

# **Bifurcations in a dynamical system describing formation of cloud droplets on atmospheric particulate matter**

Sylwester Arabas

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Nonlinear Processes  
in Geophysics



Open Access

## On the CCN (de)activation nonlinearities

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# atmospheric particulate matter (PM) and clouds



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# atmospheric particulate matter (PM) and clouds

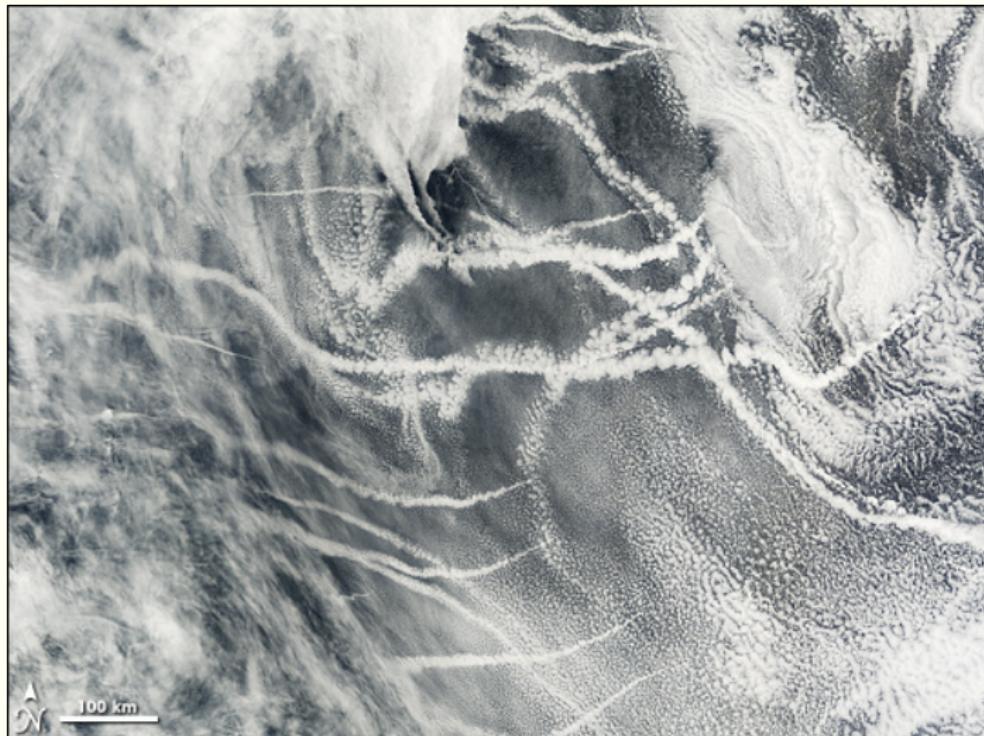


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no particles  $\rightsquigarrow$  no clouds

<https://www.youtube.com/watch?v=EneDwu0HrVg>

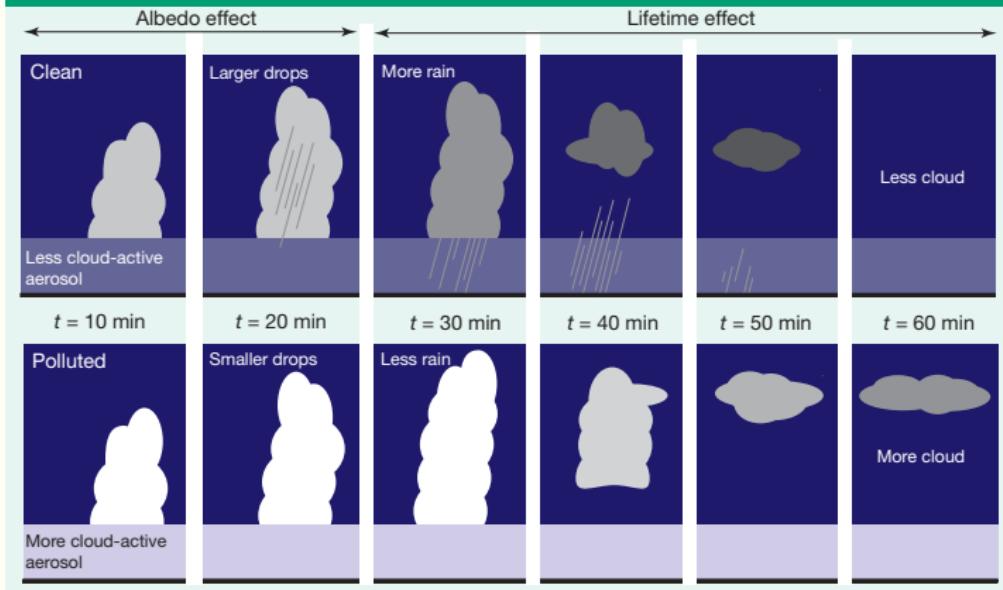
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NASA (<https://earthobservatory.nasa.gov/NaturalHazards/view.php?id=20248>)

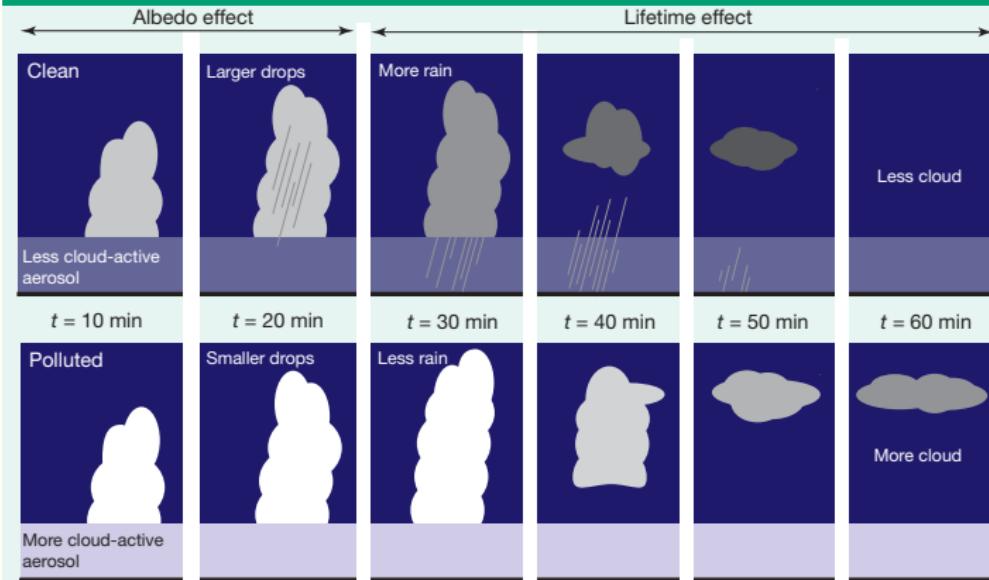
# atmospheric particulate matter (PM) and clouds

Stevens and Feingold, 2009 (Nature)



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Stevens and Boucher, 2012 (Nature)

*"there is something captivating about the idea that fine particulate matter, suspended almost invisibly in the atmosphere, holds the key to some of the greatest mysteries of climate science"*



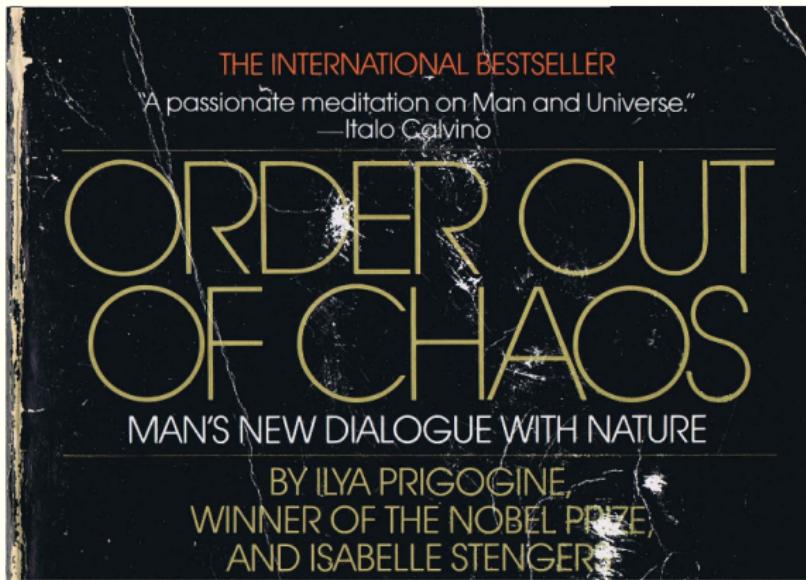
THE INTERNATIONAL BESTSELLER

"A passionate meditation on Man and Universe."  
—Italo Calvino

# ORDER OUT OF CHAOS

MAN'S NEW DIALOGUE WITH NATURE

BY ILYA PRIGOGINE,  
WINNER OF THE NOBEL PRIZE  
AND ISABELLE STENGER



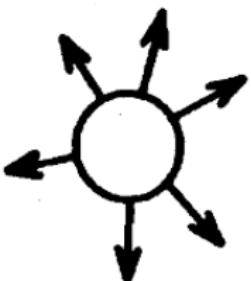
## Prigogine and Stengers 1984

*"Much of this book has centered around the relation between the microscopic and the macroscopic. One of the most important problems in evolutionary theory is the eventual feedback between macroscopic structures and microscopic events: macroscopic structures emerging from microscopic events would in turn lead to a modification of the microscopic mechanisms."*

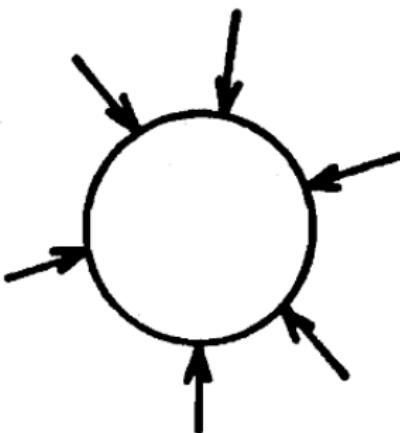
# regime-transition (bifurcation) example from P&S 1984

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ORDER OUT OF CHAOS 188



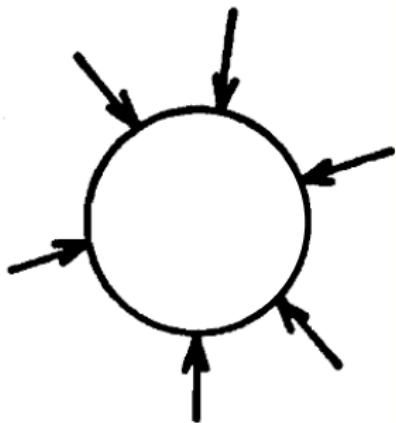
(a)



(b)

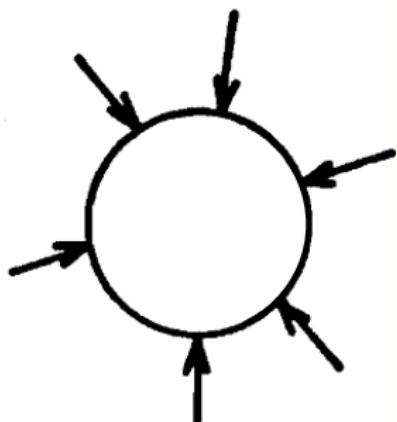
Figure 19. Nucleation of a liquid droplet in a supersaturated vapor. (a) droplet smaller than the critical size; (b) droplet larger than the critical size. The existence of the threshold has been experimentally verified for dissipative structures.

