

# $^3\text{He}$ library

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## Usage

This library provides constant and functions for various  $^3\text{He}$  properties. Supported Languages:

- Fortran-77. You should use `he3.fh` include file and `libhe3` library for fortran programs. All functions and arguments are `real*8`. Use `-fno-range-check` compiler flag to allow NaN values.
- Fortran-90. You should include `he3.f90h` instead.
- C. You should use `he3.h` include file and `libhe3` library in C programs. Lowercase names with underscore should be used (like `he3_pf_`). Type of arguments is always `double*`, type of returned value is `double`.
- Matlab, Octave. `mex` files are located in the `matlab` folder. Lowercase names should be used. In function arguments you can mix numbers with arrays or matrices of the same size. Example:

```
ttc=0:0.1:1; p=10;
I = he3_nu_b(p, ttc);
```

To build `mex` files you must run `make octave`, `make matlab` or `make matlab64`. Fixed `rpath` is used for the library. For building this documentation octave files are needed.

- **Command line.** You can use all the functions via a command line interface. To get a list of all functions run the program `he3` without arguments:

```
> he3
```

To get information about specific function run the program with the function name as an argument:

```
> he3 he3_nu_b
```

You can use matlab-style vectors (`value1:step:value2`) to get a table of values (if step is missing then 20 points will be printed):

```
> he3 he3_nu_b 0.1:0.1:1 0
```

will give you Leggett frequency vs temperature at zero pressure.

## Location

GIT: <https://github.com/slazav/he3lib>

## Constants

<code>const_na</code>	Avogadro constant $N_A = 6.02214129 \cdot 10^{23}$ [1/mol]
<code>const_kb</code>	Boltsman constant $k_B = 1.3806488 \cdot 10^{-16}$ [erg/K]
<code>const_r</code>	R-gas constant $R = 8.314472 \cdot 10^7$ [sgs]
<code>const_h</code>	Plank constant $h = 6.62606957 \cdot 10^{27}$ [g cm <sup>2</sup> /s]
<code>const_hbar</code>	$\hbar = h/2\pi = 1.054571726 \cdot 10^{27}$ [g cm <sup>2</sup> /s]
<code>const_mu0</code>	Vacuum permeability $\mu_0 = 1.2566370614$ [G*cm/A]
<code>const_pi</code>	$\pi = 3.1415926535897932$
<code>const_2pi</code>	$2\pi$
<code>he3_amass</code>	<sup>3</sup> He atom mass, $m_3 = 5.0079 \cdot 10^{-24}$ [g]
<code>he3_mmass</code>	<sup>3</sup> He molar mass, $\mu_3 = N_A m_3 = 3.0158$ [g/mol]
<code>he3_gyro</code>	<sup>3</sup> He gyromagnetic ratio $\gamma = 20378$ [(G s) <sup>-1</sup> ],

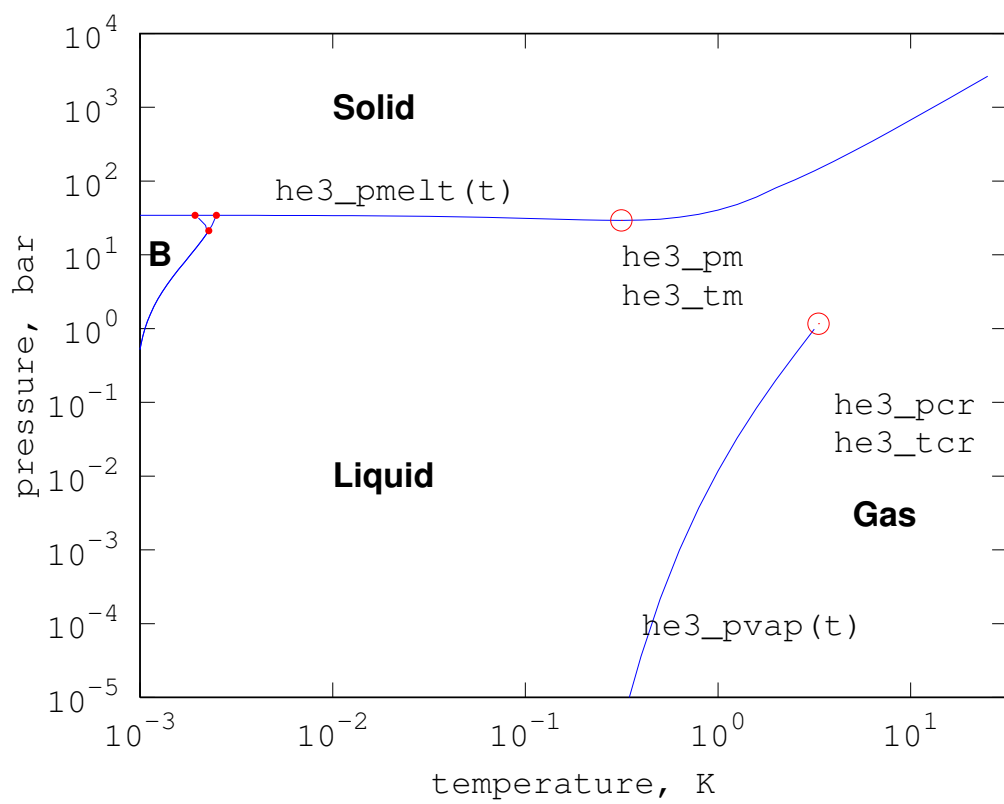
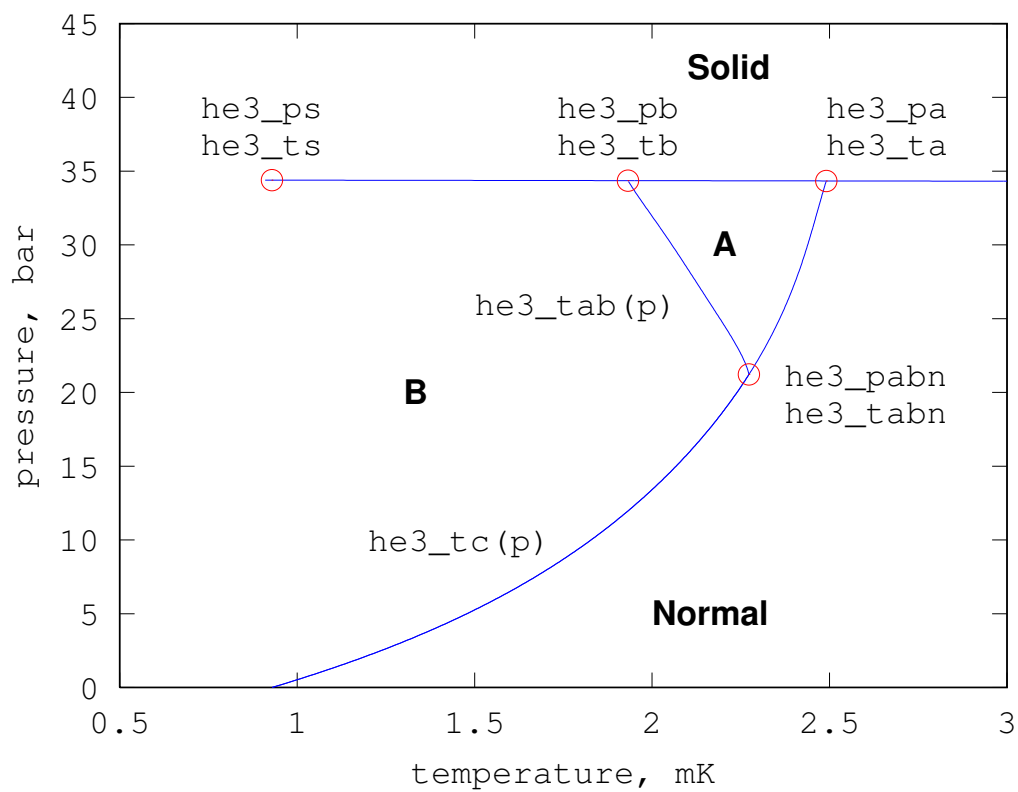
## Phase diagram

Vapor pressure and critical point are from the 1962  $^3\text{He}$  scale of temperatures. Melting curve  $P_m(T)$  at 0.9 – 250 mK,  $T_c(P)$ ,  $T_{AB}(P)$  and all tricritical points are from Greywall-86 temperature scale. Melting curve minimum is from PLTS-2000 temperature scale. Melting curve for higher temperatures is from Osborne, Abraham, Weinstock-1951 and Mills, Grilly-1955 with interpolation in the regions 0.25–0.5K and 1.5–2.0K. PLTS-2000 melting curve and critical points on it are also available. See more information in the source file `he3_phase.f`.

