^{3}He library

vladislav.zavyalov@aalto.fi

December 9, 2015

Usage

This library provides constant and functions for various ^{3}He properties. Supported Languages:

- Fortran-77. You should use `he3.fh` include file and `libhe3` library for fortran programs. All functions and arguments are `real*8`. Use `-fno-range-check` compiler flag to allow `NaN` values.
- Fortran-90. You should include he3.f90h instead.
- **C.** You should use `he3.h` include file and `libhe3` library in C programs. Lowercase names with underscore should be used (like `he3_pf_`). Type of arguments is always `double*`, type of returned value is `double`.
- **Matlab, Octave.** `mex` files are located in the `matlab` folder. Lowercase names should be used. In function arguments you can mix numbers with arrays or matrices of the same size. Example:

  ```
  ttc=0:0.1:1; p=10;
  I = he3_nu_b(p, ttc);
  ```

  To build `mex` files you must run `make octave`, `make matlab` or `make matlab64`. Fixed rpath is used for the library. For building this documentation octave files are needed.
- **Command line.** You can use all the functions via a command line interface. To get a list of all functions run the program `he3` without arguments:

  ```
  > he3
  ```

  To get information about specific function run the program with the function name as an argument:

  ```
  > he3 he3_nu_b
  ```

  You can use matlab-style vectors (value1:step:value2) to get a table of values (if step is missing then 20 points will be printed):

  ```
  > he3 he3_nu_b 0.1:0.1:1 0
  ```

  will give you Leggett frequency vs temperature at zero pressure.

Location

GIT: https://github.com/slazav/he3lib

Constants

`const_na`	Avogadro constant $N_A = 6.02214129 \cdot 10^{23}$ [1/mol]
`const_kb`	Boltsman constant $k_B = 1.3806488 \cdot 10^{-16}$ [erg/K]
`const_r`	R-gas constant $R = 8.314472 \cdot 10^{7}$ [sgs]
`const_h`	Plank constant $h = 6.62606957 \cdot 10^{27}$ [g cm^2/s]
`const_hbar`	$\hbar = h/2\pi = 1.054571726 \cdot 10^{27}$ [g cm^2/s]
`const_mu0`	Vacuum permeability $\mu_0 = 1.2566370614$ [G*cm/A]
`const_pi`	$\pi = 3.1415926535897932$
`const_2pi`	2π
`he3_amass`	^{3}He atom mass, $m_3 = 5.0079 \cdot 10^{-24}$ [g]
`he3_mmass`	^{3}He molar mass, $\mu_3 = N_A m_3 = 3.0158$ [g/mol]
`he3_gyro`	^{3}He gyromagnetic ratio $\gamma = 20378$ [(G s)$^{-1}$],

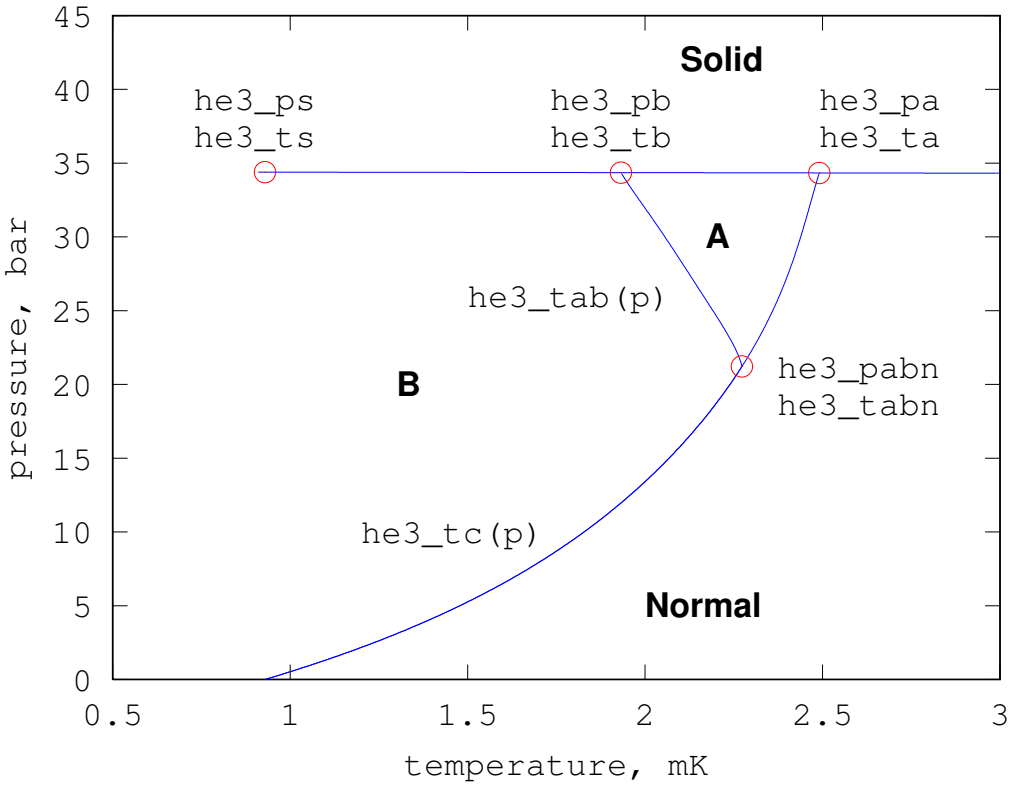
Phase diagram

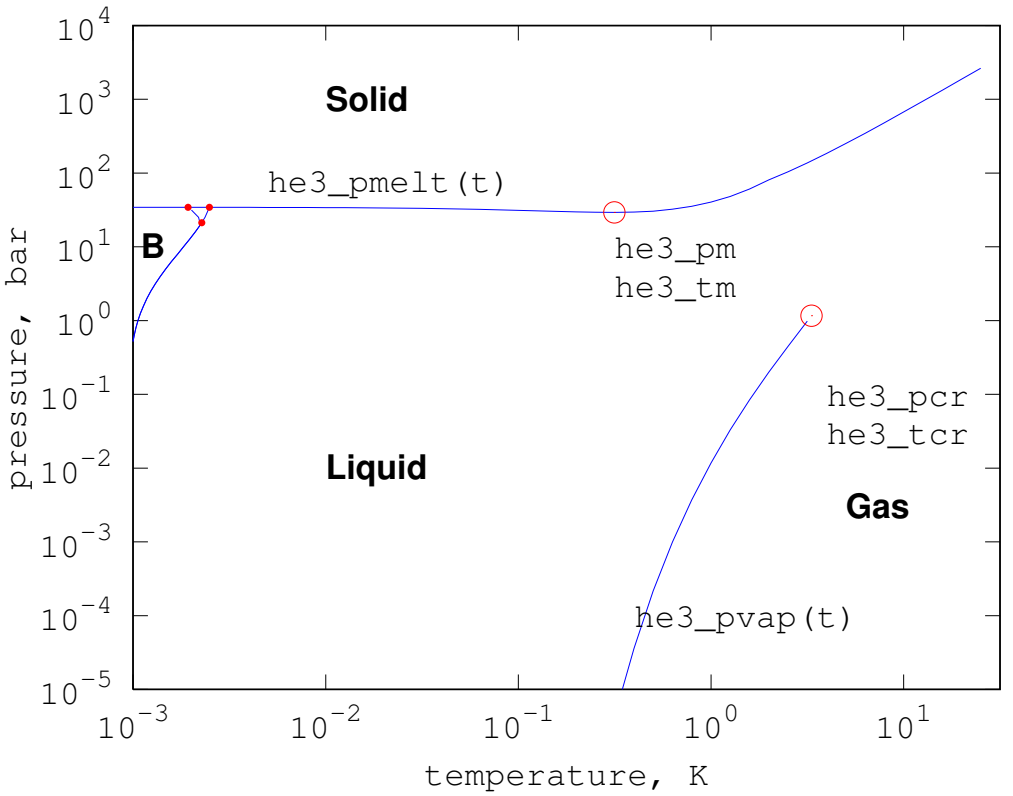
Vapor pressure and critical point are from the 1962 ^{3}He scale of temperatures. Melting curve $P_m(T)$ at 0.9 – 250 mK, $T_c(P)$, $T_{AB}(P)$ and all tricritical points are from Greywall-86 temperature scale. Melting curve minimum is from PLTS-2000 temperature scale. Melting curve for higher temperatures is from Osborne, Abraham, Weinstock-1951 and Mills, Grilly-1955 with interpolation in the regions 0.25–0.5K and 1.5–2.0K. PLTS-2000 melting curve and critical points on it are also available. See more information in the source file `he3_phase.f`.

