

# Immersion freezing in particle-based cloud $\mu$ -physics models

S. Arabas<sup>1</sup>, J.H. Curtis<sup>2</sup>, I. Silber<sup>3</sup>, A. Fridlind<sup>4</sup>, D.A. Knopf<sup>5</sup>, M. West<sup>2</sup> & N. Riemer<sup>2</sup>



funding:



University of California Davis, Sep 3<sup>rd</sup> 2025



## Immersion Freezing in Particle-Based Aerosol-Cloud Microphysics: A Probabilistic Perspective on Singular and Time-Dependent Models

Sylwester Arabas<sup>1</sup> , Jeffrey H. Curtis<sup>2</sup> , Israel Silber<sup>3,4</sup> , Ann M. Fridlind<sup>5</sup> ,  
Daniel A. Knopf<sup>6</sup> , Matthew West<sup>7</sup> , and Nicole Riemer<sup>2</sup> 

10.1029/2024MS004770

# Aerosol-cloud interactions: a conceptual picture

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background image: vitsly / Hokusai

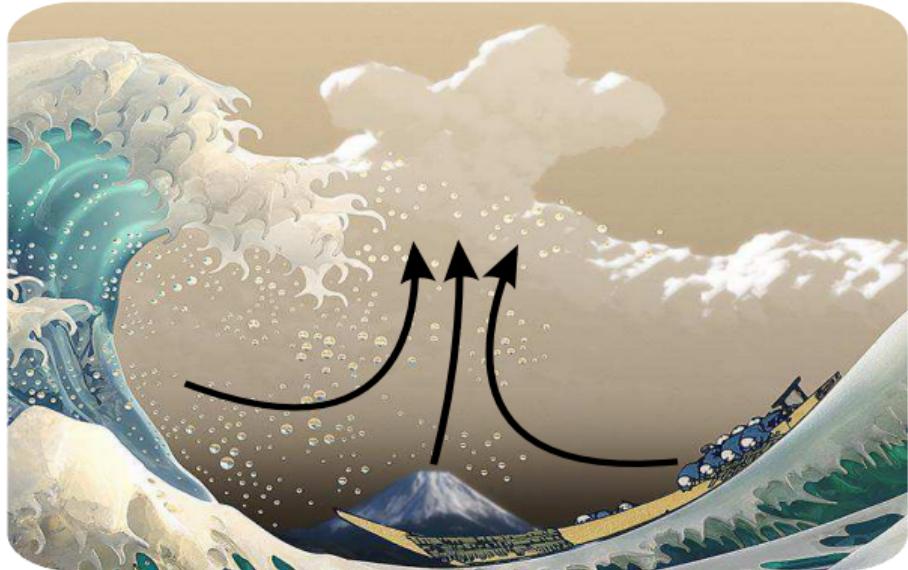
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two-way interactions:

- aerosol characteristics influence cloud microstructure
- cloud processes influence aerosol size and composition

background image: vitsly / Hokusai

# Aerosol-cloud interactions: $\mu$ -physics models

JAMES

Journal of Advances in  
Modeling Earth Systems

## COMMISSIONED MANUSCRIPT

10.1029/2019MS001689

### Key Points:

- Microphysics is an important component of weather and climate models, but its representation in current models is highly uncertain

## Confronting the Challenge of Modeling Cloud and Precipitation Microphysics

Hugh Morrison<sup>1</sup> , Marcus van Lier-Walqui<sup>2</sup> , Ann M. Fridlind<sup>3</sup> , Wojciech W. Grabowski<sup>1</sup> , Jerry Y. Harrington<sup>4</sup>, Corinna Hoose<sup>5</sup> , Alexei Korolev<sup>6</sup> , Matthew R. Kumjian<sup>4</sup> , Jason A. Milbrandt<sup>7</sup>, Hanna Pawlowska<sup>8</sup> , Derek J. Posselt<sup>9</sup>, Olivier P. Prat<sup>10</sup>, Karly J. Reimel<sup>4</sup>, Shin-Ichiro Shima<sup>11</sup> , Bastiaan van Diedenhoven<sup>2</sup> , and Lulin Xue<sup>1</sup> 

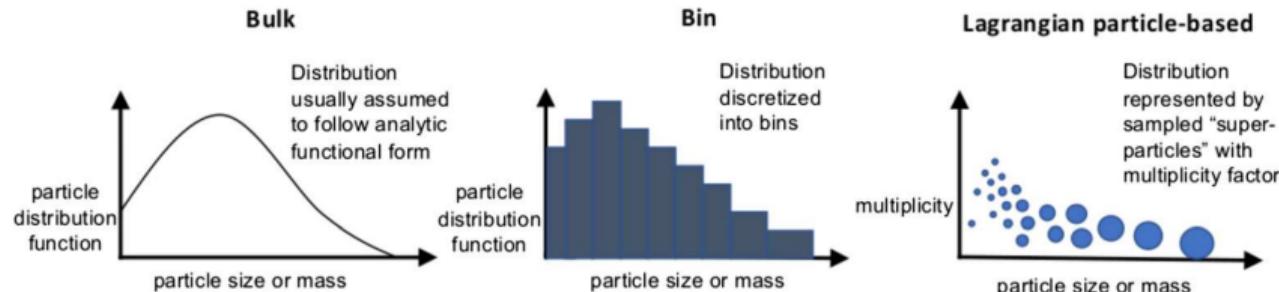
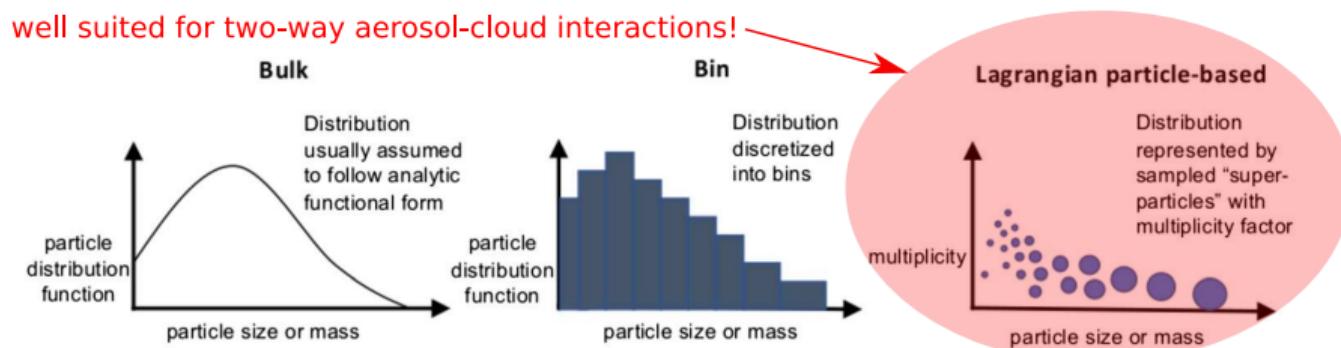


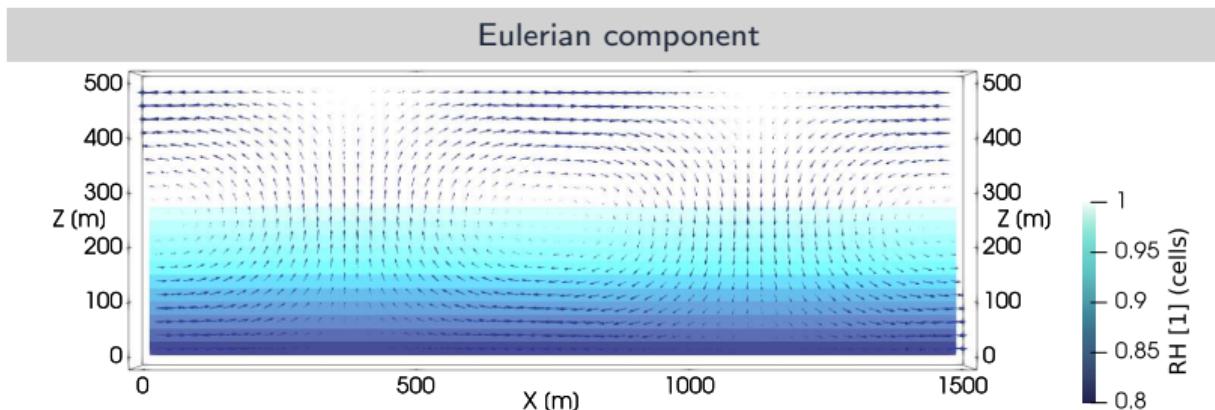
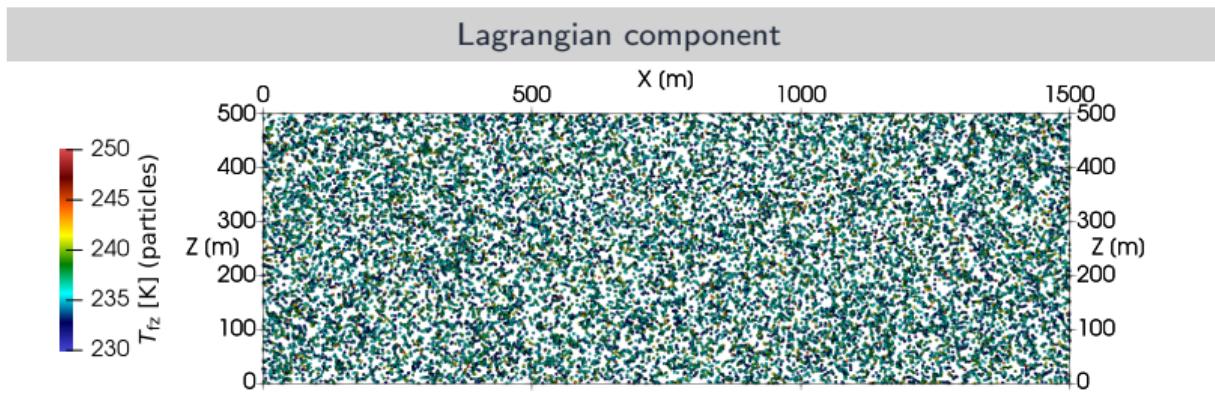
Figure 3. Representation of cloud and precipitation particle distributions in the three main types of microphysics

# Aerosol-cloud interactions: $\mu$ -physics models



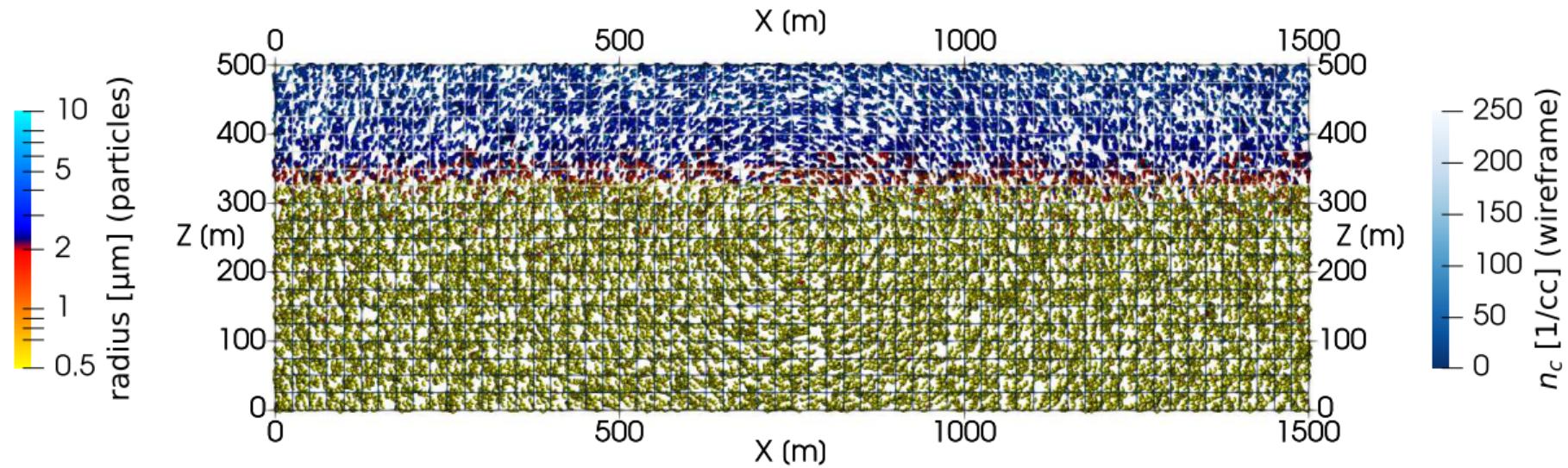
**Figure 3.** Representation of cloud and precipitation particle distributions in the three main types of microphysics

# Particle-based $\mu$ -physics + prescribed-flow: model state



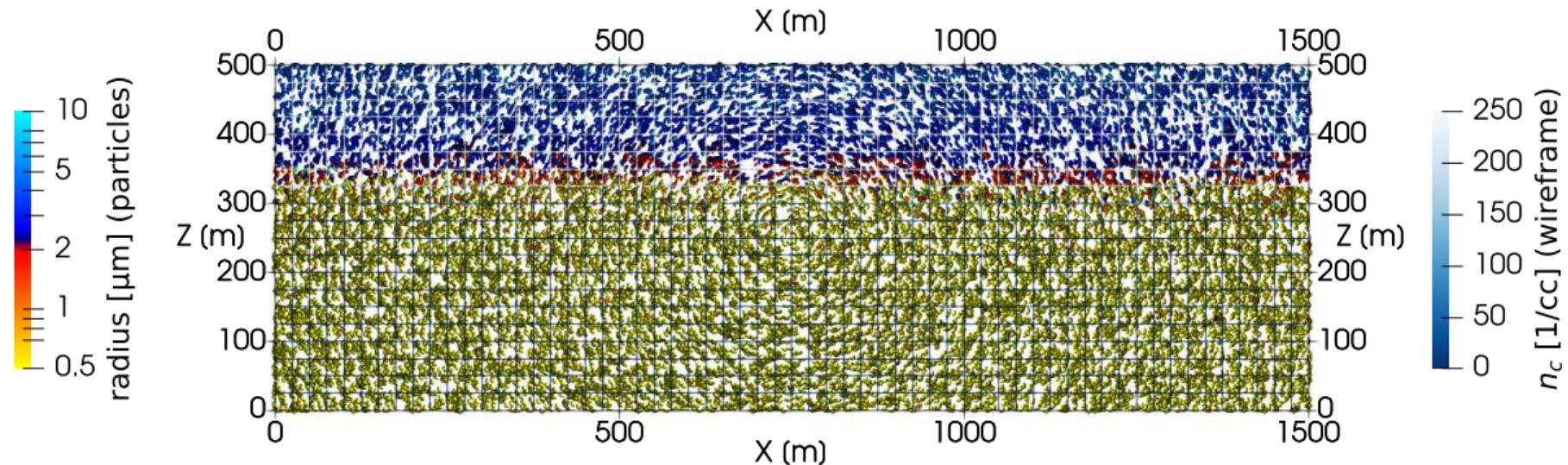
# Particle-based $\mu$ -physics + prescribed-flow: spin-up

Time: 30 s (spin-up till 600.0 s)



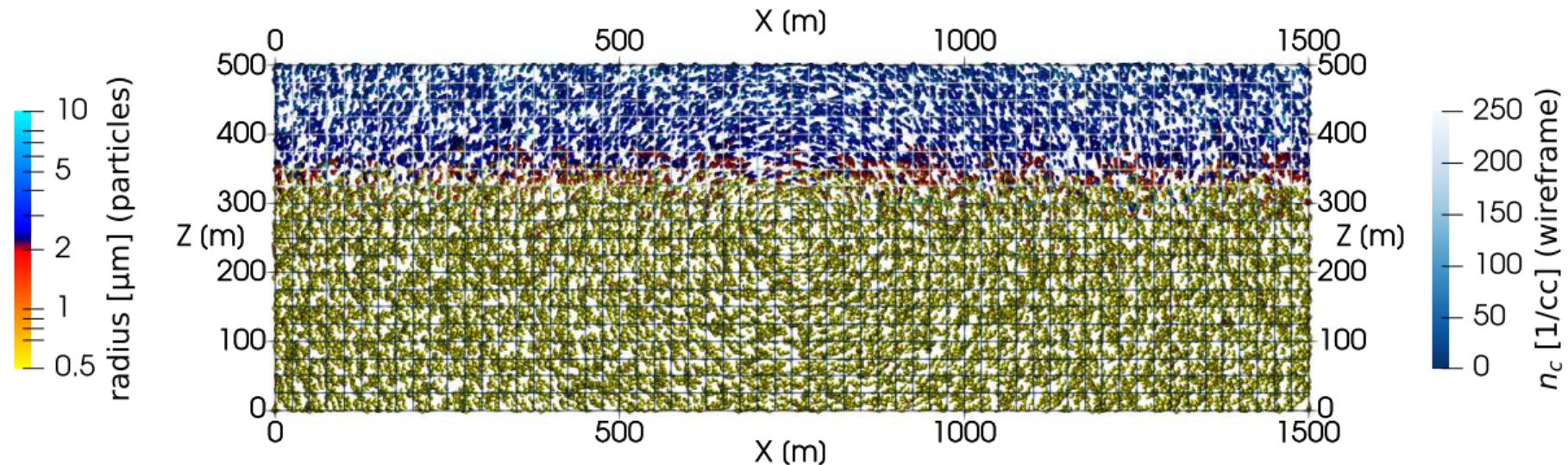
# Particle-based $\mu$ -physics + prescribed-flow: spin-up

Time: 60 s (spin-up till 600.0 s)



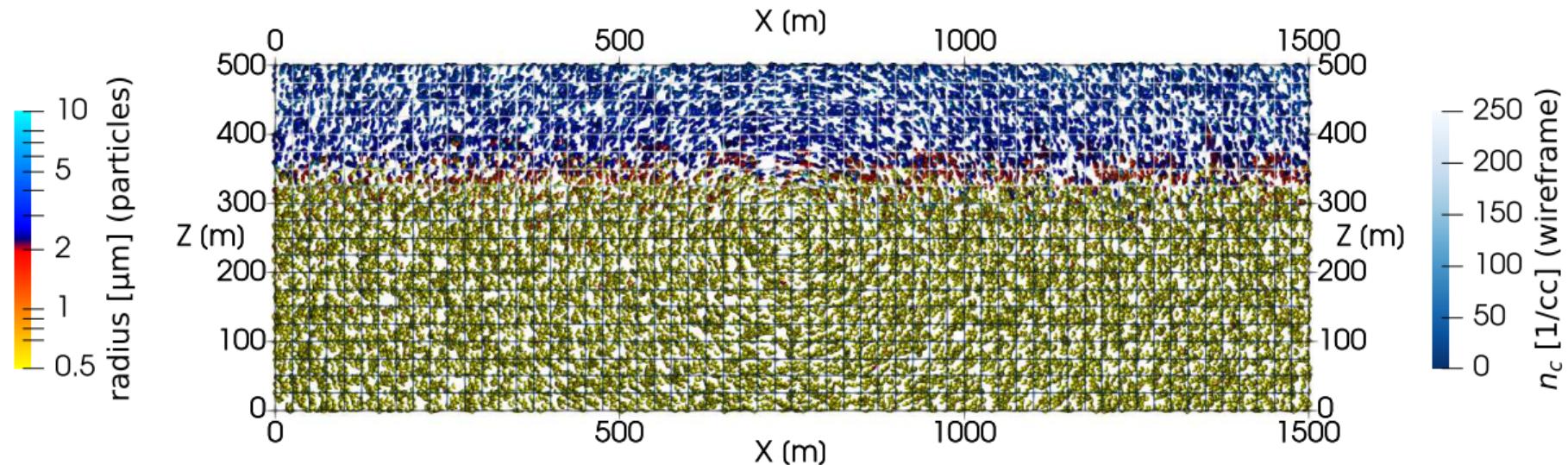
# Particle-based $\mu$ -physics + prescribed-flow: spin-up

Time: 90 s (spin-up till 600.0 s)



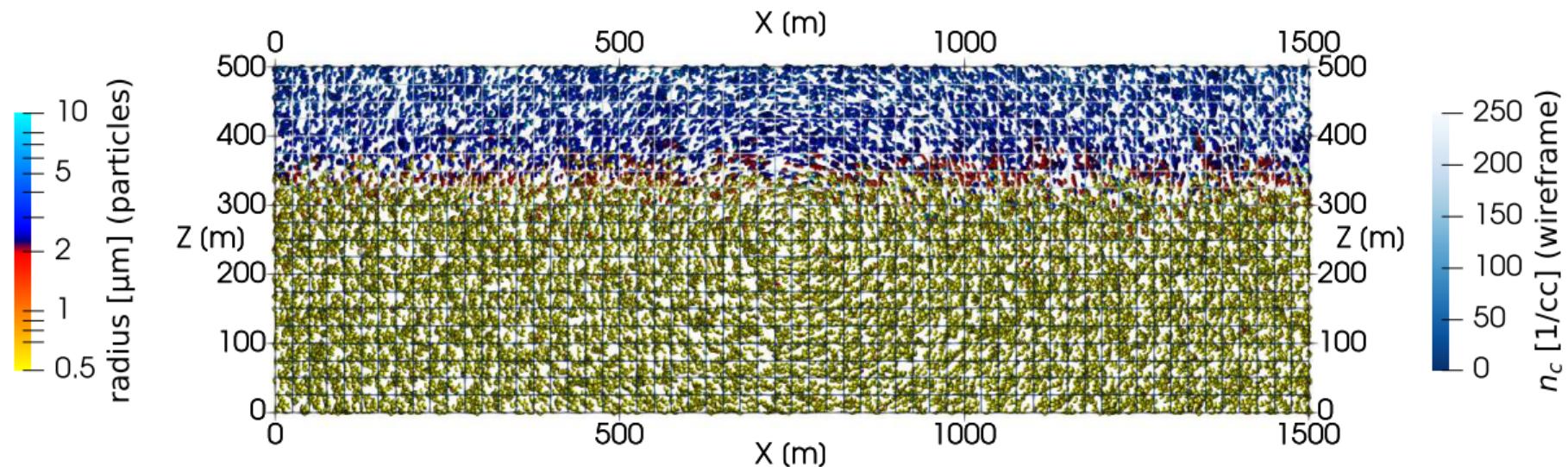
# Particle-based $\mu$ -physics + prescribed-flow: spin-up

Time: 120 s (spin-up till 600.0 s)



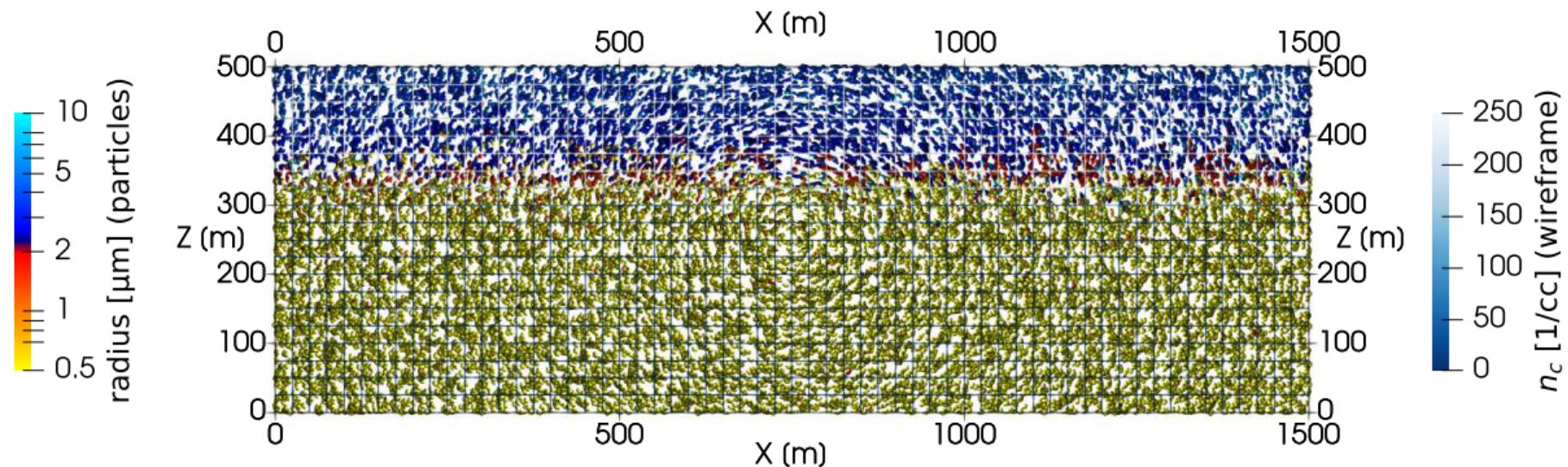
# Particle-based $\mu$ -physics + prescribed-flow: spin-up

Time: 150 s (spin-up till 600.0 s)