## droplet growth laws in a nutshell: mass and heat diffusion



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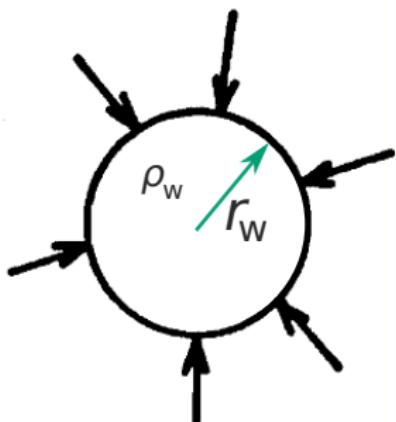
Fick's and Fourier's laws combined  
spherical geometry



$$\dot{r}_w = \frac{1}{r_w} \frac{D_{\text{eff}}}{\rho_w} (\rho_v - \rho_o)$$

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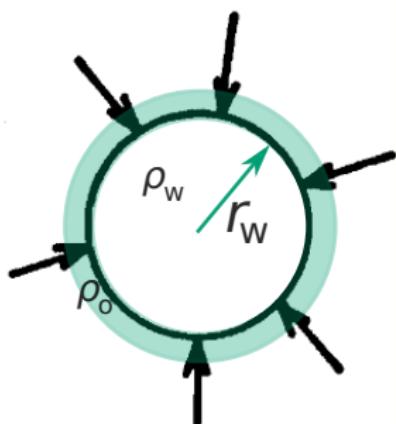
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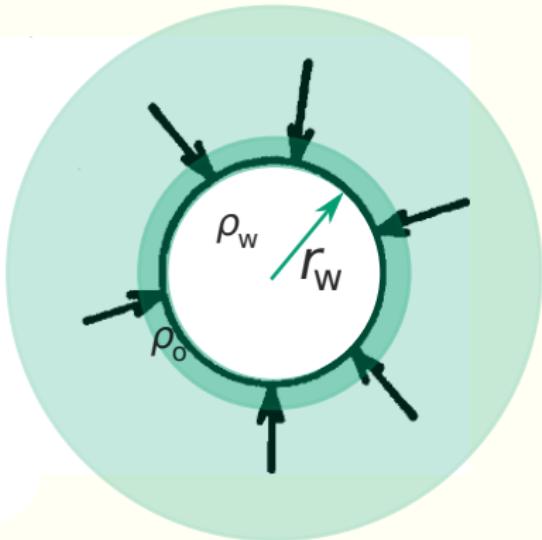


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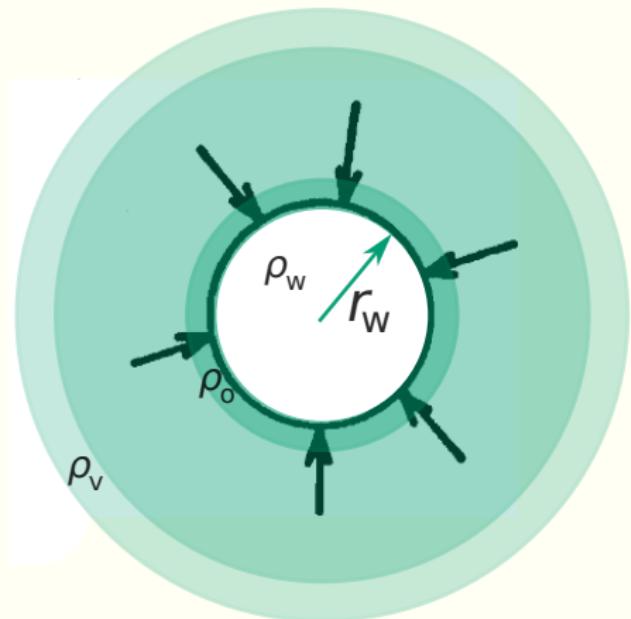
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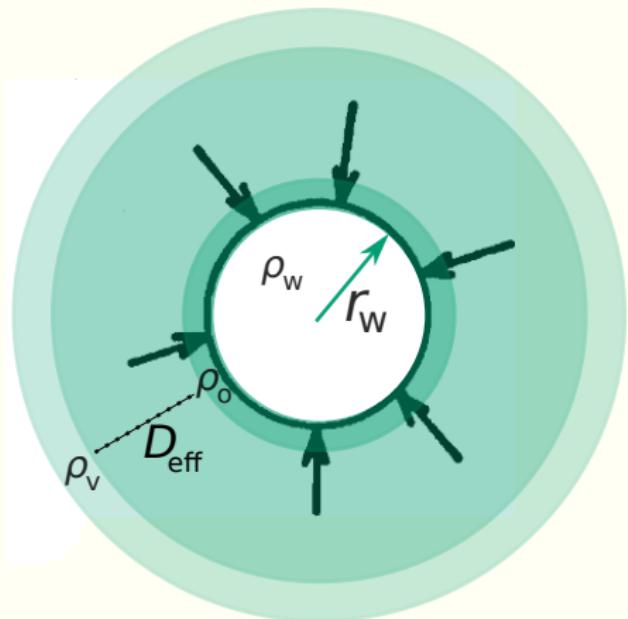
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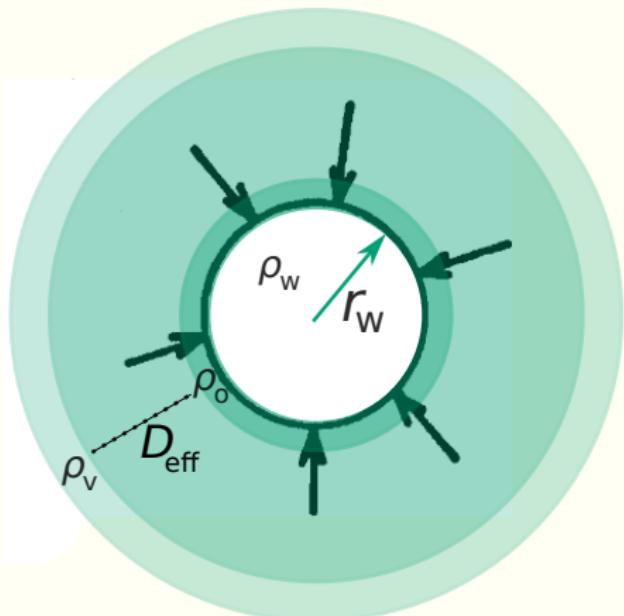
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non-dimensional numbers:

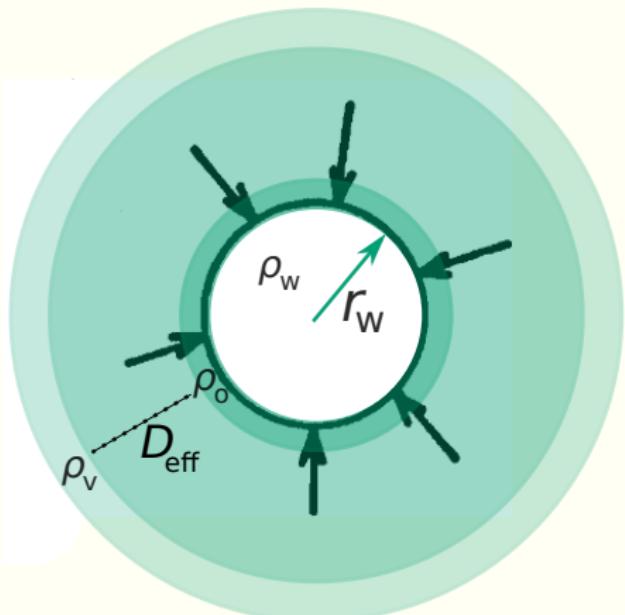
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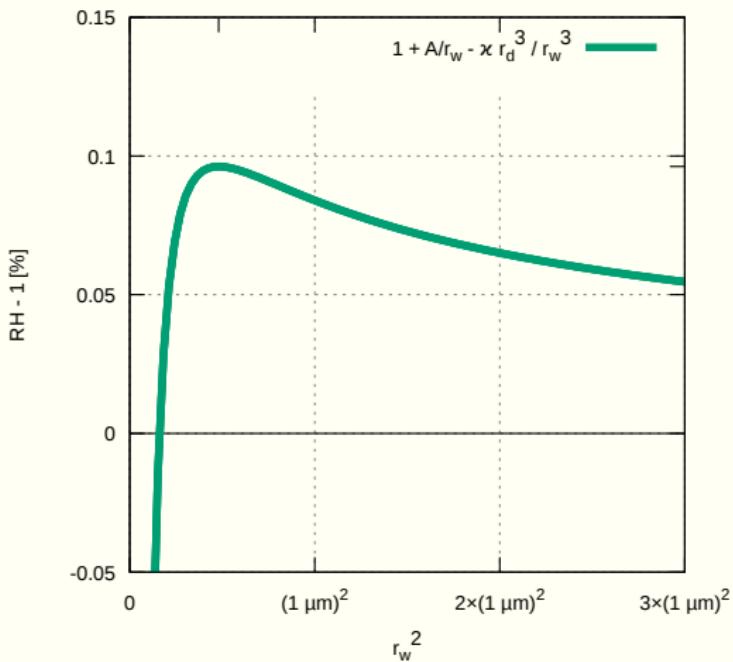
$$\dot{r}_w = \frac{1}{r_w} D_{\text{eff}} \frac{\rho_{vs}}{\rho_w} (\text{RH} - \text{RH}_{\text{eq}})$$
$$\text{RH}_{\text{eq}} = \frac{r_w^3 - r_d^3}{r_w^3 - r_d^3(1 - \kappa)} \exp\left(\frac{A}{r_w}\right)$$
$$\approx 1 + \frac{A}{r_w} - \frac{\kappa r_d^3}{r_w^3}$$

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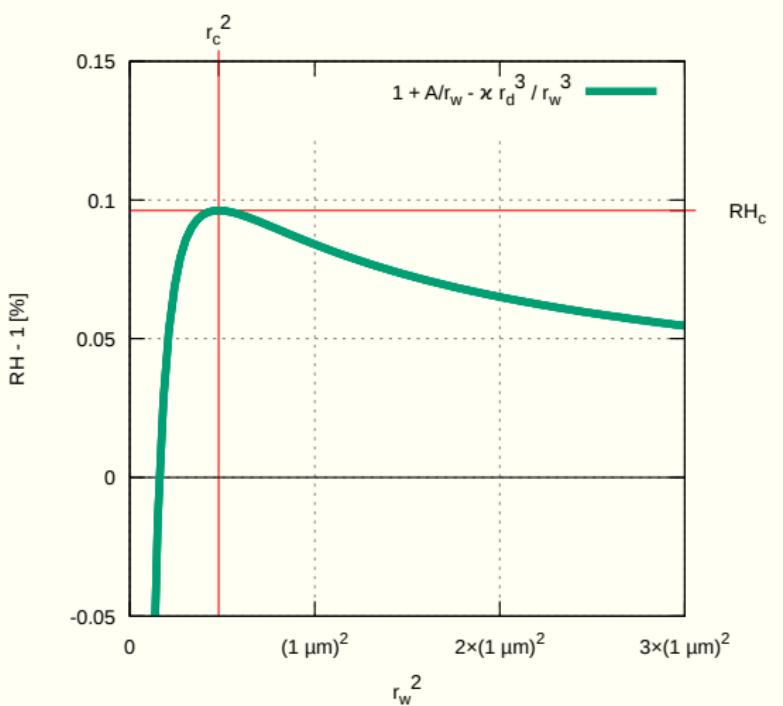