`he3_pvap(T)`	Vapor pressure [bar] vs temperature [K], $T = 0.2 - 3.324$ K
`he3_pcr`	Gas-liquid critical point pressure, 1.16317 [bar]
`he3_tcr`	Gas-liquid critical point temperature, 3.324 [K]
`he3_pmelt(T)`	Melting pressure [bar] vs temperature [K], $T = 0.0009 - 31$ K
`he3_pm`	Melting curve minimum pressure, 29.3113 [bar]
`he3_tm`	Melting curve minimum temperature, 0.31524 [K]
`he3_pa`	Superfluid trans. at melting curve, pressure, 34.3380 [bar]
`he3_ta`	Superfluid trans. at melting curve, temp., 2.491 [mK]
`he3_pb`	A-B trans. at melting curve, pressure, 34.3580 [bar]
`he3_tb`	A-B trans. at melting curve, temp., 1.932 [mK]
`he3_ps`	Neel transition at melting curve, pressure, 34.3905 [bar]
`he3_ts`	Neel transition at melting curve, temp., 0.9291 [mK]
`he3_tc(P)`	Superfluid transition temperature [mK] vs pressure [bar], $P = 0 - 34.358$ bar
`he3_tab(P)`	A-B transition temperature [mK] vs pressure [bar], $P = 0 - 34.3609$ bar, below 21.22 bar is equal to `he3_tc`
`he3_tabn`	A-B-Normal point temperature, 2.2311 [mK]
`he3_pabn`	A-B-Normal point pressure, 21.22 [bar]

PLTS-2000 temperature scale:

`he3_pmelt_plts(T)`	PLTS-2000 melting presure [bar] vs temperature [K].
`he3_pa_plts`	Superfluid trans. at melting curve, pressure, 34.3407 [bar]
`he3_ta_plts`	Superfluid trans. at melting curve, temp., 2.444 [mK]
`he3_pb_plts`	A-B trans. at melting curve, pressure, 34.3609 [bar]
`he3_tb_plts`	A-B trans. at melting curve, temp., 1.896 [mK]
`he3_ps_plts`	Neel transition at melting curve, pressure, 34.3934 [bar]
`he3_ts_plts`	Neel transition at melting curve, temp., 0.902 [mK]


Solid
he3_ps
he3_ts
he3_pb
he3_tb
he3_pa
he3_ta
A
he3_tab(p)
B
he3_pabn
he3_tabn
he3_tc(p)
Normal
pressure, bar
temperature, mK
45
40
35
30
25
20
15
10
5
0
0.5
1
1.5
2
2.5
3


Solid
he3_pmelt(t)
B
he3_pm
he3_tm
he3_pcr
he3_tcr
Liquid
Gas
he3_pvap(t)
pressure, bar
temperature, K
10^{4}
10^{3}
10^{2}
10^{1}
10^{0}
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-3}
10^{-2}
10^{-1}
10^{0}
10^{1}

Fermi-liquid parameters

Argument is pressure, [bar] in the range 0 – 34.4. See more information in the source file `he3_fermi.f`.

Molar volume and heat capacity in normal ^{3}He:

`he3_vm(P)` Molar volume v_m, [cm^3/mol], Graywall-86 (from Wheatley-75)

`he3_gammaf(P)` R-gas constant $\gamma_f = C_V/RT$, [1/(K mol)], Greywall-86

Derived values, F_1^s parameter:

`he3_rho(P)` Density $\rho = \mu_3/v_m$, [g/cm^3]

`he3_2n0(P)` $2N(0) = \dfrac{\gamma_f}{v_m}\ \dfrac{3N_A}{k_B\pi^2}$, [1/(erg cm^3)]

`he3_pf(P)` $p_F = h\left(\dfrac{3}{8\pi}\ \dfrac{N_A}{v_m}\right)^{1/3}$, [g cm/s]

`he3_meff(P)` $m^\star = \dfrac{h^3}{8\pi}\ \dfrac{2N(0)}{p_F}$, [g]

`he3_mm(P)` $m^\star/m_3$

`he3_vf(P)` $v_F = p_F/m^\star$, [cm/s]

`he3_f1s(P)` $F_1^s = 3(m^\star/m_3 - 1)$

`he3_a(P)` Average atomic spacing, $a = (v_m/N_A)^{1/3}$, Å

`he3_gdk(P)` Average dipolar coupling enegy, $g_d/k_B = \dfrac{2\pi\gamma^2\hbar^2}{3v_m k_B}$, [K]

`he3_tfeff(P)` Effective Fermi temperature, $T_{F_{eff}} = \dfrac{\pi^2}{2\gamma_f}$, [K]

Sound velocity in normal ^{3}He and F_0^s parameter:

`he3_c1(P)` First sound velocity, c_1, [cm/s], measured, Wheatley-75

`he3_f0s(P)` $F_0^s = 3\ m^\star m_3\ c_1^2/p_F^2 - 1$

Magnetic susceptibility in normal ^{3}He and F_0^a parameter:

`he3_f0a(P)` $F_0^a, Z_0/4$, from magnetic susceptibility measurements, Hensley-1993

`he3_chi_n(P)` $\chi_N = \dfrac{2N(0)(\gamma\hbar)^2}{4(1+F_0^a)}$.

Other fermi-liquid parameters:

`he3_f1a(P)` F_1^a, calculated from spin-wave velocities in ^{3}He-B, Zavjalov-2015

`he3_f2a(P)` F_2^a, currently is 0

`he3_f2s(P)` F_2^s, currently is 0

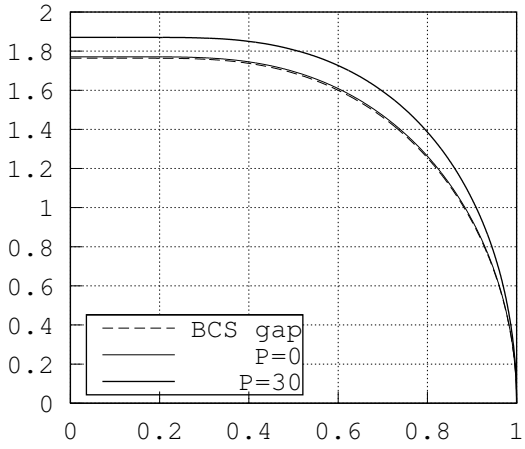
Pressure-dependent values:

