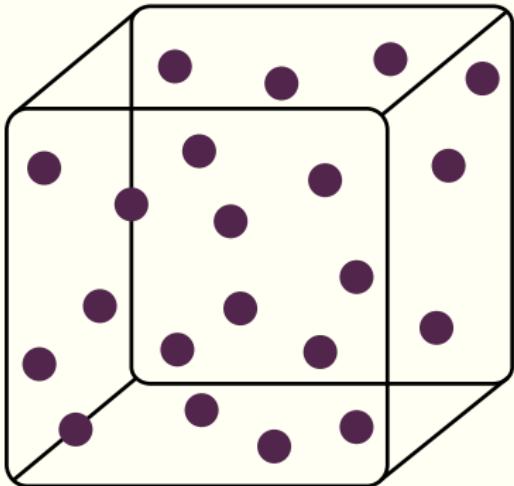
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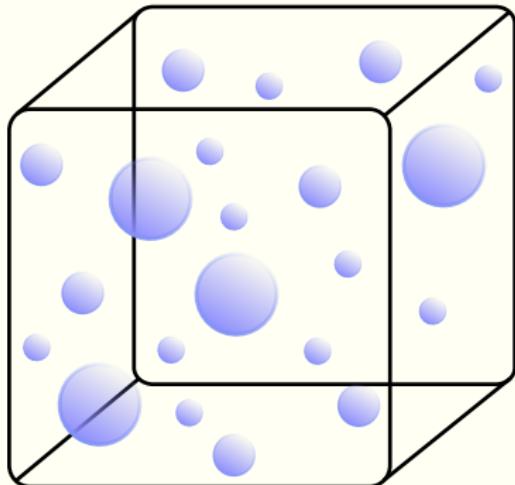
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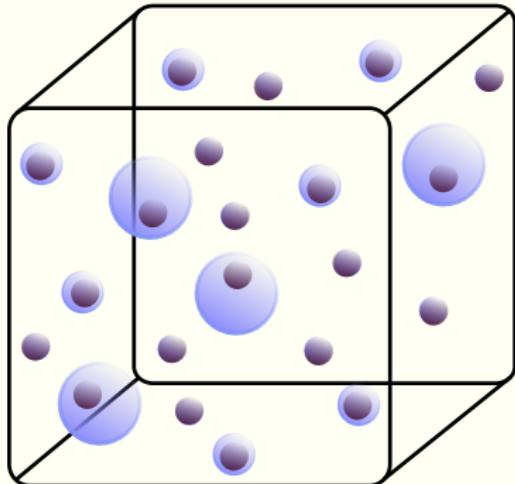
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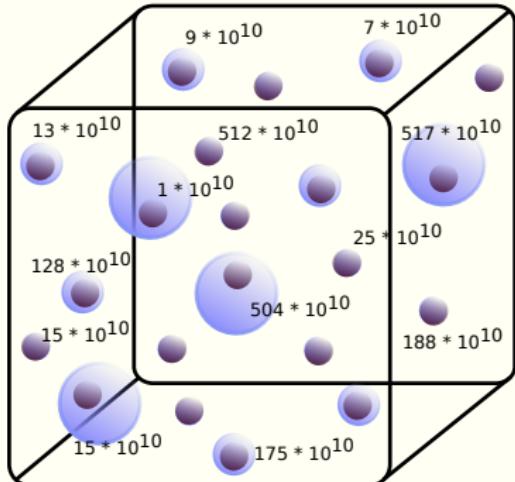
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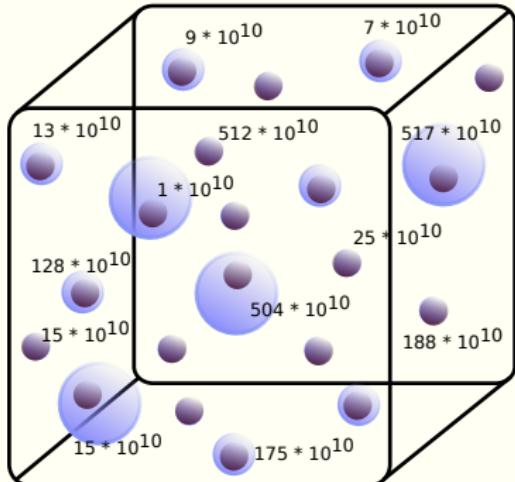
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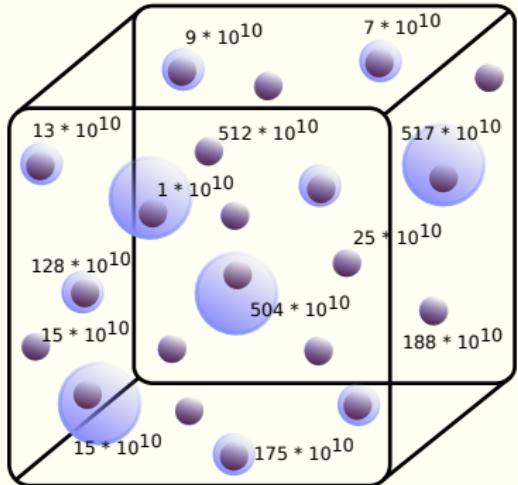
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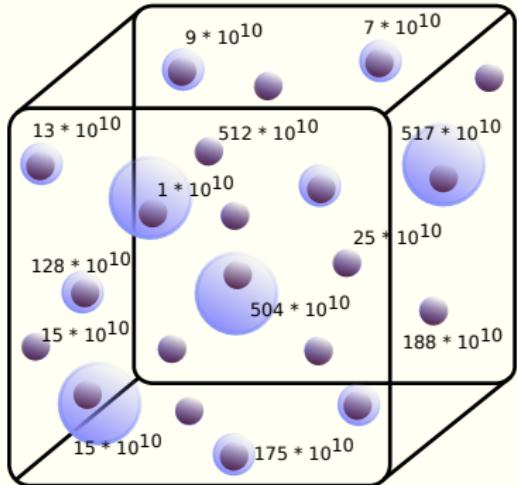
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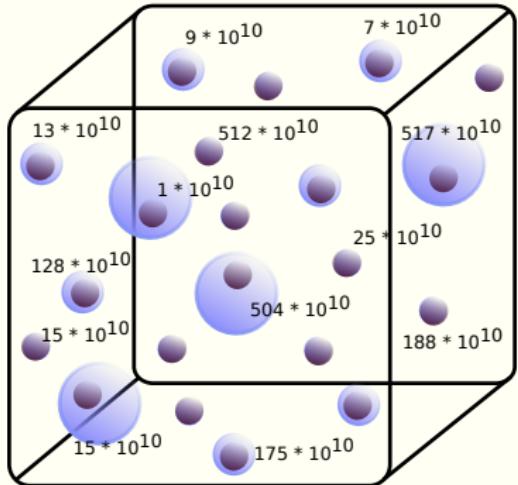
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- ☒ scalability in no. of attributes  
**(aqueous chemistry!)**
- ☒ each particle: **monodisperse!**
- ☒ each timestep: **constant RH!**