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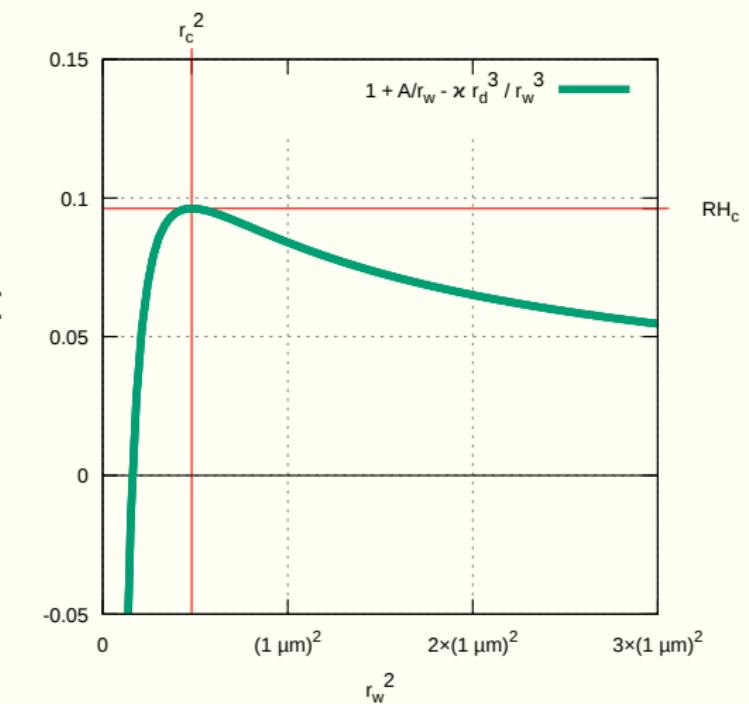


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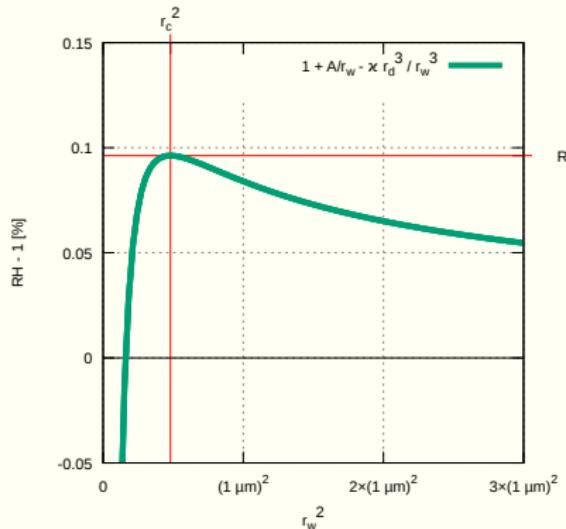


maximum at  $(r_c, \text{RH}_c)$ :

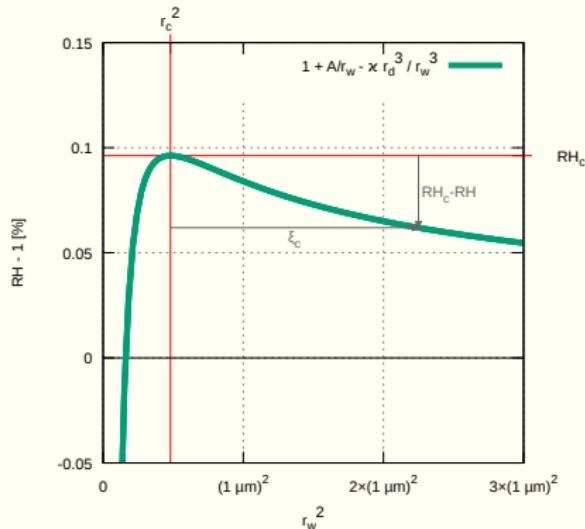
$$r_c = \sqrt{3\kappa r_d^3 / A}$$

$$\text{RH}_c = 1 + \frac{2A}{3r_c}$$

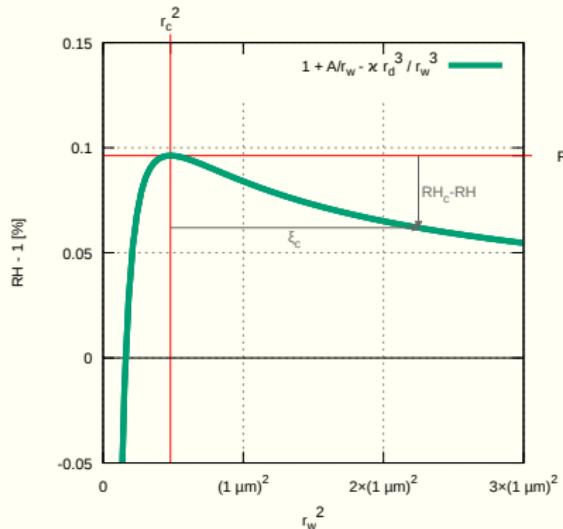
# phase portrait of the system: flipped Köhler curve



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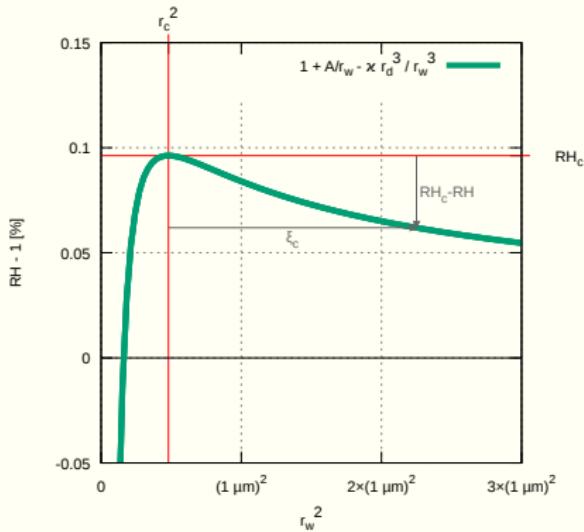
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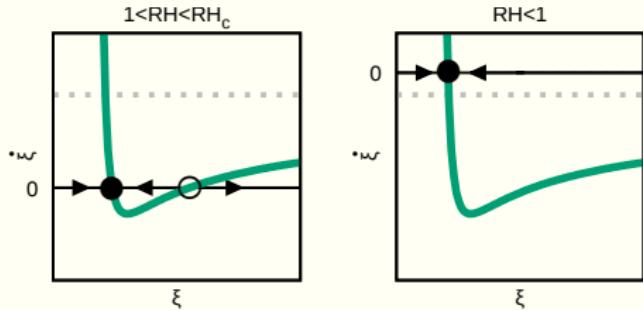
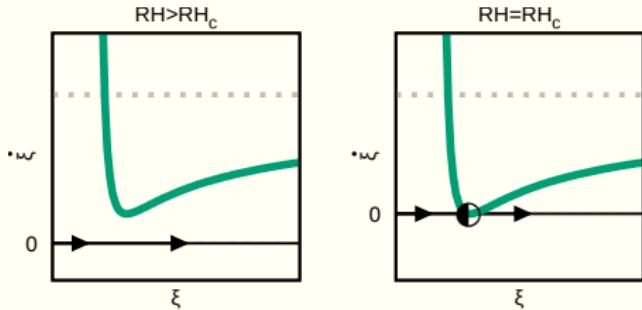
$$\dot{\xi} = 2D_{\text{eff}} \frac{\rho_{\text{vs}}}{\rho_w} (\text{RH} - \text{RH}_{\text{eq}}(\xi))$$

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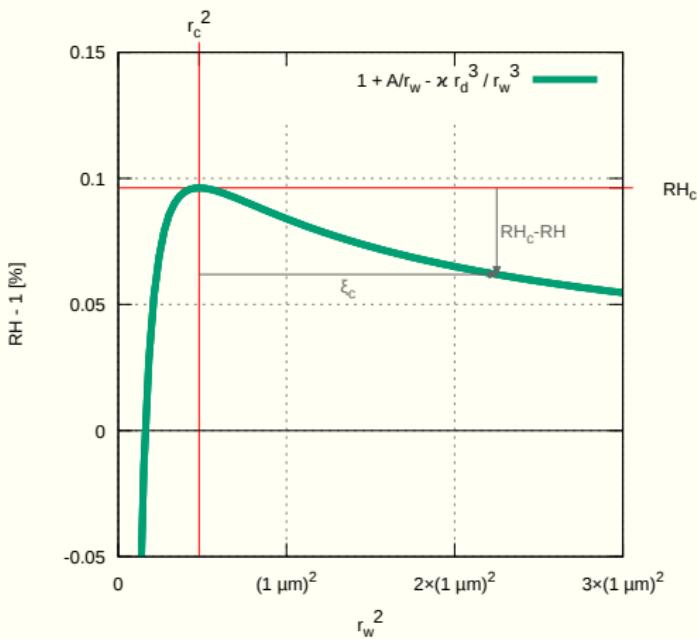
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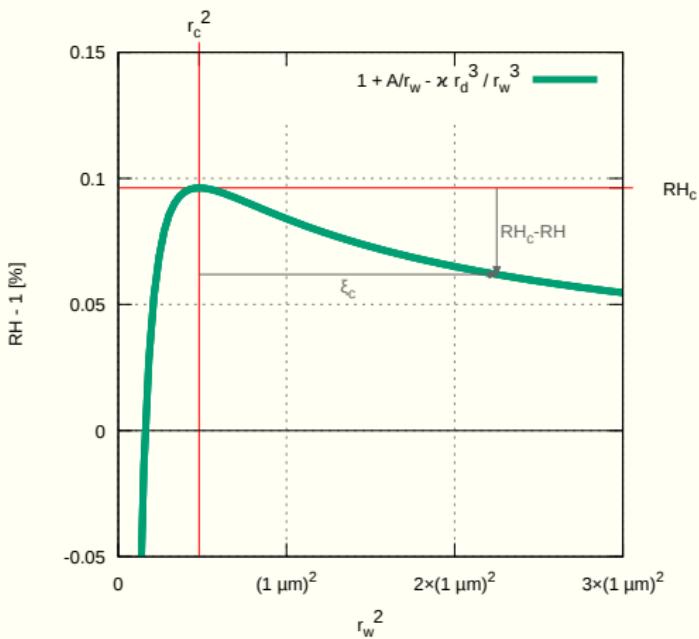
# saddle-node bifurcation at Köhler curve maximum