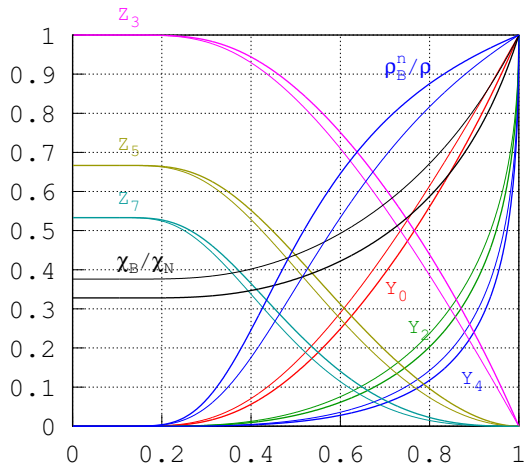
P, bar	0	3	6	9	12	15	18	21	24	27	30	33
T_c, mK	0.929	1.290	1.560	1.769	1.934	2.067	2.177	2.267	2.339	2.395	2.438	2.474
T_{AB}, mK	–	–	–	–	–	–	–	–	2.217	2.137	2.056	1.969
v_m, cm^3/mol	36.84	33.95	32.03	30.71	29.71	28.89	28.18	27.55	27.01	26.56	26.17	25.75
c_1, m/s	182.9	227.4	259.9	285.8	307.9	327.3	344.8	360.6	375.2	389.2	402.9	416.1
γ_f, 1/(K mol)	2.784	2.981	3.158	3.321	3.475	3.625	3.772	3.918	4.065	4.211	4.355	4.496
ρ, g/cm^3	0.082	0.089	0.094	0.098	0.102	0.104	0.107	0.109	0.112	0.114	0.115	0.117
$2N(0), 10^{38}$	1.002	1.164	1.307	1.434	1.551	1.663	1.774	1.885	1.995	2.102	2.207	2.315
$p_F, 10^{-20}$ g cm/s	8.280	8.509	8.675	8.798	8.896	8.979	9.054	9.122	9.182	9.234	9.280	9.330
$m^\star, 10^{-23}$ g	1.401	1.584	1.744	1.887	2.018	2.144	2.269	2.392	2.515	2.635	2.753	2.872
$m^\star/m_3$	2.797	3.162	3.482	3.767	4.030	4.282	4.530	4.777	5.022	5.261	5.497	5.735
v_F, m/s	59.11	53.73	49.74	46.64	44.08	41.87	39.91	38.13	36.51	35.05	33.71	32.49
$\chi_N, 10^{-9}$	38.4	48.6	56.9	63.9	70.5	77.0	83.4	89.4	94.7	99.7	104.5	109.5
F_0^s	9.27	15.99	22.52	28.91	35.32	41.81	48.43	55.17	62.10	69.32	76.95	84.82
F_1^s	5.39	6.49	7.45	8.30	9.09	9.85	10.59	11.33	12.07	12.78	13.49	14.21
F_0^a	-0.70	-0.72	-0.73	-0.74	-0.75	-0.75	-0.75	-0.76	-0.76	-0.76	-0.76	-0.76
F_1^a	-0.60	-0.60	-0.61	-0.62	-0.62	-0.63	-0.64	-0.64	-0.65	-0.66	-0.66	-0.67
a, Å	3.940	3.834	3.761	3.708	3.668	3.634	3.604	3.577	3.553	3.533	3.516	3.497
g_d/k_B, μK	114.5	124.3	131.7	137.4	142.0	146.0	149.7	153.1	156.2	158.8	161.2	163.9
$T_{F_{eff}}$, K	1.773	1.656	1.563	1.486	1.420	1.361	1.308	1.260	1.214	1.172	1.133	1.098
$<W>$	84.3	107.3	119.8	127.6	134.5	141.3	146.8	150.1	150.8	150.1	149.4	149.0
$<W_I>$	11.39	14.25	15.45	16.03	16.55	17.11	17.54	17.66	17.46	17.09	16.76	16.48
$<W_D>$	24.83	31.59	35.27	37.55	39.54	41.47	43.05	43.96	44.12	43.86	43.63	43.46
$<W_L>$	-9.38	-16.79	-20.57	-22.68	-24.53	-26.41	-27.85	-28.34	-27.81	-26.70	-25.66	-24.73
γ_0	0.135	0.133	0.129	0.126	0.123	0.121	0.119	0.118	0.116	0.114	0.112	0.111
δ_0	0.295	0.294	0.294	0.294	0.294	0.294	0.293	0.293	0.293	0.292	0.292	0.292
w_0	1.204	1.206	1.208	1.210	1.212	1.213	1.214	1.214	1.215	1.216	1.217	1.218
λ_1^a	-0.111	-0.157	-0.172	-0.178	-0.182	-0.187	-0.190	-0.189	-0.184	-0.178	-0.172	-0.166
$\tau_N(0, T_c)$, μs	0.603	0.230	0.133	0.092	0.070	0.056	0.047	0.040	0.036	0.034	0.032	0.030
$\Delta_{WC+}(T=0)/T_c$	1.771	1.795	1.813	1.825	1.834	1.843	1.850	1.856	1.861	1.866	1.870	1.875
$\chi_B(T=0)/\chi_N$	0.376	0.356	0.347	0.341	0.337	0.333	0.329	0.328	0.327	0.328	0.328	0.328
$\nu_B(T=0), kHz$	125.1	161.9	188.1	208.6	226.0	241.6	256.1	269.9	283.3	296.1	308.5	320.6
$c_\perp(T=0), cm/s$	2170	1940	1781	1661	1563	1479	1404	1338	1280	1228	1180	1136
$c_\parallel(T=0), cm/s$	2484	2220	2039	1901	1789	1692	1607	1531	1464	1404	1349	1299
P, bar	0	3	6	9	12	15	18	21	24	27	30	33

Energy gap

BCS energy gap + trivial strong coupling correction + some values derived from energy gap. See more information in the source file `he3_gap.f`.

`he3_bcsgap(ttc)`	BCS gap for 3He-B in T_c units, $\Delta_{\rm BCS}/k_BT_c$
`he3_bcsgap_fast(ttc)`	Einzel approximation for BCS gap (0.5% accuracy, 70 times faster) (Einzel-1991, f.68)
`he3_trivgap(ttc, P)`	Trivial strong-coupling correction to the BCS gap
`he3_todogap(ttc, P)`	Gap based on Todoschenko's measurements: linear interpolation in density between BCS value at zero bar and measured value 1.99 at melting pressure + temperature behaviour as in `he3_trivgap`
`he3_gap(ttc, P)`	Wrapper for the gap used everywhere in the lib (trivgap by default)
`he3_yosida(ttc, gap, n)`	Yosida functions $Y_n(T/T_c,\Delta)=\int_{-\infty}^{\infty}\left(\frac{\xi_k}{E_k}\right)^n\frac{1}{2T/T_c}\,\mathrm{ch}^{-2}\left(\frac{E_k}{2T/T_c}\right)d\xi_k$ Low temperature limit: $Y_n(T,\Delta)=2\Gamma\left(\frac{n+1}{2}\right)\left(\frac{T}{\Delta}\right)^{\frac{n-1}{2}}\exp\left(-\frac{\Delta}{T}\right)$ Note: type of n parameter in fortran should be real*8 (this was done for standard function handling). If n is integer, result is unpredictable!
`he3_yosida_par(ttc, gap)`	$Y_q^{\parallel}(T/T_c,\Delta)$
`he3_yosida_perp(ttc, gap)`	$Y_q^{\perp}(T/T_c,\Delta)$
`he3_z3(ttc, gap)`	
`he3_z5(ttc, gap)`	
`he3_z7(ttc, gap)`	
`he3_lambda(ttc, gap)`	
`he3_rho_nb(ttc, p)`	B-phase normal component density: $\frac{\rho_B^n}{\rho_N}=\frac{(3+F_1^s)Y_0}{3+F_1^sY_0}$
`he3_chi_b(ttc, p)`	B-phase susceptibility (ratio of he3_chi_n): $\frac{\chi_B}{\chi_N}=\frac{(1+F_0^a)(2+Y_0)}{3+F_0^a(2+Y_0)}$
`he3_chi_bp(ttc, P)`	B-phase Cooper pair susceptibility (ratio of he3_chi_b): $\frac{\chi_B^p}{\chi_B}=\frac{2(1-Y_2)}{(2+Y_0)}$





Dipole energy and Legget frequency

Values are given according to Thuneberg's paper JLTP 122, p657 (2001). Temperature independent g_d parameter is restored from experimental data for Ω_B, measured in transverse and longitudinal NMR. See more information in the source file `he3_dipole.f`.