# particle-based $\mu$ -physics for LES

Seminal works: Shima et al. 2009, Andrejczuk et al. 2010  
(3D simulations of atmospheric aerosol-cloud-precipitation system)

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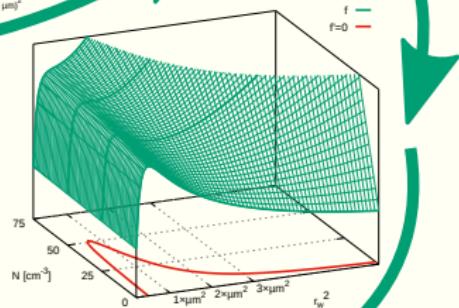
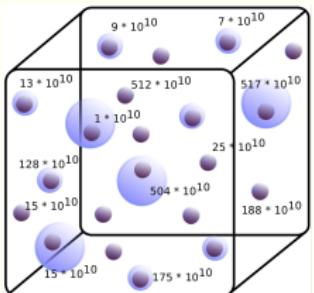
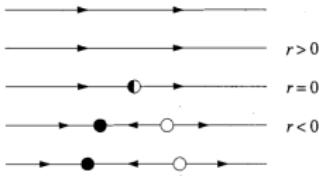
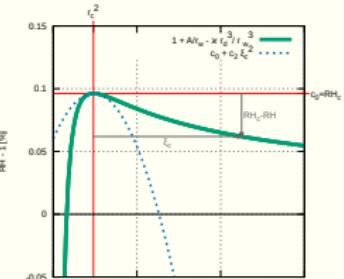
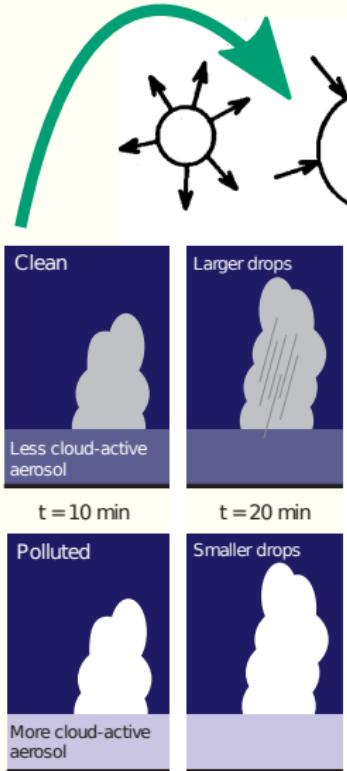
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- extensions: fluctuations, bi-/poly-disperse spectra, non-cloud appl.

# Thank you for your attention!

<https://doi.org/10.5194/npg-24-535-2017>