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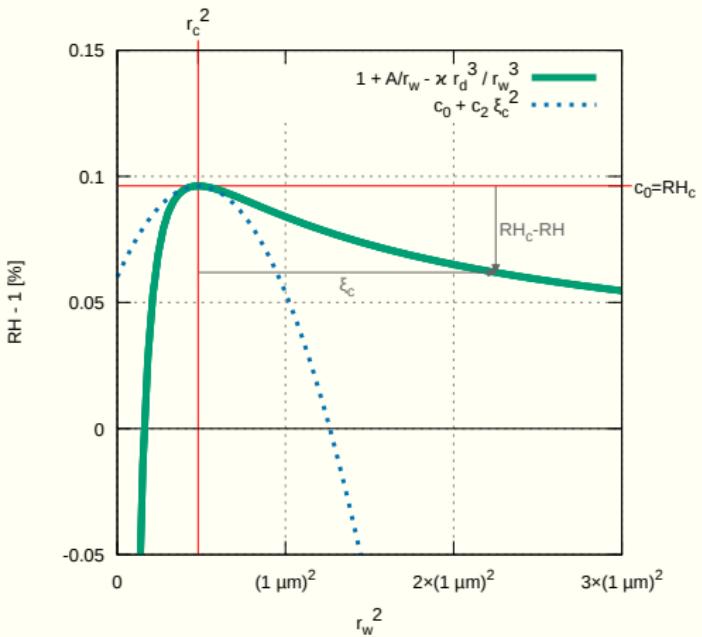
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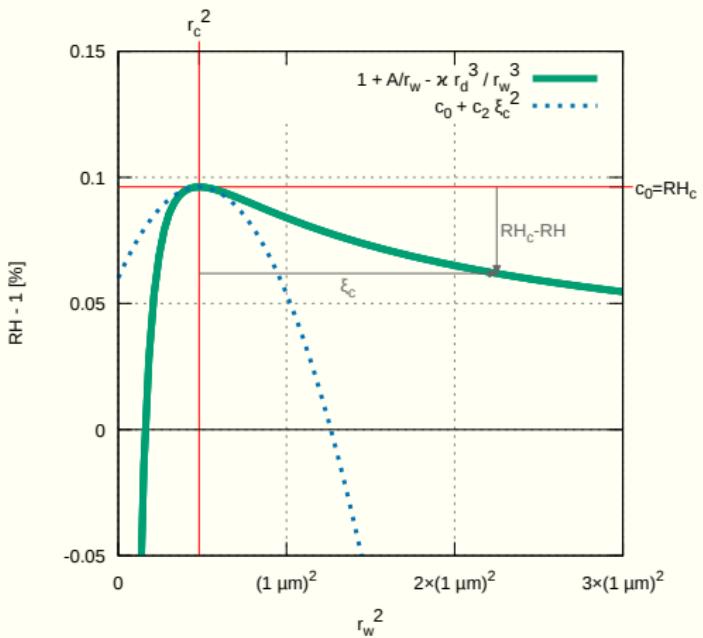
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$$RH_{eq}(\xi_c) = c_0 + c_1 \xi_c + c_2 \xi_c^2 + \dots$$

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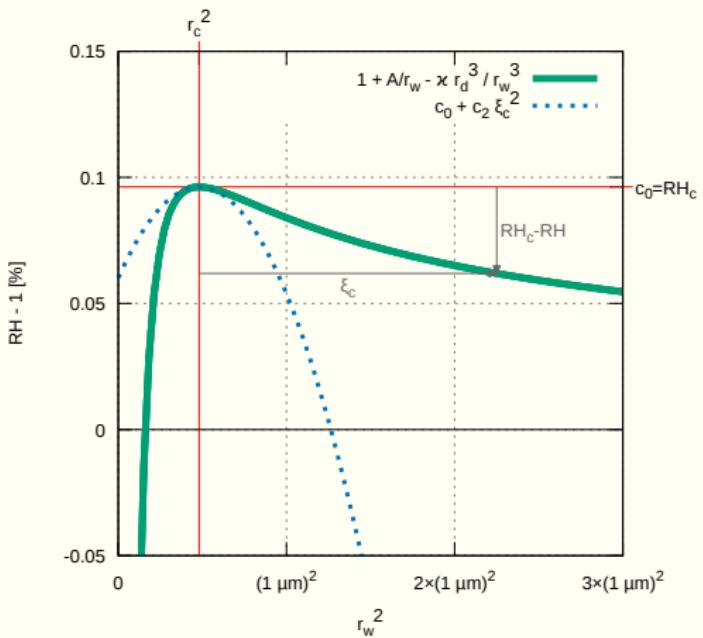


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$$\dot{x} = r + x^2$$



# coalescence in the saddle-node bottleneck

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*key observation: time of passage through the parabolic *bottleneck* dominates all other timescales*

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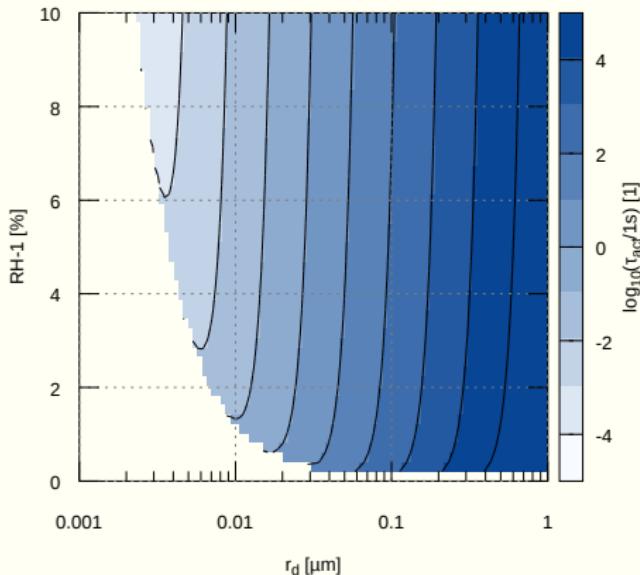
$$\begin{aligned}\tau_{act} &\approx \int_{-\infty}^{+\infty} \frac{d\xi_c}{\dot{\xi}_c} \\ &= \frac{r_c^{5/2}}{\sqrt{A}} \frac{\rho_w/\rho_{vs}}{D_{\text{eff}}} \frac{\pi}{\sqrt{RH - RH_c}}\end{aligned}$$

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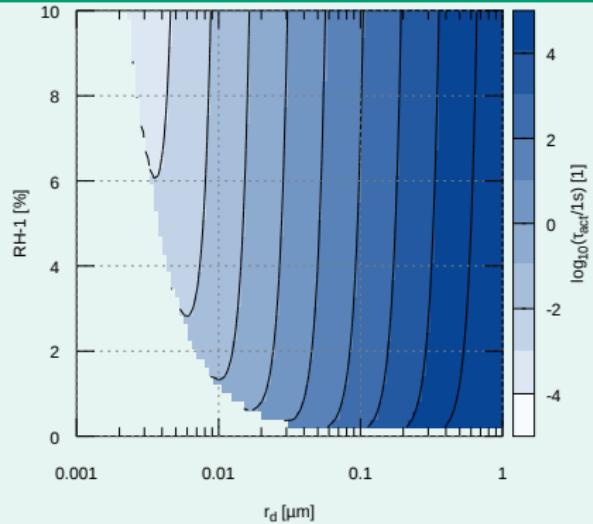
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# activation timescale: analytic vs. numerical

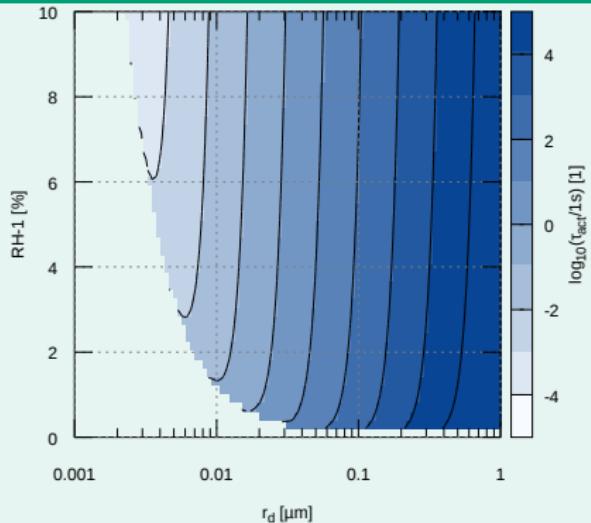
Arabas & Shima 2017



note: axes ranges vs. close-to-equilibrium assumption

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Arabas & Shima 2017



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Hoffmann, 2016 (MWR)

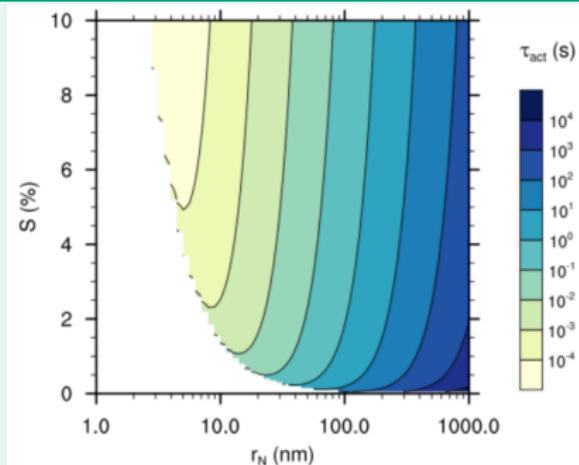


FIG. 2. The activation time scale  $\tau_{\text{act}}$  as a function of dry aerosol radius  $r_N$  and supersaturation  $S$ . For values of  $S < S_{\text{crit}}$  (white areas),  $\tau_{\text{act}}$  does not exist.

$$r \frac{dr}{dt} = \left( S - \frac{A}{r} + \frac{Br_N^3}{r^3} \right) / (F_k + F_D), \quad (10)$$

The second time scale is associated with the activation of particles, for which Köhler theory is essential. This makes an analytic solution for (10) impossible. Numerically calculated values of  $\tau_{\text{act}}$  measuring the time needed for a wetted aerosol to grow beyond its critical radius  $r_{\text{crit}} = \sqrt{3Br_N^3/A}$  are given in Fig. 2 as a function of

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simple moisture budget (const T,p):

$$\dot{RH} \approx \frac{\dot{\rho}_v}{\rho_{vs}} = -N \underbrace{\frac{4\pi\rho_w}{3\rho_{vs}}}_{\alpha} 3r_w^2 \dot{r}_w$$

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new phase portrait:

$$\dot{\xi} \sim (RH_0 - 1) - \underbrace{\left( \frac{A}{\xi^{\frac{1}{2}}} - \frac{\kappa r_d^3}{\xi^{\frac{3}{2}}} + \alpha N \xi^{\frac{3}{2}} \right)}_f$$