Dipolar energy:

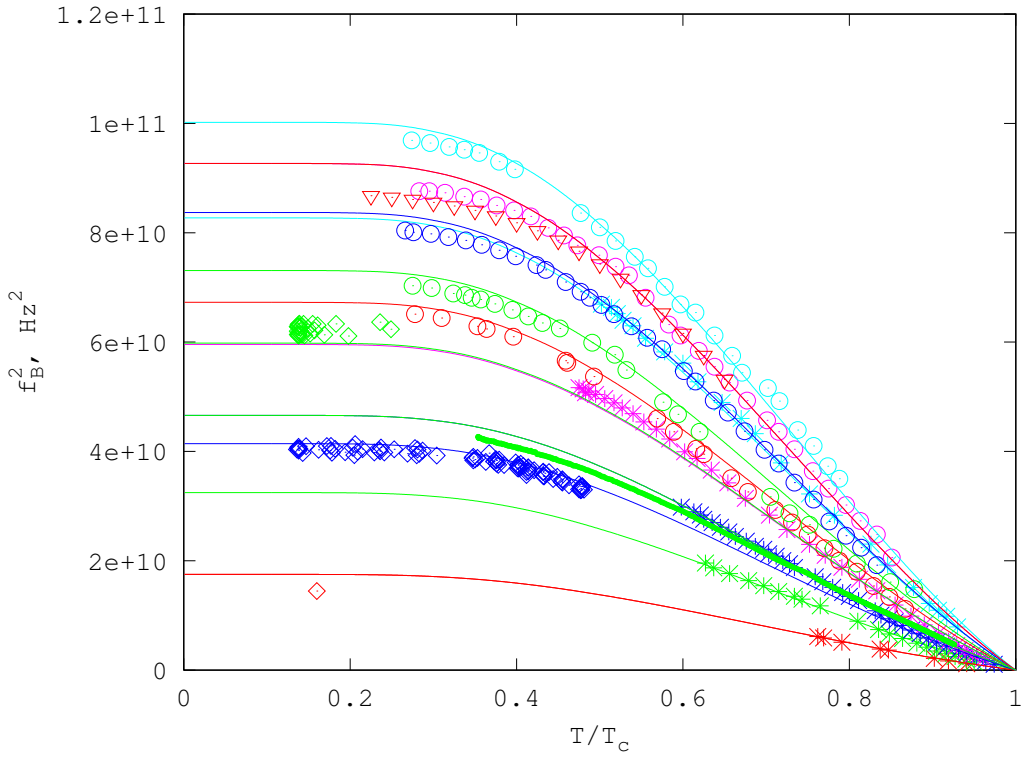
$$F_D = \lambda_D \int (R_{ii}R_{jj} + R_{ij}R_{ji})\ d^3r \quad = \quad 4\lambda_D \int \cos\theta(1+2\cos\theta)\ d^3r$$

$$\lambda_D = \Delta^2 g_d, \qquad \Omega_B = \gamma\sqrt{15\lambda_D/\chi_B}$$

Note: In old papers (Leggett, ...) $g_D/5$ is used instead of λ_G.

`he3_gd(p)`	g_d, [1/(erg cm^3)]
`he3_ld(p)`	$\lambda_D = \Delta^2 g_d$, [erg/cm^3]
`he3_nu_b(ttc, p)`	B-phase Leggett frequency $\nu_B = \frac{\gamma}{2\pi}\sqrt{15\Delta^2 g_d/\chi_B}$, Hz
`he3_nu_b1(ttc, p)`	Less accurate formula without using g_d, Hz

$$\nu_B = \frac{1}{2\pi}\sqrt{\frac{3\pi}{2\chi}}\ \frac{\gamma^2\hbar}{2}\ N(0)\ \Delta \log\frac{e_f}{\Delta}$$



Gradient energy and spin wave velocity

Gradient energy coefficients and derived values, including spin wave velocities. See more information in the source file `he3_grad.f`.

$$F_\nabla = \frac{1}{2}\Delta^2 \left[K_1(\nabla_j R_{ak})(\nabla_j R_{ak}) + K_2(\nabla_j R_{ak})(\nabla_k R_{aj}) + K_3(\nabla_j R_{aj})(\nabla_k R_{ak})\right]$$

`he3_grad_K0(ttc, p)` $K_1 = K_2 = K_3$ without fermi-liquid corrections, see VW7.23m

`he3_grad_c(ttc, p)`
`he3_grad_delta(ttc, p)` c and δ paramters calculated with fermi-liquid corrections (Cross-1975), see VW7.25. These values are used to calculate all other things:

$$c = -\frac{\rho_s}{10}\,\frac{3+F_1^a}{3+F_1^s}\,\frac{1}{1+F_1^a(5-3\rho_s/\rho)/15}, \qquad \delta = \frac{F_1^a \rho_s/\rho}{3+F_1^s(1-\rho_s/\rho)}$$

`he3_grad_K12(ttc, p)`
`he3_grad_K3(ttc, p)`

$$K_1 = K_2 = -\frac{2}{\Delta^2}\left(\frac{\hbar}{2m}\right)^2 c, \qquad K_3 = -\frac{2}{\Delta^2}\left(\frac{\hbar}{2m}\right)^2 (1+\delta)c$$

`he3_grad_K(ttc, p)`
`he3_grad_Kp(ttc, p)`

$$K = 2K_1 + K_2 + K_3, \qquad K' = K_2 + K_3$$

`he3_grad_lg1(ttc, p)`
`he3_grad_lg2(ttc, p)`
`he3_grad_lsgb(ttc, p)` Thunebergs λ_{G1}, λ_{G2} and λ^b_{SG}:

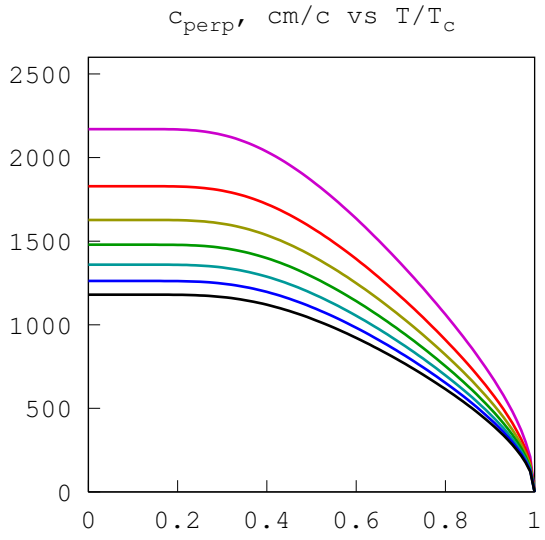
$$\lambda_{G1} = \frac{1}{2}\Delta^2(K_2+K_3), \qquad \lambda_{G2} = \frac{1}{2}\Delta^2 K_1, \qquad \lambda^b_{SG} = \Delta^2 K_2$$

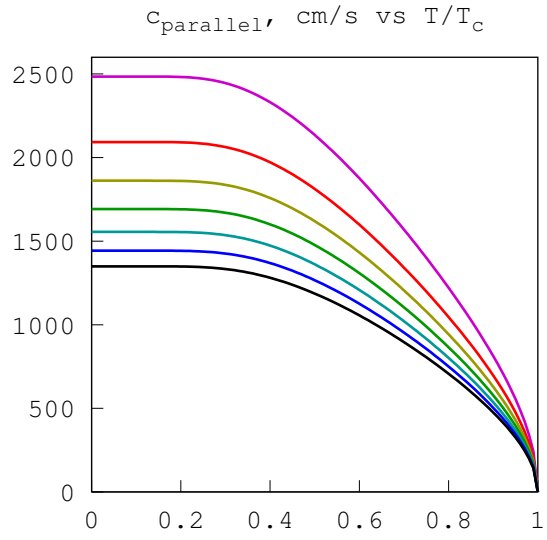
`he3_cpar(ttc, p)`
`he3_cperp(ttc, p)` velocity of transverse spin waves parallel and perpendicular to the l direction [cm/s]:

$$c_\parallel^2 = \frac{\gamma^2\Delta^2}{\chi_B}K, \qquad c_\perp^2 = \frac{\gamma^2\Delta^2}{\chi_B}(K-K'/2), \qquad c_\parallel^2/c_\perp^2 \approx 4/3$$

`he3_clpar(ttc, p)`
`he3_clperp(ttc, p)` same for longitudinal waves [cm/s]:

$$C_\parallel^2 = \frac{\gamma^2\Delta^2}{\chi_B}(K-K'), \qquad C_\perp^2 = \frac{\gamma^2\Delta^2}{\chi_B}K, \qquad C_\parallel^2/C_\perp^2 \approx 1/2$$





Other textural parameters

Values are given according to Thuneberg's paper JLTP 122, p657 (2001). High order Fermi liquid parameters (F_2^a, F_3^a) are set to 0. See more information in the source file `he3_text.f`.