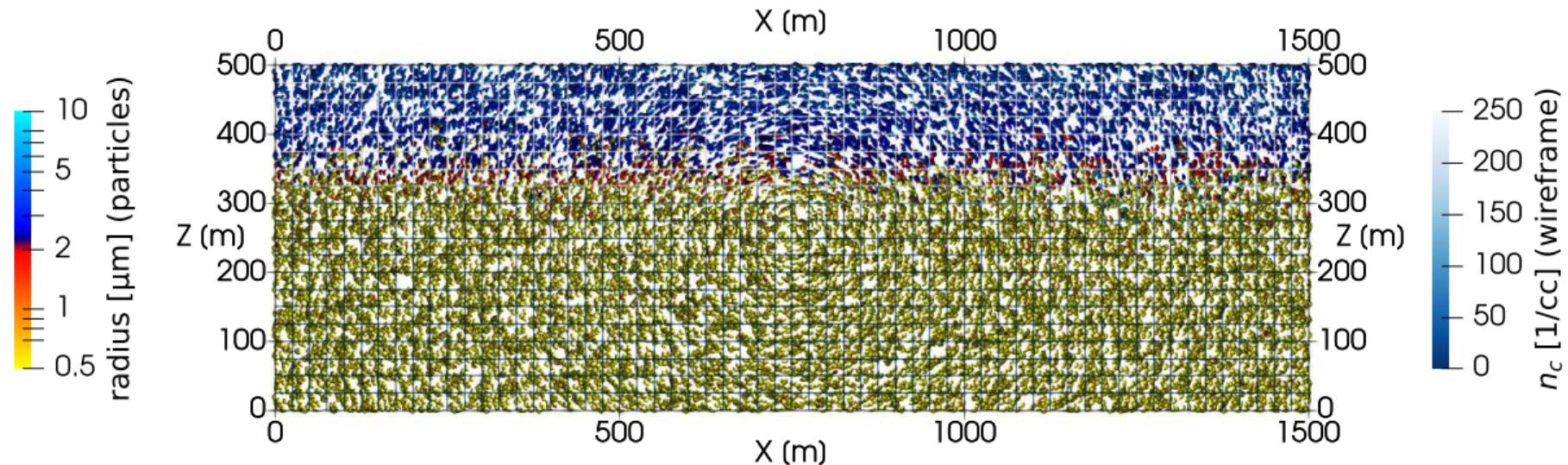
# Particle-based $\mu$ -physics + prescribed-flow: spin-up

Time: 180 s (spin-up till 600.0 s)



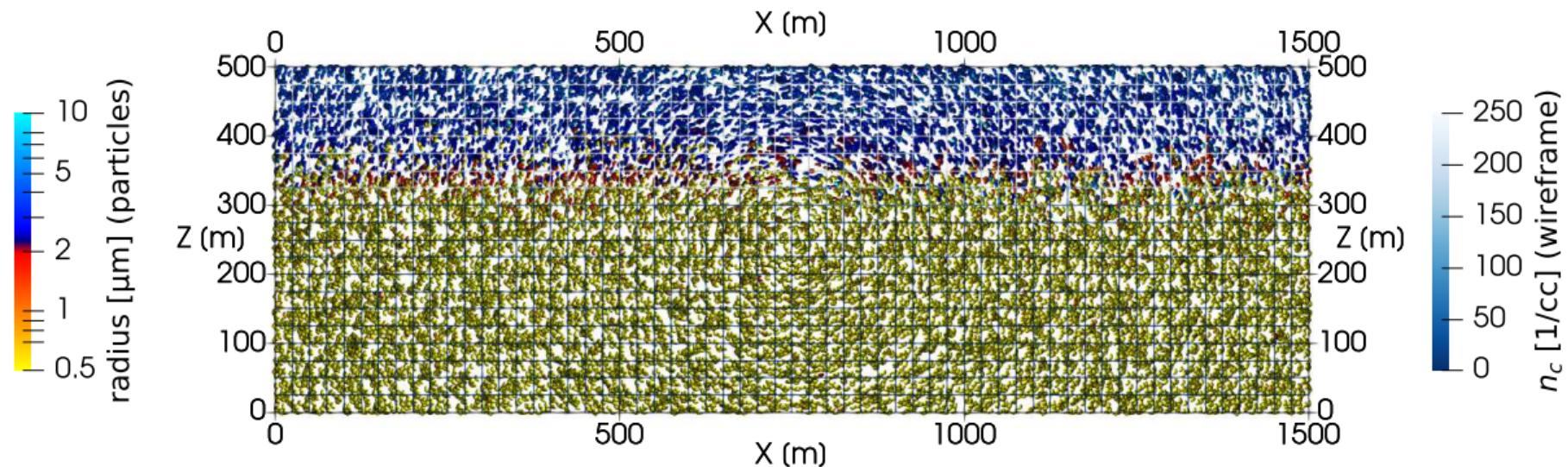
# Particle-based $\mu$ -physics + prescribed-flow: spin-up

Time: 210 s (spin-up till 600.0 s)



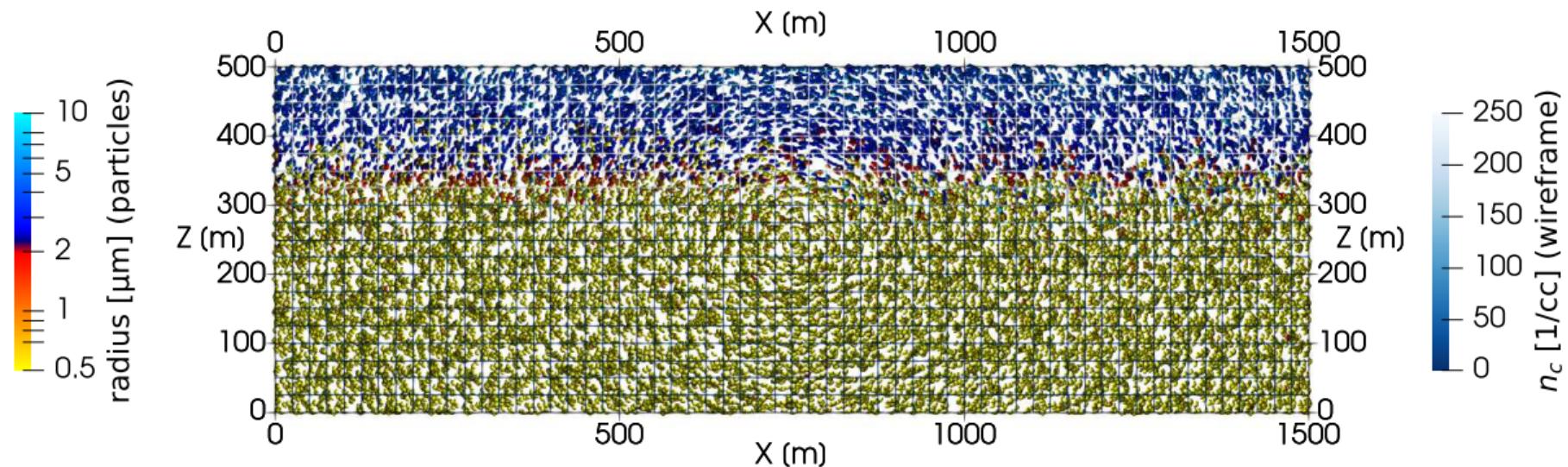
# Particle-based $\mu$ -physics + prescribed-flow: spin-up

Time: 240 s (spin-up till 600.0 s)



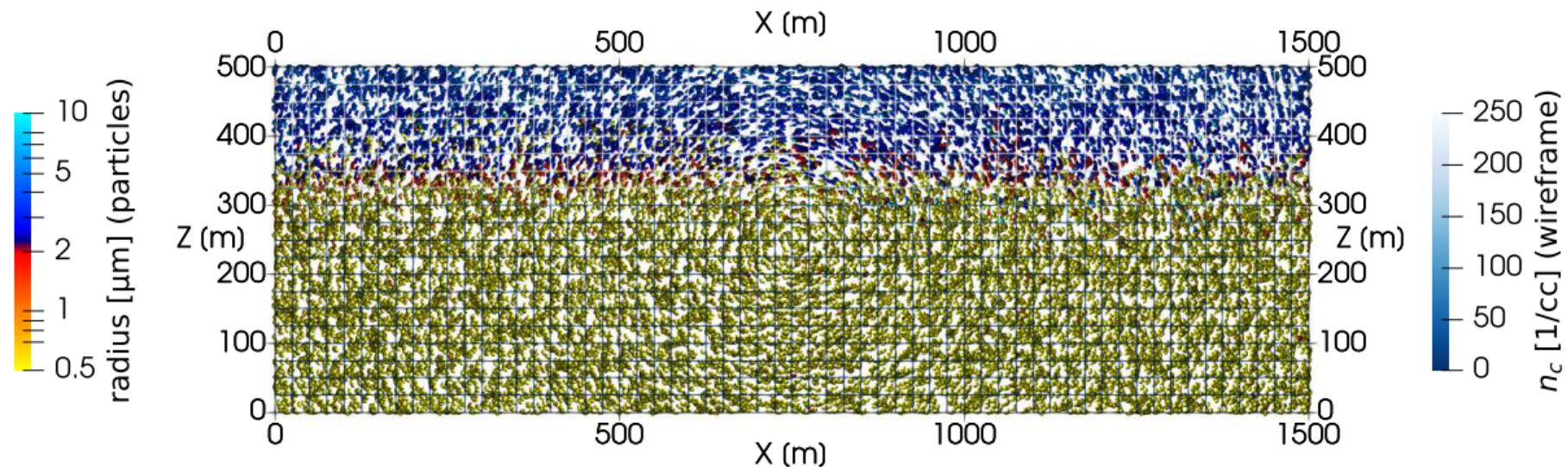
# Particle-based $\mu$ -physics + prescribed-flow: spin-up

Time: 270 s (spin-up till 600.0 s)



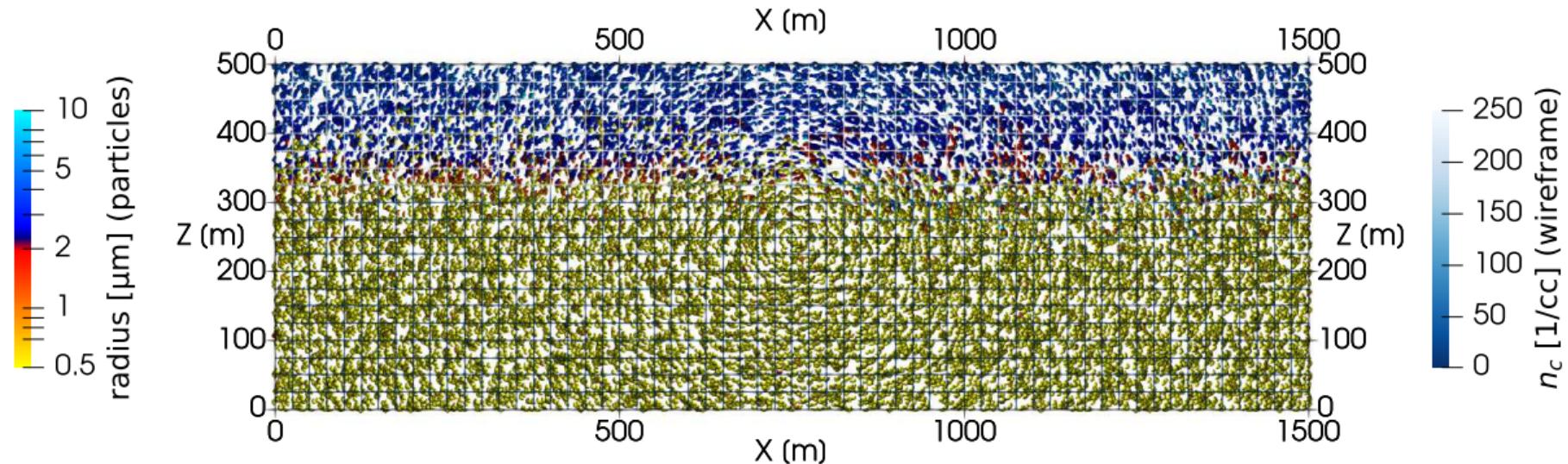
# Particle-based $\mu$ -physics + prescribed-flow: spin-up

Time: 300 s (spin-up till 600.0 s)



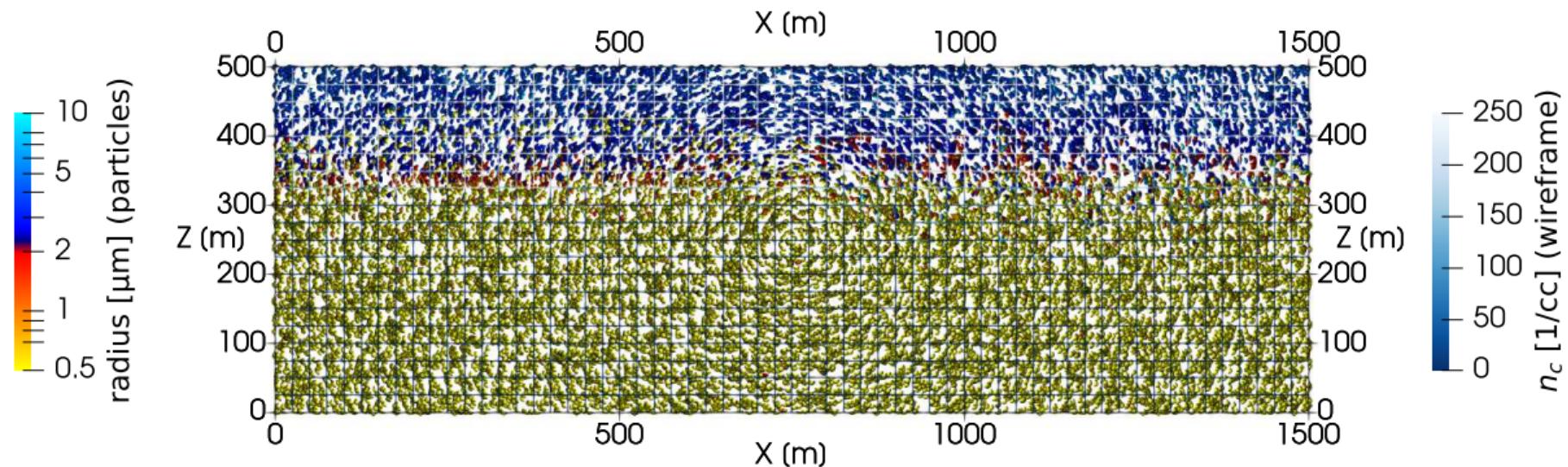
# Particle-based $\mu$ -physics + prescribed-flow: spin-up

Time: 330 s (spin-up till 600.0 s)



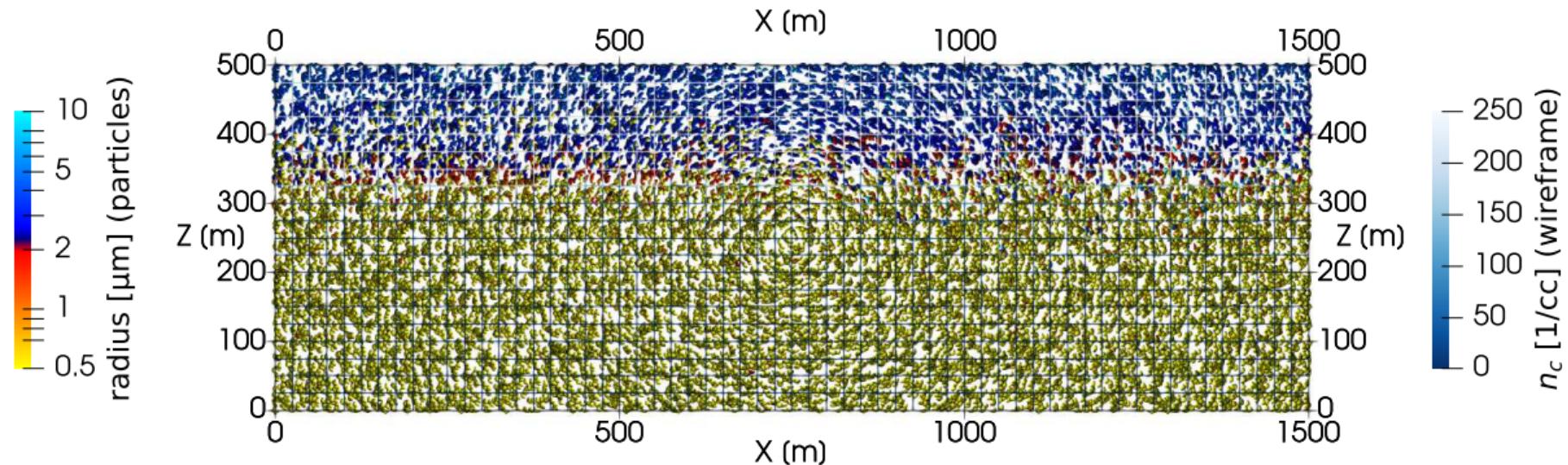
# Particle-based $\mu$ -physics + prescribed-flow: spin-up

Time: 360 s (spin-up till 600.0 s)



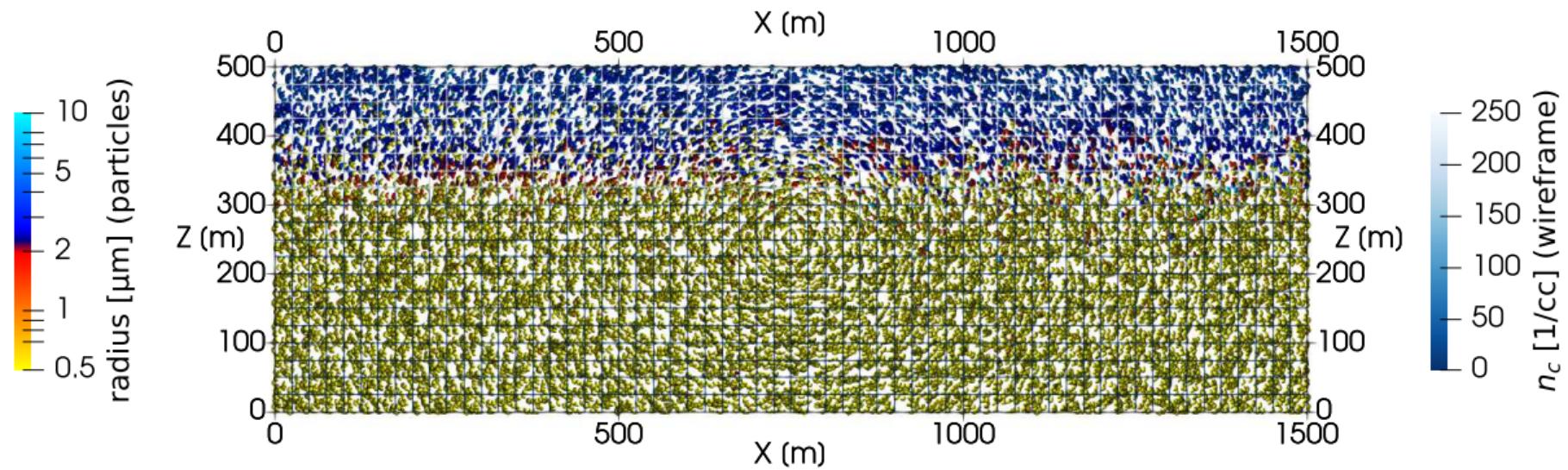
# Particle-based $\mu$ -physics + prescribed-flow: spin-up

Time: 390 s (spin-up till 600.0 s)



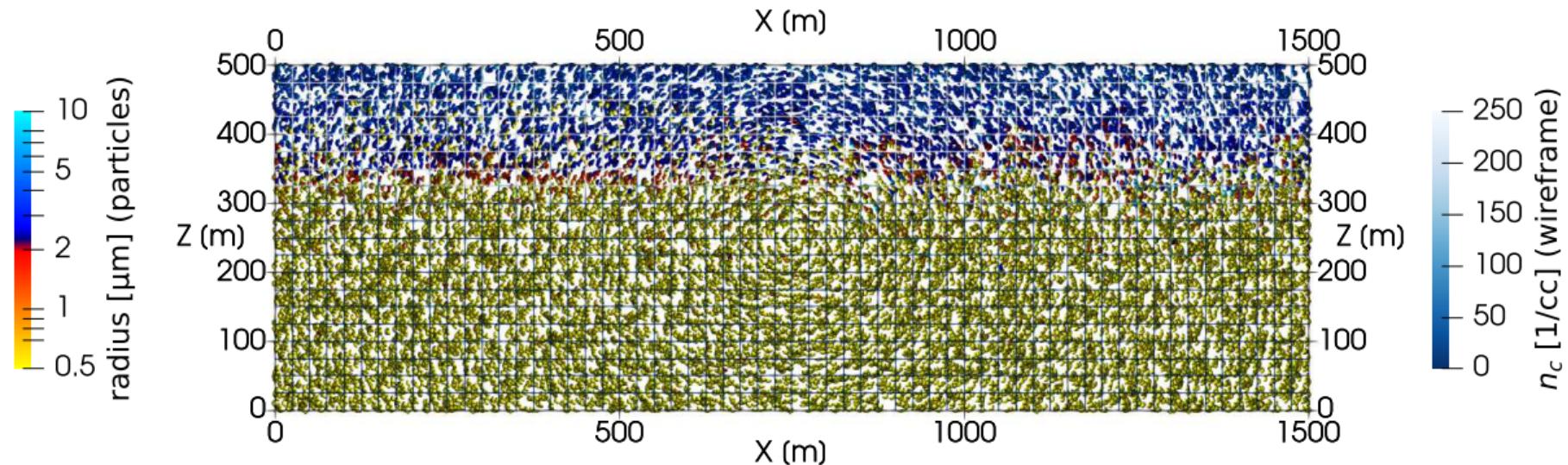
# Particle-based $\mu$ -physics + prescribed-flow: spin-up

Time: 420 s (spin-up till 600.0 s)



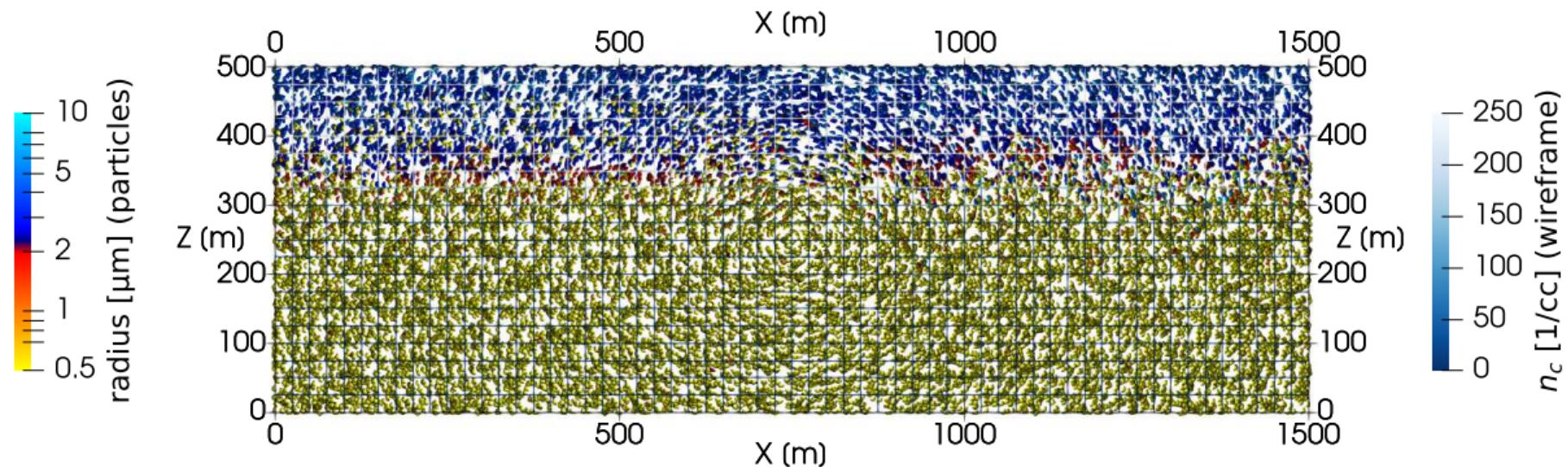
# Particle-based $\mu$ -physics + prescribed-flow: spin-up

Time: 450 s (spin-up till 600.0 s)



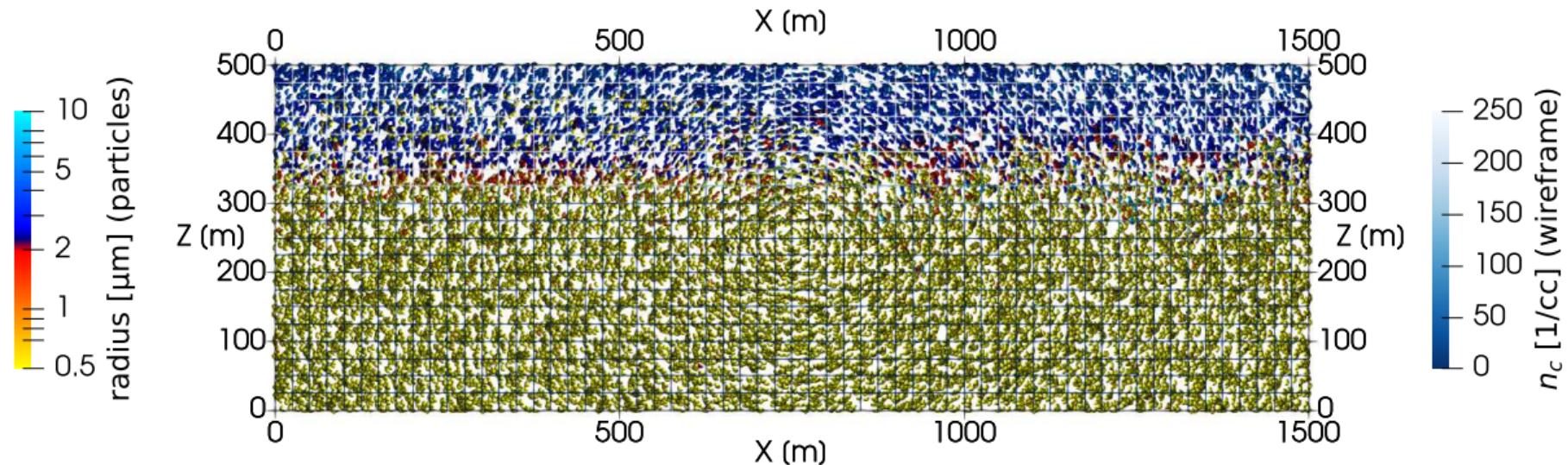
# Particle-based $\mu$ -physics + prescribed-flow: spin-up