<code>he3_pvap(T)</code>	Vapor pressure [bar] vs temperature [K], $T = 0.2 - 3.324$ K
<code>he3_pcr</code>	Gas-liquid critical point pressure, 1.16317 [bar]
<code>he3_tcr</code>	Gas-liquid critical point temperature, 3.324 [K]
<code>he3_pmelt(T)</code>	Melting pressure [bar] vs temperature [K], $T = 0.0009 - 31$ K
<code>he3_pm</code>	Melting curve minimum pressure, 29.3113 [bar]
<code>he3_tm</code>	Melting curve minimum temperature, 0.31524 [K]
<code>he3_pa</code>	Superfluid trans. at melting curve, pressure, 34.3380 [bar]
<code>he3_ta</code>	Superfluid trans. at melting curve, temp., 2.491 [mK]
<code>he3_pb</code>	A-B trans. at melting curve, pressure, 34.3580 [bar]
<code>he3_tb</code>	A-B trans. at melting curve, temp., 1.932 [mK]
<code>he3_ps</code>	Neel transition at melting curve, pressure, 34.3905 [bar]
<code>he3_ts</code>	Neel transition at melting curve, temp., 0.9291 [mK]
<code>he3_tc(P)</code>	Superfluid transition temperature [mK] vs pressure [bar], $P = 0 - 34.358$ bar
<code>he3_tab(P)</code>	A-B transition temperature [mK] vs pressure [bar], $P = 0 - 34.3609$ bar, below 21.22 bar is equal to <code>he3_tc</code>
<code>he3_tabn</code>	A-B-Normal point temperature, 2.2311 [mK]
<code>he3_pabn</code>	A-B-Normal point pressure, 21.22 [bar]

PLTS-2000 temperature scale:

<code>he3_pmelt_plts(T)</code>	PLTS-2000 melting pressure [bar] vs temperature [K].
<code>he3_pa_plts</code>	Superfluid trans. at melting curve, pressure, 34.3407 [bar]
<code>he3_ta_plts</code>	Superfluid trans. at melting curve, temp., 2.444 [mK]
<code>he3_pb_plts</code>	A-B trans. at melting curve, pressure, 34.3609 [bar]
<code>he3_tb_plts</code>	A-B trans. at melting curve, temp., 1.896 [mK]
<code>he3_ps_plts</code>	Neel transition at melting curve, pressure, 34.3934 [bar]
<code>he3_ts_plts</code>	Neel transition at melting curve, temp., 0.902 [mK]



## Fermi-liquid parameters

Argument is pressure, [bar] in the range 0 – 34.4. See more information in the source file `he3_fermi.f`.

### Molar volume and heat capacity in normal $^3\text{He}$ :

`he3_vm(P)` Molar volume  $v_m$ , [ $\text{cm}^3/\text{mol}$ ], Graywall-86 (from Wheatley-75)

`he3_gammaf(P)` R-gas constant  $\gamma_f = C_V/RT$ , [ $1/(\text{K mol})$ ], Greywall-86

### Derived values, $F_1^s$ parameter:

`he3_rho(P)` Density  $\rho = \mu_3/v_m$ , [ $\text{g}/\text{cm}^3$ ]

`he3_2n0(P)`  $2N(0) = \frac{\gamma_f}{v_m} \frac{3N_A}{k_B\pi^2}$ , [ $1/(\text{erg cm}^3)$ ]

`he3_pf(P)`  $p_F = h \left( \frac{3}{8\pi} \frac{N_A}{v_m} \right)^{1/3}$ , [ $\text{g cm/s}$ ]

`he3_meff(P)`  $m^* = \frac{h^3}{8\pi} \frac{2N(0)}{p_F}$ , [g]

`he3_mm(P)`  $m^*/m_3$

`he3_vf(P)`  $v_F = p_F/m^*$ , [ $\text{cm/s}$ ]

`he3_f1s(P)`  $F_1^s = 3(m^*/m_3 - 1)$

`he3_a(P)` Average atomic spacing,  $a = (v_m/N_A)^{1/3}$ , Å

`he3_gdk(P)` Average dipolar coupling enegy,  $g_d/k_B = \frac{2\pi\gamma^2\hbar^2}{3v_mk_B}$ , [K]

`he3_tfeff(P)` Effective Fermi temperature,  $T_{F_{eff}} = \frac{\pi^2}{2\gamma_f}$ , [K]

### Sound velocity in normal $^3\text{He}$ and $F_0^s$ parameter:

`he3_c1(P)` First sound velocity,  $c_1$ , [ $\text{cm/s}$ ], measured, Wheatley-75

`he3_f0s(P)`  $F_0^s = 3 m^*m_3 c_1^2/p_F^2 - 1$

### Magnetic susceptibility in normal $^3\text{He}$ and $F_0^a$ parameter:

`he3_f0a(P)`  $F_0^a, Z_0/4$ , from magnetic susceptibility measurements, Hensley-1993

`he3_chi_n(P)`  $\chi_N = \frac{2N(0)(\gamma\hbar)^2}{4(1 + F_0^a)}$ .

### Other fermi-liquid parameters:

`he3_f1a(P)`  $F_1^a$ , calculated from spin-wave velocities in  $^3\text{He-B}$ , Zavjalov-2015

`he3_f2a(P)`  $F_2^a$ , currently is 0

`he3_f2s(P)`  $F_2^s$ , currently is 0

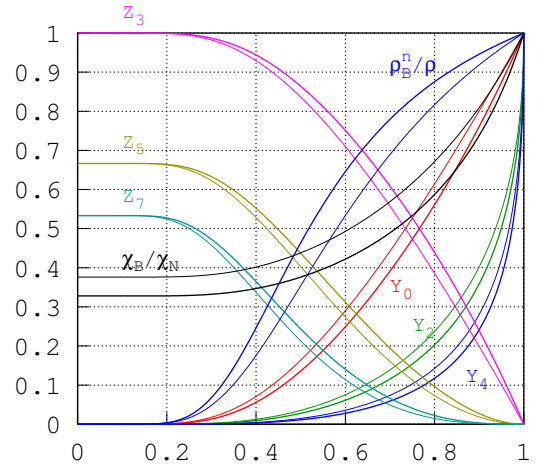
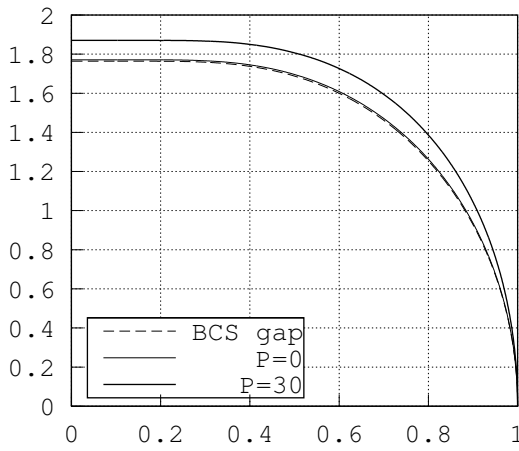
Pressure-dependent values:

P, bar	0	3	6	9	12	15	18	21	24	27	30	33
$T_c$ , mK	0.929	1.290	1.560	1.769	1.934	2.067	2.177	2.267	2.339	2.395	2.438	2.474
$T_{AB}$ , mK	—	—	—	—	—	—	—	—	2.217	2.137	2.056	1.969
$v_m$ , cm <sup>3</sup> /mol	36.84	33.95	32.03	30.71	29.71	28.89	28.18	27.55	27.01	26.56	26.17	25.75
$c_1$ , m/s	182.9	227.4	259.9	285.8	307.9	327.3	344.8	360.6	375.2	389.2	402.9	416.1
$\gamma_f$ , 1/(K mol)	2.784	2.981	3.158	3.321	3.475	3.625	3.772	3.918	4.065	4.211	4.355	4.496
$\rho$ , g/cm <sup>3</sup>	0.082	0.089	0.094	0.098	0.102	0.104	0.107	0.109	0.112	0.114	0.115	0.117
$2N(0)$ , 10 <sup>38</sup>	1.002	1.164	1.307	1.434	1.551	1.663	1.774	1.885	1.995	2.102	2.207	2.315
$p_F$ , 10 <sup>-20</sup> g cm/s	8.280	8.509	8.675	8.798	8.896	8.979	9.054	9.122	9.182	9.234	9.280	9.330
$m^*$ , 10 <sup>-23</sup> g	1.401	1.584	1.744	1.887	2.018	2.144	2.269	2.392	2.515	2.635	2.753	2.872
$m^*/m_3$	2.797	3.162	3.482	3.767	4.030	4.282	4.530	4.777	5.022	5.261	5.497	5.735
$v_F$ , m/s	59.11	53.73	49.74	46.64	44.08	41.87	39.91	38.13	36.51	35.05	33.71	32.49
$\chi_N$ , 10 <sup>-9</sup>	38.4	48.6	56.9	63.9	70.5	77.0	83.4	89.4	94.7	99.7	104.5	109.5
$F_0^s$	9.27	15.99	22.52	28.91	35.32	41.81	48.43	55.17	62.10	69.32	76.95	84.82
$F_1^s$	5.39	6.49	7.45	8.30	9.09	9.85	10.59	11.33	12.07	12.78	13.49	14.21
$F_0^a$	-0.70	-0.72	-0.73	-0.74	-0.75	-0.75	-0.75	-0.76	-0.76	-0.76	-0.76	-0.76
$F_1^a$	-0.60	-0.60	-0.61	-0.62	-0.62	-0.63	-0.64	-0.64	-0.65	-0.66	-0.66	-0.67
$a$ , Å	3.940	3.834	3.761	3.708	3.668	3.634	3.604	3.577	3.553	3.533	3.516	3.497
$g_d/k_B$ , $\mu$ K	114.5	124.3	131.7	137.4	142.0	146.0	149.7	153.1	156.2	158.8	161.2	163.9
$T_{F_{eff}}$ , K	1.773	1.656	1.563	1.486	1.420	1.361	1.308	1.260	1.214	1.172	1.133	1.098
$\langle W \rangle$	84.3	107.3	119.8	127.6	134.5	141.3	146.8	150.1	150.8	150.1	149.4	149.0
$\langle W_I \rangle$	11.39	14.25	15.45	16.03	16.55	17.11	17.54	17.66	17.46	17.09	16.76	16.48
$\langle W_D \rangle$	24.83	31.59	35.27	37.55	39.54	41.47	43.05	43.96	44.12	43.86	43.63	43.46
$\langle W_L \rangle$	-9.38	-16.79	-20.57	-22.68	-24.53	-26.41	-27.85	-28.34	-27.81	-26.70	-25.66	-24.73
$\gamma_0$	0.135	0.133	0.129	0.126	0.123	0.121	0.119	0.118	0.116	0.114	0.112	0.111
$\delta_0$	0.295	0.294	0.294	0.294	0.294	0.294	0.293	0.293	0.293	0.292	0.292	0.292
$w_0$	1.204	1.206	1.208	1.210	1.212	1.213	1.214	1.214	1.215	1.216	1.217	1.218
$\lambda_1^a$	-0.111	-0.157	-0.172	-0.178	-0.182	-0.187	-0.190	-0.189	-0.184	-0.178	-0.172	-0.166
$\tau_N(0, T_c)$ , $\mu$ s	0.603	0.230	0.133	0.092	0.070	0.056	0.047	0.040	0.036	0.034	0.032	0.030
$\Delta_{WC+}(T=0)/T_c$	1.771	1.795	1.813	1.825	1.834	1.843	1.850	1.856	1.861	1.866	1.870	1.875
$\chi_B(T=0)/\chi_N$	0.376	0.356	0.347	0.341	0.337	0.333	0.329	0.328	0.327	0.328	0.328	0.328
$\nu_B(T=0)$ , kHz	125.1	161.9	188.1	208.6	226.0	241.6	256.1	269.9	283.3	296.1	308.5	320.6
$c_\perp(T=0)$ , cm/s	2170	1940	1781	1661	1563	1479	1404	1338	1280	1228	1180	1136
$c_\parallel(T=0)$ , cm/s	2484	2220	2039	1901	1789	1692	1607	1531	1464	1404	1349	1299
P, bar	0	3	6	9	12	15	18	21	24	27	30	33

## Energy gap

BCS energy gap + trivial strong coupling correction + some values derived from energy gap. See more information in the source file `he3_gap.f`.

<code>he3_bcsgap(ttc)</code>	BCS gap for 3He-B in $T_c$ units, $\Delta_{\text{BCS}}/k_B T_c$
<code>he3_bcsgap_fast(ttc)</code>	Einzel approximation for BCS gap (0.5% accuracy, 70 times faster) (Einzel-1991, f.68)
<code>he3_trivgap(ttc, P)</code>	Trivial strong-coupling correction to the BCS gap
<code>he3_todogap(ttc, P)</code>	Gap based on Todoschenko's measurements: linear interpolation in density between BCS value at zero bar and measured value 1.99 at melting pressure + temperature behaviour as in <code>he3_trivgap</code>
<code>he3_gap(ttc, P)</code>	Wrapper for the gap used everywhere in the lib (trivgap by default)
<code>he3_yosida(ttc, gap, n)</code>	Yosida functions $Y_n(T/T_c, \Delta) = \int_{-\infty}^{\infty} \left( \frac{\xi_k}{E_k} \right)^n \frac{1}{2T/T_c} \text{ch}^{-2} \left( \frac{E_k}{2T/T_c} \right) d\xi_k$ Low temperature limit: $Y_n(T, \Delta) = 2\Gamma \left( \frac{n+1}{2} \right) \left( \frac{T}{\Delta} \right)^{\frac{n-1}{2}} \exp \left( -\frac{\Delta}{T} \right)$ Note: type of n parameter in fortran should be real*8 (this was done for standard function handling). If n is integer, result is unpredictable!
<code>he3_yosida_par(ttc, gap)</code>	$Y_q^{\parallel}(T/T_c, \Delta)$
<code>he3_yosida_perp(ttc, gap)</code>	$Y_q^{\perp}(T/T_c, \Delta)$
<code>he3_z3(ttc, gap)</code>	
<code>he3_z5(ttc, gap)</code>	
<code>he3_z7(ttc, gap)</code>	
<code>he3_lambda(ttc, gap)</code>	
<code>he3_rho_nb(ttc, p)</code>	B-phase normal component density: $\frac{\rho_B^n}{\rho_N} = \frac{(3 + F_1^s)Y_0}{3 + F_1^s Y_0}$
<code>he3_chi_b(ttc, p)</code>	B-phase susceptibility (ratio of <code>he3_chi_n</code> ): $\frac{\chi_B}{\chi_N} = \frac{(1 + F_0^a)(2 + Y_0)}{3 + F_0^a(2 + Y_0)}$
<code>he3_chi_bp(ttc, P)</code>	B-phase Cooper pair susceptibility (ratio of <code>he3_chi_b</code> ): $\frac{\chi_B^p}{\chi_B} = \frac{2(1 - Y_2)}{(2 + Y_0)}$



## Dipole energy and Legget frequency

Values are given according to Thuneberg's paper JLTP 122, p657 (2001). Temperature independent  $g_d$  parameter is restored from experimental data for  $\Omega_B$ , measured in transverse and longitudinal NMR. See more information in the source file `he3_dipole.f`.