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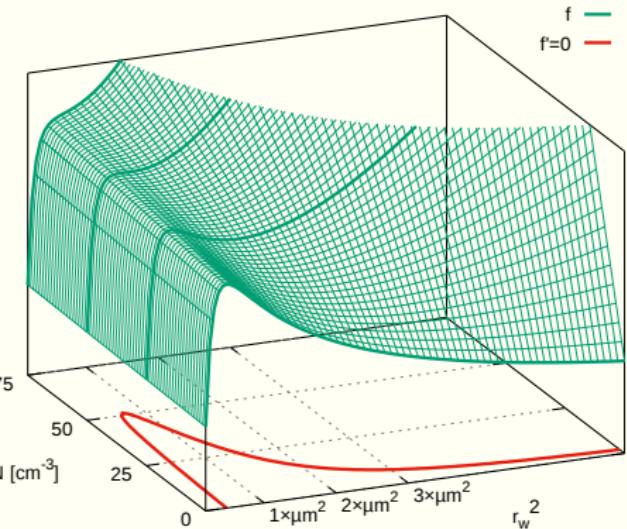
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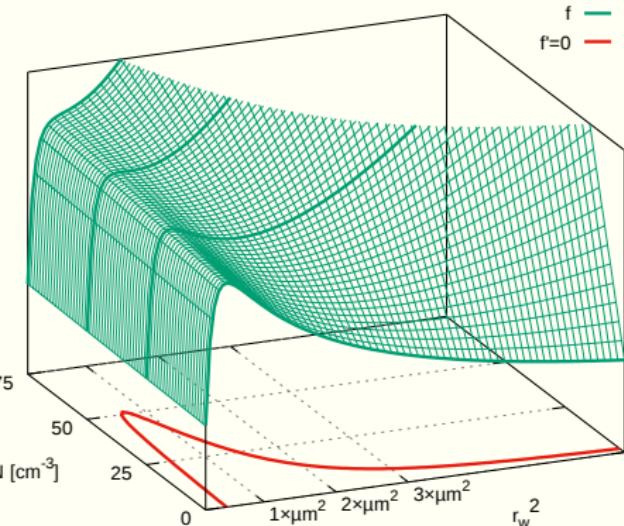
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regime-controlling params: RH, N



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simple moisture budget (const T,p):

$$\dot{RH} \approx \frac{\dot{\rho_v}}{\rho_{vs}} = -N \underbrace{\frac{4\pi\rho_w}{3\rho_{vs}}}_{\alpha} 3r_w^2 \dot{r}_w$$

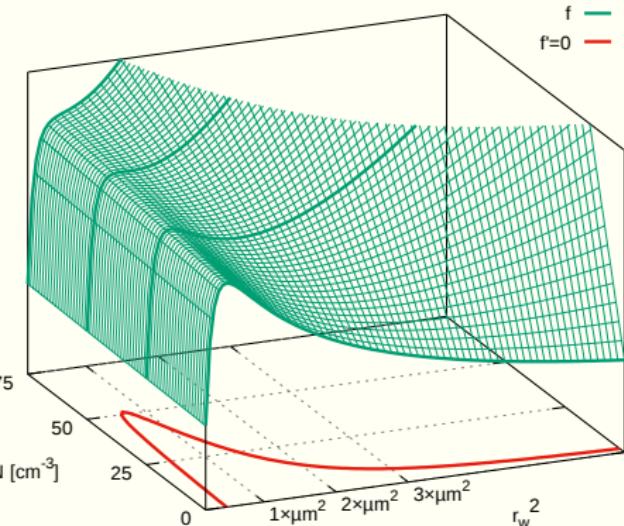
integrating in time:

$$RH = RH_0 - \alpha N r_w^3$$

new phase portrait:

$$\dot{\xi} \sim (RH_0 - 1) - \underbrace{\left( \frac{A}{\xi^{\frac{1}{2}}} - \frac{\kappa r_d^3}{\xi^{\frac{3}{2}}} + \alpha N \xi^{\frac{3}{2}} \right)}_f$$

regime-controlling params: RH, N



$$\text{sgn}(f') = \text{sgn}\left(\kappa r_d^3 - \frac{A}{3} r_w + \alpha N r_w^3\right)$$

# bifurcations (and catastrophe) in the RH-coupled system

Prigogine & Stengers 1984

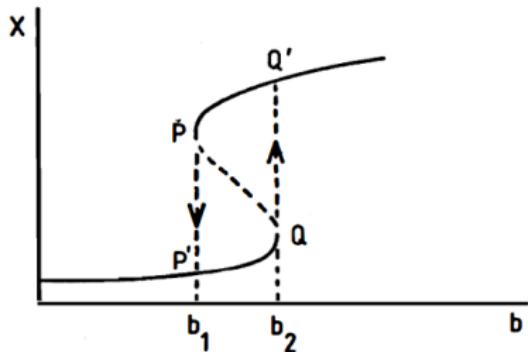


Figure 15. This figure shows how a "hysteresis" phenomenon occurs if we have the value of the bifurcation parameter  $b$  first growing and then diminishing. If the system is initially in a stationary state belonging to the lower branch, it will stay there while  $b$  grows. But at  $b = b_2$ , there will be a discontinuity: The system jumps from  $Q$  to  $Q'$ , on the higher branch. Inversely, starting from a state on the higher branch, the system will remain there till  $b = b_1$ , when it will jump down to  $P$ . Such types of bistable behavior are observed in many fields, such as lasers, chemical reactions or biological membranes.

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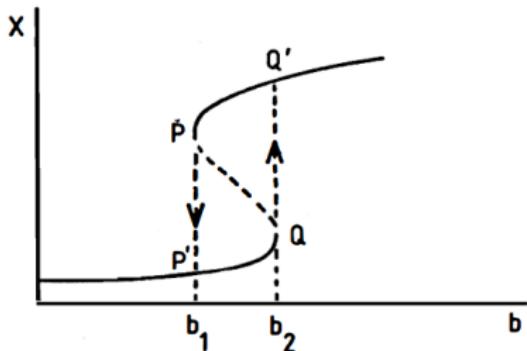
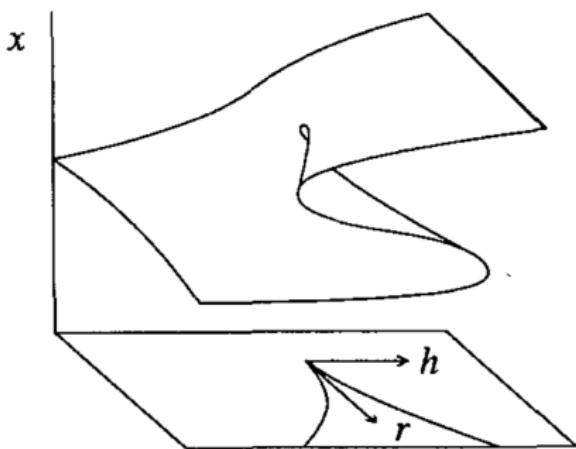


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Strogatz 2014



"cusp catastrophe"

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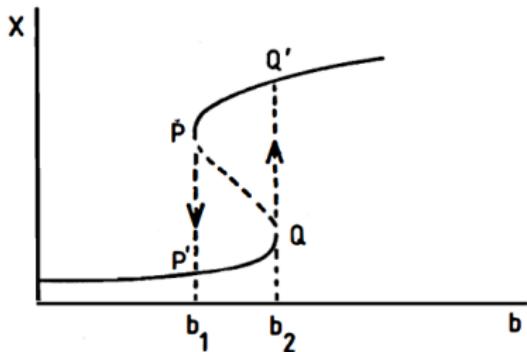
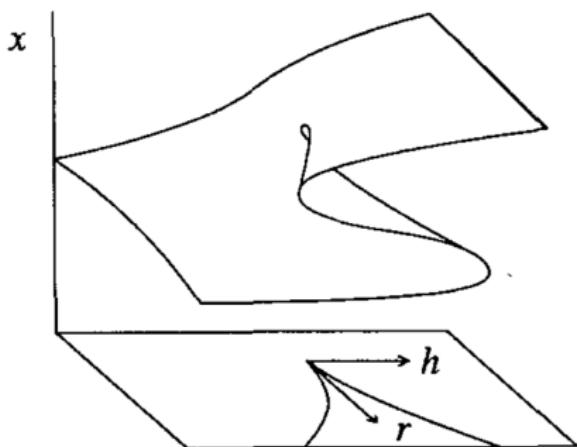


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"cusp catastrophe"

↔ "jumps", hysteretic behaviour ( $r_w$ , RH) for small enough  $N$ , close to equilibrium (slow process)

# hysteresis: activation/deactivation cycle



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✚ nomenclature:

# hysteresis: activation/deactivation cycle



## ‣ nomenclature:

- CCN activation
- (heterogeneous) nucleation

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## ✚ significance:

- aerosol processing by clouds (aqueous chemistry, coalescence)

## lifting the constant T-p assumptions: parcel model

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vertically displaced (velocity  $w$ , hydrostatic background) adiabatic parcel:  
( $q$ : mixing ratio,  $p_d$ : bgnd pressure,  $\rho_d$  bgnd density,  $g, l_v, c_{pd}$ : constants)

$$\begin{bmatrix} \dot{p}_d \\ \dot{T} \\ \dot{r}_w \end{bmatrix} = \begin{bmatrix} -\rho_d g w \\ (\dot{p}_d/\rho_d - \dot{q} l_v)/c_{pd} \\ (D_{\text{eff}}/\rho_w)(\rho_v - \rho_o)/r_w \end{bmatrix}$$

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- $w \rightarrow 0$  (and hence  $\dot{p}_d \approx 0$ ) i.e., slow, close-to-equilibrium evolution of the system relevant to fixed-point analysis (by some means pertinent to formation of non-convective clouds such as fog)

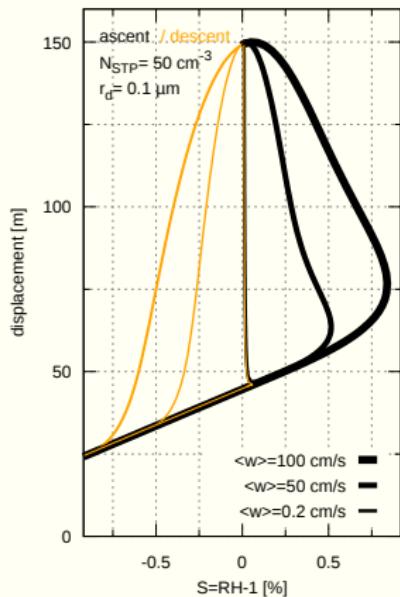
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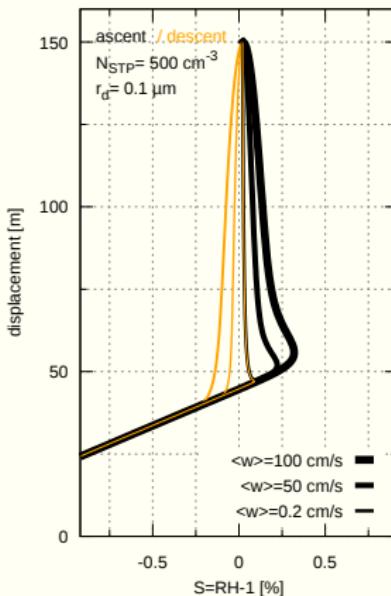
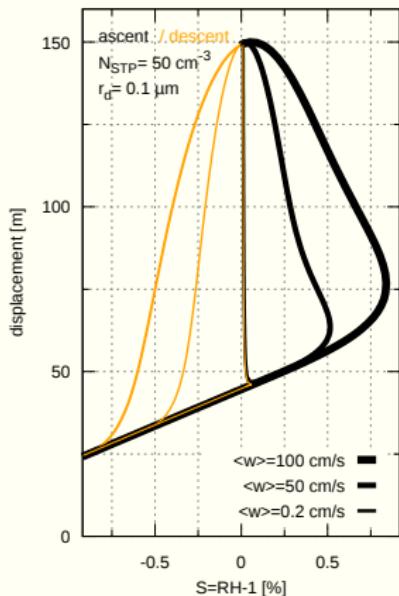
- $w \rightarrow 0$  (and hence  $\dot{p}_d \approx 0$ ) i.e., slow, close-to-equilibrium evolution of the system relevant to fixed-point analysis (by some means pertinent to formation of non-convective clouds such as fog)
- $N \rightarrow 0$  (and hence  $\dot{q} \approx 0$ ) i.e., weak coupling between particle size evolution and ambient thermodynamics (pertinent to the case of low particle concentration).