Bulk terms:

$$\begin{aligned} F_{DH} &= -a \int_V (\mathbf{n}\cdot\mathbf{H})^2 \; d^3r \\ F_{DV} &= -\lambda_{DV} \int_V [\mathbf{n}\cdot(\mathbf{v_s}-\mathbf{v_n})]^2 \; d^3r \qquad \ll F_{HV} \text{ at } H \approx 200 \text{ Oe} \\ F_{HV} &= -\lambda_{HV} \int_V [\mathbf{H}\cdot R\cdot(\mathbf{v_s}-\mathbf{v_n})]^2 \; d^3r \end{aligned}$$

Surface terms:

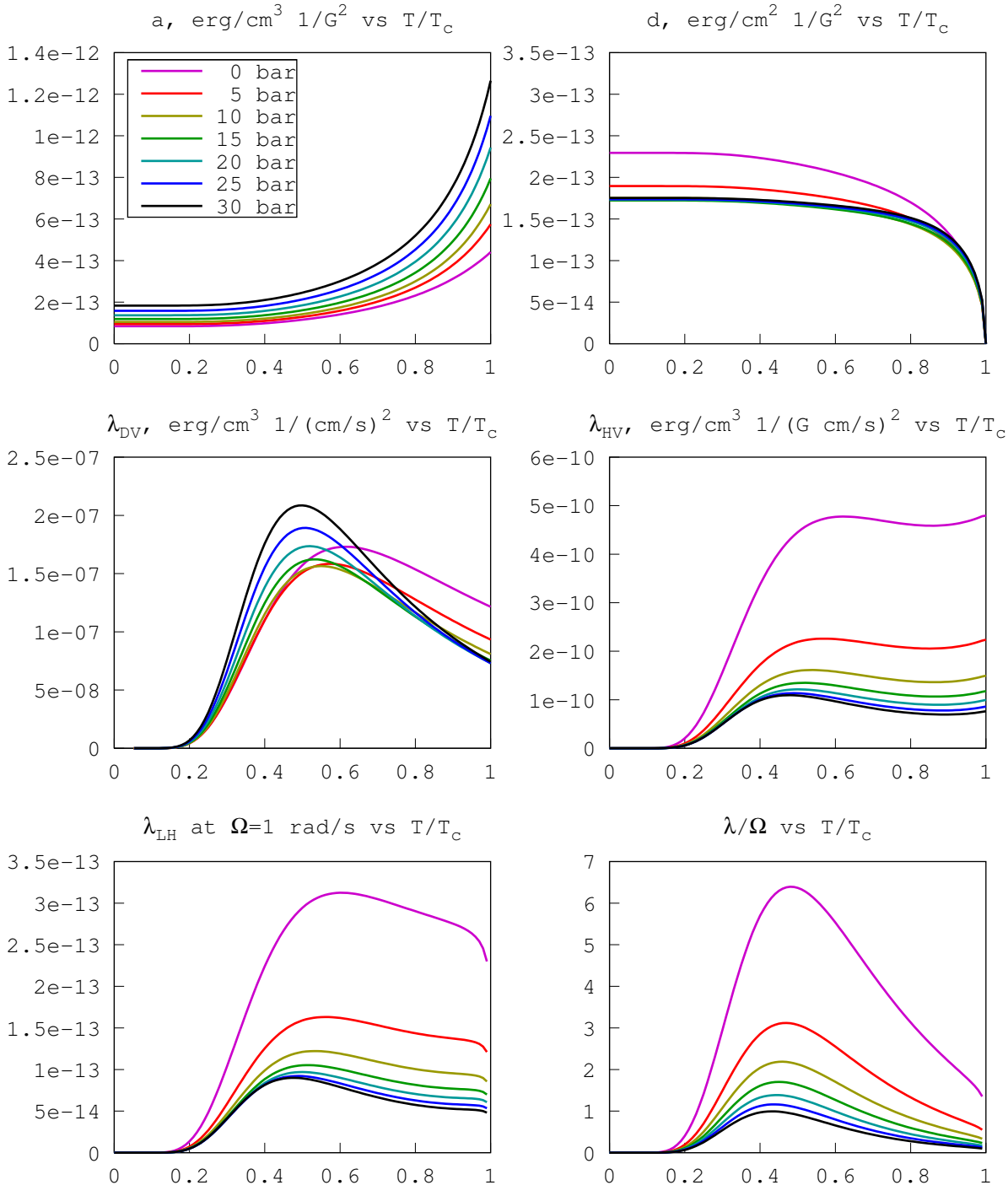
$$\begin{aligned} F_{SH} &= -d \int_S [\mathbf{H}\cdot R\cdot\mathbf{s}]^2 \; d^2r \\ F_{SG} &= -\lambda_{SG} \int_S s_j R_{\alpha j} \frac{\partial R_{\alpha i}}{\partial r_i} \; d^2r \end{aligned}$$

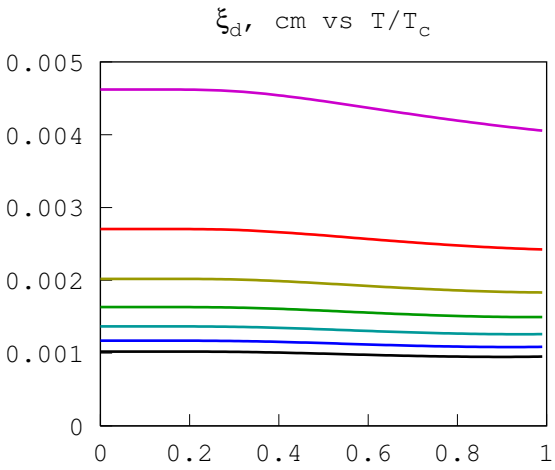
Vortex term:

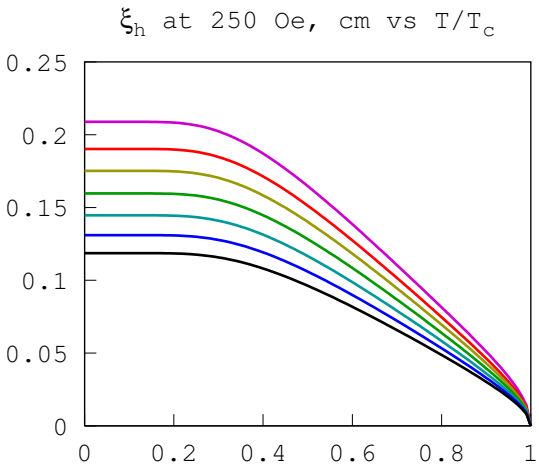
$$F_{LH} = \frac{\lambda_{LH}}{2\Omega} \int_L |\omega_s| [\mathbf{H}\cdot R\cdot\mathbf{l}]^2 \; d^3r$$

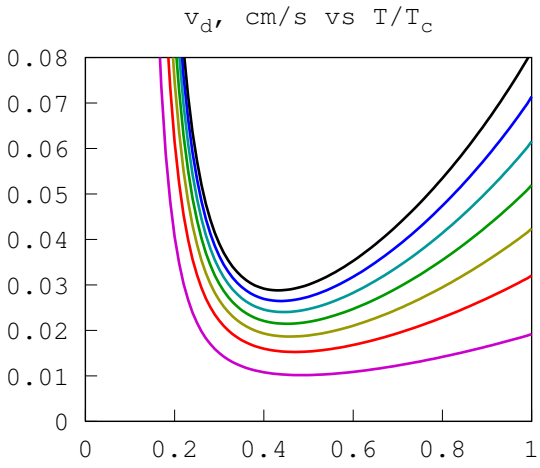
`he3_text_a(ttc, p)`	$a = \frac{5g_d}{2}\left[\frac{\hbar\gamma/2}{1+F_0^a(2+Y_0)/3}\right]^2\left(5-3\frac{Z_3}{Z_5}\right)$, [erg/cm^3 G^{-2}]
`he3_text_d(ttc, p)`	d, [erg/(cm^2 G^2)] – GL extrapolation!
`he3_text_ldv(ttc, p)`	$\lambda_{DV} = 5g_d\left(\frac{m^* v_F}{1+F_1^s Y_0/3}\right)^2\left(1-\frac{3Z_5}{2Z_3}\right)$, [erg/cm^3 (cm/s)$^{-2}$]
`he3_text_lhv(ttc, p)`	$\lambda_{HV} = \frac{\rho}{\Delta^2}\frac{1+F_1^s/3}{(1+F_1^s Y_0/3)^2}\left(\frac{\hbar\gamma/2}{1+F_0^a(2+Y_0)/3)}\right)^2\left(Z_3-\frac{9}{10}Z_5+\frac{9}{10}\frac{Z_5^2}{Z_3}-\frac{3}{2}Z_7\right)$
`he3_text_llh(ttc, p, omega)`	$\lambda_{LH} = \frac{\hbar}{2m}\Omega\lambda_{HV}\left(\ln\frac{R}{r}-\frac{3}{4}\right)$, [erg/(cm^3 G^2 s)] – counterflow part only!
`he3_text_lo(ttc, p, omega)`	lambda/omega used in the texture library: $\frac{\lambda}{\Omega} = \frac{5\lambda_{LH}}{2a\Omega}$, (rad/s)$^{-1}$