Dipolar energy:

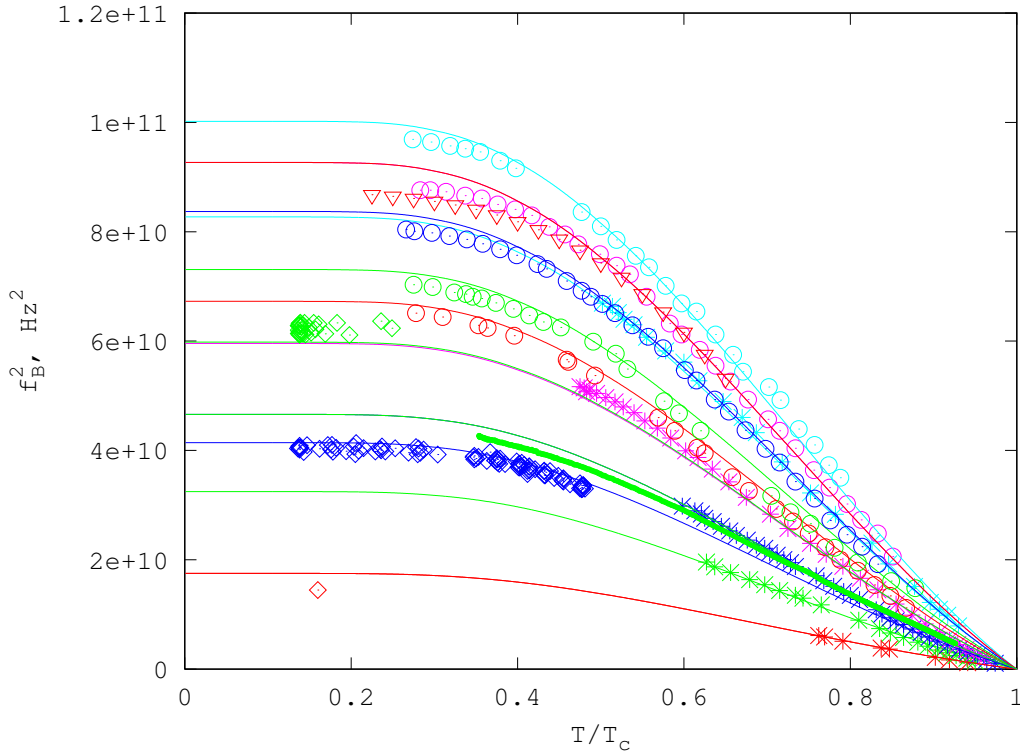
$$F_D = \lambda_D \int (R_{ii}R_{jj} + R_{ij}R_{ji}) d^3r = 4\lambda_D \int \cos\theta(1 + 2\cos\theta) d^3r$$

$$\lambda_D = \Delta^2 g_d, \quad \Omega_B = \gamma \sqrt{15\lambda_D/\chi_B}$$

Note: In old papers (Leggett, ...)  $g_D/5$  is used instead of  $\lambda_G$ .

- `he3_gd(p)`  $g_d$ , [1/(erg cm<sup>3</sup>)]  
`he3_ld(p)`  $\lambda_D = \Delta^2 g_d$ , [erg/cm<sup>3</sup>]  
`he3_nu_b(ttc, p)` B-phase Leggett frequency  $\nu_B = \frac{\gamma}{2\pi} \sqrt{15\Delta^2 g_d/\chi_B}$ , Hz  
`he3_nu_b1(ttc, p)` Less accurate formula without using  $g_d$ , Hz  

$$\nu_B = \frac{1}{2\pi} \sqrt{\frac{3\pi}{2\chi}} \frac{\gamma^2 \hbar}{2} N(0) \Delta \log \frac{e_f}{\Delta}$$



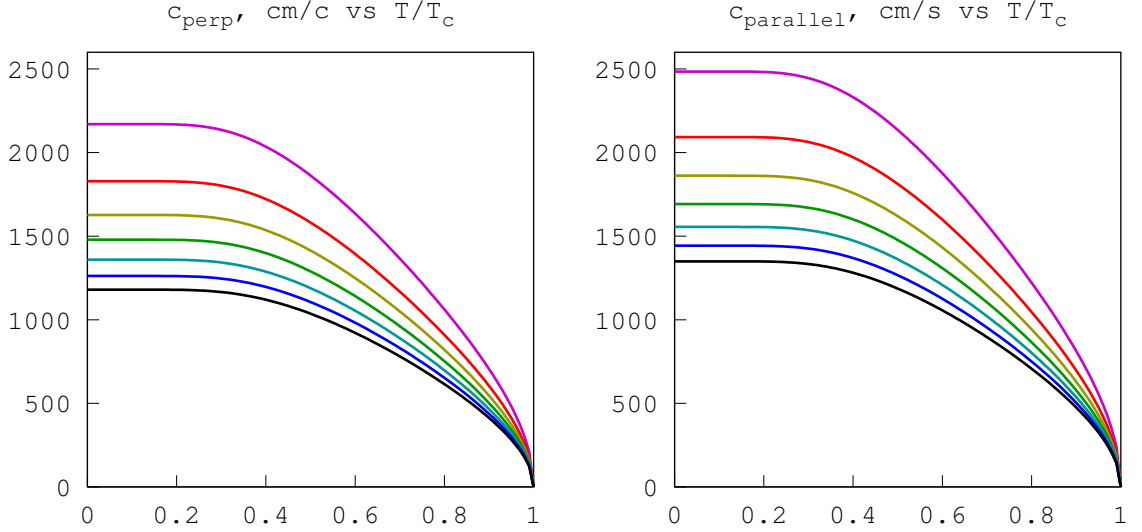


## Gradient energy and spin wave velocity

Gradient energy coefficients and derived values, including spin wave velocities. See more information in the source file `he3_grad.f`.

$$F_{\nabla} = \frac{1}{2}\Delta^2 [K_1(\nabla_j R_{ak})(\nabla_j R_{ak}) + K_2(\nabla_j R_{ak})(\nabla_k R_{aj}) + K_3(\nabla_j R_{aj})(\nabla_k R_{ak})]$$

<code>he3_grad_K0(ttc, p)</code>	$K_1 = K_2 = K_3$ without fermi-liquid corrections, see VW7.23m
<code>he3_grad_c(ttc, p)</code>	$c$ and $\delta$ paramters calculated with fermi-liquid corrections (Cross-1975),
<code>he3_grad_delta(ttc, p)</code>	see VW7.25. These values are used to calculate all other things: $c = -\frac{\rho_s}{10} \frac{3 + F_1^a}{3 + F_1^s} \frac{1}{1 + F_1^a(5 - 3\rho_s/\rho)/15}, \quad \delta = \frac{F_1^a \rho_s/\rho}{3 + F_1^s(1 - \rho_s/\rho)}$
<code>he3_grad_K12(ttc, p)</code> <code>he3_grad_K3(ttc, p)</code>	$K_1 = K_2 = -\frac{2}{\Delta^2} \left(\frac{\hbar}{2m}\right)^2 c, \quad K_3 = -\frac{2}{\Delta^2} \left(\frac{\hbar}{2m}\right)^2 (1 + \delta)c$
<code>he3_grad_K(ttc, p)</code> <code>he3_grad_Kp(ttc, p)</code>	$K = 2K_1 + K_2 + K_3, \quad K' = K_2 + K_3$
<code>he3_grad_lg1(ttc, p)</code> <code>he3_grad_lg2(ttc, p)</code> <code>he3_grad_lsgb(ttc, p)</code>	Thunebergs $\lambda_{G1}$ , $\lambda_{G2}$ and $\lambda_{SG}^b$ : $\lambda_{G1} = \frac{1}{2}\Delta^2(K_2 + K_3), \quad \lambda_{G2} = \frac{1}{2}\Delta^2 K_1, \quad \lambda_{SG}^b = \Delta^2 K_2$
<code>he3_cpar(ttc, p)</code> <code>he3_cperp(ttc, p)</code>	velocity of transverse spin waves parallel and perpendicular to the $l$ direction [cm/s]: $c_{\parallel}^2 = \frac{\gamma^2 \Delta^2}{\chi_B} K, \quad c_{\perp}^2 = \frac{\gamma^2 \Delta^2}{\chi_B} (K - K'/2), \quad c_{\parallel}^2/c_{\perp}^2 \approx 4/3$
<code>he3_clpar(ttc, p)</code> <code>he3_clperp(ttc, p)</code>	same for longitudinal waves [cm/s]: $C_{\parallel}^2 = \frac{\gamma^2 \Delta^2}{\chi_B} (K - K'), \quad C_{\perp}^2 = \frac{\gamma^2 \Delta^2}{\chi_B} K, \quad C_{\parallel}^2/C_{\perp}^2 \approx 1/2$



## Other textural parameters

Values are given according to Thuneberg's paper JLTP 122, p657 (2001). High order Fermi liquid parameters ( $F_2^a$ ,  $F_3^a$ ) are set to 0. See more information in the source file `he3_text.f`.