Time: 480 s (spin-up till 600.0 s)



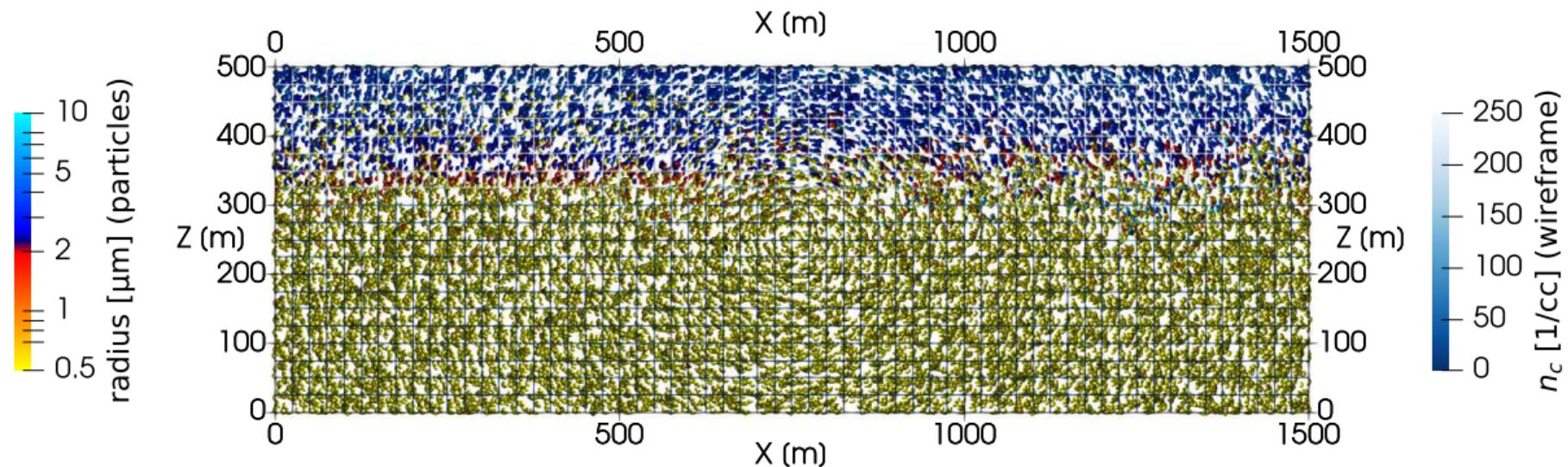
# Particle-based $\mu$ -physics + prescribed-flow: spin-up

Time: 510 s (spin-up till 600.0 s)



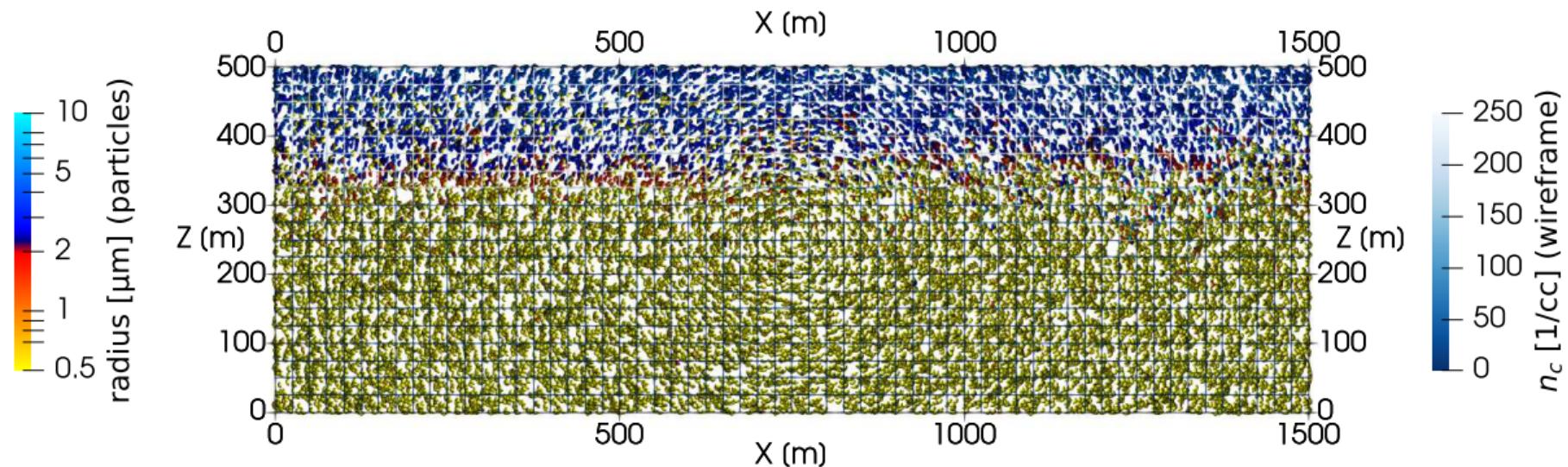
# Particle-based $\mu$ -physics + prescribed-flow: spin-up

Time: 540 s (spin-up till 600.0 s)



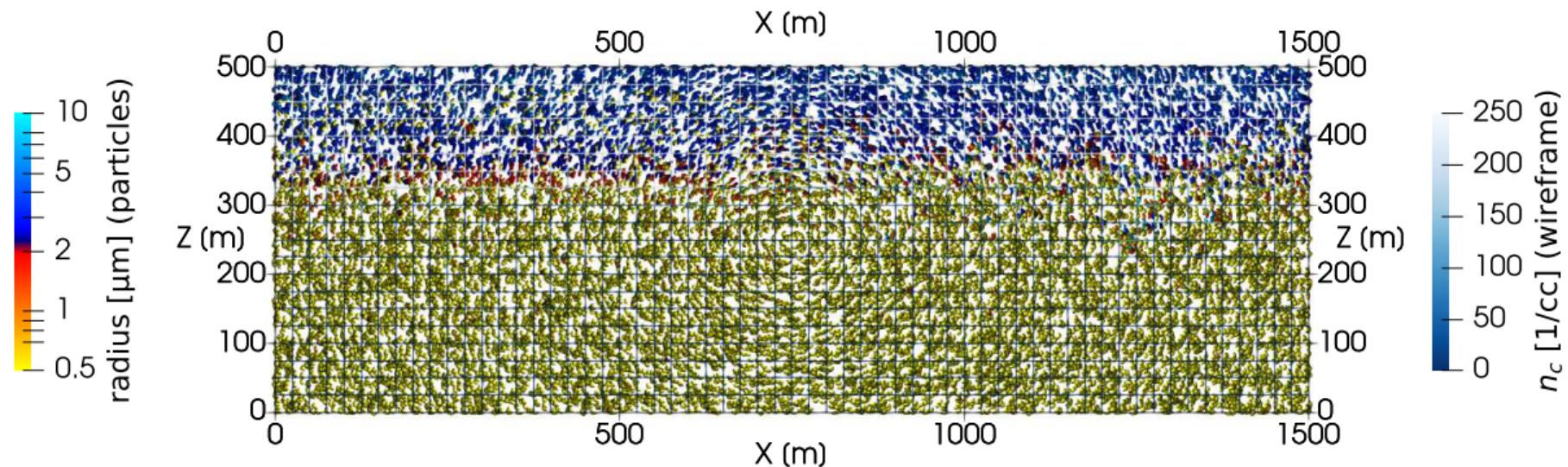
# Particle-based $\mu$ -physics + prescribed-flow: spin-up

Time: 570 s (spin-up till 600.0 s)



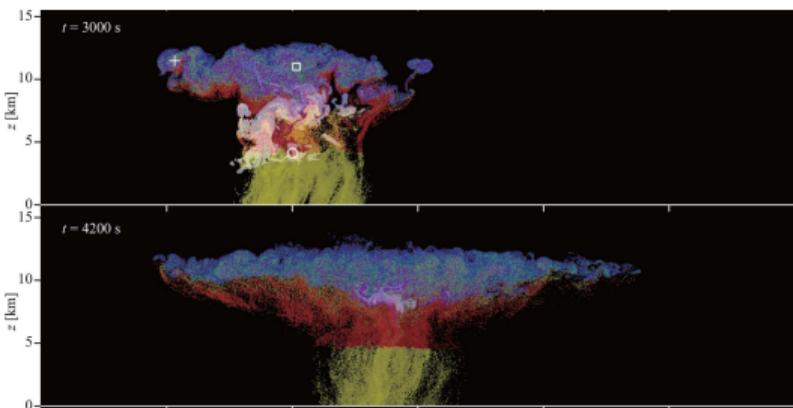
# Particle-based $\mu$ -physics + prescribed-flow: spin-up

Time: 600 s (spin-up till 600.0 s)



Shima, Sato, Hashimoto & Misumi 2020 (GMD):

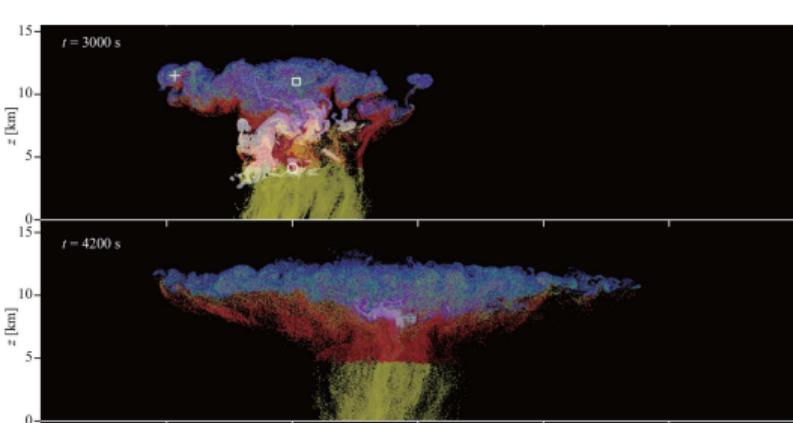
*Predicting the morphology of ice particles in deep convection using the super-droplet method*



**Figure 1.** Typical realization of CTRL cloud spatial structures at  $t = 2040, 2460, 3000, 4200$ , and  $5400$  s. The mixing ratio of cloud water, rainwater, cloud ice, graupel, and snow aggregates are plotted in fading white, yellow, blue, red, and green, respectively. The symbols indicate examples of unrealistic predicted ice particles (Sects. 7.3 and 9.1). See also Movie 1 in the video supplement.

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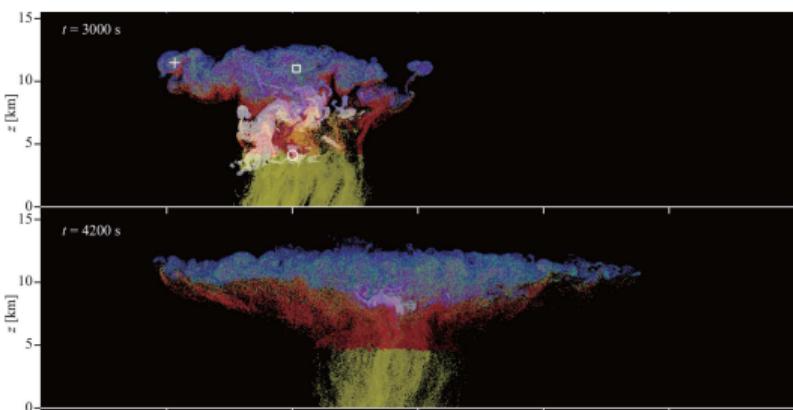


► Eulerian component: momentum, heat, moisture budget

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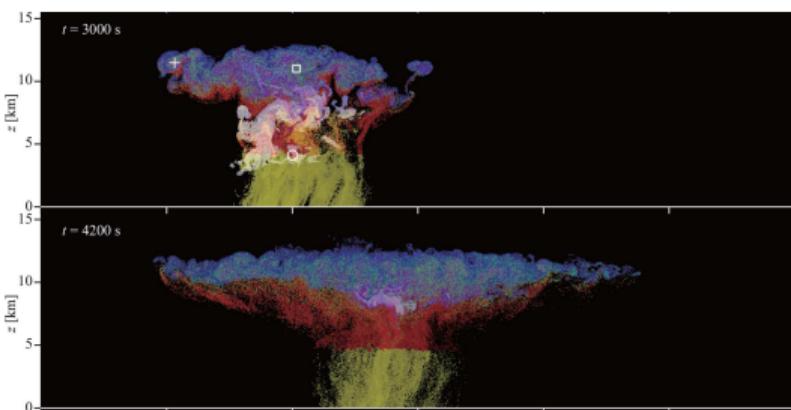


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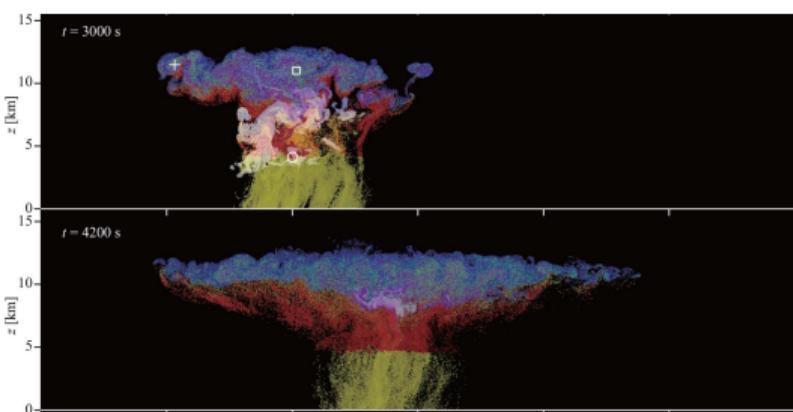


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- ▶ particle-resolved processes:
  - advection and sedimentation
  - homogeneous and immersion freezing (singular)
  - melting
  - condensation and evaporation (incl. CCN [de]activation)
  - deposition and sublimation
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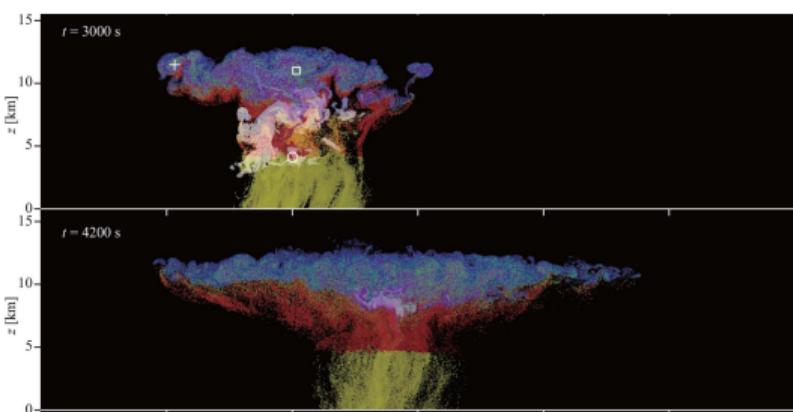


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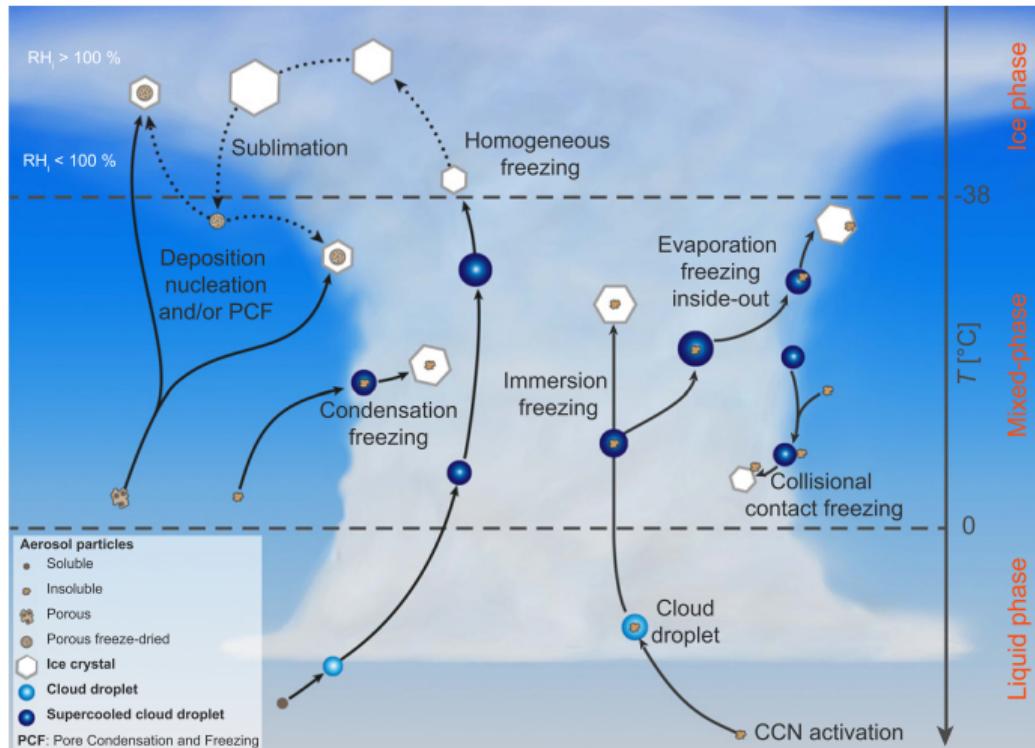
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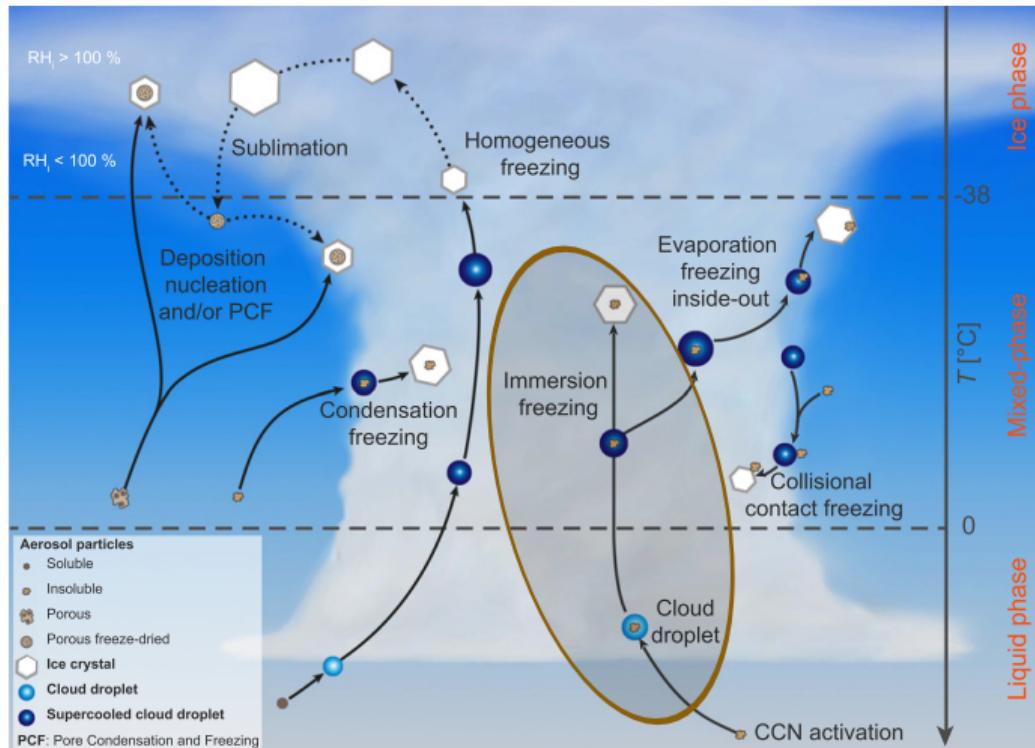
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# Immersion freezing and other ice crystal formation pathways in clouds



Kanji et al. 2017, graphics F. Mahrt, <https://doi.org/10.1175/AMSMONOGRAPHS-D-16-0006.1>

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# Immersion freezing: bacteria and the Olympics

## Journal of Geophysical Research: Atmospheres

### RESEARCH ARTICLE

10.1002/2016JD025251

#### Key Points:

- Very ice active Snomax protein aggregates are fragile and their ice nucleation ability decreases over months of freezer storage
  - Partitioning of ice active protein aggregates into the immersion oil reduces the droplet's measured freezing temperature
- Freezing is measured in the core of

### The unstable ice nucleation properties of Snomax® bacterial particles

Michael Polen<sup>1</sup>, Emily Lawlis<sup>1</sup>, and Ryan C. Sullivan<sup>1</sup>

<sup>1</sup>Center for Atmospheric Particle Studies, Carnegie Mellon University, Pittsburgh, Pennsylvania, USA

**Abstract** Snomax® is often used as a surrogate for biological ice nucleating particles (INPs) and has recently been proposed as an INP standard for evaluating ice nucleation methods. We have found the immersion freezing properties of Snomax particles to be substantially unstable, observing a loss of ice nucleation ability

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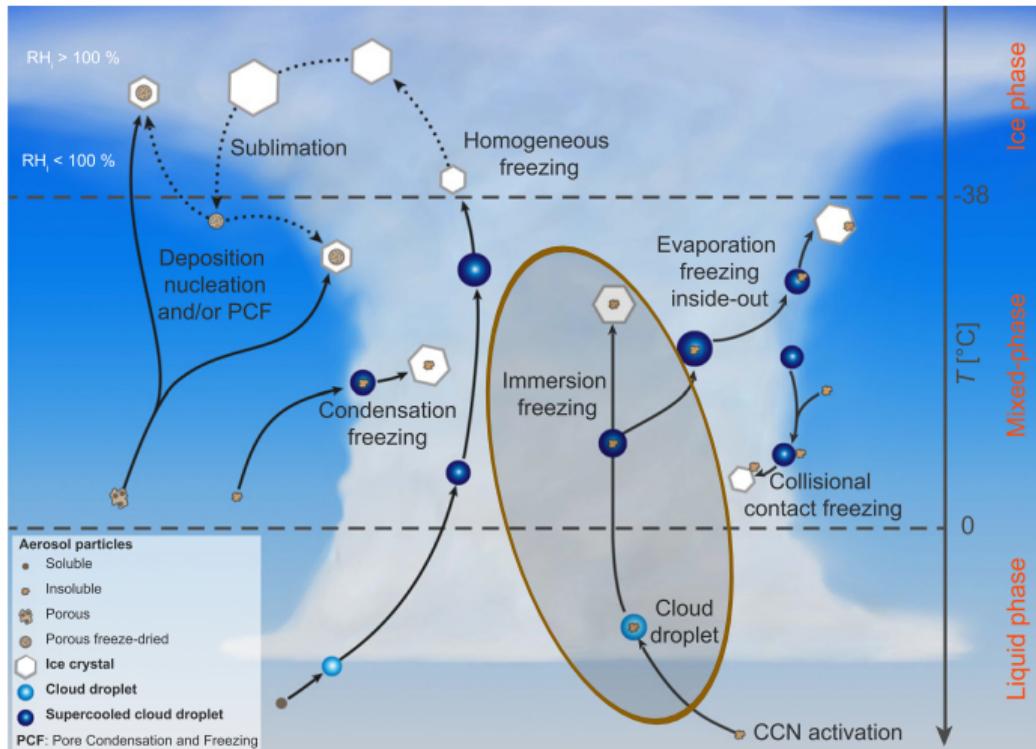
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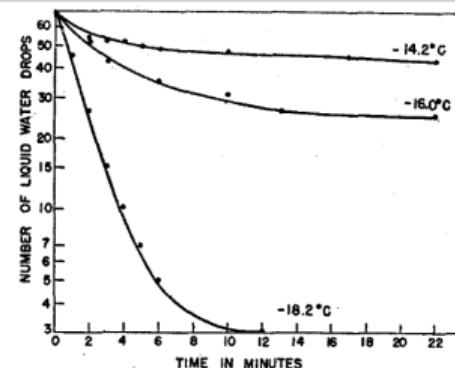


<https://www.reuters.com/markets/commodities/making-snow-stick-wind-challenges-winter-games-slope-makers-2021-11-29/>

# Immersion freezing and other ice crystal formation pathways in clouds

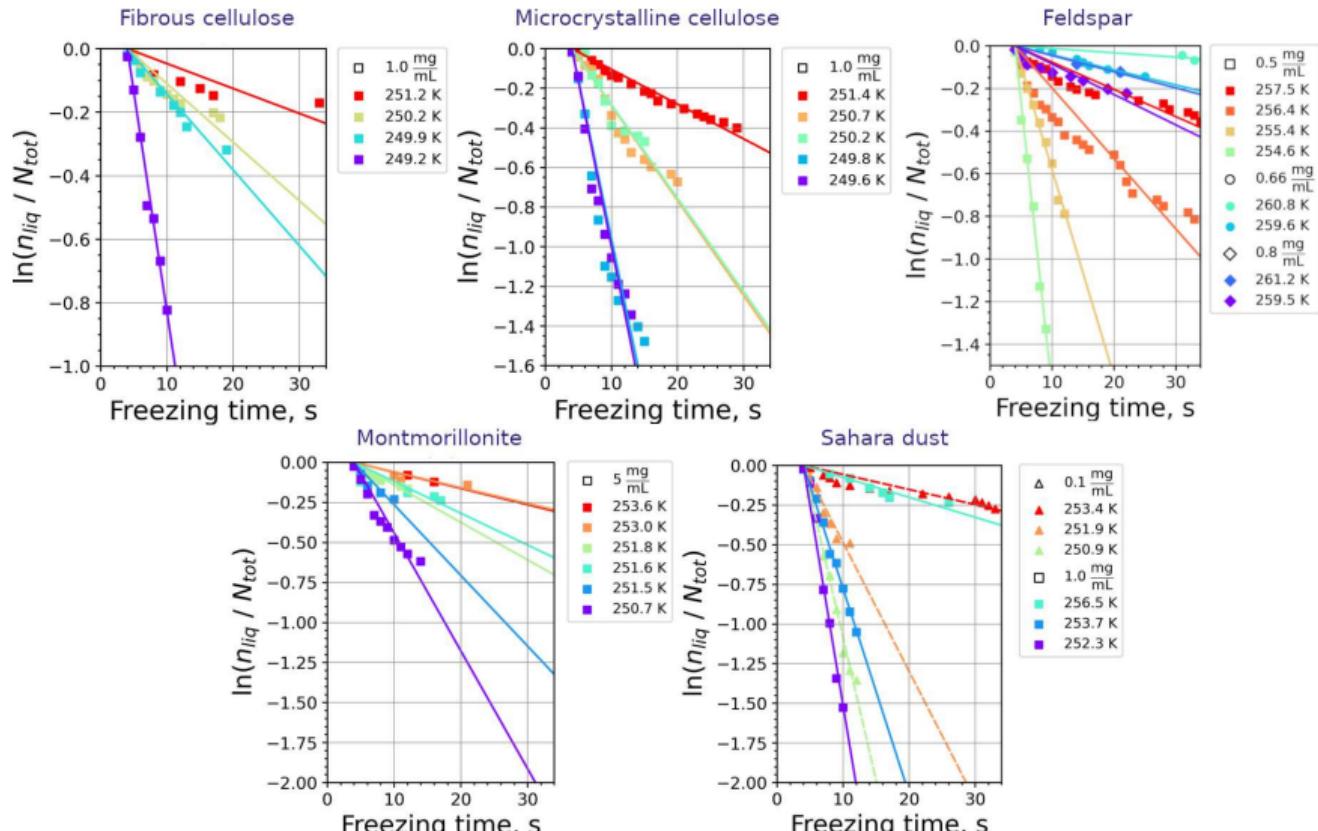


Vonnegut 1948 (J. Colloid Sci.)