`he3_text_xih(ttc, p, h)`	Magnetic length $\xi_H = \sqrt{65\lambda_{G2}/(8aH^2)}$
`he3_text_xid(ttc, p)`	Dipolar length (according to Hakonen-1989) $\xi_D = \sqrt{13\lambda_{G2}/12\lambda_D}$. Note: in Thuneberg's paper $\xi_D = \sqrt{\lambda_{G2}/\lambda_D}$ is used.
`he3_text_vd(ttc, p)`	Dipolar velocity (according to Thuneberg-2001) $v_D = \sqrt{2a/5\lambda_{HV}}$


a, erg/cm^3 1/G^2 vs T/T_c
0 bar
5 bar
10 bar
15 bar
20 bar
25 bar
30 bar
1.4e-12
1.2e-12
1e-12
8e-13
6e-13
4e-13
2e-13
0
0 0.2 0.4 0.6 0.8 1
d, erg/cm^2 1/G^2 vs T/T_c
3.5e-13
3e-13
2.5e-13
2e-13
1.5e-13
1e-13
5e-14
0
0 0.2 0.4 0.6 0.8 1
λ_DV, erg/cm^3 1/(cm/s)^2 vs T/T_c
2.5e-07
2e-07
1.5e-07
1e-07
5e-08
0
0 0.2 0.4 0.6 0.8 1
λ_HV, erg/cm^3 1/(G cm/s)^2 vs T/T_c
6e-10
5e-10
4e-10
3e-10
2e-10
1e-10
0
0 0.2 0.4 0.6 0.8 1
λ_LH at Ω=1 rad/s vs T/T_c
3.5e-13
3e-13
2.5e-13
2e-13
1.5e-13
1e-13
5e-14
0
0 0.2 0.4 0.6 0.8 1
λ/Ω vs T/T_c
7
6
5
4
3
2
1
0
0 0.2 0.4 0.6 0.8 1


ξ_d, cm vs T/T_c
0.005
0.004
0.003
0.002
0.001
0
0
0.2
0.4
0.6
0.8
1


ξ_h at 250 Oe, cm vs T/T_c
0.25
0.2
0.15
0.1
0.05
0
0
0.2
0.4
0.6
0.8
1


v_d, cm/s vs T/T_c
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0
0
0.2
0.4
0.6
0.8
1

Transport properties in the normal phase

See more information in the source file `he3_transp_n.f`.

Crossections and scattering factors:

`he3_scatt_w(P)`	Scattering crossection $<W>$ (Einzel-1978, f.82)
`he3_scatt_wi(P)`	$<W_I>$ (Einzel-1978, f.82)
`he3_scatt_wd(P)`	$<W_D>$ (Einzel-1978, f.82)
`he3_scatt_wl(P)`	$<W_L>$ (Einzel-1978, f.71)
`he3_scatt_g0(P)`	Scattering factor $\gamma_0 =< W_I > / < W >$ (Einzel-1978, f.66)
`he3_scatt_d0(P)`	Scattering factor $\delta_0 =< W_D > / < W >$ (Einzel-1978, f.67)
`he3_scatt_w0(P)`	$w_0 = 1 - 3/2\gamma_0 + \delta_0$ (Einzel-1978, f.79)
`he3_scatt_l1a(P)`	Scattering factor λ_1^a used in spin diffusion transport time. Can be neglected. (Einzel-1978, f.74, p.350).

`he3_tau_n0(ttc,p)`	Normal state quasiparticle lifetime at the Fermi level, s. $\tau_N(0,T) = \tau_N(0,T_c)\left(\frac{T_c}{T}\right)^2 = \frac{32E_F\hbar}{<W>(\pi k_B T)^2}$ (Einzel-1991, p.325)
`he3_tau_n_av(ttc,p)`	Thermal average of normal state quasiparticle lifetime, s. $\bar{\tau}_N = \left\langle \frac{\tau_N(0,T)}{1+(\xi_k/\pi T)^2} \right\rangle = \frac{3}{4}\tau_N(0,T)$ (Einzel-1991, f.4-5)
`he3_tau_nd(ttc,p)`	Spin diffusion transport time for a normal Fermi-liquid, s $\tau_{ND} = \tau_1 = \bar{\tau}_N/(1-\lambda_1^a)$ (Einzel-1991, p.328)
`he3_diffn_hydr(ttc, p)`	Hydrodynamic spin diffusion in normal liquid, D_0, cm^2/s (Einzel JLTP84 (1991) f.23)
`he3_diffn_perp(ttc, p, nu0)`	Perpendicular component of spin diffusion, reduced because of the Leggett-Rice effect, $D_N^{\perp}$, cm^2/s (Einzel JLTP84 (1991) f.22)

TODO: Temperature is measured in T_c units which is strange for the normal phase. Maybe it is better to use K of T_F units here.

Transport properties in the B phase

Values for `he3_tau0, he3_tau_av, he3_diff_*` are extended to $T > T_c$ region using normal phase functions. See more information in the source file `he3_transp_b.f` and in `doc_tech/spin_diff.pdf`.

Collision integral for Bogoliubov quasiparticles:

`he3_coll_int(xi, ttc, gap, g0, d0)`	Full temperature range approximation (Einzel, Wolfle, Hirschfeld, JLTP80 (1990), p.66)
`he3_coll_int_lt(xi, ttc, gap, g0, d0)`	Collision integral for low temp (good for $< 0.7T_c$) (Einzel, JLTP84 (1991), f.76)
`he3_coll_int_ht(xi, ttc, gap, g0, d0)`	Collision integral for high temp (Einzel, JLTP84 (1991), f.75)

Bogoliubov quasiparticles lifetime and mean free path:

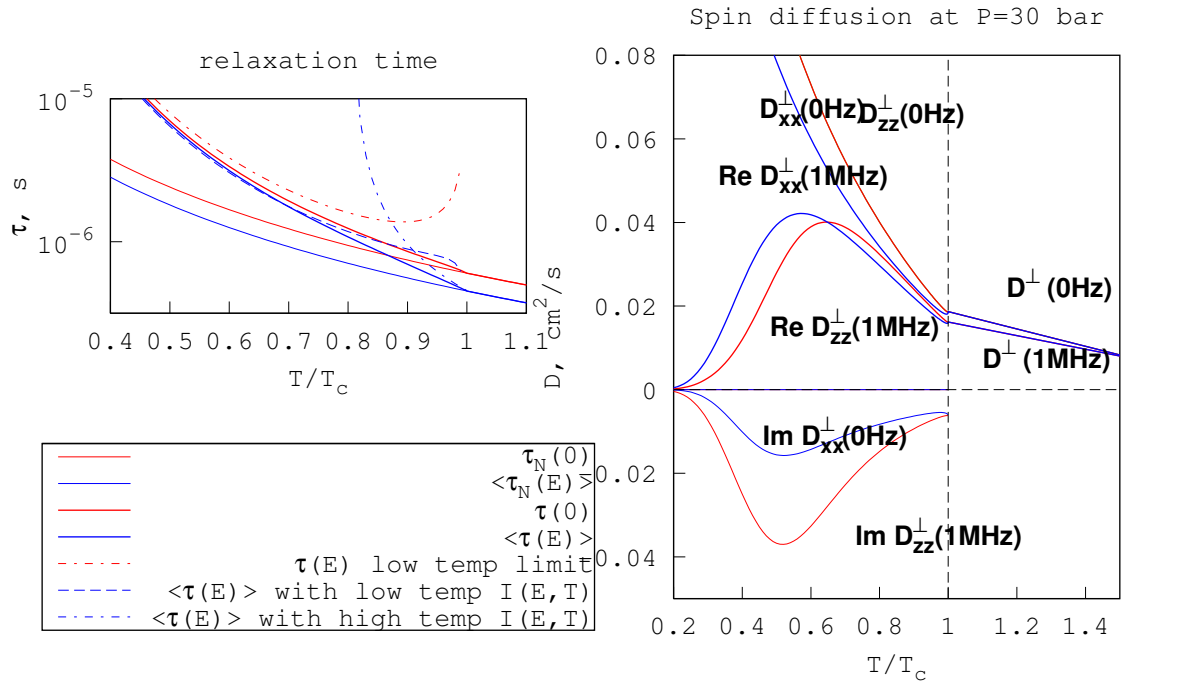
`he3_tau0(ttc, p)`	Lifetime at the Fermi level, s. $1/\tau(0) = \dfrac{I(0,T)}{\tau_N(0,T)}$, (Einzel-1991, f.74,76. Einzel-1990 f.A1)
`he3_tau0lt(ttc, p)`	$1/\tau, T \to 0$ limit (does not depend on energy). (Einzel-1978, f.79)
`he3_tau_av(ttc, p)`	Thermal average lifetime, s. $1/\bar{\tau} = \dfrac{1}{Y_0}\displaystyle\int_{-\infty}^{\infty} \dfrac{\phi_k \; d\xi}{\tau}$ (Einzel-1991, f.77)
`he3_fpath(ttc, p)`	Mean free path, cm (Einzel JLTP32 (1978) f.84)
`he3_tau_dperp(ttc, p)` `he3_tau_dpar(ttc, p)`	Spin diff. transport time, s. $\tau_D^{\perp,\parallel} = \bar{\tau}/(1 - \lambda_1^a Y^{\perp,\parallel})$ (Einzel-1991, f.90,96)

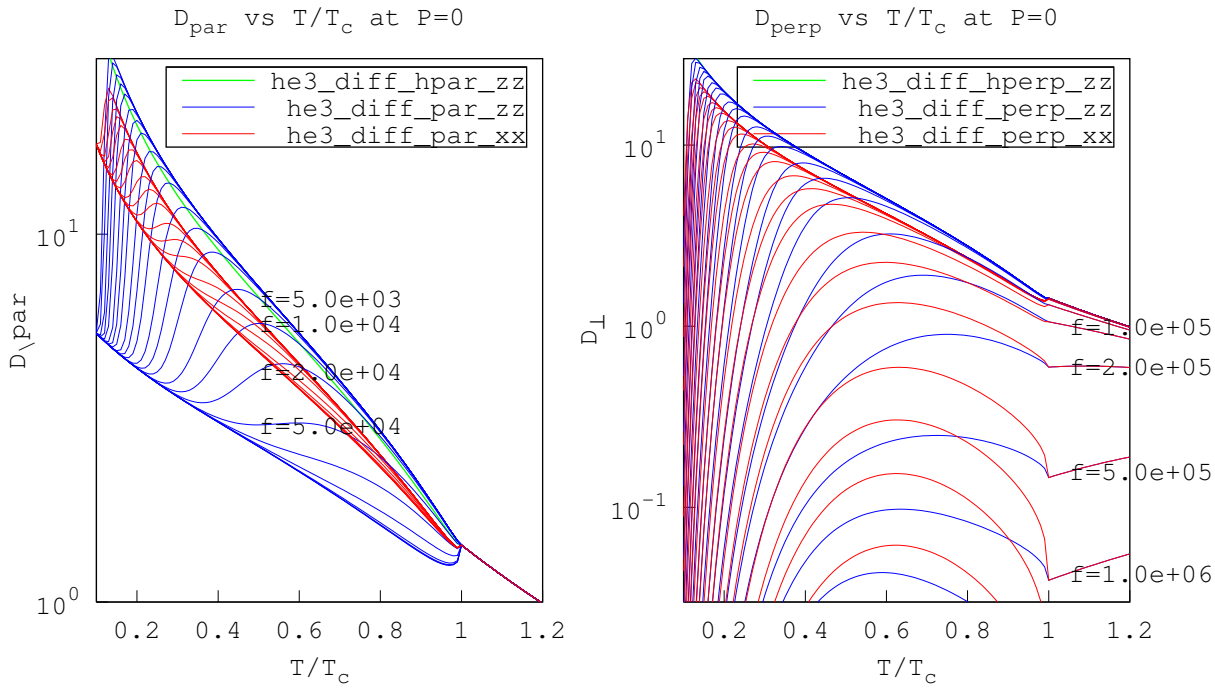
Spin diffusion:

`he3_diff_hperp_zz(ttc, p)` `he3_diff_hpar_zz(ttc, p)`	Spin diffusion in hydrodynamic limit ($\omega_L\tau \ll 1$), $D_{zz}^{\perp,\parallel}(\omega_L = 0)$, cm^2/s. According to Einzel's paper (see f.105) nonhydrodynamic effects do not affect $D^{\parallel}$. In Mukharsky paper they do. (Einzel JLTP84 (1991) f.102)

`he3_diff_perp_xx(ttc, p, nu0)`
`he3_diff_perp_zz(ttc, p, nu0)`
`he3_diff_perp_xx_im(ttc, p, nu0)`
`he3_diff_perp_zz_im(ttc, p, nu0)`
`he3_diff_par_xx(ttc, p, nu0)`
`he3_diff_par_zz(ttc, p, nu0)`
	Components of spin diffusion tenzor, cm^2/s
	(Bunkov et al, 1990 f.3; Einzel, 1991, f.108;
	Markelov, Mukharsky, 1992, f.7,8)

Problem with $D^{\parallel}$. According to Einzel-1991 (see f.105) nonhydrodynamic effects do not affect $D^{\parallel}$ and you should use he3_diff_hpar_zz for any frequency. In Markelov-Mukharsky paper the result for $D^{\parallel}$ is completely different (it is used in he3_diff_par_xx, _zz). $D^{\perp}$ is the same in all papers.
TODO:
Exchange coupling strength is in the $F_1^a = 0$ limit. (small difference from normal phase values)
How does `he3_diff_perp_??_im` extends to normal phase?


relaxation time
τ, s
T/T_c
D, cm^2/s
τ_N(0)
<τ_N(E)>
τ(0)
<τ(E)>
τ(E) low temp limit
<τ(E)> with low temp I(E,T)
<τ(E)> with high temp I(E,T)
Spin diffusion at P=30 bar
D^⊥_xx(0Hz) D^⊥_zz(0Hz)
Re D^⊥_xx(1MHz)
D^⊥ (0Hz)
Re D^⊥_zz(1MHz)
D^⊥ (1MHz)
Im D^⊥_xx(0Hz)
Im D^⊥_zz(1MHz)
T/T_c


D_par vs T/T_c at P=0
he3_diff_hpar_zz
he3_diff_par_zz
he3_diff_par_xx
f=5.0e+03
f=1.0e+04
f=2.0e+04
f=5.0e+04
D\par
T/T_c
D_perp vs T/T_c at P=0
he3_diff_hperp_zz
he3_diff_perp_zz
he3_diff_perp_xx
f=1.0e+05
f=2.0e+05
f=5.0e+05
f=1.0e+06
D_⊥
T/T_c

B_2-phase

Data for magnetic field of the A-B transition came from Inseob Hahn PhD thesis (see also). An approximation problem was fixed near $P = 21.2$ bar. Gap distortion and spin polarization is calculated according with Ashida and Nagai paper (Progr.Theor.Phys. 74 949 (1985)). See more information in the source file `he3_b2.f`.

he3_b2hcr(ttc,T)	Magnetic field of AB transition
he3_b2tab(p,H)	Temperature of AB transition in non-zero field
he3_b2gap1(ttc, p, H)	$\Delta_{\parallel}/\Delta_0$
he3_b2gap2(ttc, p, H)	$\Delta_{\perp}/\Delta_0$
he3_b2heff(ttc, p, H)	effective field
he3_b2mag(ttc, p, H)	magnetization
he3_b2rho_npar(ttc, p, H)	normal fluid density
he3_b2rho_nper(ttc, p, H)	normal fluid density
he3_b2rhoab_npar(ttc, p)	normal fluid density at the A-B boundary
he3_b2rhoab_nper(ttc, p)	normal fluid density at the A-B boundary
he3_b2magab(ttc, p, H)	magnetization an the A-B boundary

Other

See more information in the source file `he3_other.f`.