Bulk terms:

$$\begin{aligned}
F_{DH} &= -a \int_V (\mathbf{n} \cdot \mathbf{H})^2 d^3r \\
F_{DV} &= -\lambda_{DV} \int_V [\mathbf{n} \cdot (\mathbf{v}_s - \mathbf{v}_n)]^2 d^3r \ll F_{HV} \text{ at } H \approx 200 \text{ Oe} \\
F_{HV} &= -\lambda_{HV} \int_V [\mathbf{H} \cdot R \cdot (\mathbf{v}_s - \mathbf{v}_n)]^2 d^3r
\end{aligned}$$

Surface terms:

$$\begin{aligned}
F_{SH} &= -d \int_S [\mathbf{H} \cdot R \cdot \mathbf{s}]^2 d^2r \\
F_{SG} &= -\lambda_{SG} \int_S s_j R_{\alpha j} \frac{\partial R_{\alpha i}}{\partial r_i} d^2r
\end{aligned}$$

Vortex term:

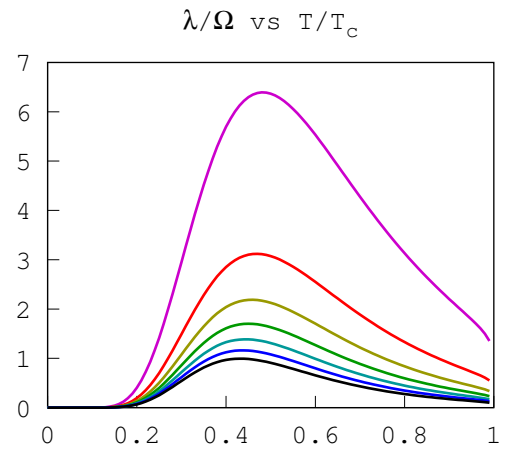
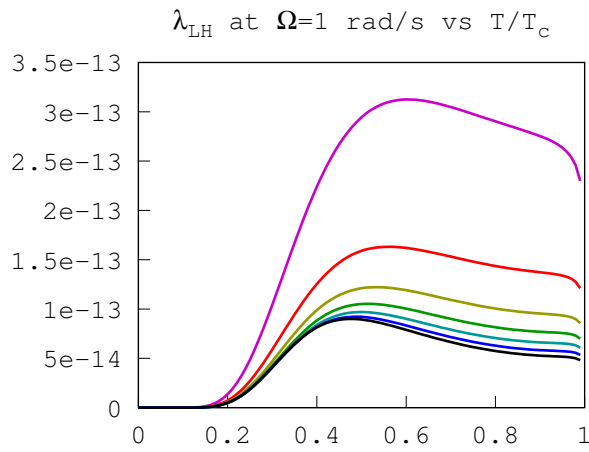
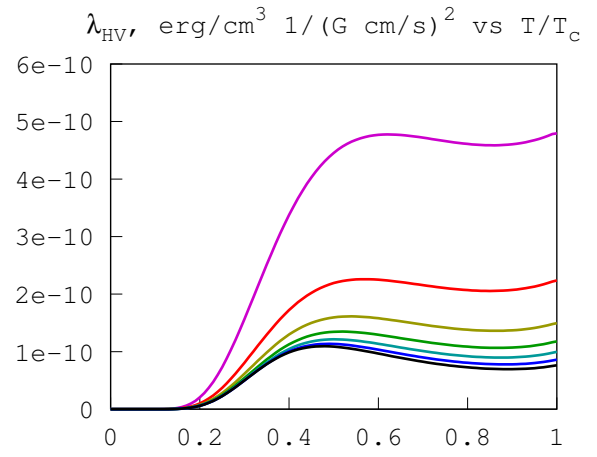
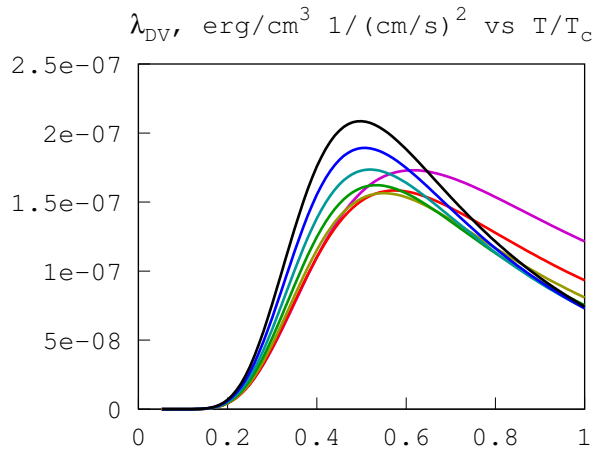
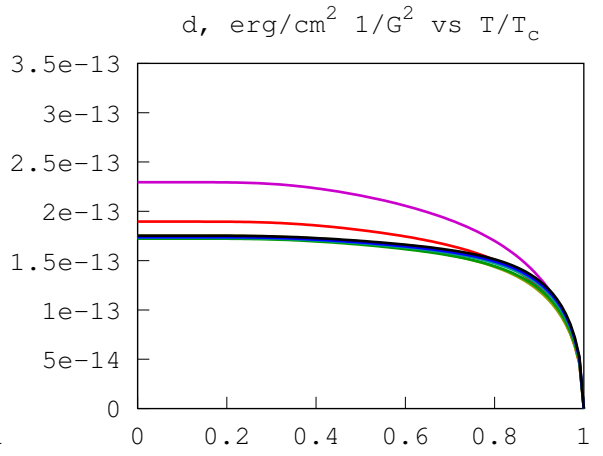
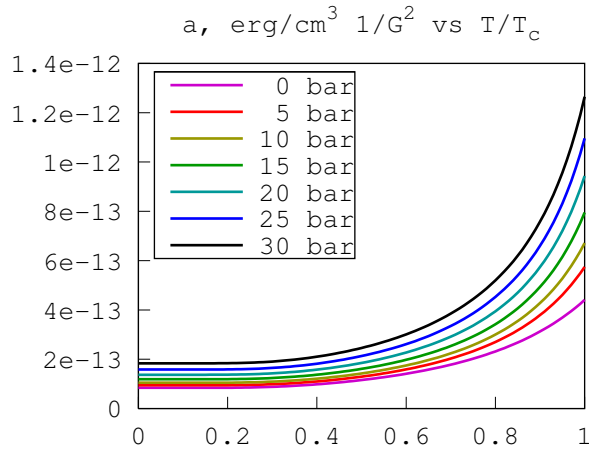
$$F_{LH} = \frac{\lambda_{LH}}{2\Omega} \int_L |\omega_s| [\mathbf{H} \cdot R \cdot \mathbf{l}]^2 d^3r$$