Fraction of water drops remaining unfrozen as a function of time.

# Szakáll et al. 2021, ACP 21: isothermal experiments



# Heterogeneous Nucleations is a Stochastic Process

by

J. S. MARSHALL

McGill University, Montreal, Canad.

*Presented at the International Congress on the Physics of Clouds (Hailstorms)  
at Verona 9-13 August 1960.*

# Poissonian model of freezing & Ice Nucleation Active Sites (INAS)

theory (in modern notation)

(Bigg '53, Langham & Mason '58, Carte '59, Marshall '61)

Poisson counting process with rate  $r$ :

$$P^*(k \text{ events in time } t) = \frac{(rt)^k \exp(-rt)}{k!}$$

$$P(\text{one or more events in time } t) = 1 - P^*(k = 0, t)$$

$$\ln(1 - P) = -rt$$

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introducing  $J_{\text{het}}(T)$ ,  $T(t)$  and INP surface  $A$ :

$$\ln(1 - P(A, t)) = -A \underbrace{\int_0^t J_{\text{het}}(T(t')) dt'}$$

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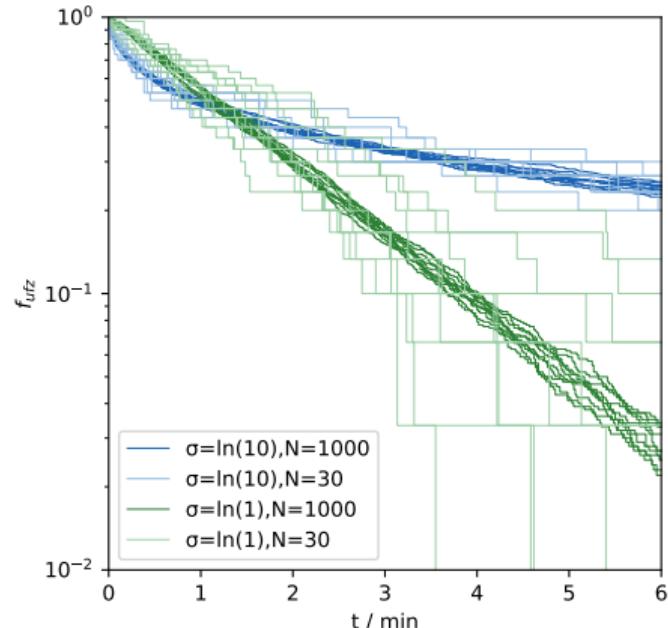
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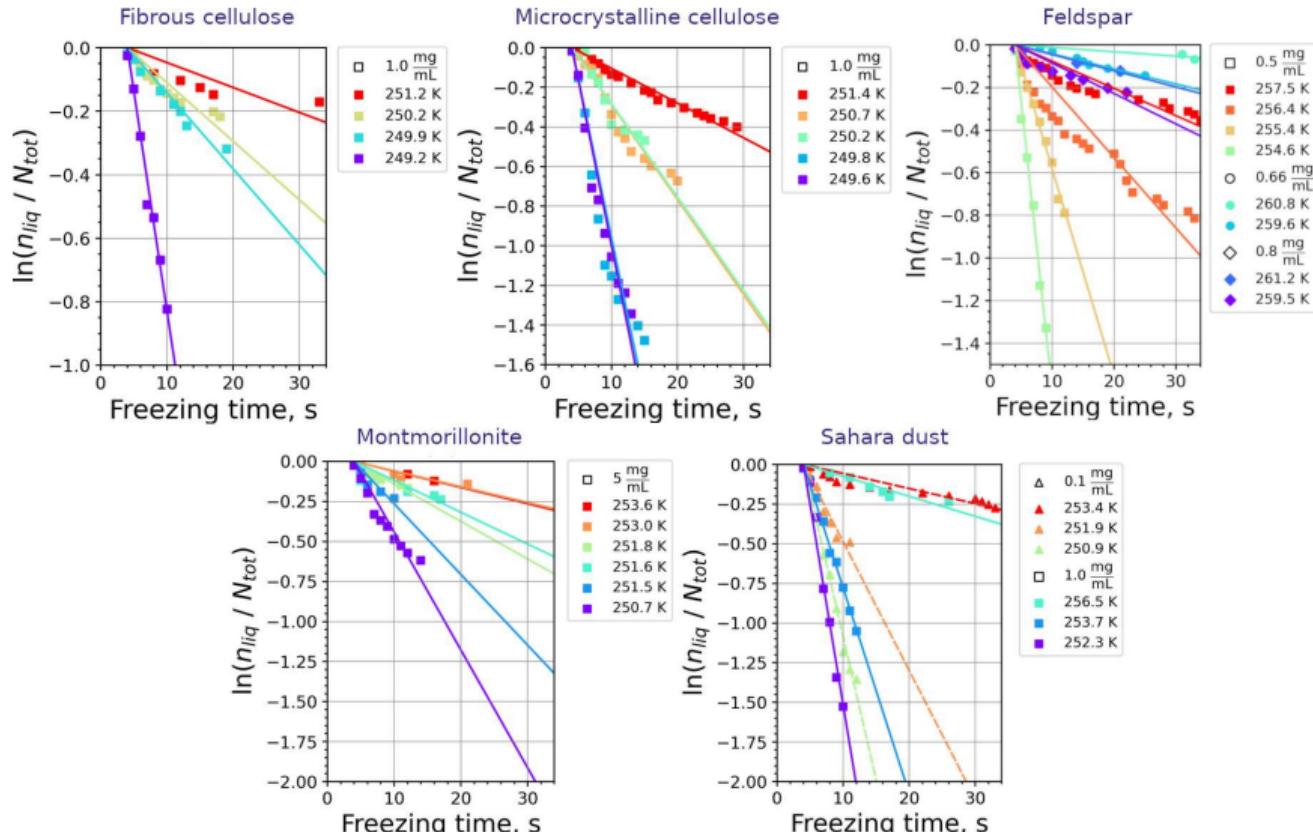
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Monte Carlo: const  $J_{\text{het}}$ , lognormal  $A$



(as in Alpert & Knopf 2016, Fig. 1a)

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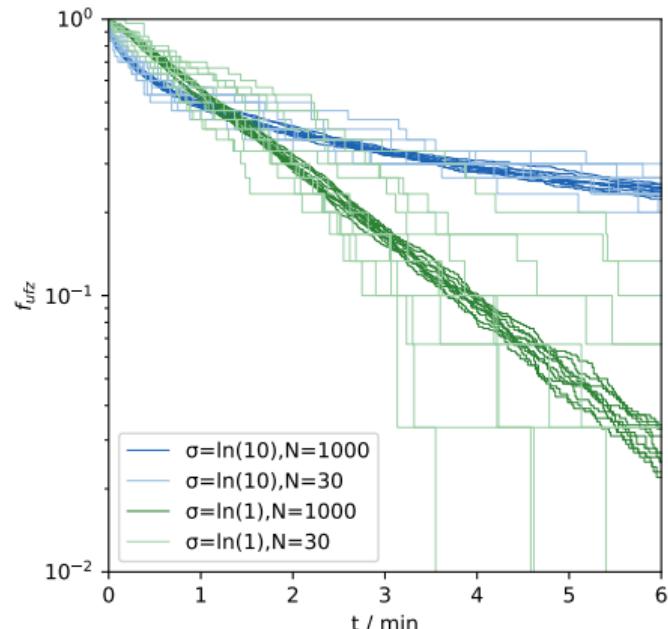
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Monte Carlo: const  $J_{\text{het}}$ , lognormal  $A$



(as in Alpert & Knopf 2016, Fig. 1a)

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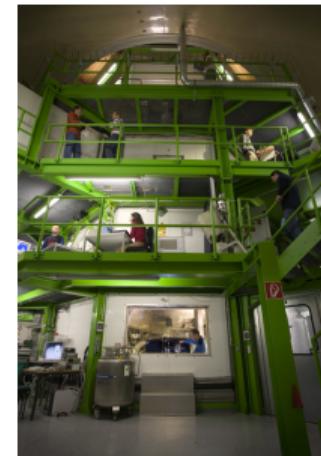
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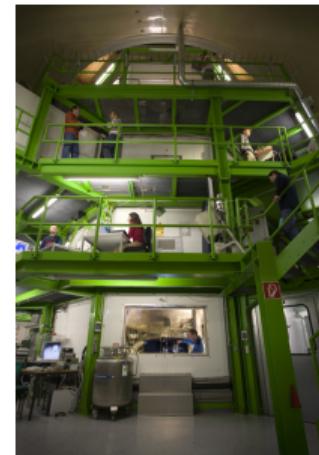
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# AIDA @ KIT



(<https://www.imk-aaf.kit.edu/>, photo: KIT/Ottmar Möhler)

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AIDA cooling rate: ca.  $0.5\text{ K}/\text{min}$

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# $J_{\text{het}}$ or $n_s$ ?

Vali 2014 (ACP)

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 Atmospheric Chemistry and Physics

**Heterogeneous ice nucleation: exploring the transition from stochastic to singular freezing behavior**

Atmos. Chem. Phys., 11, 8767–8775, 2011  
www.atmos-chem-phys.net/11/8767/2011/  
doi:10.5194/acp-11-8767-2011  
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**npj | climate and atmospheric science**

Article | [Open access](#) | Published: 17 January 2020

**Stochastic nucleation processes and substrate abundance explain time-dependent freezing in supercooled droplets**

D. Niedermeyer<sup>1</sup>, R. A. Shaw<sup>2</sup>, S. Hartmann<sup>1</sup>, H. Wex<sup>1</sup>, T. Clauss<sup>1</sup>, J. Voigtlander<sup>1</sup>, and F. Stratmann<sup>1</sup>

<sup>1</sup>Leibniz Institute for Tropospheric Research, 04318 Leipzig, Germany  
<sup>2</sup>Dept. of Physics, Michigan Technological University, Houghton, Michigan 49931, USA

Received: 24 June 2011 – Accepted: 19 August 2011 – Published in Atmos. Chem. Phys. Discuss.: 28 January 2011

Revised: 24 June 2011 – Accepted: 19 August 2011 – Published in Atmos. Chem. Phys. Discuss.: 30 August 2011

**Minimal cooling rate dependence of ice nuclei activity in the immersion mode**

JOURNAL OF GEOPHYSICAL RESEARCH: ATMOSPHERES, VOL. 118, 10,201–10,213, 2013

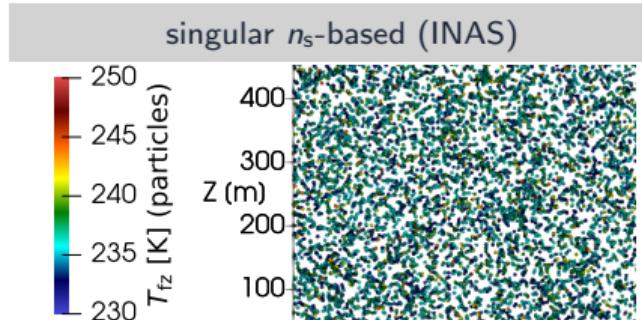
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Timothy P. Wright,<sup>1</sup> Markus D. Petters,<sup>1</sup> John D. Hader,<sup>1</sup> Travis Morton,<sup>1</sup> and Amara L. Holder,<sup>1</sup>

Received 12 April 2013; revised 4 September 2013; accepted 5 September 2013; published 23 September 2013.

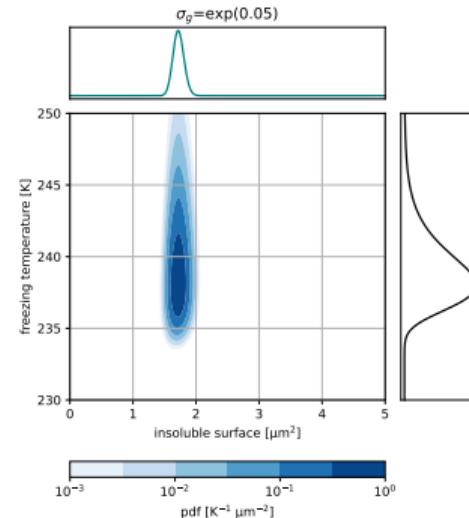
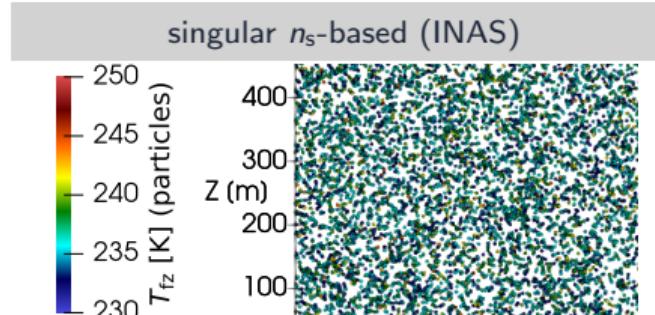
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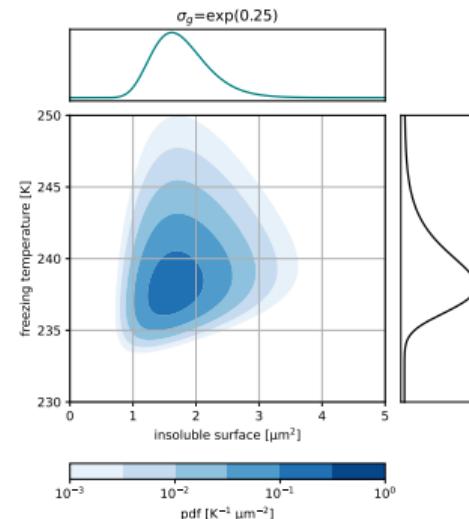
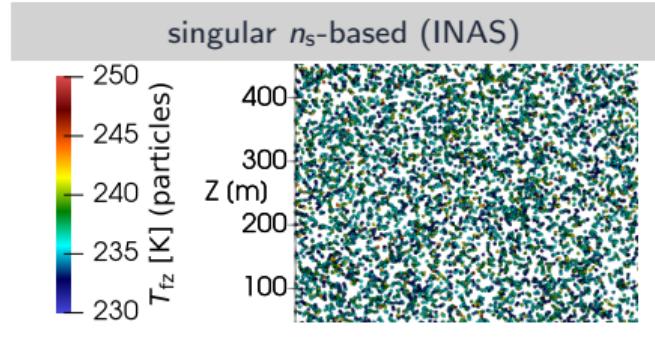
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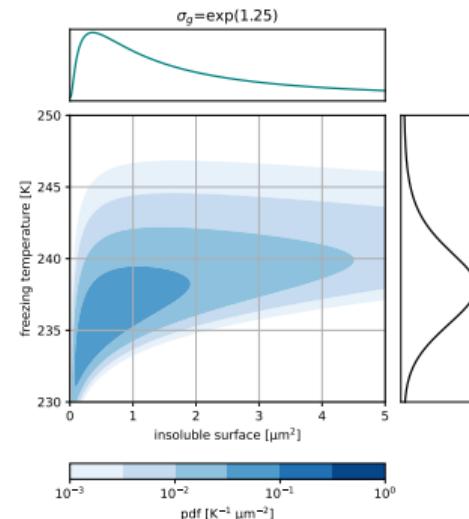
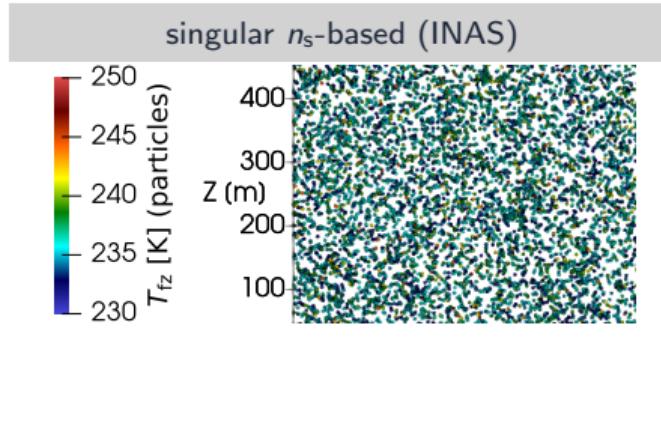
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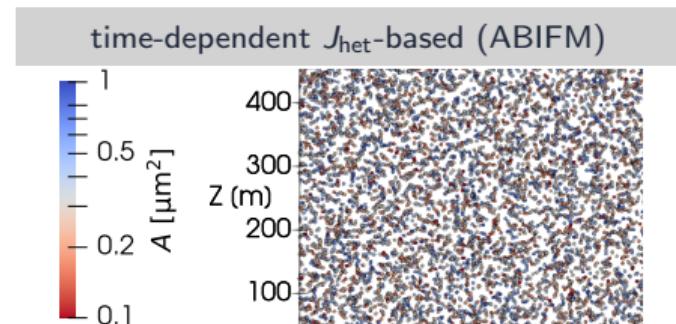
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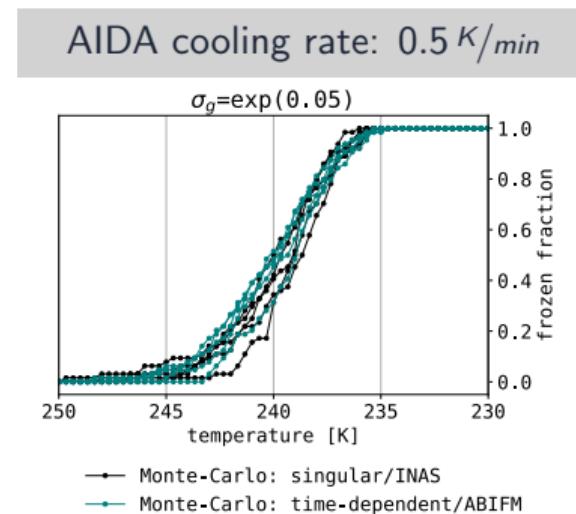
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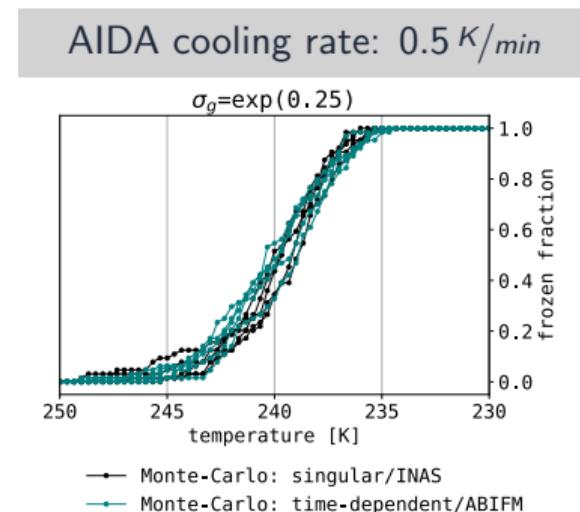
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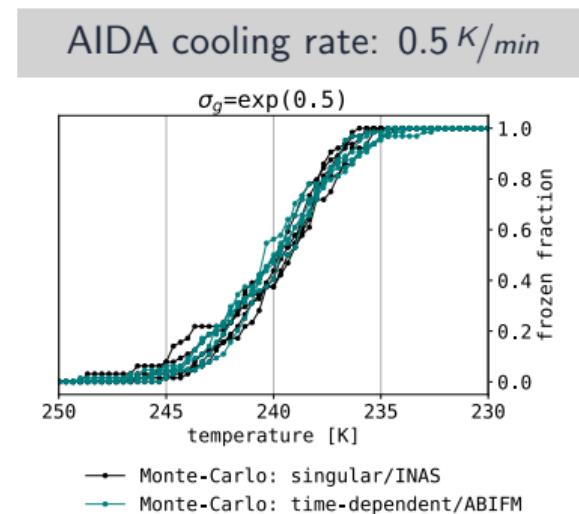
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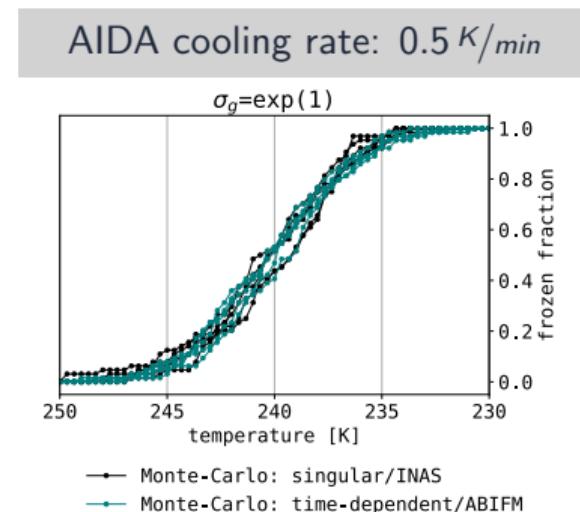
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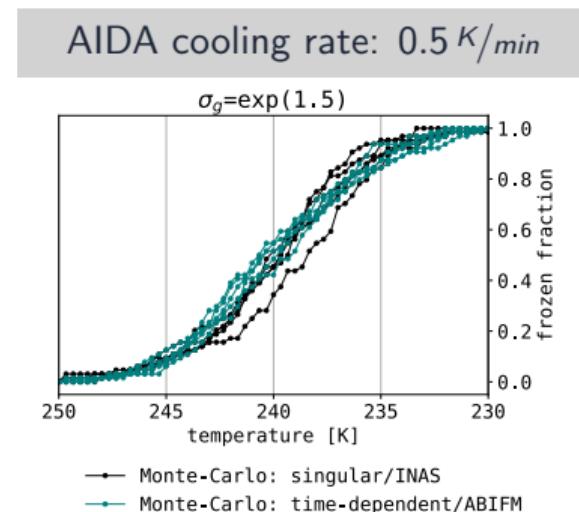
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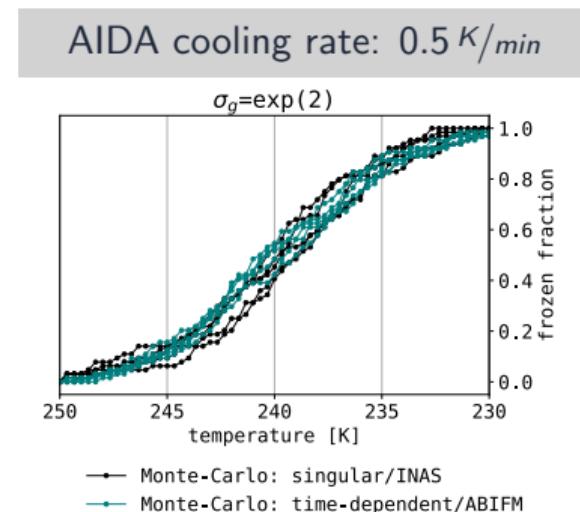
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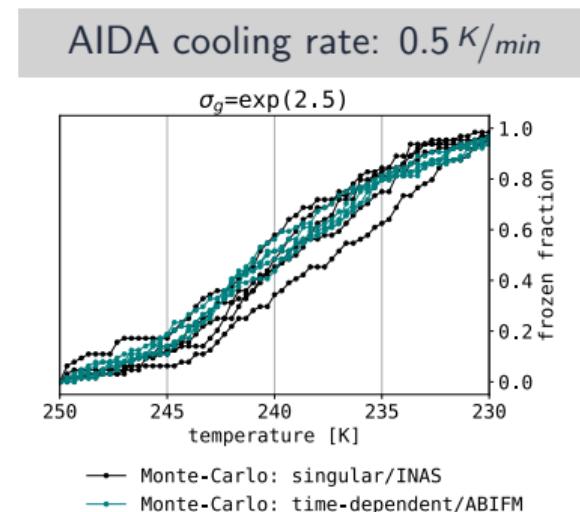
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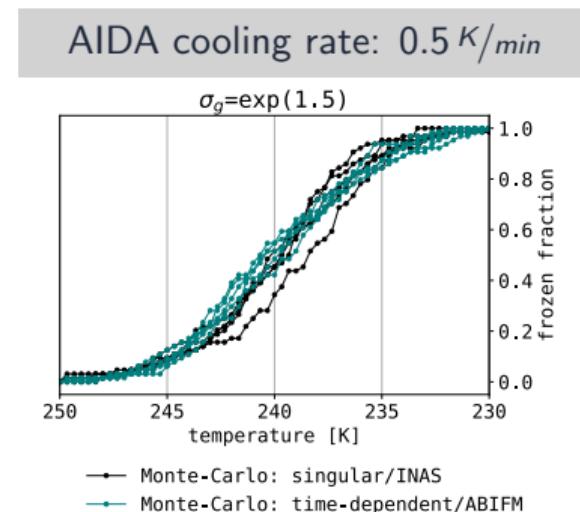
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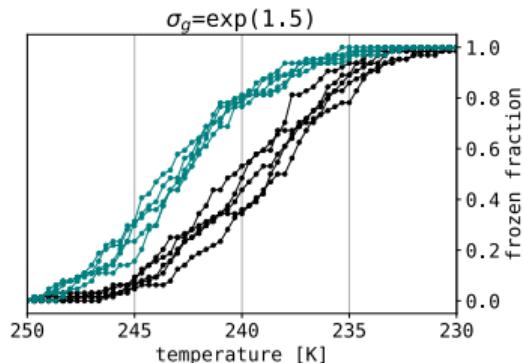