# parcel model: numerical integration (sinusoidal $w$ )



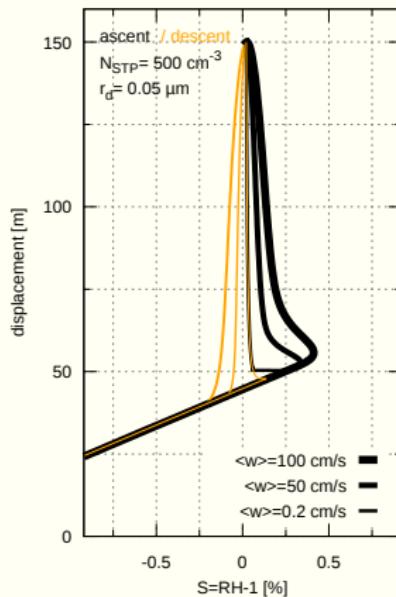
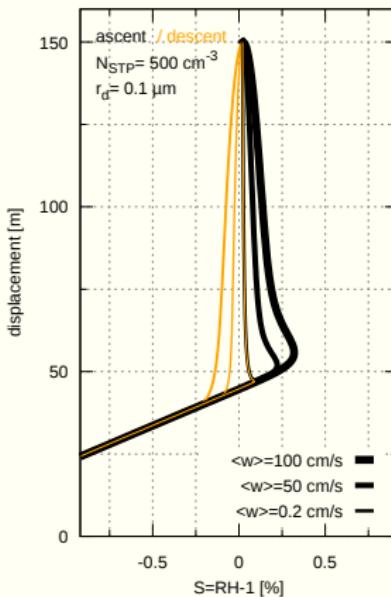
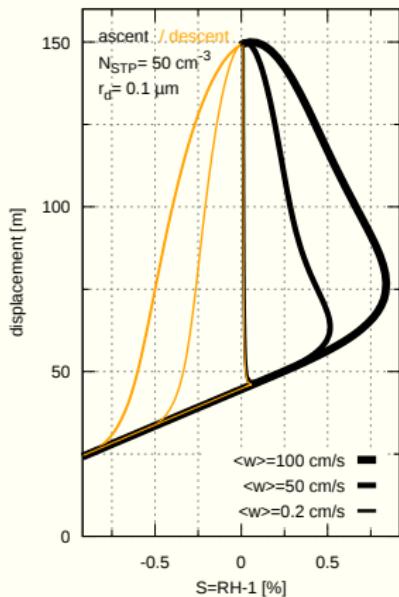
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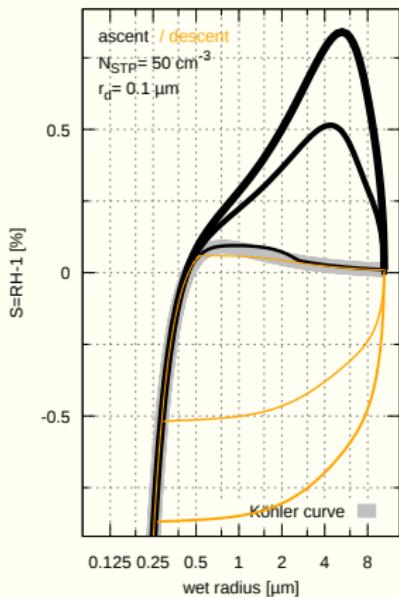
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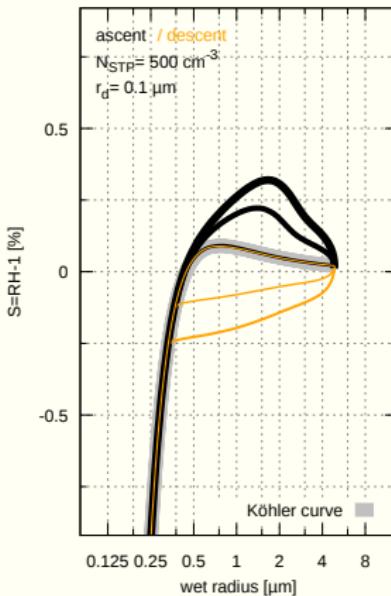
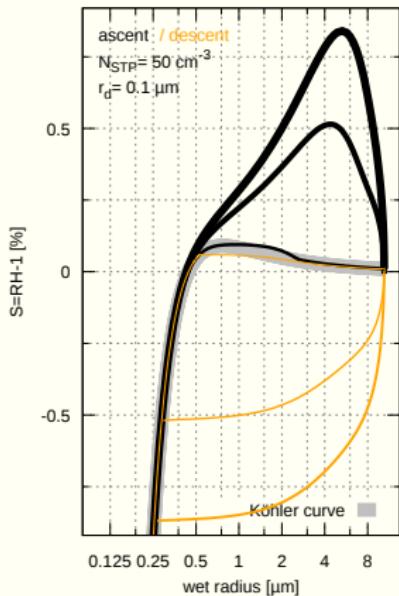
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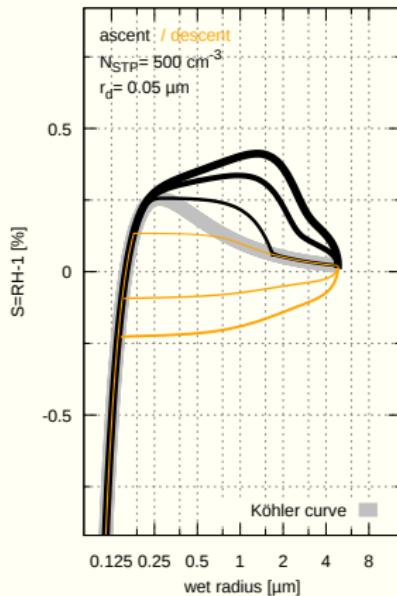
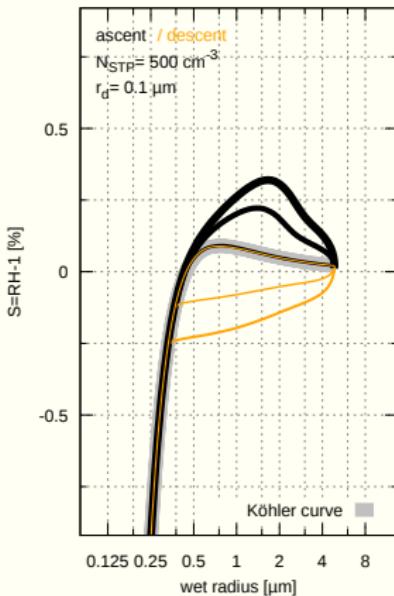
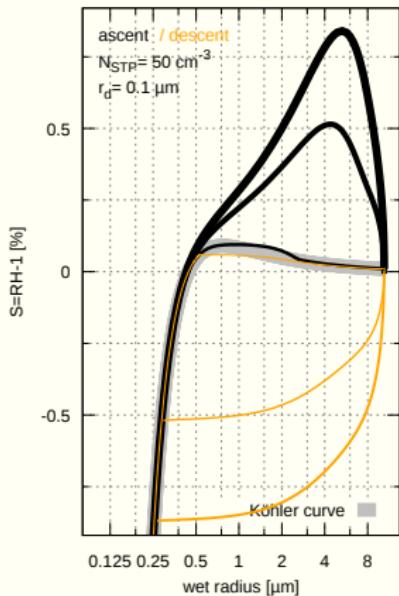
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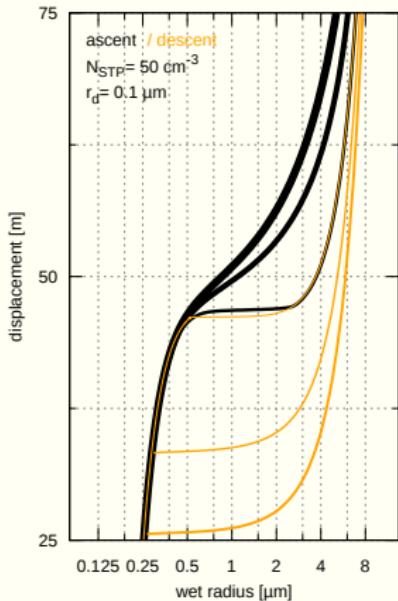
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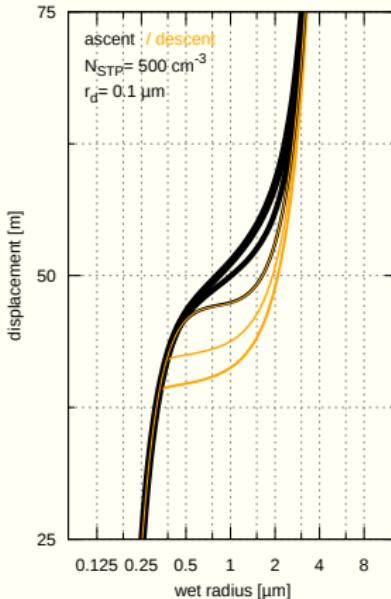
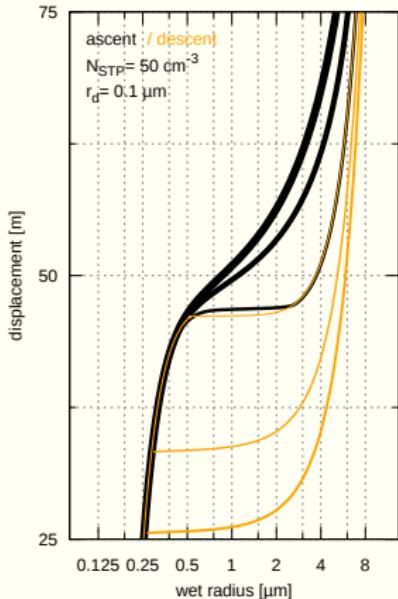
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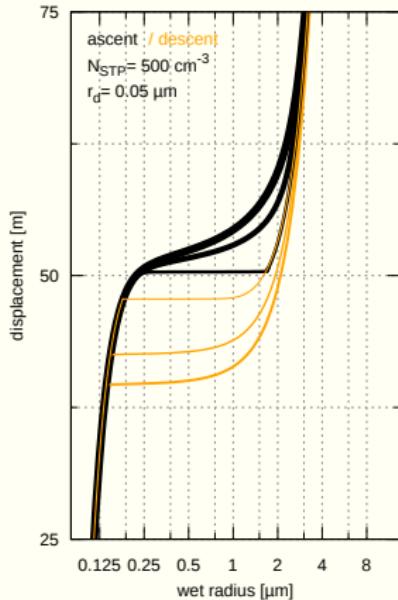
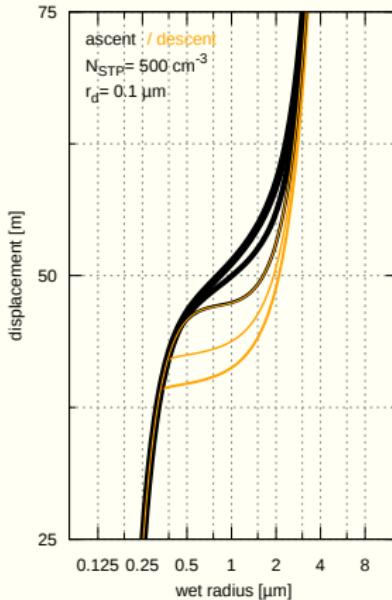
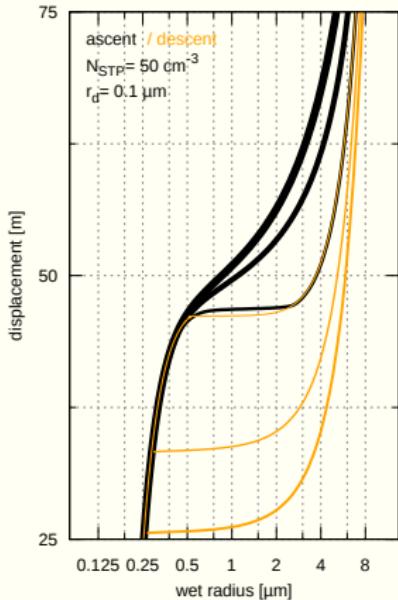
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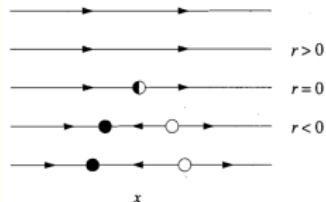
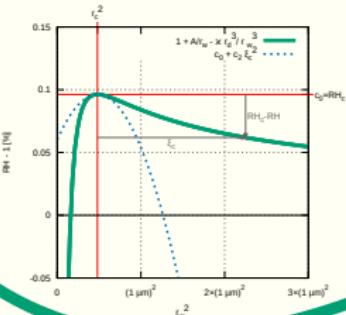
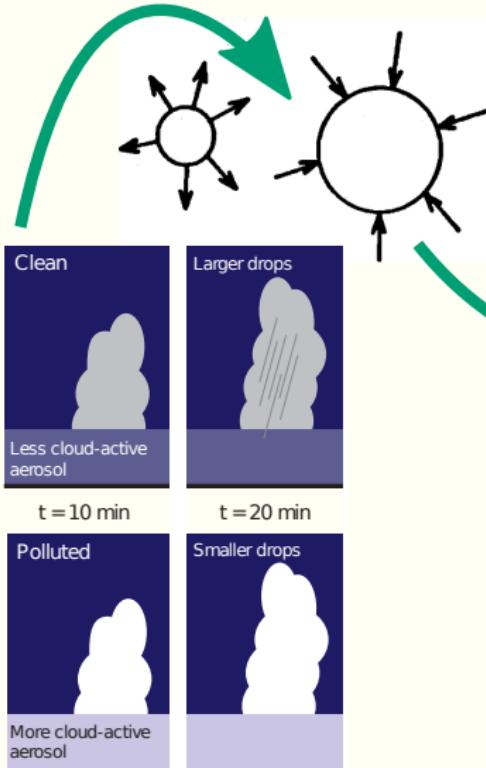
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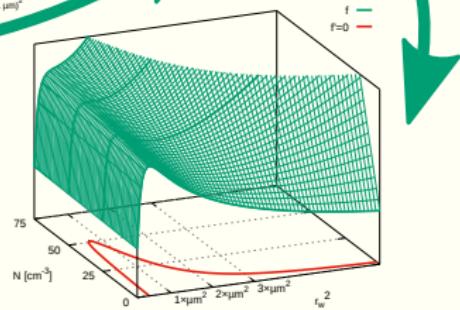
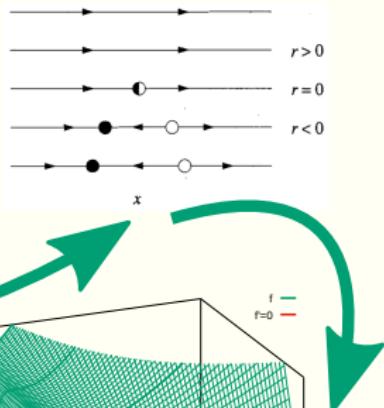
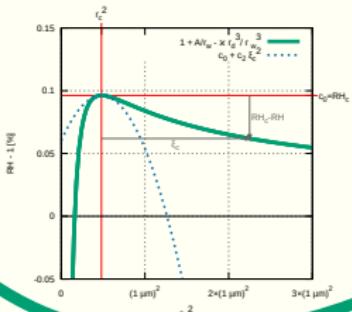
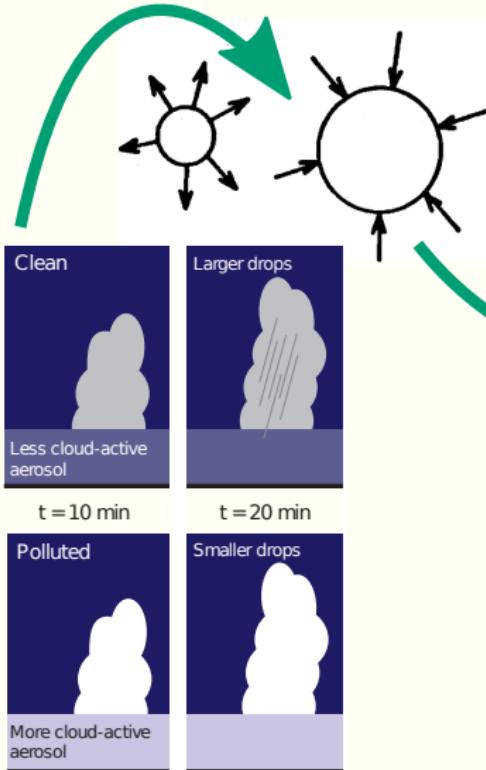
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connecting the dots ...