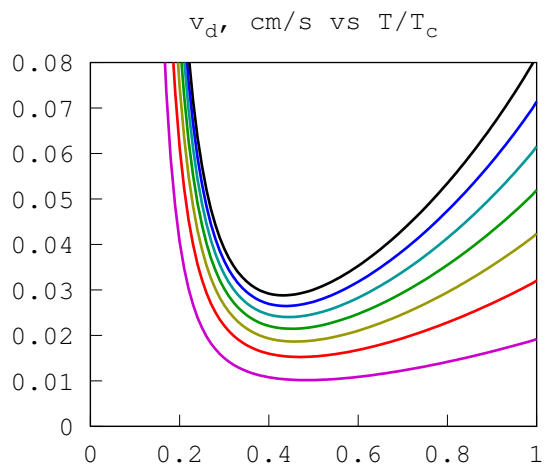
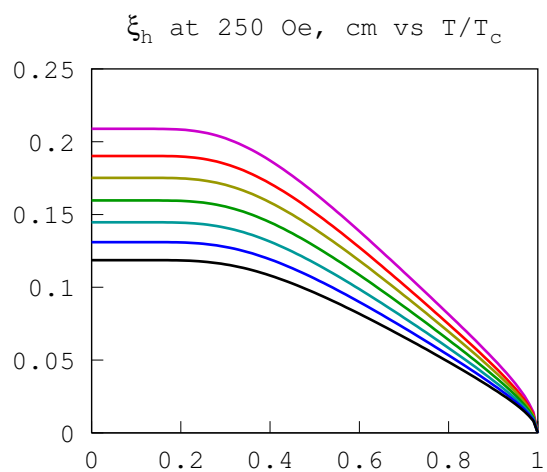
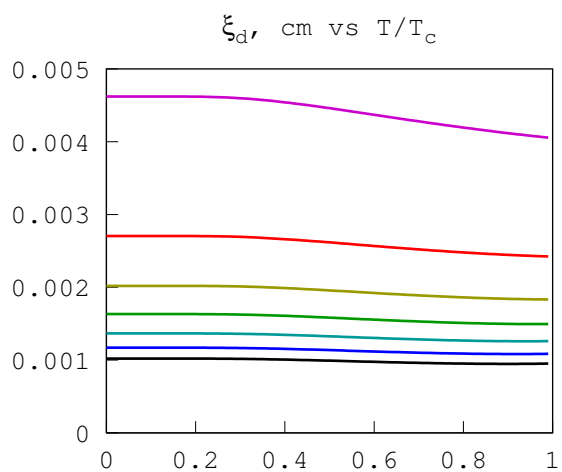
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<code>he3_text_a(ttc, p)</code>	$a = \frac{5g_d}{2} \left[ \frac{\hbar\gamma/2}{1 + F_0^a(2 + Y_0)/3} \right]^2 \left( 5 - 3\frac{Z_3}{Z_5} \right), [\text{erg}/\text{cm}^3 \text{ G}^{-2}]$
<code>he3_text_d(ttc, p)</code>	$d, [\text{erg}/(\text{cm}^2 \text{ G}^2)] - \text{GL extrapolation!}$
<code>he3_text_ldv(ttc, p)</code>	$\lambda_{DV} = 5g_d \left( \frac{m^*v_F}{1 + F_1^s Y_0/3} \right)^2 \left( 1 - \frac{3Z_5}{2Z_3} \right), [\text{erg}/\text{cm}^3 (\text{cm}/\text{s})^{-2}]$
<code>he3_text_lhv(ttc, p)</code>	$\lambda_{HV} = \frac{\rho}{\Delta^2} \frac{1 + F_1^s/3}{(1 + F_1^s Y_0/3)^2} \left( \frac{\hbar\gamma/2}{1 + F_0^a(2 + Y_0)/3} \right)^2 \left( Z_3 - \frac{9}{10}Z_5 + \frac{9}{10} \frac{Z_5^2}{Z_3} - \frac{3}{2}Z_7 \right)$
<code>he3_text_llh(ttc, p, omega)</code>	$\lambda_{LH} = \frac{\hbar}{2m} \Omega \lambda_{HV} \left( \ln \frac{R}{r} - \frac{3}{4} \right), [\text{erg}/(\text{cm}^3 \text{ G}^2 \text{ s})] - \text{counterflow part only!}$
<code>he3_text_lo(ttc, p, omega)</code>	lambda/omega used in the texture library: $\frac{\lambda}{\Omega} = \frac{5\lambda_{LH}}{2a\Omega}, (\text{rad}/\text{s})^{-1}$

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<code>he3_text_xih(ttc, p, h)</code>	Magnetic length $\xi_H = \sqrt{65\lambda_{G2}/(8aH^2)}$
<code>he3_text_xid(ttc, p)</code>	Dipolar length (according to Hakonen-1989) $\xi_D = \sqrt{13\lambda_{G2}/12\lambda_D}$ . Note: in Thuneberg's paper $\xi_D = \sqrt{\lambda_{G2}/\lambda_D}$ is used.
<code>he3_text_vd(ttc, p)</code>	Dipolar velocity (according to Thuneberg-2001) $v_D = \sqrt{2a/5\lambda_{HV}}$





## Transport properties in the normal phase

See more information in the source file `he3_transp.n.f`.

Crossections and scattering factors:

<code>he3_scatt_w(P)</code>	Scattering crossection $\langle W \rangle$ (Einzel-1978, f.82)
<code>he3_scatt_wi(P)</code>	$\langle W_I \rangle$ (Einzel-1978, f.82)
<code>he3_scatt_wd(P)</code>	$\langle W_D \rangle$ (Einzel-1978, f.82)
<code>he3_scatt_wl(P)</code>	$\langle W_L \rangle$ (Einzel-1978, f.71)
<code>he3_scatt_g0(P)</code>	Scattering factor $\gamma_0 = \langle W_I \rangle / \langle W \rangle$ (Einzel-1978, f.66)
<code>he3_scatt_d0(P)</code>	Scattering factor $\delta_0 = \langle W_D \rangle / \langle W \rangle$ (Einzel-1978, f.67)
<code>he3_scatt_w0(P)</code>	$w_0 = 1 - 3/2\gamma_0 + \delta_0$ (Einzel-1978, f.79)
<code>he3_scatt_l1a(P)</code>	Scattering factor $\lambda_1^a$ used in spin diffusion transport time. Can be neglected. (Einzel-1978, f.74, p.350).

<code>he3_tau_n0(ttc,p)</code>	Normal state quasiparticle lifetime at the Fermi level, s. $\tau_N(0, T) = \tau_N(0, T_c) \left( \frac{T_c}{T} \right)^2 = \frac{32E_F\hbar}{\langle W \rangle (\pi k_B T)^2}$ (Einzel-1991, p.325)
<code>he3_tau_n_av(ttc,p)</code>	Thermal average of normal state quasiparticle lifetime, s. $\bar{\tau}_N = \left\langle \frac{\tau_N(0, T)}{1 + (\xi_k/\pi T)^2} \right\rangle = \frac{3}{4} \tau_N(0, T) \quad (\text{Einzel-1991, f.4-5})$
<code>he3_tau_nd(ttc,p)</code>	Spin diffusion transport time for a normal Fermi-liquid, s $\tau_{ND} = \tau_1 = \bar{\tau}_N / (1 - \lambda_1^a)$ (Einzel-1991, p.328)
<code>he3_diffn_hydr(ttc, p)</code>	Hydrodynamic spin diffusion in normal liquid, $D_0$ , cm <sup>2</sup> /s (Einzel JLTP84 (1991) f.23)
<code>he3_diffn_perp(ttc, p, nu0)</code>	Perpendicular component of spin diffusion, reduced because of the Leggett-Rice effect, $D_N^\perp$ , cm <sup>2</sup> /s (Einzel JLTP84 (1991) f.22)

TODO: Temperature is measured in  $T_c$  units which is strange for the normal phase. Maybe it is better to use K of  $T_F$  units here.

## Transport properties in the B phase

Values for `he3_tau0`, `he3_tau_av`, `he3_diff_*` are extended to  $T > T_c$  region using normal phase functions. See more information in the source file `he3_transp_b.f` and in `doc_tech/spin.diff.pdf`.