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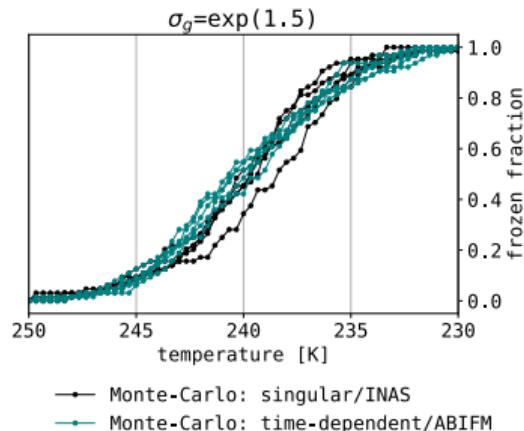
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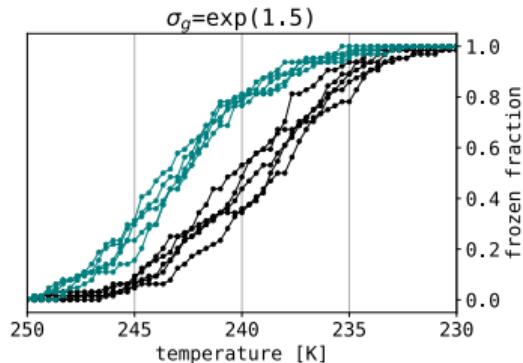


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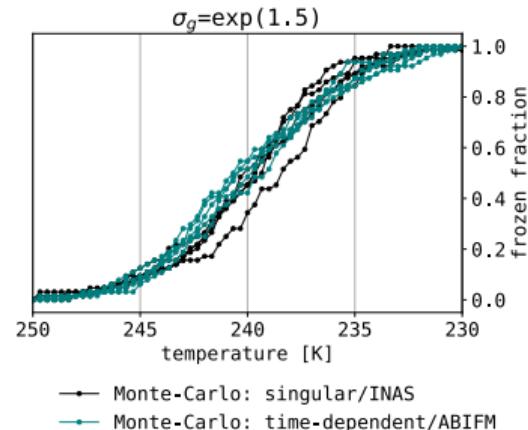
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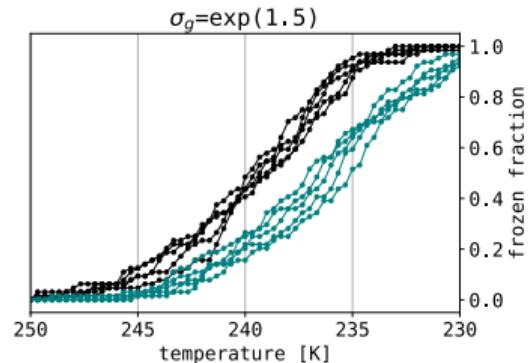
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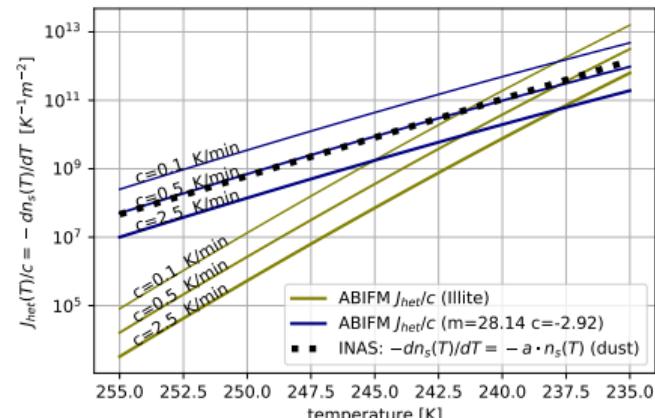
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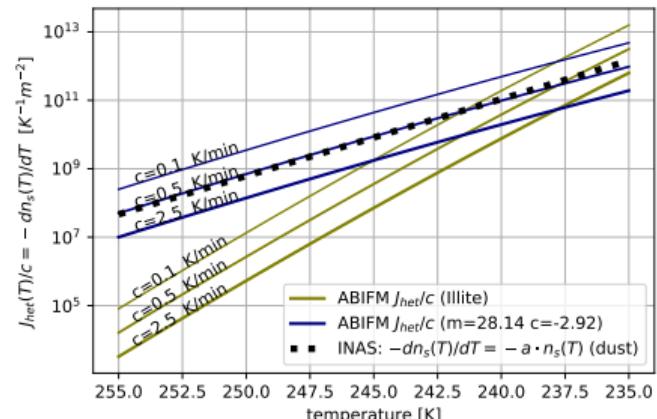
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$$-\frac{1}{c} J_{\text{het}}(T) = \frac{dn_s(T)}{dT} = a \cdot n_s(T)$$

experimental fits: INAS  $n_s$  (Niemand et al. '12)  
ABIFM  $J_{\text{het}}$  (Knopf & Alpert '13)



cf. Vali & Stansbury '66; modified singular model (Vali '94, Murray et al. '11)  
but the singular ansatz limitation of sampling  $T_{\text{fz}}$  at  $t=0$  remains

# Poissonian model of freezing & Ice Nucleation Active Sites (INAS)

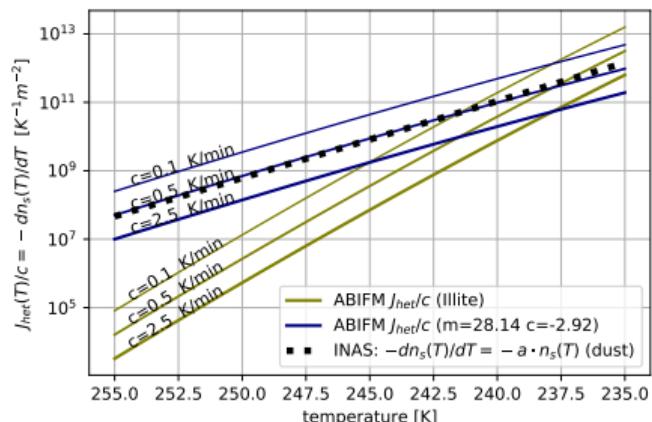
for a constant cooling rate  $c = dT/dt$ :

$$\ln(1 - P(A, t \rightsquigarrow T_{\text{fz}})) = -\frac{A}{c} \int_{T_0}^{T_0 + ct} J_{\text{het}}(T') dT' = -A \cdot n_s(T_{\text{fz}})$$

$$-\frac{1}{c} J_{\text{het}}(T) = \frac{dn_s(T)}{dT} = a \cdot n_s(T)$$

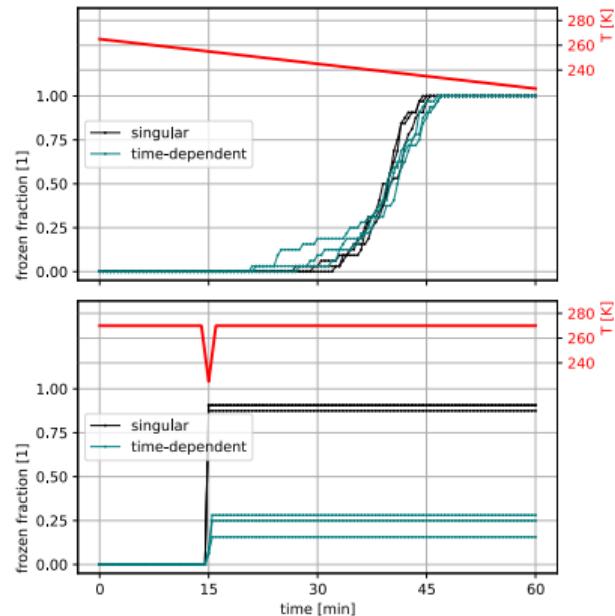
experimental fits: INAS  $n_s$  (Niemand et al. '12)  
ABIFM  $J_{\text{het}}$  (Knopf & Alpert '13)

## Is it a problem?

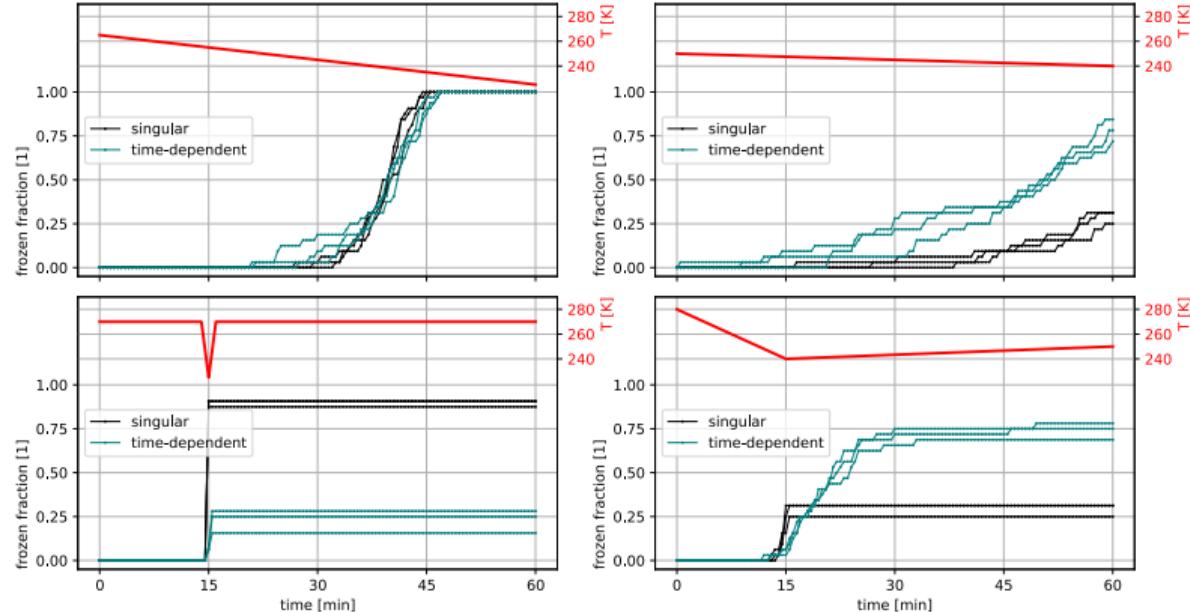


cf. Vali & Stansbury '66; modified singular model (Vali '94, Murray et al. '11)  
but the singular ansatz limitation of sampling  $T_{\text{fz}}$  at  $t=0$  remains

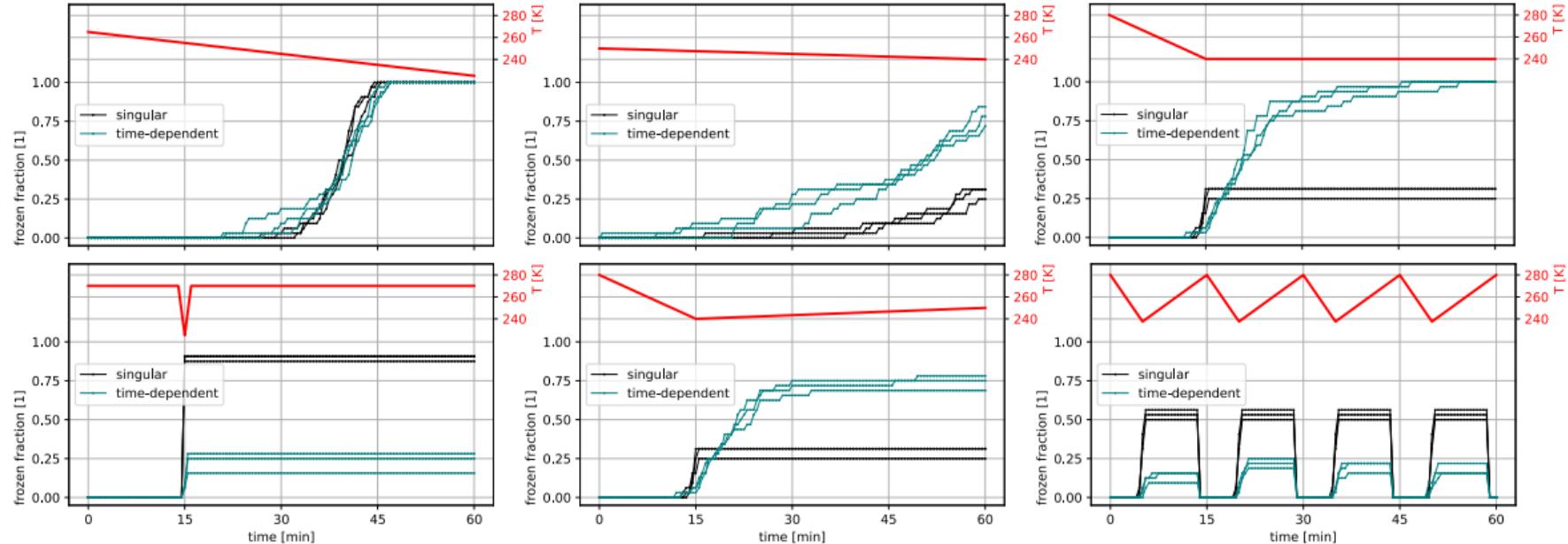
# Testing different cooling-rate profiles in a box model



# Testing different cooling-rate profiles in a box model

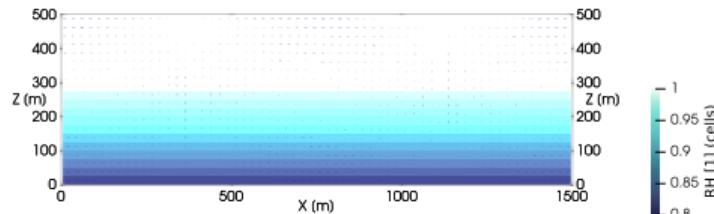


# Testing different cooling-rate profiles in a box model



# Testing three flow regimes and two immersion freezing representations

$w_{\max} \approx 1/3 \text{ m/s}$

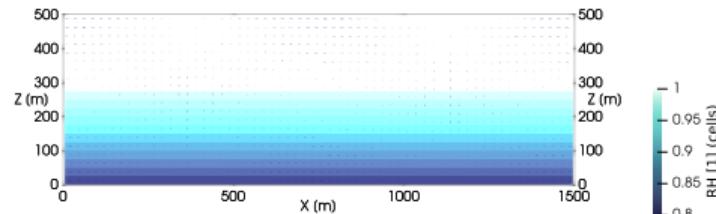


$w_{\max} \approx 1 \text{ m/s}$

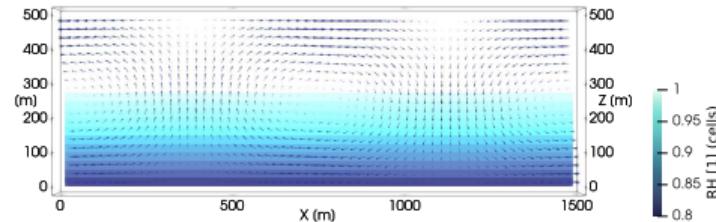
$w_{\max} \approx 3 \text{ m/s}$

# Testing three flow regimes and two immersion freezing representations

$w_{\max} \approx 1/3 \text{ m/s}$



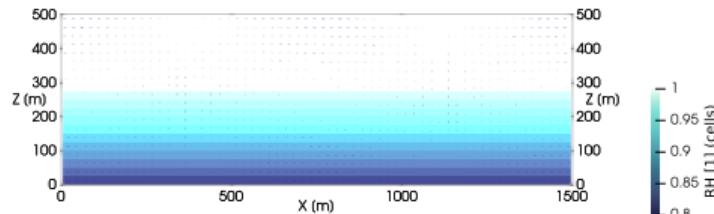
$w_{\max} \approx 1 \text{ m/s}$



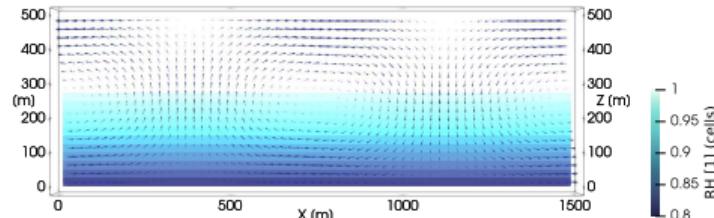
$w_{\max} \approx 3 \text{ m/s}$

# Testing three flow regimes and two immersion freezing representations

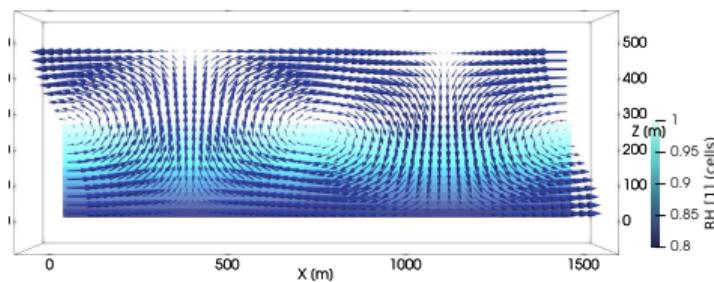
$w_{\max} \approx 1/3 \text{ m/s}$



$w_{\max} \approx 1 \text{ m/s}$

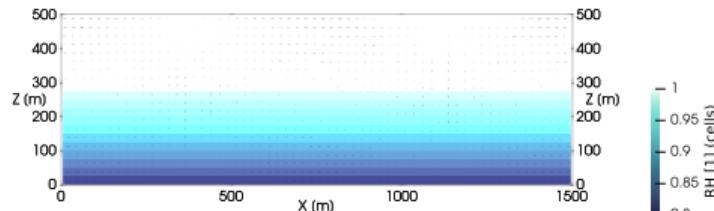


$w_{\max} \approx 3 \text{ m/s}$

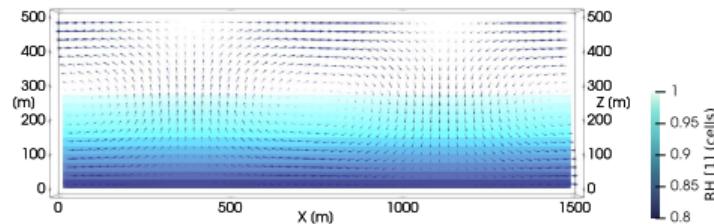


# Testing three flow regimes and two immersion freezing representations

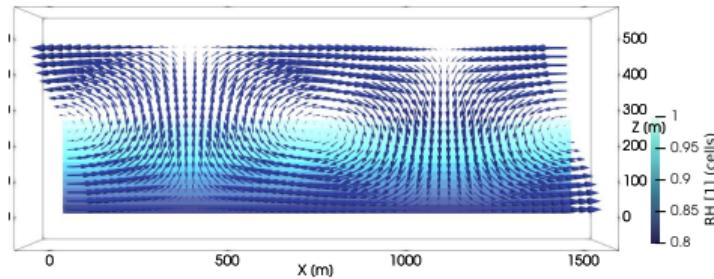
$w_{\max} \approx 1/3 \text{ m/s}$



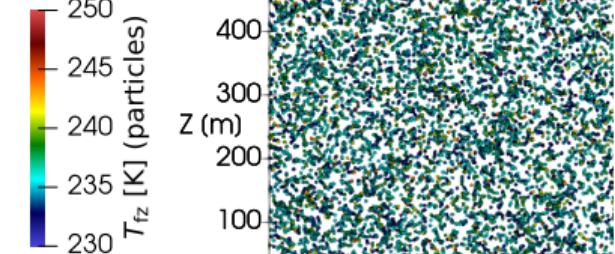
$w_{\max} \approx 1 \text{ m/s}$



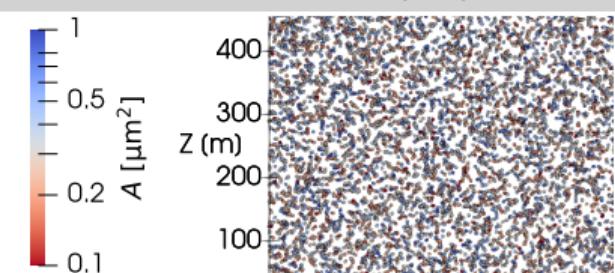
$w_{\max} \approx 3 \text{ m/s}$



singular (INAS)

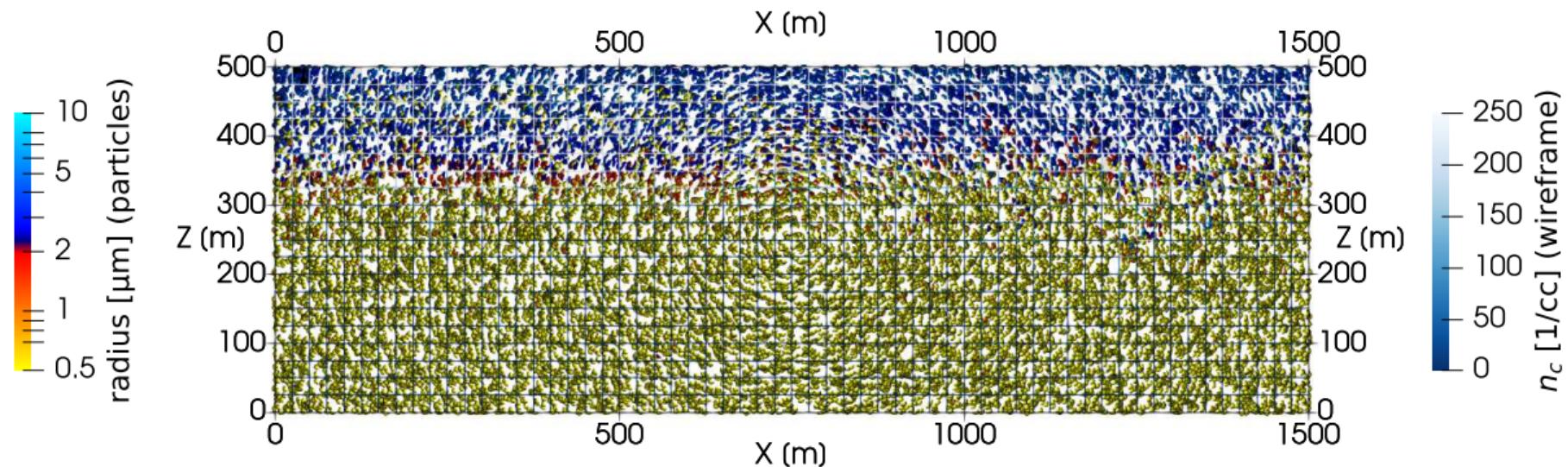


time-dependent ( $J_{\text{het}}$ )