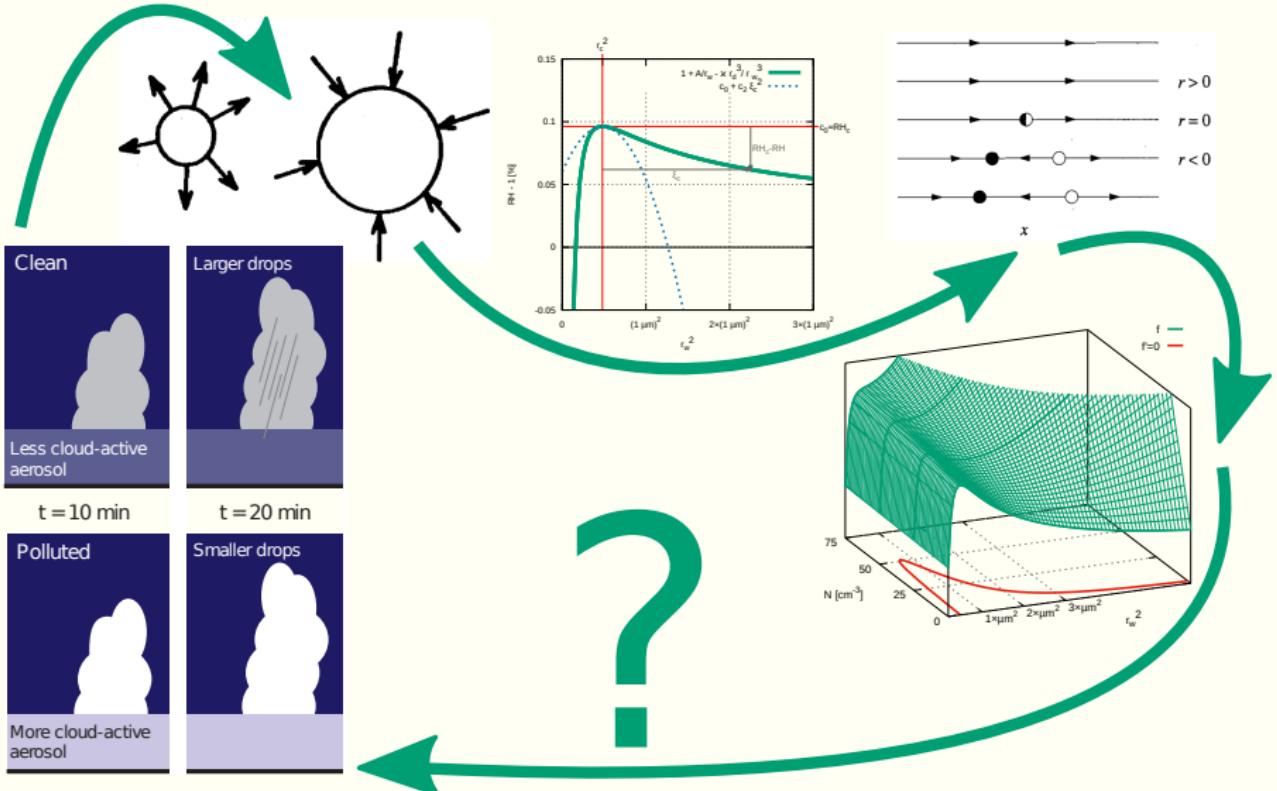
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# connecting the dots ...