Collision integral for Bogoliubov quasiparticles:

<code>he3_coll_int(xi, ttc, gap, g0, d0)</code>	Full temperature range approximation (Einzel, Wolffe, Hirschfeld, JLTP80 (1990), p.66)
<code>he3_coll_int_lt(xi, ttc, gap, g0, d0)</code>	Collision integral for low temp (good for $< 0.7T_c$ ) (Einzel, JLTP84 (1991), f.76)
<code>he3_coll_int_ht(xi, ttc, gap, g0, d0)</code>	Collision integral for high temp (Einzel, JLTP84 (1991), f.75)

Bogoliubov quasiparticles lifetime and mean free path:

<code>he3_tau0(ttc, p)</code>	Lifetime at the Fermi level, s. $1/\tau(0) = \frac{I(0, T)}{\tau_N(0, T)}$ , (Einzel-1991, f.74,76. Einzel-1990 f.A1)
<code>he3_tau0lt(ttc, p)</code>	$1/\tau, T \rightarrow 0$ limit (does not depend on energy). (Einzel-1978, f.79)
<code>he3_tau_av(ttc, p)</code>	Thermal average lifetime, s. $1/\bar{\tau} = \frac{1}{Y_0} \int_{-\infty}^{\infty} \frac{\phi_k d\xi}{\tau}$ (Einzel-1991, f.77)
<code>he3_fpath(ttc, p)</code>	Mean free path, cm (Einzel JLTP32 (1978) f.84)
<code>he3_tau_dperp(ttc, p)</code>	Spin diff. transport time, s. $\tau_D^{\perp, \parallel} = \bar{\tau}/(1 - \lambda_1^q Y^{\perp, \parallel})$
<code>he3_tau_dpar(ttc, p)</code>	(Einzel-1991, f.90,96)

Spin diffusion:

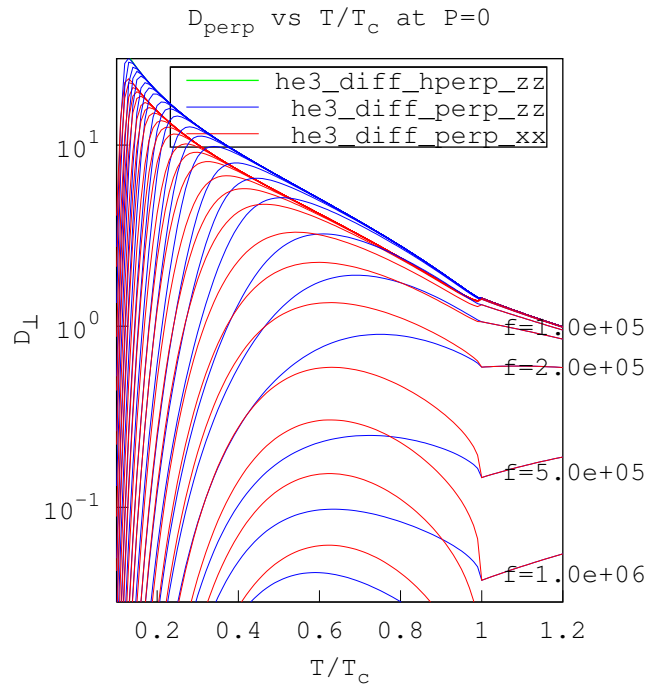
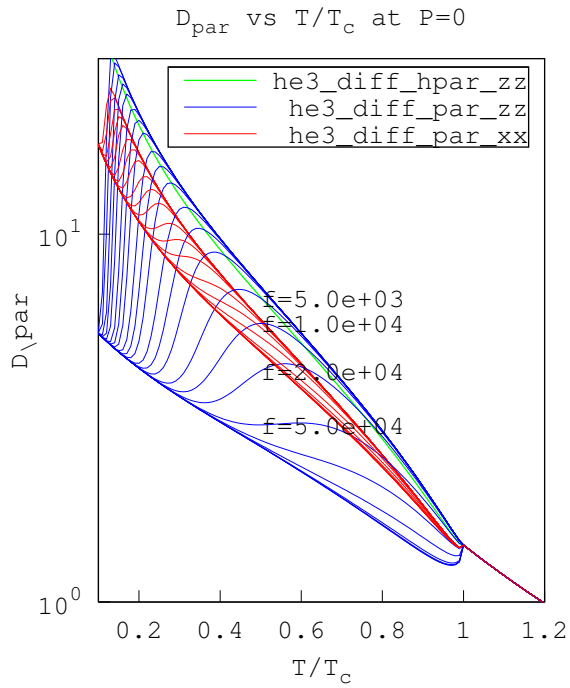
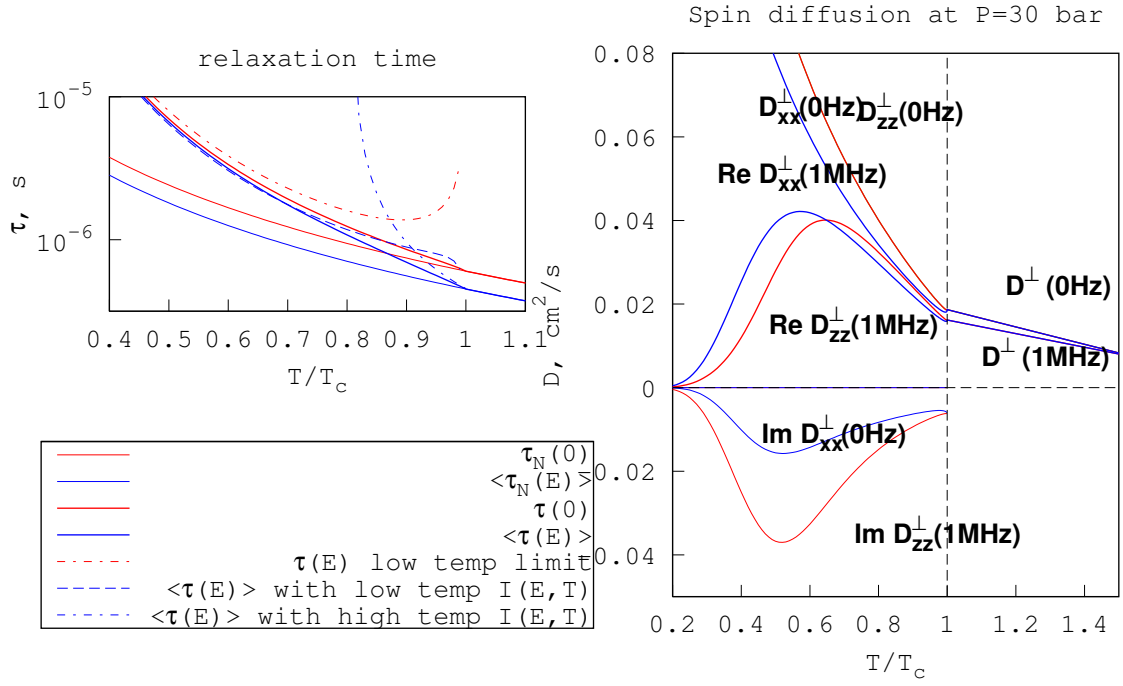
<code>he3_diff_hperp_zz(ttc, p)</code>	Spin diffusion in hydrodynamic limit ( $\omega_L \tau \ll 1$ ), $D_{zz}^{\perp, \parallel}(\omega_L = 0)$ , cm <sup>2</sup> /s. According to Einzel's paper (see f.105) nonhydrodynamic effects do not affect $D^{\parallel}$ . In Mukharsky paper they do. (Einzel JLTP84 (1991) f.102)
<code>he3_diff_hpar_zz(ttc, p)</code>	
<code>he3_diff_perp_xx(ttc, p, nu0)</code>	Components of spin diffusion tensor, cm <sup>2</sup> /s
<code>he3_diff_perp_zz(ttc, p, nu0)</code>	(Bunkov et al, 1990 f.3; Einzel, 1991, f.108;
<code>he3_diff_perp_xx_im(ttc, p, nu0)</code>	Markelov, Mukharsky, 1992, f.7,8)
<code>he3_diff_perp_zz_im(ttc, p, nu0)</code>	
<code>he3_diff_par_xx(ttc, p, nu0)</code>	
<code>he3_diff_par_zz(ttc, p, nu0)</code>	

Problem with  $D^{\parallel}$ . According to Einzel-1991 (see f.105) nonhydrodynamic effects do not affect  $D^{\parallel}$  and you should use `he3_diff_hpar_zz` for any frequency. In Markelov-Mukharsky paper the result for  $D^{\parallel}$  is completely different (it is used in `he3_diff_par_xx, zz`).  $D^{\perp}$  is the same in all papers.