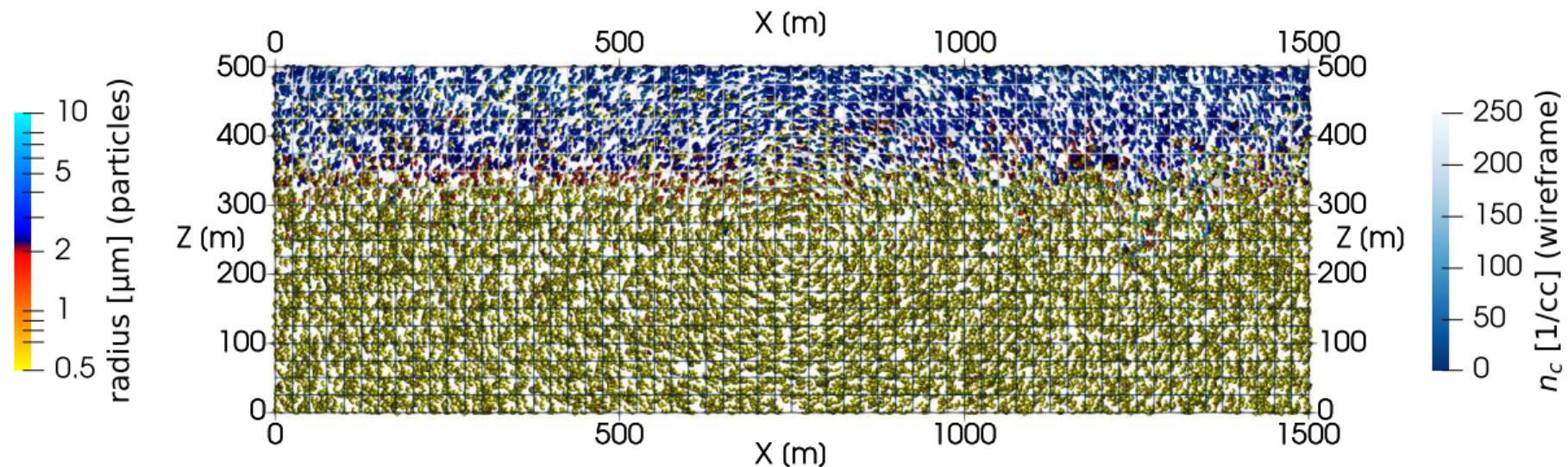
# Particle-based $\mu$ -physics + prescribed-flow: glaciation

Time: 630 s (spin-up till 600.0 s)



# Particle-based $\mu$ -physics + prescribed-flow: glaciation

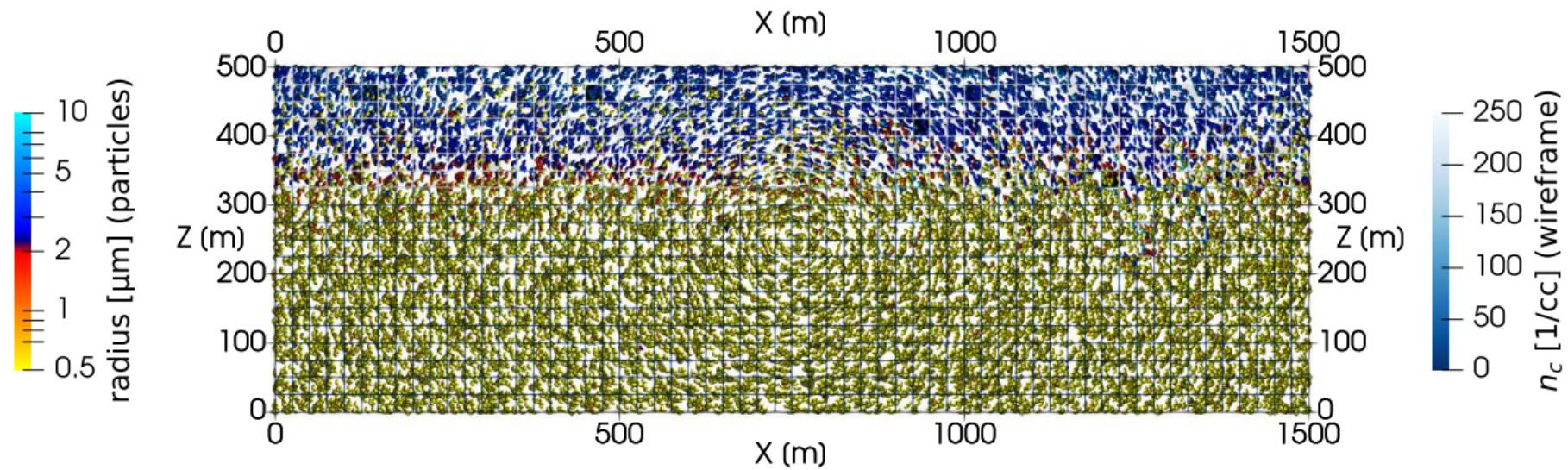
Time: 660 s (spin-up till 600.0 s)



16+16 super-particles/cell for INP-rich + INP-free particles  
 $N_{\text{aer}} = 300/\text{cc}$  (two-mode lognormal)     $N_{\text{INP}} = 150/L$  (lognormal,  $D_g = 0.74 \mu\text{m}$ ,  $\sigma_g = 2.55$ )  
spin-up = freezing off; subsequently frozen particles act as tracers

# Particle-based $\mu$ -physics + prescribed-flow: glaciation

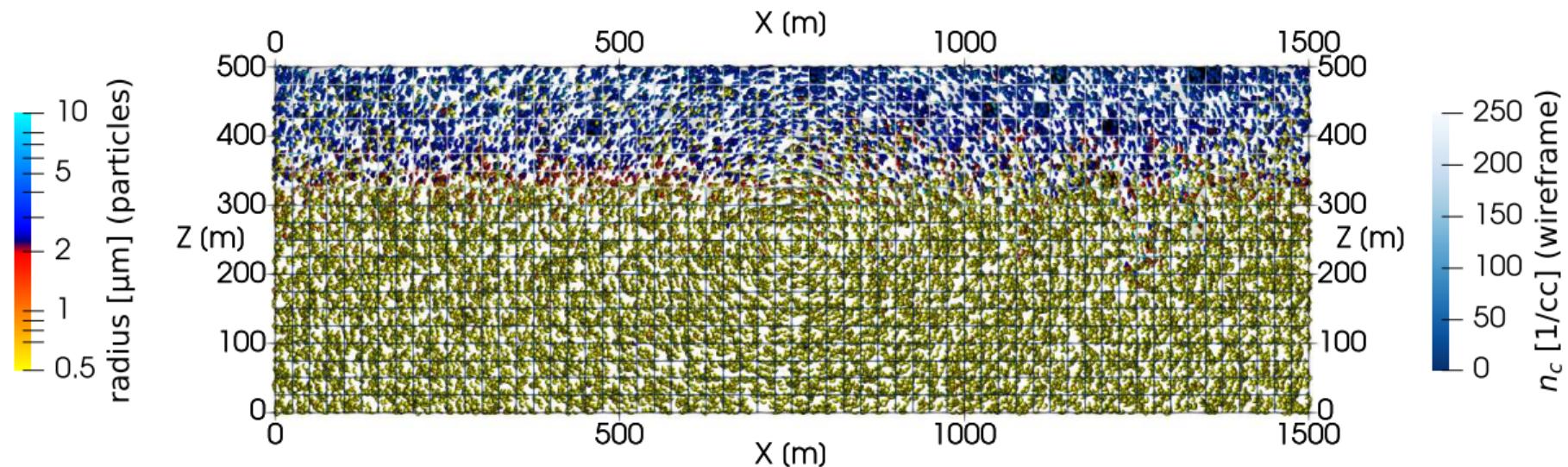
Time: 690 s (spin-up till 600.0 s)



16+16 super-particles/cell for INP-rich + INP-free particles  
 $N_{\text{aer}} = 300/\text{cc}$  (two-mode lognormal)     $N_{\text{INP}} = 150/L$  (lognormal,  $D_g = 0.74 \text{ } \mu\text{m}$ ,  $\sigma_g = 2.55$ )  
spin-up = freezing off; subsequently frozen particles act as tracers

# Particle-based $\mu$ -physics + prescribed-flow: glaciation

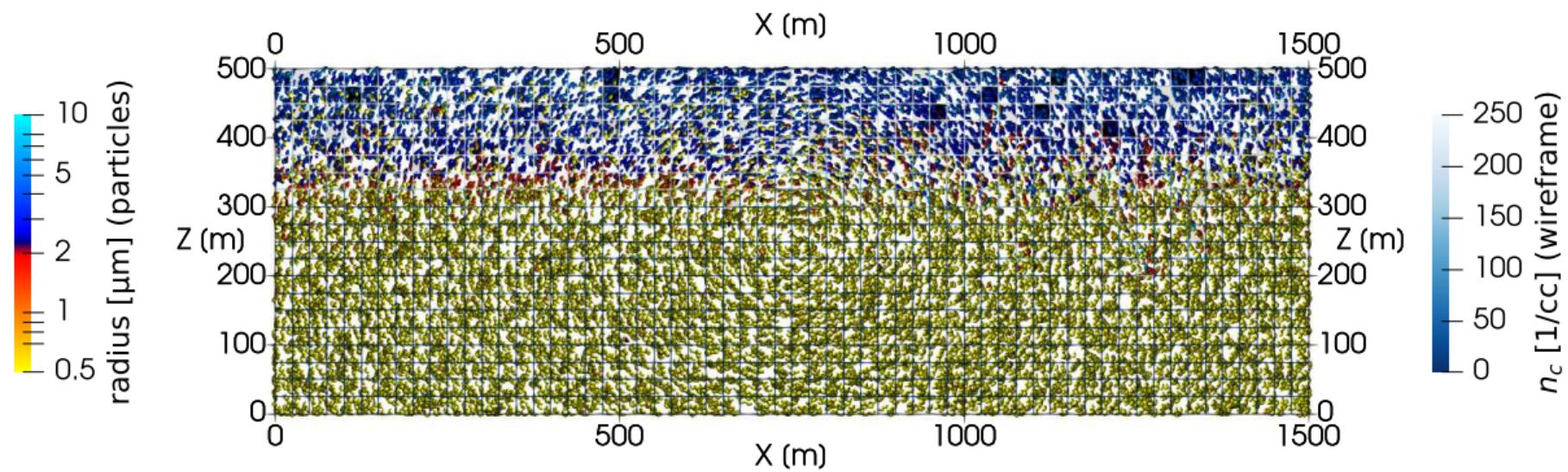
Time: 720 s (spin-up till 600.0 s)



16+16 super-particles/cell for INP-rich + INP-free particles  
 $N_{\text{aer}} = 300/\text{cc}$  (two-mode lognormal)     $N_{\text{INP}} = 150/L$  (lognormal,  $D_g = 0.74 \mu\text{m}$ ,  $\sigma_g = 2.55$ )  
spin-up = freezing off; subsequently frozen particles act as tracers

# Particle-based $\mu$ -physics + prescribed-flow: glaciation

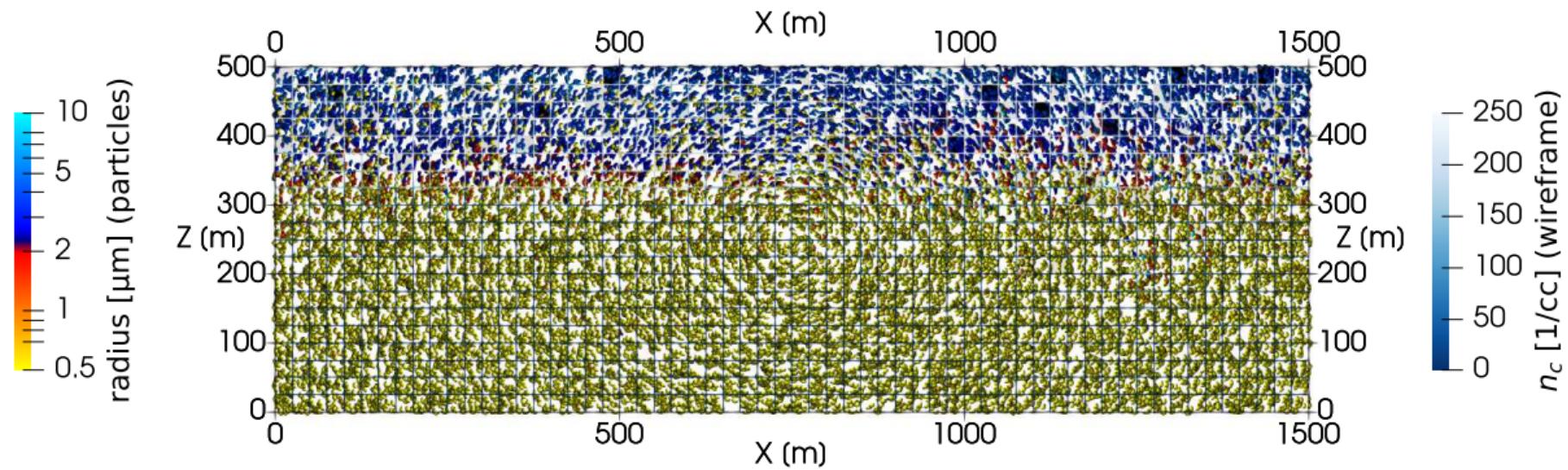
Time: 750 s (spin-up till 600.0 s)



16+16 super-particles/cell for INP-rich + INP-free particles  
 $N_{\text{aer}} = 300/\text{cc}$  (two-mode lognormal)     $N_{\text{INP}} = 150/L$  (lognormal,  $D_g = 0.74 \text{ } \mu\text{m}$ ,  $\sigma_g = 2.55$ )  
spin-up = freezing off; subsequently frozen particles act as tracers

# Particle-based $\mu$ -physics + prescribed-flow: glaciation

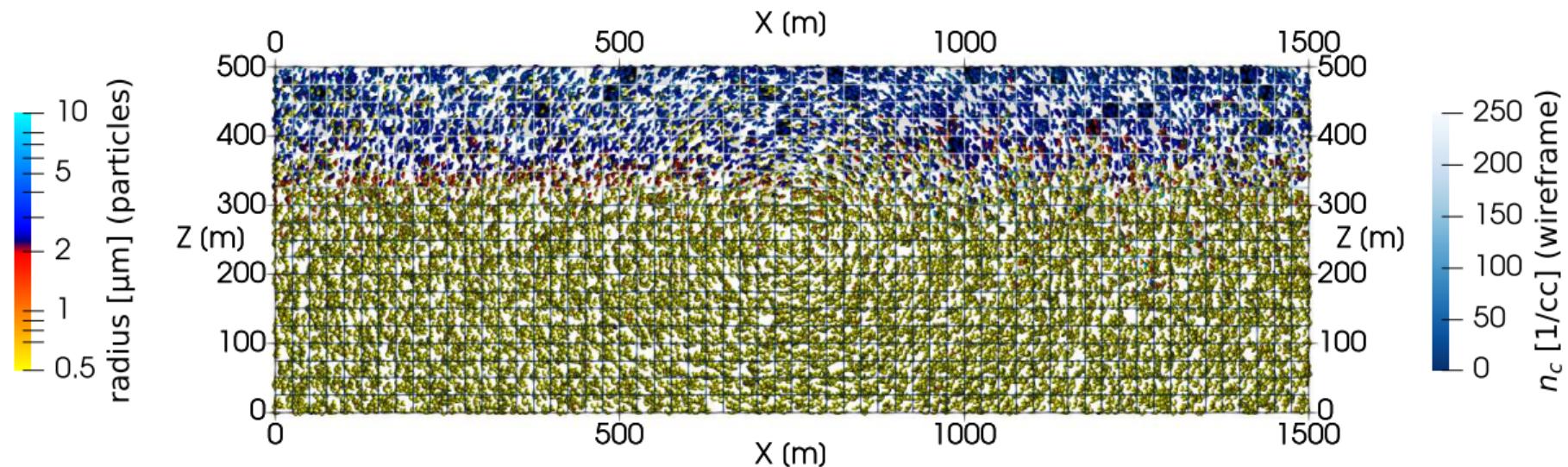
Time: 780 s (spin-up till 600.0 s)



16+16 super-particles/cell for INP-rich + INP-free particles  
 $N_{\text{aer}} = 300/\text{cc}$  (two-mode lognormal)     $N_{\text{INP}} = 150/L$  (lognormal,  $D_g = 0.74 \mu\text{m}$ ,  $\sigma_g = 2.55$ )  
spin-up = freezing off; subsequently frozen particles act as tracers

# Particle-based $\mu$ -physics + prescribed-flow: glaciation

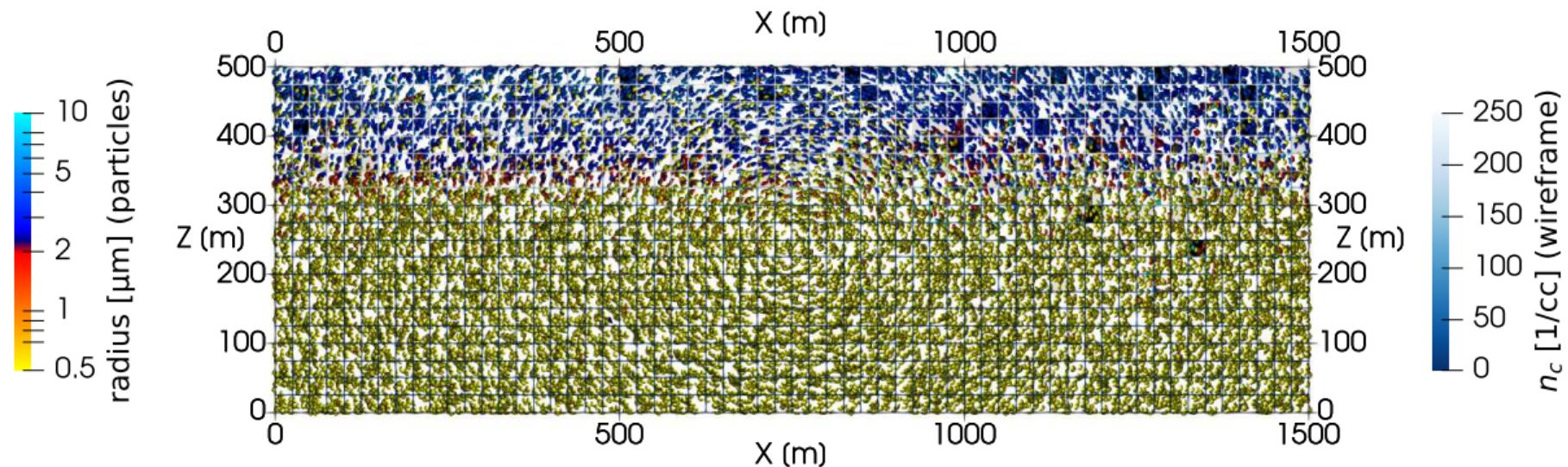
Time: 810 s (spin-up till 600.0 s)



16+16 super-particles/cell for INP-rich + INP-free particles  
 $N_{\text{aer}} = 300/\text{cc}$  (two-mode lognormal)     $N_{\text{INP}} = 150/L$  (lognormal,  $D_g = 0.74 \mu\text{m}$ ,  $\sigma_g = 2.55$ )  
spin-up = freezing off; subsequently frozen particles act as tracers

# Particle-based $\mu$ -physics + prescribed-flow: glaciation

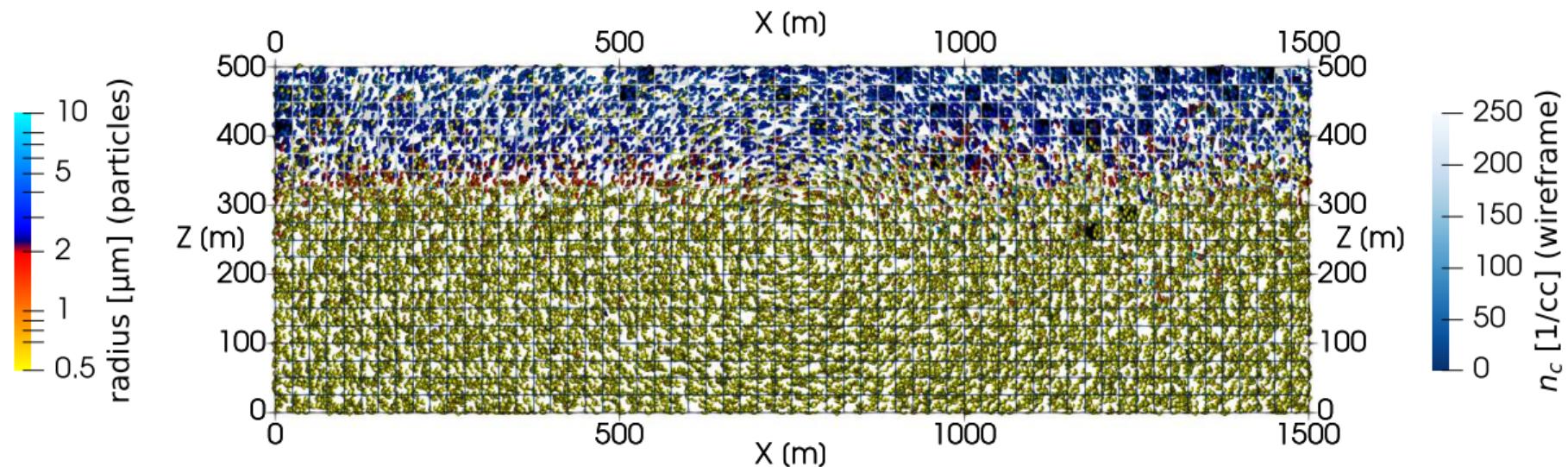
Time: 840 s (spin-up till 600.0 s)



16+16 super-particles/cell for INP-rich + INP-free particles  
 $N_{\text{aer}} = 300/\text{cc}$  (two-mode lognormal)     $N_{\text{INP}} = 150/L$  (lognormal,  $D_g = 0.74 \text{ } \mu\text{m}$ ,  $\sigma_g = 2.55$ )  
spin-up = freezing off; subsequently frozen particles act as tracers

# Particle-based $\mu$ -physics + prescribed-flow: glaciation

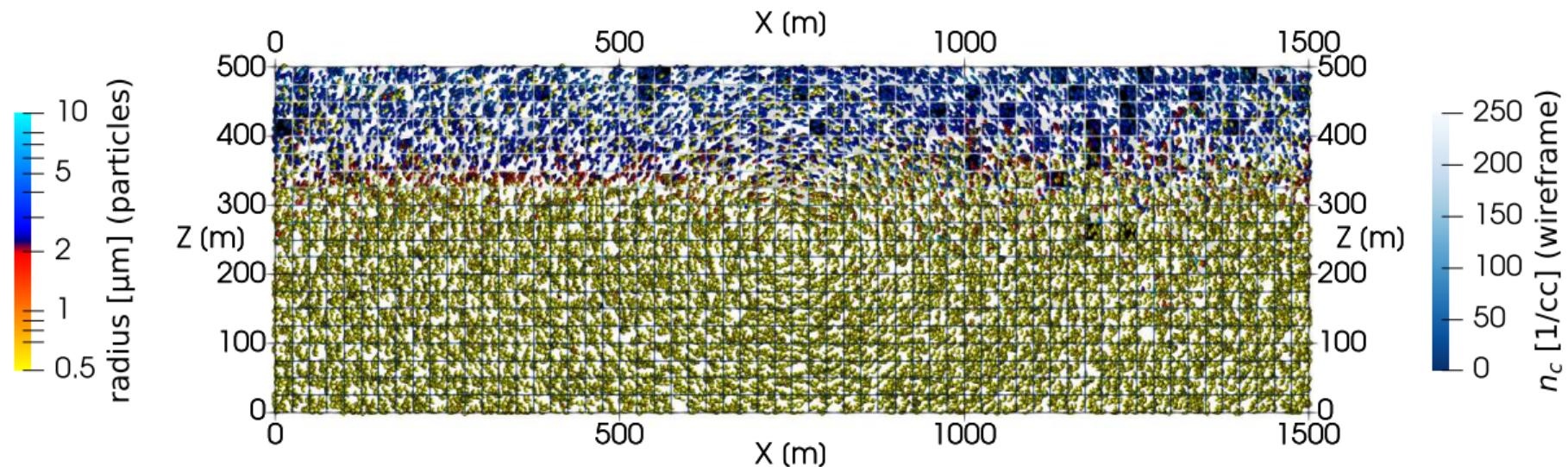
Time: 870 s (spin-up till 600.0 s)



16+16 super-particles/cell for INP-rich + INP-free particles  
 $N_{\text{aer}} = 300/\text{cc}$  (two-mode lognormal)     $N_{\text{INP}} = 150/L$  (lognormal,  $D_g = 0.74 \mu\text{m}$ ,  $\sigma_g = 2.55$ )  
spin-up = freezing off; subsequently frozen particles act as tracers