`he3_xigl(ttc, p)`	Extrapolated GL coherence length, cm
`he3_vneq(ttc, p, omega, r)`	Equilibrium vortex number

Normal liquid beyond zero temperature limit

See more information in the source file `he3_normal.f`.

`he3_cv_n(t, p)` Normal phase heat capacity C_v, Greywall-1983

TODO: V_m

ROTA-specific functions and constants

See more information in the source files `he3_rota.f` and `he3_const.f`

`rota_c_ns(t, i)`	Nuclear stage heat capacity [J/K] vs T[K] and I[A] $C_{ns} = 9.66 \cdot 10^{-5}$ [J/T^2] (0.113 [T/A] $I/T)^2$
`rota_fork_cal(w, p, n)`	Calibration of fork n, T/T_c, vs fork width w [Hz] and pressure P [bar] $w = a_n \exp\left(-\dfrac{\Delta_{\mathrm{WC}+}(P, T/T_c)}{T/T_c}\right)$, $a = \alpha_n \; p_F^4(P)$, where α_n is a geometrical factor $N = 1:$ Fork K, calibration 30.4.2010, 29 bar, a = 11700 $N = 2:$ Fork E, calibration 30.4.2010, 29 bar, a = 17543
`rota_rcell`	cell radius
`rota_nmra`	field/current in nmrA solenoid [G/A]
`rota_nmrb`	field/current in nmrB solenoid [G/A]
`rota_nmra_q(f0)`	Q value of the nmrA spectrometer vs frequency
`rota_nmra_f(n)`	frequencies of nmrA spectrometer,kHz for n=1..8
`rota_hmina_r`	effective radius of the HminA coil [cm]
`rota_hmina_n`	number of turns of the HminA coil, 4
`rota_hmina`	field/current in the center of HminA coil [G/A]
`rota_hmina_mr`	quadratic radial term of the HminA field, [G/A/cm^2]
`rota_hmina_i0i`	effectve HminA coil current divided by NMR current
`rota_hmina_i0f`	effectve HminA coil current divided by NMR frequency
`rota_rrda`	radiation damping constant R_{RD} for the nmrA spectrometer
`rota_Bza(I,Imin,r,z)`	Bz field profile of A spectrometer

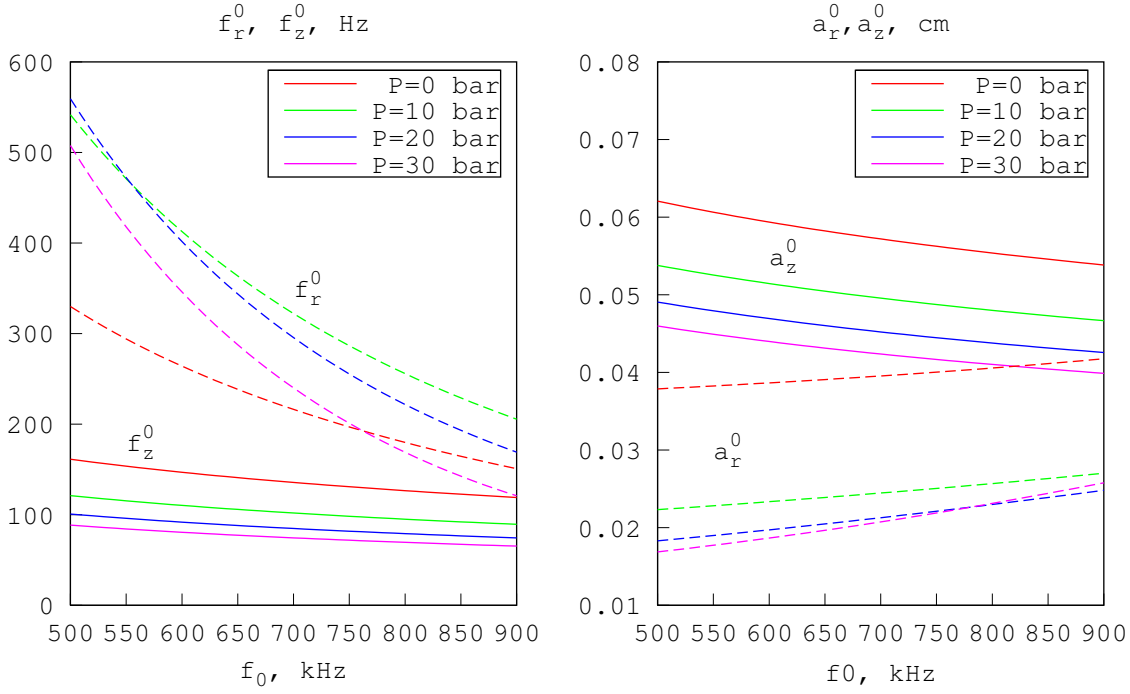
Q-balls in the zero temperature limit

See more information in the source file `he3_qball.f`

`qball_nu_b(P)`	Leggett frequency [Hz]
`qball_cpar(P)` `qball_cper(P)`	Measured $c_\parallel$, $c_\perp$, [cm/s]
`qball_lg1(P)` `qball_lg2(P)`	λ_{G1}, λ_{G2} calculated using measured spin wave velocities.
`qball_text_a(P)`	measured textural parameter a [erg/cm^3 1/G^2]
`qball_text_d(P)`	measured textural parameter d [erg/(cm^2 G^2)]
`qball_text_lsg(P)`	measured textural parameter λ_{SG}
`qball_trd(P,f0,fr,fz)`	τ_{RD} for the magnon condensate with given radial and axial frequencies (rota-specific) [s]

A model of the zero-temperature, zero-rotation condensate, based on spectra measurements (rota-specific):

`qball_dbetan(P, f0)`	Derivative of the textural angle β_N in the center of the cell, [rad/cm].
`qball_fz0(P,f0,Imin)` `qball_fr0(P,f0,Imin)`	1/2 of the distance between visible axial and radial levels, [Hz]
`qball_az0(P,f0,Imin)` `qball_ar0(P,f0,Imin)`	Axial and radial sizes of the magnon condensate, [cm]
`qball_trd0(P,f0,Imin)`	τ_{RD} for the magnon condensate, [s]



Math

See more information in the source file `he3_math.f`

math_ele(x) math_elk(y)	Complete elliptic integrals E(x) and K(x)
loop_bz(rloop,r,z) loop_br(rloop,r,z)	Magnetic field of a current loop, Bz, Br

These function have only fortran interface. Use `he3_math.fh` include file. Examples are in `test_int` folder.

real*8 math_dint(func, xmin, xmax, nx)
– simple integration of real*8 function (Gauss 2pt)

complex*16 math_cint(func, xmin, xmax, nx)
– integration of complex*16 function

real*8 math_dint2d(func, xmin, xmax, nx, ymin, ymax, ny)
– integration of 2D real*8 function

complex*16 math_cint2d(func, xmin, xmax, nx, ymin, ymax, ny)
– integration of 2D complex*16 function

real*8 math_dint_gk(func, xmin, xmax, nx, aerr)
– integrate real*8 function using Gauss-7pk+Kronrod-13pt

real*8 math_dint_gka(func, xmin, xmax, aerr_lim, rerr_lim)
– adaptive integration

real*8 math_dint2d_gk(func, xmin, xmax, nx, ymin, ymax, ny, aerr)
– integrate 2D real*8 function using Gauss-7pk+Kronrod-13pt