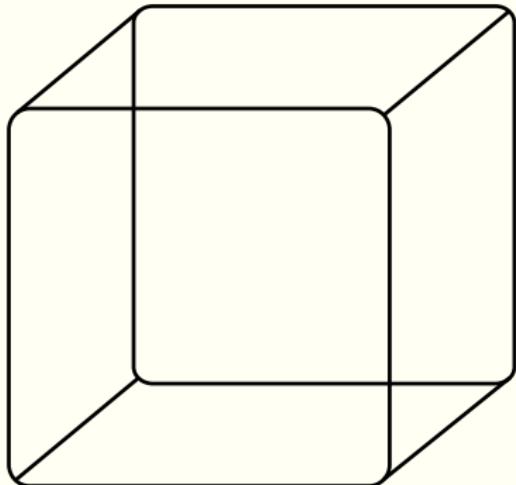
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# particle-based $\mu$ -physics for LES

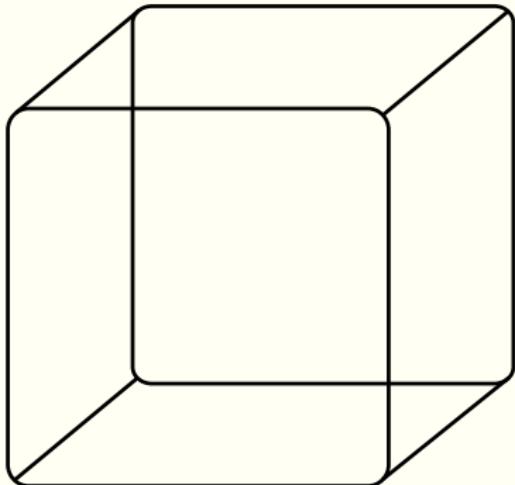
- “information carriers” in LES domain

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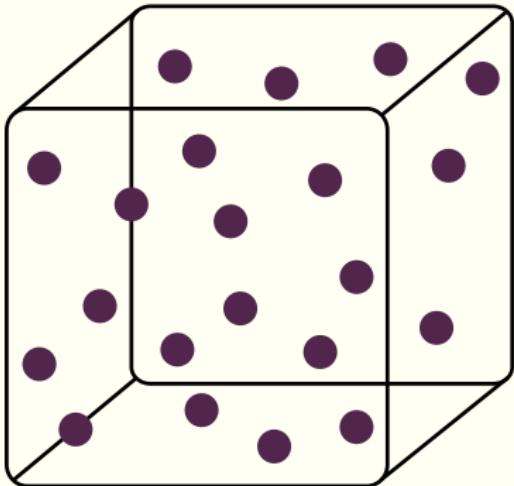
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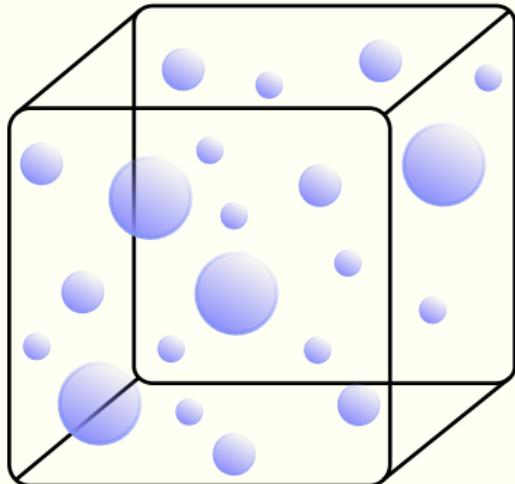
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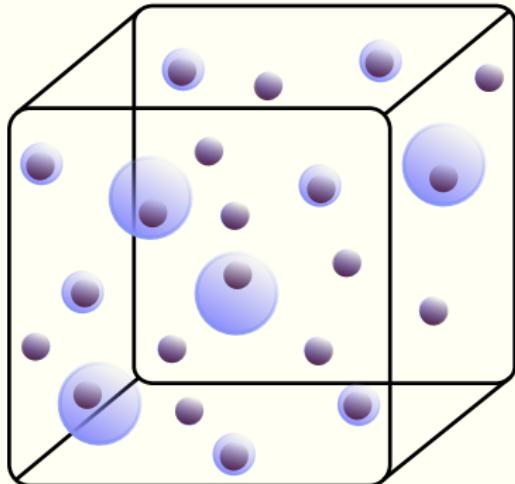
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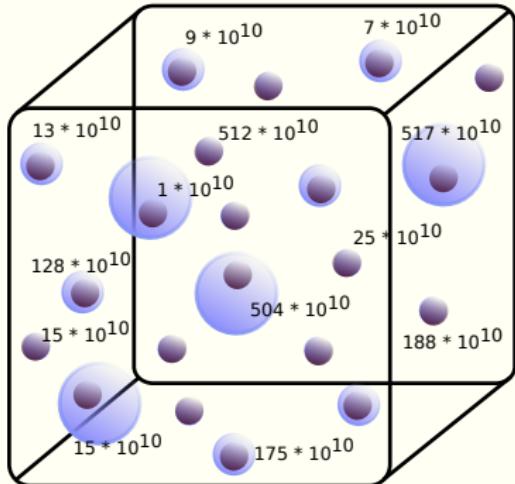
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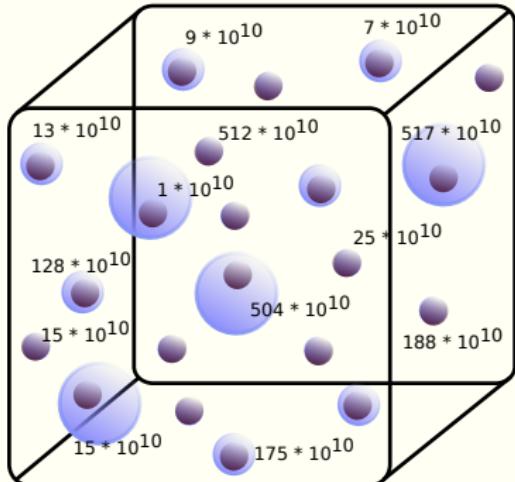
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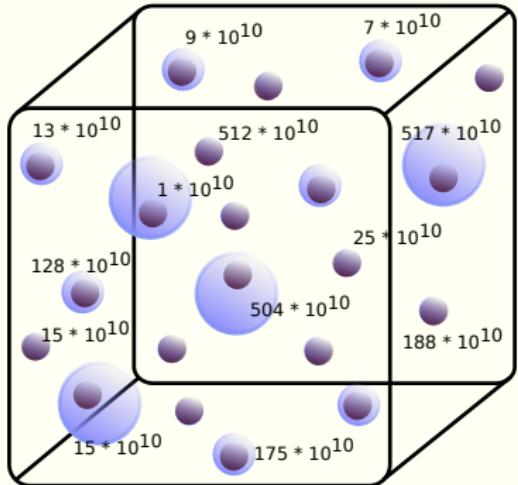
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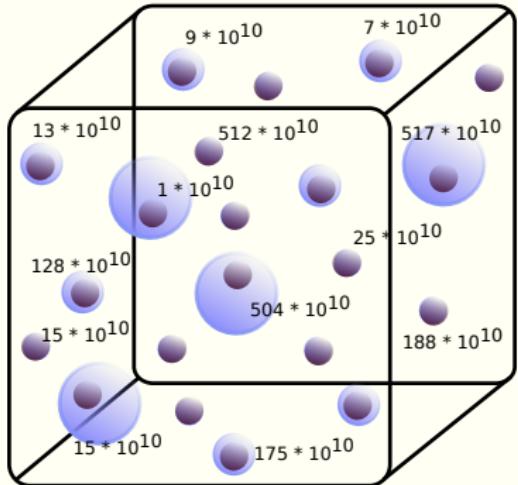
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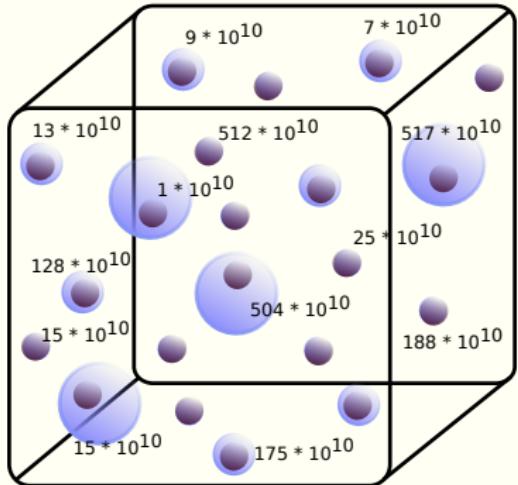
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- ☒ each timestep: **constant RH!**

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<sup>1</sup><http://www.mmm.ucar.edu/eulag/>

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<sup>4</sup><http://github.com/uclales>

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<sup>7</sup>

# particle-based $\mu$ -physics for LES

Seminal works: Shima et al. 2009, Andrejczuk et al. 2010  
(3D simulations of atmospheric aerosol-cloud-precipitation system)

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- SCALE<sup>6</sup> from RIKEN (Sato et al. 2017),
- UWLCM<sup>7</sup> from Univ. Warsaw (Grabowski et al. 2018).

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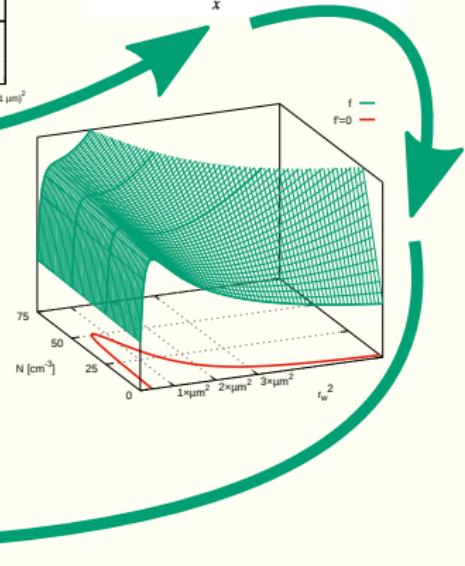
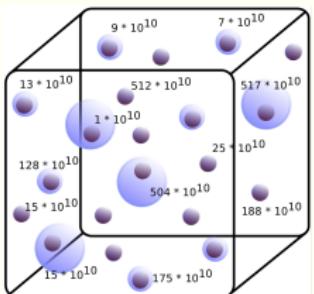
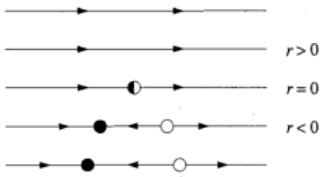
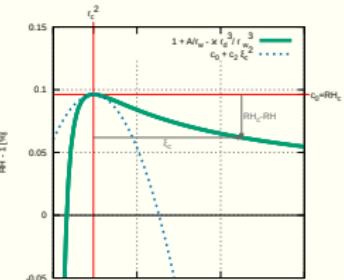
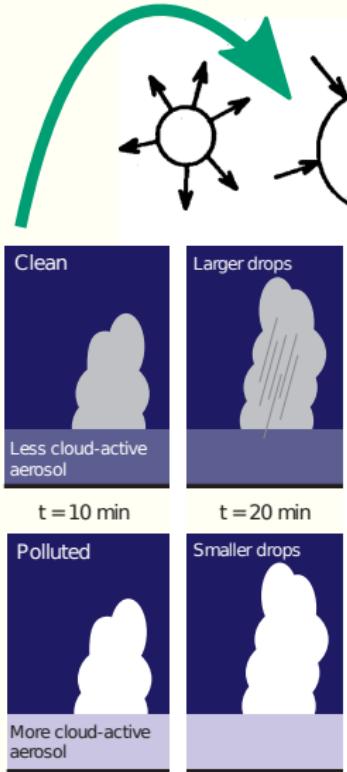
<sup>5</sup><http://pencil-code.nordita.org>

<sup>6</sup><http://scale.aics.riken.jp/>

<sup>7</sup><http://github.com/igfwu/UWLCM>

connecting the dots ...

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  - analytical results: conditions for hysteretic behaviour, timescales
  - guidance for numerical scheme design (particle-based  $\mu$ -physics)

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- extensions:
  - bi-/poly- modal/disperse spectra (spectrum width!),
  - activated/unactivated partitioning (excitable behaviour!),
  - beyond Köhler curve (charge, surfactants, non-soluble aerosol, ...)

# Thank you for your attention!

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