# Particle-based $\mu$ -physics + prescribed-flow: glaciation

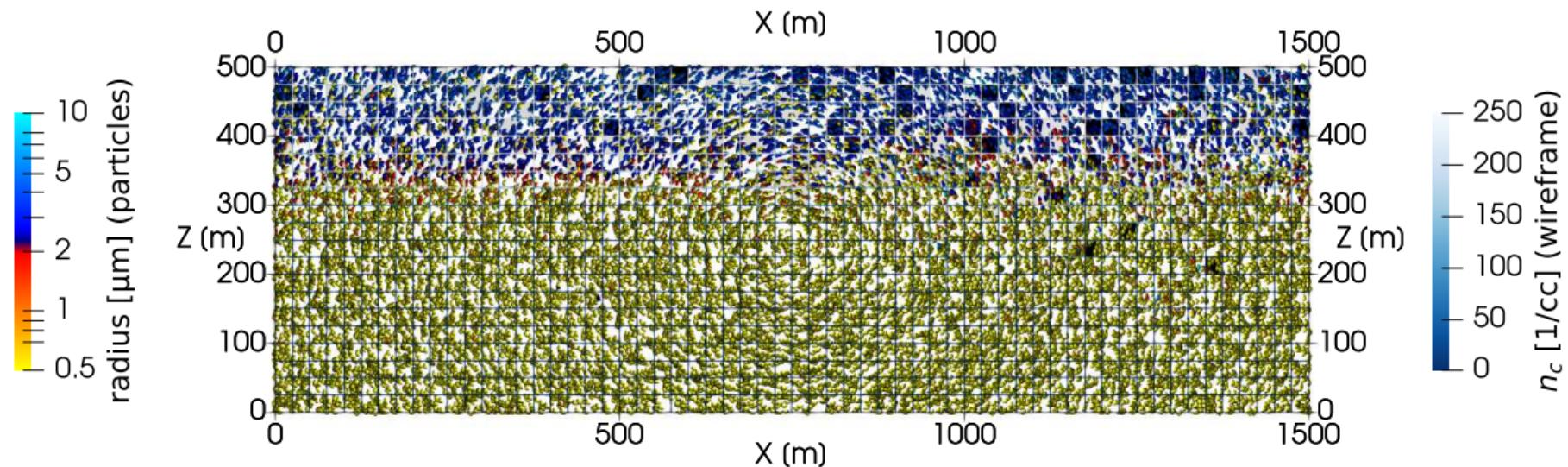
Time: 900 s (spin-up till 600.0 s)



16+16 super-particles/cell for INP-rich + INP-free particles  
 $N_{\text{aer}} = 300/\text{cc}$  (two-mode lognormal)     $N_{\text{INP}} = 150/L$  (lognormal,  $D_g = 0.74 \mu\text{m}$ ,  $\sigma_g = 2.55$ )  
spin-up = freezing off; subsequently frozen particles act as tracers

# Particle-based $\mu$ -physics + prescribed-flow: glaciation

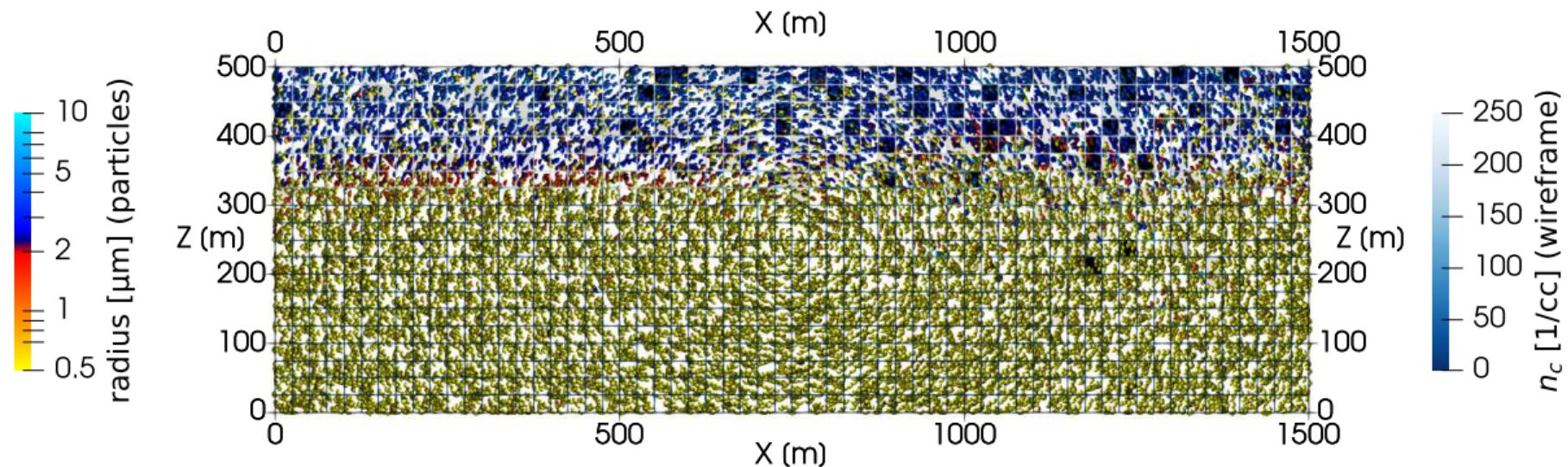
Time: 930 s (spin-up till 600.0 s)



16+16 super-particles/cell for INP-rich + INP-free particles  
 $N_{\text{aer}} = 300/\text{cc}$  (two-mode lognormal)     $N_{\text{INP}} = 150/L$  (lognormal,  $D_g = 0.74 \text{ } \mu\text{m}$ ,  $\sigma_g = 2.55$ )  
spin-up = freezing off; subsequently frozen particles act as tracers

# Particle-based $\mu$ -physics + prescribed-flow: glaciation

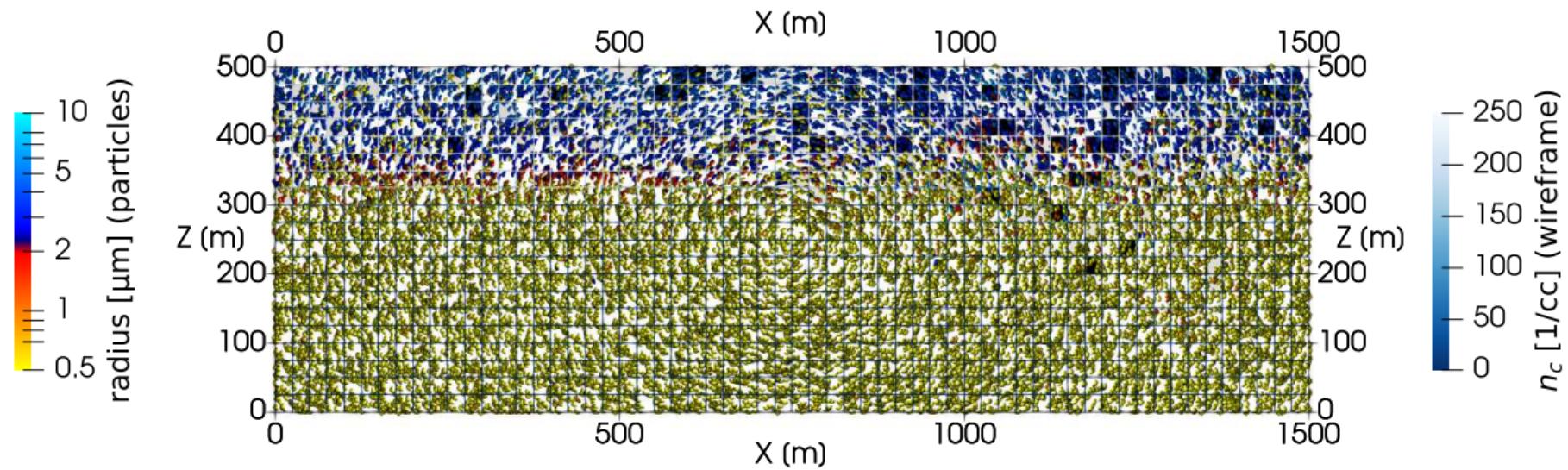
Time: 960 s (spin-up till 600.0 s)



16+16 super-particles/cell for INP-rich + INP-free particles  
 $N_{\text{aer}} = 300/\text{cc}$  (two-mode lognormal)     $N_{\text{INP}} = 150/L$  (lognormal,  $D_g = 0.74 \text{ } \mu\text{m}$ ,  $\sigma_g = 2.55$ )  
spin-up = freezing off; subsequently frozen particles act as tracers

# Particle-based $\mu$ -physics + prescribed-flow: glaciation

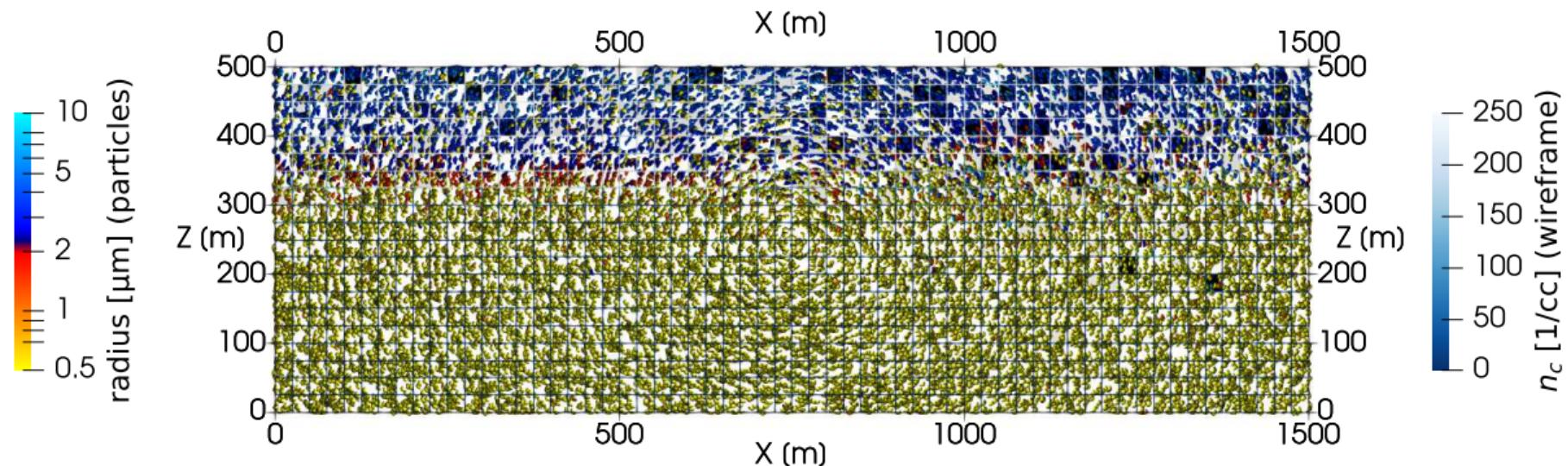
Time: 990 s (spin-up till 600.0 s)



16+16 super-particles/cell for INP-rich + INP-free particles  
 $N_{\text{aer}} = 300/\text{cc}$  (two-mode lognormal)     $N_{\text{INP}} = 150/L$  (lognormal,  $D_g = 0.74 \mu\text{m}$ ,  $\sigma_g = 2.55$ )  
spin-up = freezing off; subsequently frozen particles act as tracers

# Particle-based $\mu$ -physics + prescribed-flow: glaciation

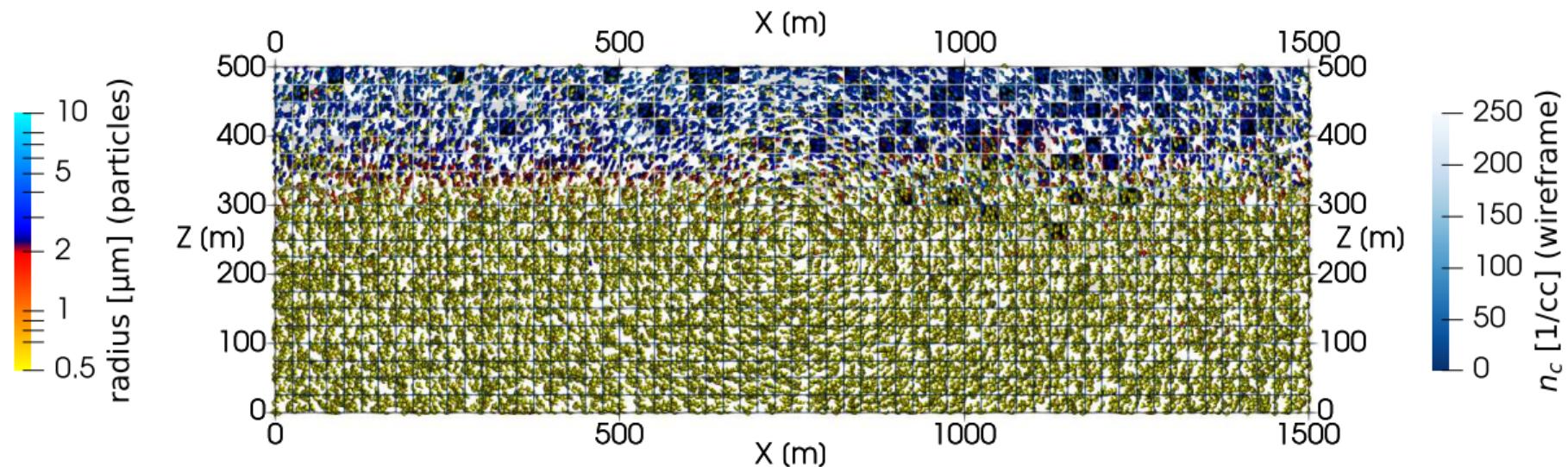
Time: 1020 s (spin-up till 600.0 s)



16+16 super-particles/cell for INP-rich + INP-free particles  
 $N_{\text{aer}} = 300/\text{cc}$  (two-mode lognormal)     $N_{\text{INP}} = 150/L$  (lognormal,  $D_g = 0.74 \mu\text{m}$ ,  $\sigma_g = 2.55$ )  
spin-up = freezing off; subsequently frozen particles act as tracers

# Particle-based $\mu$ -physics + prescribed-flow: glaciation

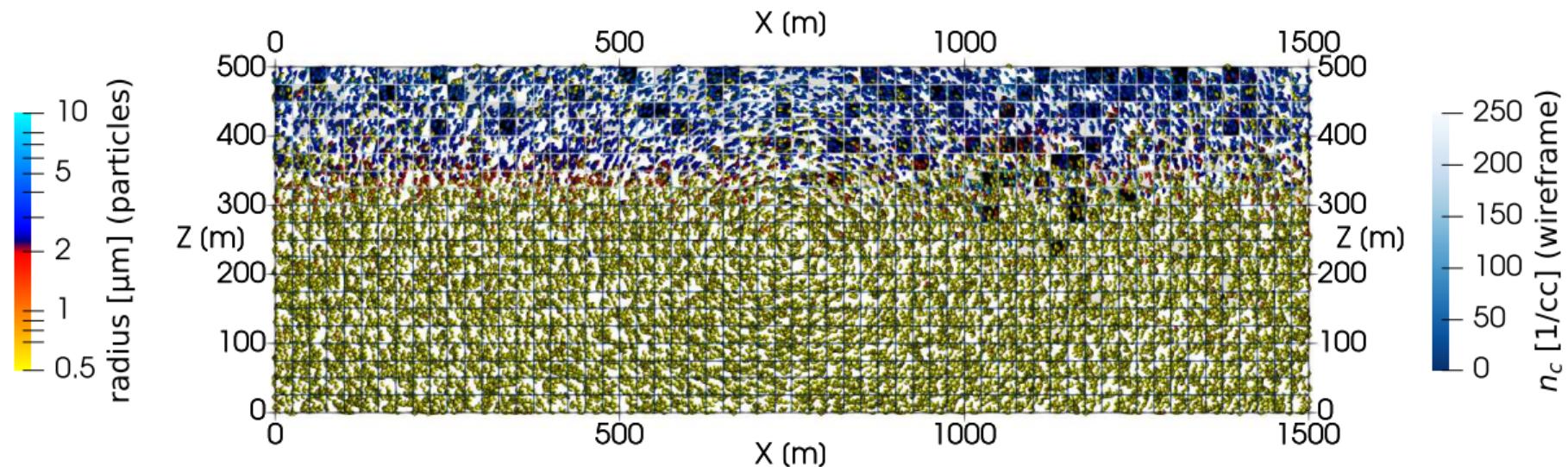
Time: 1050 s (spin-up till 600.0 s)



16+16 super-particles/cell for INP-rich + INP-free particles  
 $N_{\text{aer}} = 300/\text{cc}$  (two-mode lognormal)     $N_{\text{INP}} = 150/L$  (lognormal,  $D_g = 0.74 \mu\text{m}$ ,  $\sigma_g = 2.55$ )  
spin-up = freezing off; subsequently frozen particles act as tracers

# Particle-based $\mu$ -physics + prescribed-flow: glaciation

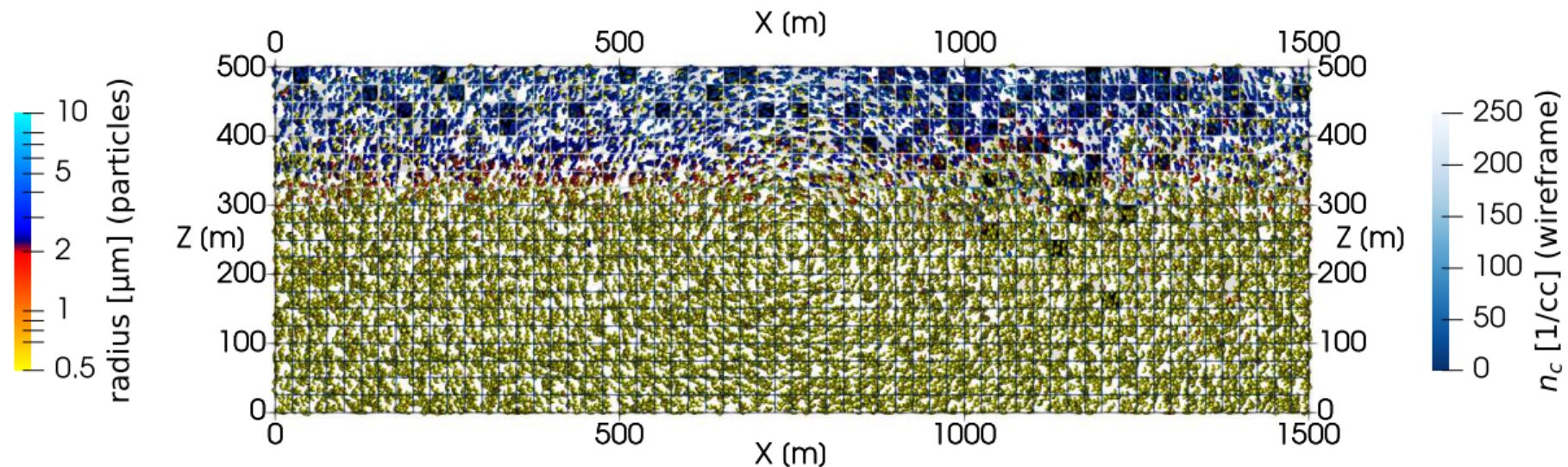
Time: 1080 s (spin-up till 600.0 s)



16+16 super-particles/cell for INP-rich + INP-free particles  
 $N_{\text{aer}} = 300/\text{cc}$  (two-mode lognormal)     $N_{\text{INP}} = 150/L$  (lognormal,  $D_g = 0.74 \mu\text{m}$ ,  $\sigma_g = 2.55$ )  
spin-up = freezing off; subsequently frozen particles act as tracers

# Particle-based $\mu$ -physics + prescribed-flow: glaciation

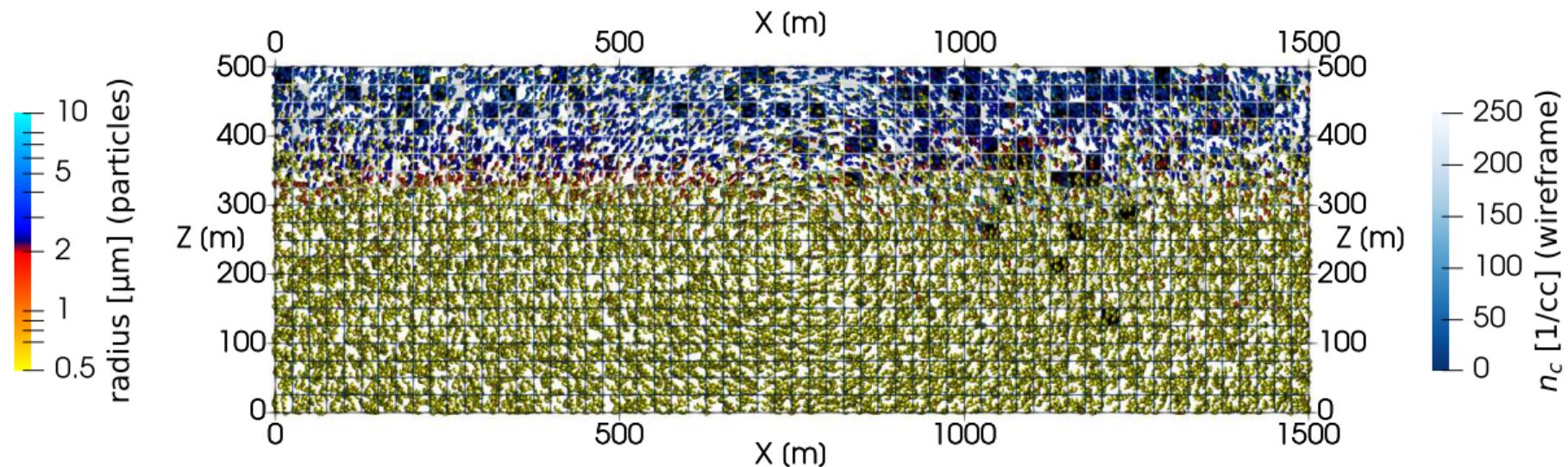
Time: 1110 s (spin-up till 600.0 s)



16+16 super-particles/cell for INP-rich + INP-free particles  
 $N_{\text{aer}} = 300/\text{cc}$  (two-mode lognormal)     $N_{\text{INP}} = 150/L$  (lognormal,  $D_g = 0.74 \text{ } \mu\text{m}$ ,  $\sigma_g = 2.55$ )  
spin-up = freezing off; subsequently frozen particles act as tracers

# Particle-based $\mu$ -physics + prescribed-flow: glaciation

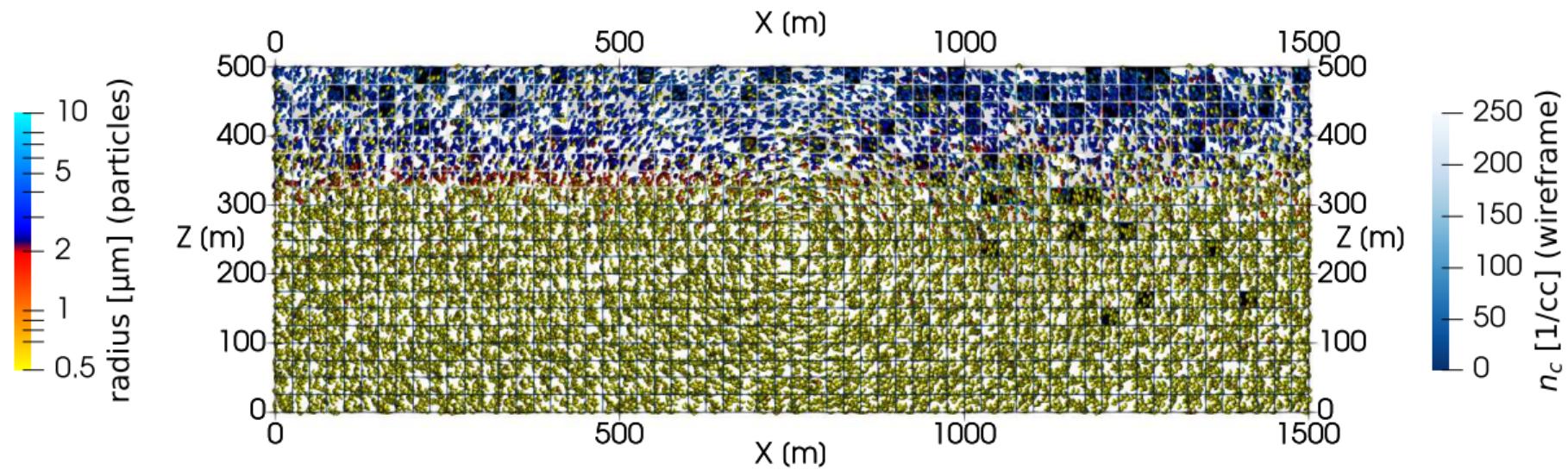
Time: 1140 s (spin-up till 600.0 s)



16+16 super-particles/cell for INP-rich + INP-free particles  
 $N_{\text{aer}} = 300/\text{cc}$  (two-mode lognormal)     $N_{\text{INP}} = 150/L$  (lognormal,  $D_g = 0.74 \mu\text{m}$ ,  $\sigma_g = 2.55$ )  
spin-up = freezing off; subsequently frozen particles act as tracers