TODO:

Exchange coupling strength is in the  $F_1^a = 0$  limit. (small difference from normal phase values)

How does `he3_diff_perp_??_im` extends to normal phase?



## Other

See more information in the source file `he3_other.f`.

`he3_xigl(ttc, p)` Extrapolated GL coherence length, cm  
`he3_vneq(ttc, p, omega, r)` Equilibrium vortex number

## Normal liquid beyond zero temperature limit

See more information in the source file `he3_normal.f`.

`he3_cv_n(t, p)` Normal phase heat capacity  $C_v$ , Greywall-1983  
TODO:  $V_m$

## ROTA-specific functions and constants

See more information in the source files `he3_rota.f` and `he3_const.f`

`rota_c_ns(t, i)` Nuclear stage heat capacity [J/K] vs T[K] and I[A]  
 $C_{ns} = 9.66 \cdot 10^{-5} \text{ [J/T}^2\text{]} (0.113 \text{ [T/A]} I/T)^2$

`rota_fork_cal(w, p, n)` Calibration of fork  $n$ ,  $T/T_c$ , vs fork width  $w$  [Hz] and pressure  $P$  [bar]  
 $w = a_n \exp\left(-\frac{\Delta_{WC+}(P, T/T_c)}{T/T_c}\right),$   
 $a = \alpha_n p_F^4(P)$ , where  $\alpha_n$  is a geometrical factor  
 $N = 1$  : Fork K, calibration 30.4.2010, 29 bar,  $a = 11700$   
 $N = 2$  : Fork E, calibration 30.4.2010, 29 bar,  $a = 17543$

`rota_rcell` cell radius

`rota_nmra` field/current in nmrA solenoid [G/A]

`rota_nmrb` field/current in nmrB solenoid [G/A]

`rota_nmra_q(f0)` Q value of the nmrA spectrometer vs frequency

`rota_nmra_f(n)` frequencies of nmrA spectrometer, kHz for  $n=1..8$

`rota_hmina_r` effective radius of the HminA coil [cm]

`rota_hmina_n` number of turns of the HminA coil, 4

`rota_hmina` field/current in the center of HminA coil [G/A]

`rota_hmina_mr` quadratic radial term of the HminA field, [G/A/cm<sup>2</sup>]

`rota_hmina_i0i` effective HminA coil current divided by NMR current

`rota_hmina_i0f` effective HminA coil current divided by NMR frequency

`rota_rrda` radiation damping constant  $R_{RD}$  for the nmrA spectrometer

`rota_Bza(I, Imin, r, z)` Bz field profile of A spectrometer



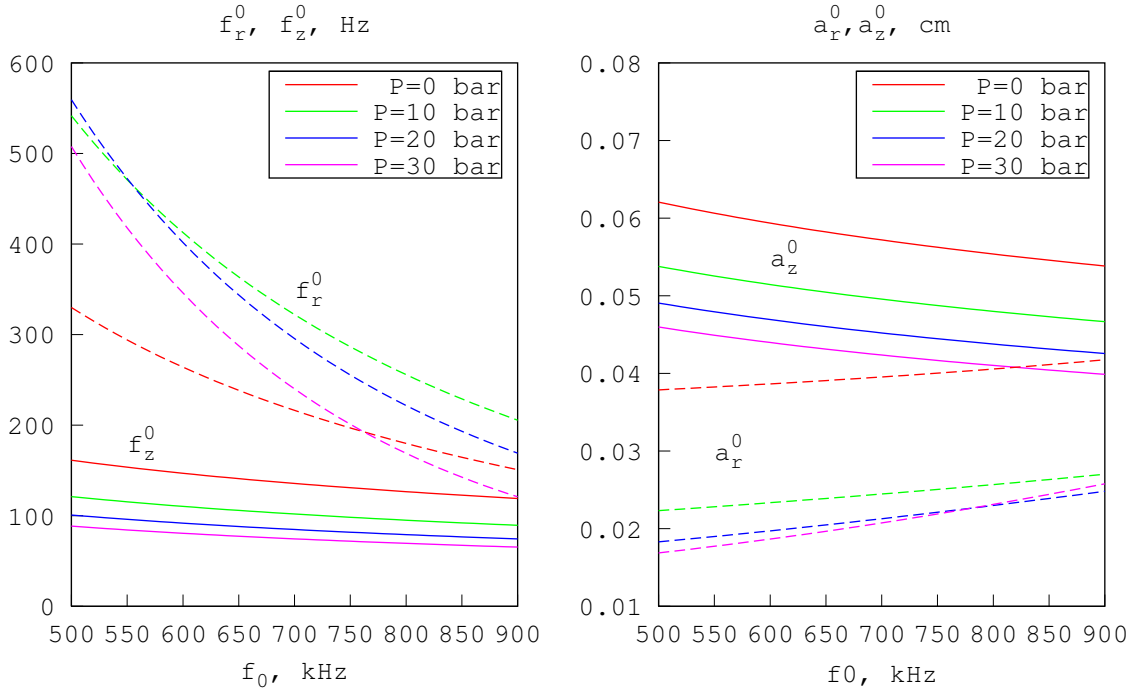
## Q-balls in the zero temperature limit

See more information in the source file `he3_qball.f`

<code>qball_nu_b(P)</code>	Leggett frequency [Hz]
<code>qball_cpar(P)</code> <code>qball_cper(P)</code>	Measured $c_{  }$ , $c_{\perp}$ , [cm/s]
<code>qball_lg1(P)</code> <code>qball_lg2(P)</code>	$\lambda_{G1}$ , $\lambda_{G2}$ calculated using measured spin wave velocities.
<code>qball_text_a(P)</code>	measured textural parameter $a$ [erg/cm <sup>3</sup> 1/G <sup>2</sup> ]
<code>qball_text_d(P)</code>	measured textural parameter $d$ [erg/(cm <sup>2</sup> G <sup>2</sup> )]
<code>qball_text_lsg(P)</code>	measured textural parameter $\lambda_{SG}$
<code>qball_trd(P,f0,fr,fz)</code>	$\tau_{RD}$ for the magnon condensate with given radial and axial frequencies (rota-specific) [s]

A model of the zero-temperature, zero-rotation condensate, based on spectra measurements (rota-specific):

<code>qball_dbetan(P, f0)</code>	Derivative of the textural angle $\beta_N$ in the center of the cell, [rad/cm].
<code>qball_fz0(P,f0,Imin)</code> <code>qball_fr0(P,f0,Imin)</code>	1/2 of the distance between visible axial and radial levels, [Hz]
<code>qball_az0(P,f0,Imin)</code> <code>qball_ar0(P,f0,Imin)</code>	Axial and radial sizes of the magnon condensate, [cm]
<code>qball_trd0(P,f0,Imin)</code>	$\tau_{RD}$ for the magnon condensate, [s]



## Math

See more information in the source file `he3_math.f`

`math_ele(x)`                      Complete elliptic integrals  $E(x)$  and  $K(x)$   
`math_elk(y)`

`loop_bz(rloop,r,z)`              Magnetic field of a current loop,  $B_z$ ,  $B_r$   
`loop_br(rloop,r,z)`

These function have only fortran interface. Use `he3_math.fh` include file. Examples are in `test_int` folder.

`real*8 math_dint(func, xmin, xmax, nx)`  
– simple integration of `real*8` function (Gauss 2pt)

`complex*16 math_cint(func, xmin, xmax, nx)`  
– integration of `complex*16` function

`real*8 math_dint2d(func, xmin, xmax, nx, ymin, ymax, ny)`  
– integration of 2D `real*8` function

`complex*16 math_cint2d(func, xmin, xmax, nx, ymin, ymax, ny)`  
– integration of 2D `complex*16` function

`real*8 math_dint_gk(func, xmin, xmax, nx, aerr)`  
– integrate `real*8` function using Gauss-7pk+Kronrod-13pt

`real*8 math_dint_gka(func, xmin, xmax, aerr_lim, rerr_lim)`  
– adaptive integration

`real*8 math_dint2d_gk(func, xmin, xmax, nx, ymin, ymax, ny, aerr)`  
– integrate 2D `real*8` function using Gauss-7pk+Kronrod-13pt