# Particle-based $\mu$ -physics + prescribed-flow: glaciation

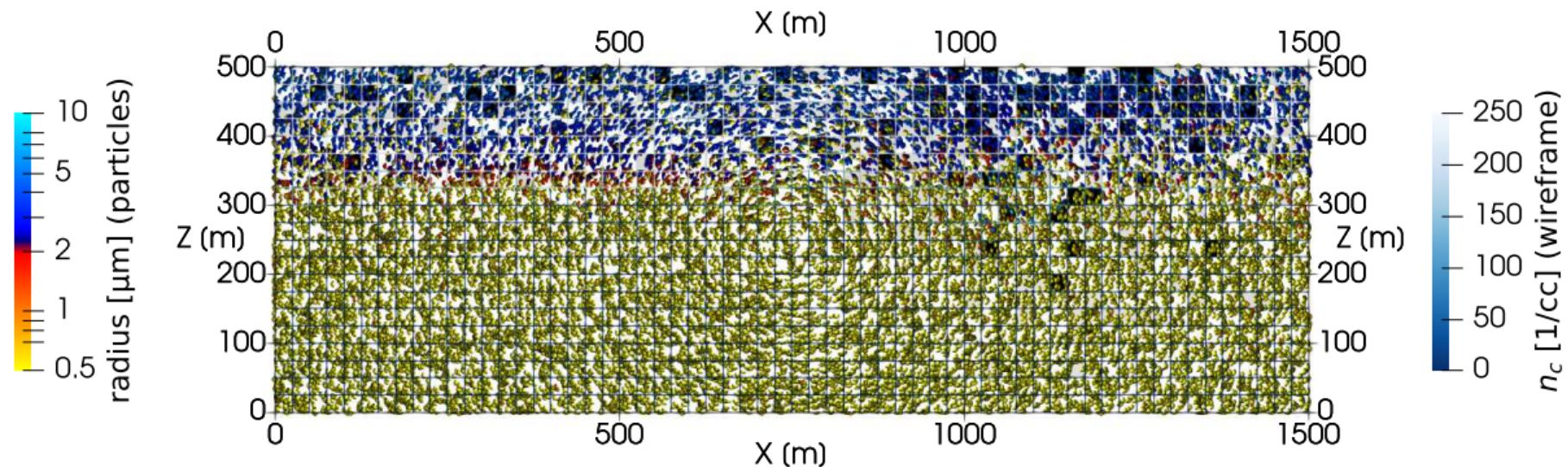
Time: 1170 s (spin-up till 600.0 s)



16+16 super-particles/cell for INP-rich + INP-free particles  
 $N_{\text{aer}} = 300/\text{cc}$  (two-mode lognormal)     $N_{\text{INP}} = 150/L$  (lognormal,  $D_g = 0.74 \mu\text{m}$ ,  $\sigma_g = 2.55$ )  
spin-up = freezing off; subsequently frozen particles act as tracers

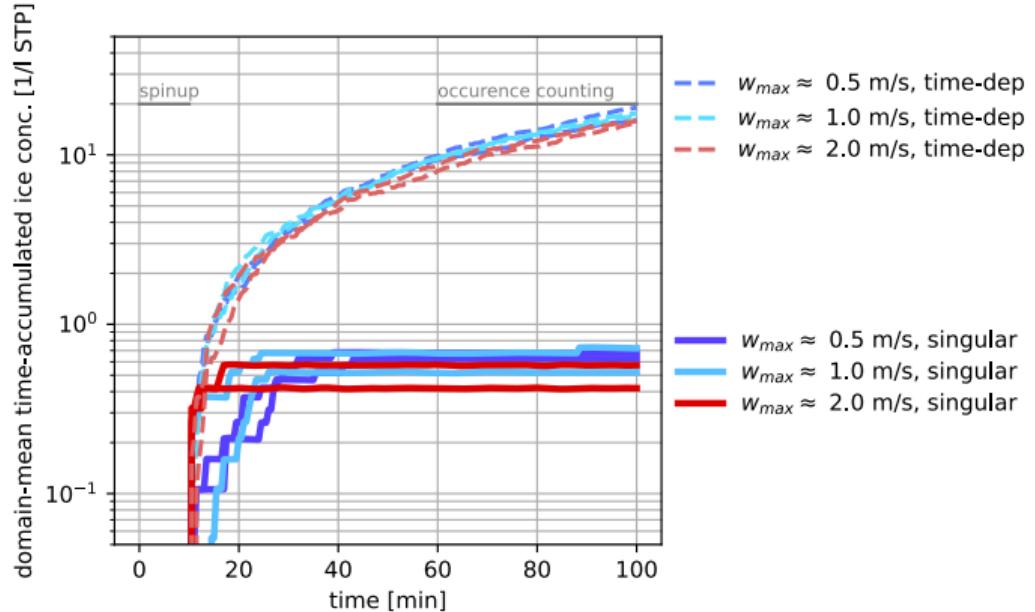
# Particle-based $\mu$ -physics + prescribed-flow: glaciation

Time: 1200 s (spin-up till 600.0 s)

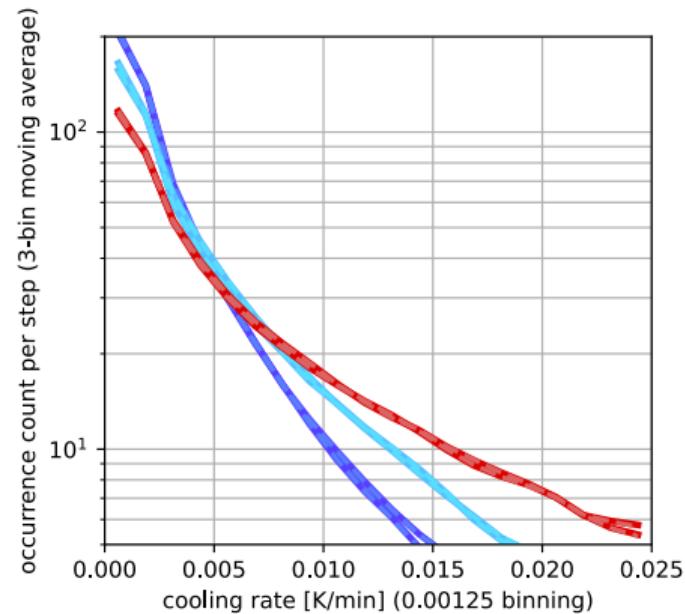
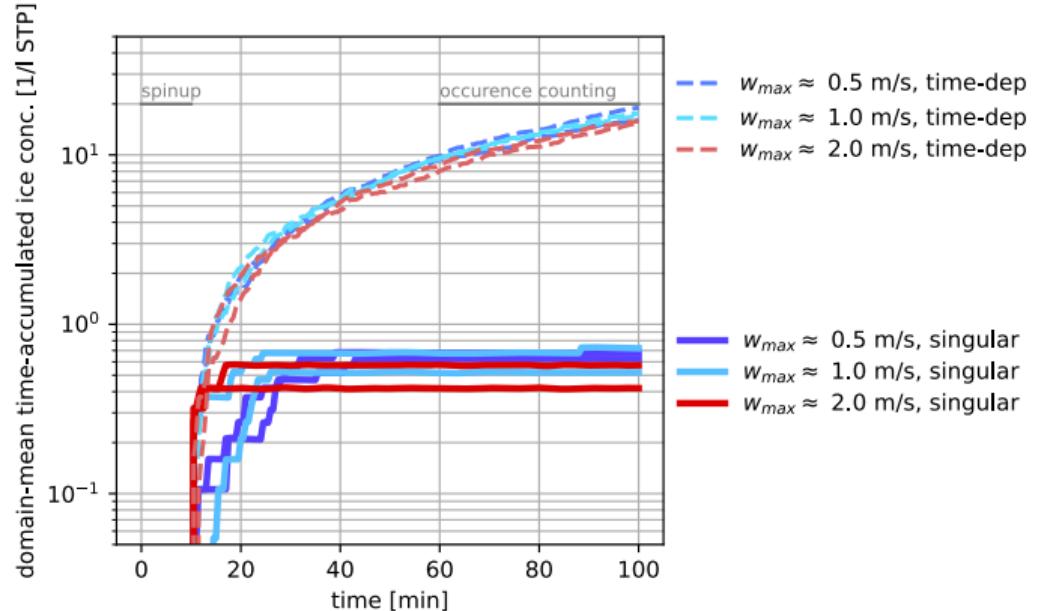


16+16 super-particles/cell for INP-rich + INP-free particles  
 $N_{\text{aer}} = 300/\text{cc}$  (two-mode lognormal)     $N_{\text{INP}} = 150/L$  (lognormal,  $D_g = 0.74 \mu\text{m}$ ,  $\sigma_g = 2.55$ )  
spin-up = freezing off; subsequently frozen particles act as tracers

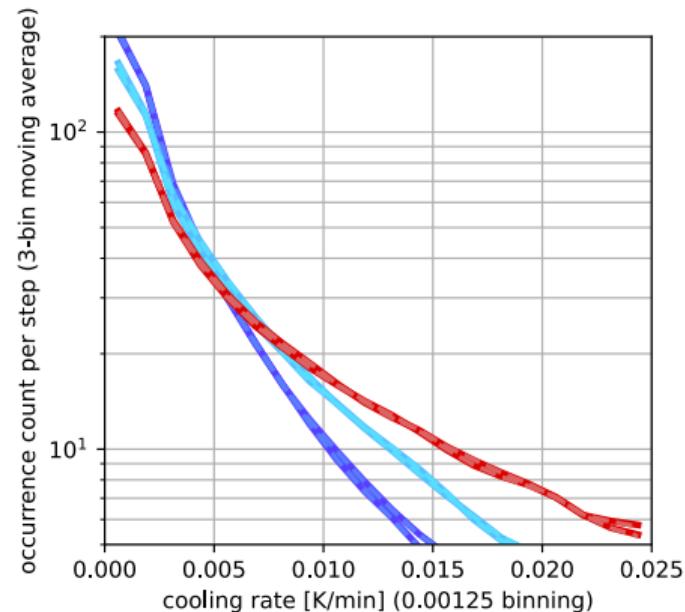
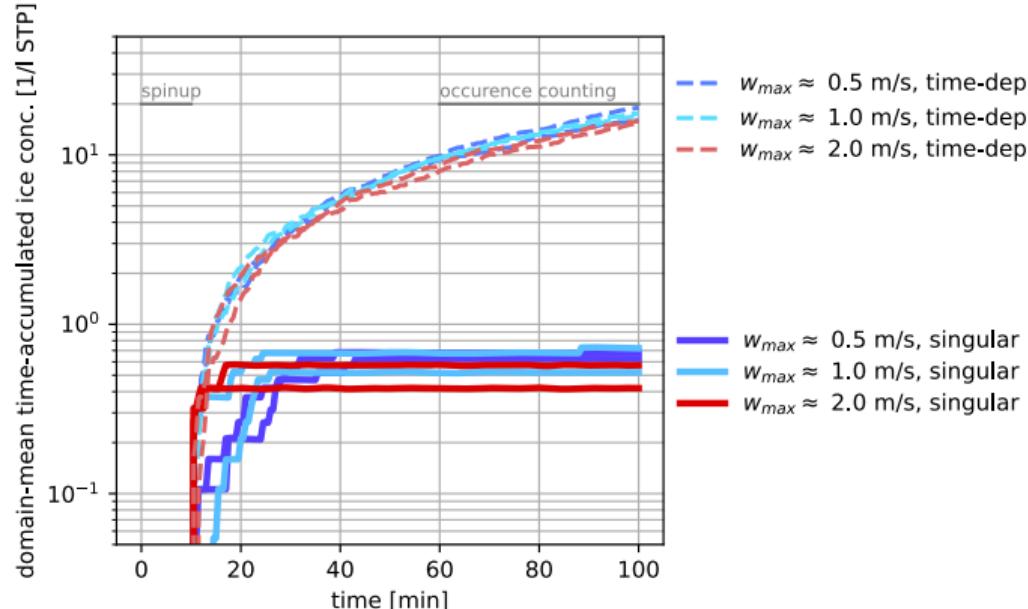
# Testing three flow regimes and two immersion freezing representations



# Testing three flow regimes and two immersion freezing representations

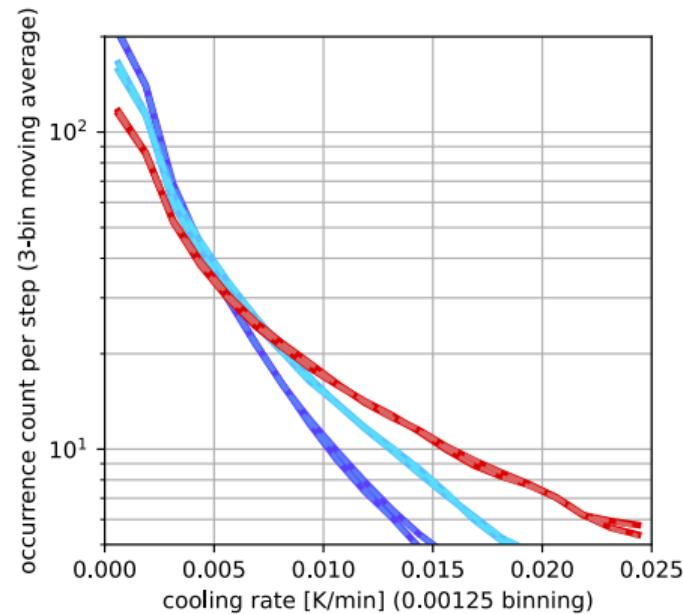
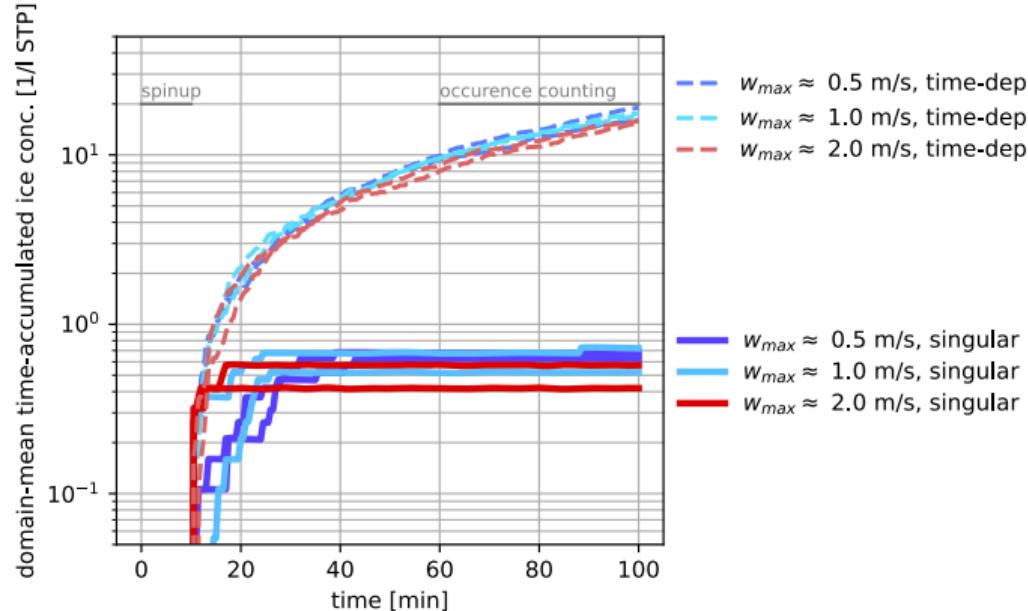


# Testing three flow regimes and two immersion freezing representations



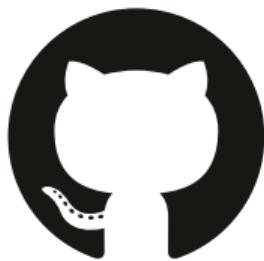
- range of cooling rates in simple flow (far from 0.5 K/min for AIDA as in Niemand et al. 2012)

# Testing three flow regimes and two immersion freezing representations



- ▶ range of cooling rates in simple flow (far from 0.5 K/min for AIDA as in Niemand et al. 2012)
- ▶ **only time-dependent scheme robust across flow regimes** (consistent with box model & theory)

100%  python™ open-source code:



/ OPEN**ATMOS** / PySDM



# $J_{\text{het}}$ or $n_s$ ?

Vali 2014 (ACP)

*"Interpretations of the experimental results face considerable difficulties ... two separate ways of interpreting the same observations; one assigned primacy to time the other emphasized the temperature-dependent impacts of the impurities ... dichotomy – the stochastic and singular models"*

 Atmospheric Chemistry and Physics

**Heterogeneous ice nucleation: exploring the transition from stochastic to singular freezing behavior**

Atmos. Chem. Phys., 11, 8767–8775, 2011  
www.atmos-chem-phys.net/11/8767/2011/  
doi:10.5194/acp-11-8767-2011  
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**npj | climate and atmospheric science**

Article | [Open access](#) | Published: 17 January 2020

**Stochastic nucleation processes and substrate abundance explain time-dependent freezing in supercooled droplets**

D. Niedermeyer<sup>1</sup>, R. A. Shaw<sup>2</sup>, S. Hartmann<sup>1</sup>, H. Wex<sup>1</sup>, T. Clauss<sup>1</sup>, J. Voigtlander<sup>1</sup>, and F. Stratmann<sup>1</sup>

<sup>1</sup>Leibniz Institute for Tropospheric Research, 04318 Leipzig, Germany  
<sup>2</sup>Dept. of Physics, Michigan Technological University, Houghton, Michigan 49931, USA

Received: 12 November 2010 – Accepted: 19 August 2011 – Published in Atmos. Chem. Phys. Discuss.: 28 January 2011  
Revised: 24 June 2011 – Accepted: 19 August 2011 – Published: 30 August 2011

**Minimal cooling rate dependence of ice nuclei activity in the immersion mode**

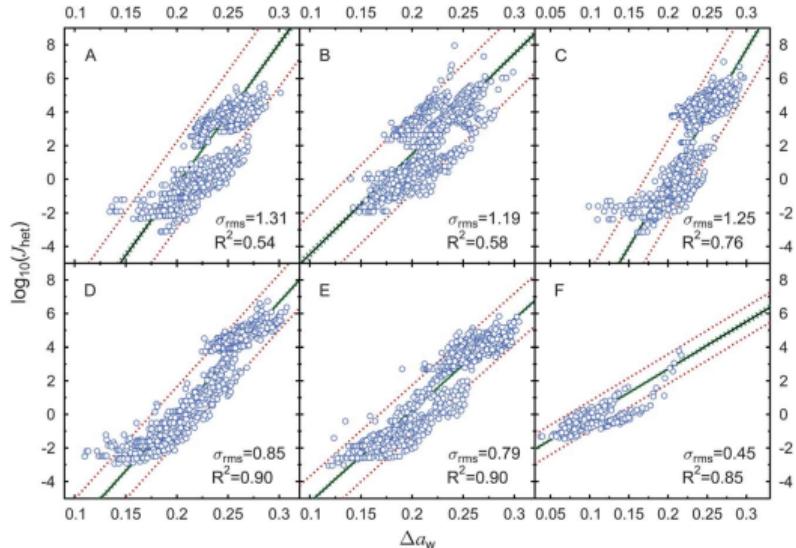
JOURNAL OF GEOPHYSICAL RESEARCH: ATMOSPHERES, VOL. 118, 10,201–10,215, 2013  
DOI: 10.1002/jgrd.50270

**Minimal cooling rate dependence of ice nuclei activity in the immersion mode**

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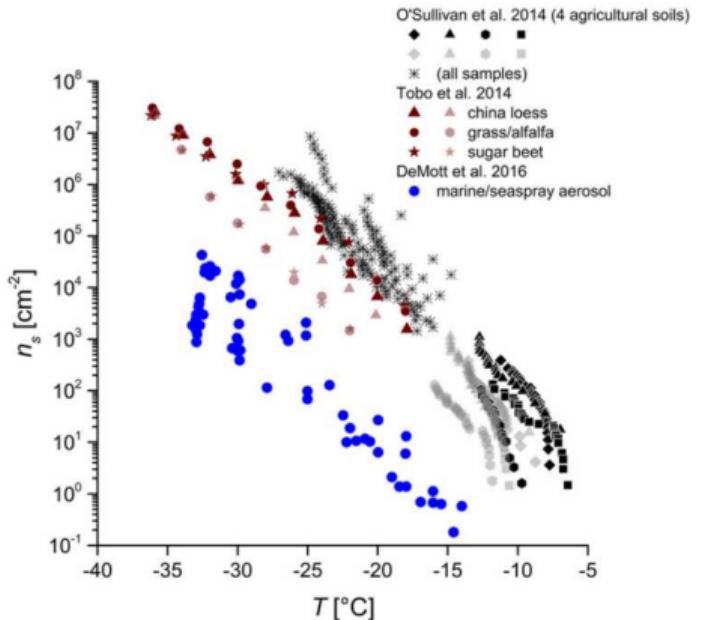
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# Knopf & Alpert '13



**Fig. 3** The decadal log of the heterogeneous ice nucleation rate coefficients,  $\log_{10}(J_{\text{het}})$ , are shown as a function of  $\Delta a_w$  for individually analysed freezing events, initiated by the different IN types investigated in this study and previous work.<sup>41,43,57,66,73,75,78,95</sup> Log<sub>10</sub>( $J_{\text{het}}$ ) are shown for (A) *Nannochloris atomus*, (B) *Thalassiosira pseudonana*, (C) Pahokee Peat, (D) Leonardite, (E) Illite, and (F) 1-nonadecanol. The solid black line is a linear fit where dashed green and red lines represent confidence intervals and prediction bands at 95% level. The root mean square error,  $\sigma_{\text{rms}}$ , and the adjusted coefficient of determination,  $R^2$ , are given in each panel.

# Kanji et al. '17



**FIG. 1-6.** Ice nucleation active site densities  $n_s$  as a function of temperature for H<sub>2</sub>O<sub>2</sub> (hydrogen peroxide) treated (lighter-shaded symbols) and untreated (dark symbols) agricultural soil dusts in comparison to the  $n_s$  of marine aerosol. Differences between various black symbols are for organic content (OC). High OC (12.7 wt%)

# $J_{\text{het}}$ or $n_s$ ?

Vali 2014 (ACP)

*"Interpretations of the experimental results face considerable difficulties ... two separate ways of interpreting the same observations; one assigned primacy to time the other emphasized the temperature-dependent impacts of the impurities ... dichotomy – the stochastic and singular models"*

stochastic or deterministic?

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DeMott 1990 (J. Appl. Meteorol.)

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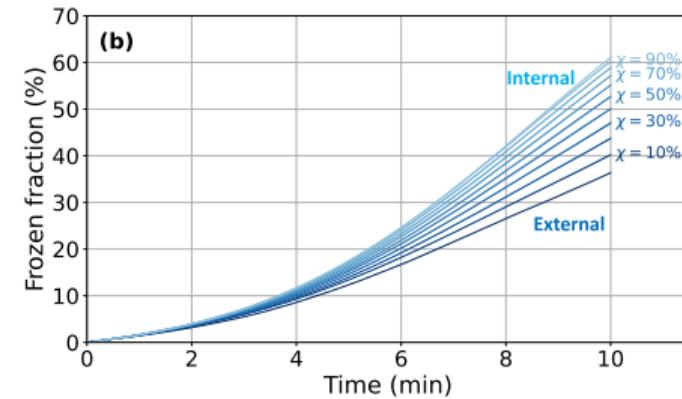
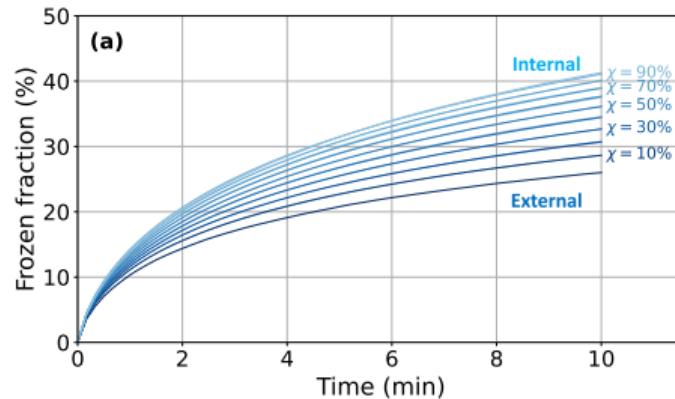
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common underlying Poissonian model

# The impact of aerosol mixing state on immersion freezing: Insights from classical nucleation theory and particle-resolved simulations

Wenhan Tang , Sylwester Arabas , Jeffrey H. Curtis , Daniel A. Knopf , Matthew West , and Nicole Riemer

**Abstract.** Immersion freezing, initiated by ice-nucleating particles (INPs) in supercooled aqueous droplets, plays an important role in the formation of ice crystals within clouds. The efficiency of immersion freezing depends strongly on INP composition and, crucially, on the mixing state—how chemical species are distributed across the particle population. Here, we quantify the impact of aerosol mixing state on immersion freezing using a combined theoretical and particle-resolved modeling approach.



(a): isothermal freezing conditions with  $-20^{\circ}\text{C}$

(b): constant cooling rate from  $-10^{\circ}\text{C}$  to  $-30^{\circ}\text{C}$  within 10 minutes





Thank you for your attention!

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**Immersion Freezing in Particle-Based Aerosol-Cloud Microphysics:  
A Probabilistic Perspective on Singular and Time-Dependent Models**

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Daniel A. Knopf<sup>6</sup> , Matthew West<sup>7</sup> , and Nicole Riemer<sup>2</sup> 

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