# <sup>3</sup>He library

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#### Usage

This library provides constant and functions for various <sup>3</sup>He properties. Supported Languages:

- Fortran-77. You should use he3.fh include file and libhe3 library for fortran programs. All functions and arguments are real\*8. Use -fno-range-check compiler flag to allow NaN values.
- Fortran-90. You should include he3.f90h instead.
- C. You should use he3.h include file and libhe3 library in C programs. Lowercase names with underscore should be used (like he3\_pf\_). Type of arguments is always double\*, type of returned value is double.
- Matlab, Octave. mex files are located in the matlab folder. Lowercase names should be used. In function arguments you can mix numbers with arrays or matrices of the same size. Example:

```
ttc=0:0.1:1; p=10;
I = he3_nu_b(p, ttc);
```

To build mex files you can use make octave, make matlab or make matlab64 in the matlab folder. Fixed rpath is used for the library. For building this documentation octave files are needed.

• Command line. You can use all the functions via a command line interface. To get a list of all functions run the program he3 without arguments:

> he3

To get information about specific function run the program with the function name as an argument:

```
> he3 he3_nu_b
```

You can use matlab-style vectors (value1:step:value2) to get a table of values (if step is missing then 20 points will be printed):

```
> he3 he3_nu_b 0.1:0.1:1 0
```

will give you Leggett frequency vs temperature at zero pressure.

#### Location

```
GIT https://github.com/slazav/he3lib
```

```
ROTA /home/slazav/he3lib/lib
/rota/programs/src/he3lib - old, "stable" version.
Use "addpath /rota/programs/src/he3lib/matlab" in Matlab.
```

# General

```
Avogadro constant N_A = 6.02214129 \cdot 10^{23} [1/\text{mol}]
const_na
                Boltsman constant k_B=1.3806488\cdot 10^{-16}~[\mathrm{erg/K}]
const\_kb
                R-gas constant R = 8.314472 \cdot 10^7 \text{ [sgs]}
const_r
                Plank constant h = 6.62606957 \cdot 10^{27} \text{ [sgs]}
const_h
const_hbar \hbar = h/2\pi = 1.054571726 \cdot 10^{27} \; [\mathrm{sgs}]
const_pi
                \pi = 3.1415926535897932
                2\pi
const_2pi
                <sup>3</sup>He atom mass, m_3 = 5.0079 \cdot 10^{-24} [g]
he3_amass
                <sup>3</sup>He molar mass, \mu = N_A m_3 = 3.0158 [g/mol]
he3\_mmass
                <sup>3</sup>He gyromagnetic ratio \gamma = 20378 [(G s)<sup>-1</sup>],
he3_gyro
```

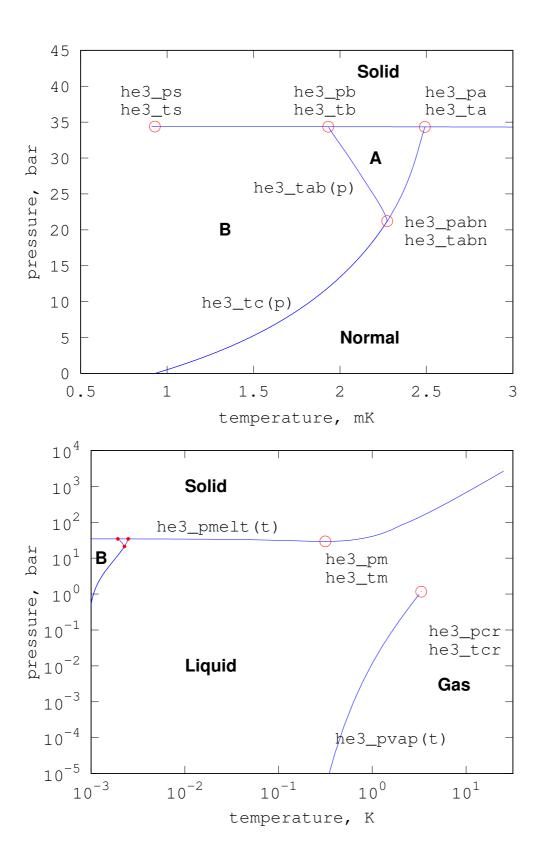
# Phase diagram

Vapor pressure and critical point are from the 1962  $^3$ He scale of temperatures. Melting curve  $P_m(T)$  at 0.9-250 mK,  $T_c(P)$ ,  $T_{AB}(P)$  and all tricritical points are from Greywall-86 temperature scale. Melting curve minimum is from PLTS-2000 temperature scale. Melting curve for higher temperatures is from Osborne, Abraham, Weinstock-1951 and Mills, Grilly-1955 with interpolation in the regions 0.25-0.5K and 1.5-2.0K. PLTS-2000 melting curve and critical points on it are also available. See more information in the source file he3\_phase.f.

he3_pvap(T)	Vapor pressure [bar] vs temperature [K], $T=0.2-3.324~\mathrm{K}$
he3_pcr	Gas-liquid critical point pressure, 1.16317 [bar]
he3_tcr	Gas-liquid critical point temperature, $3.324~[\mathrm{K}]$
he3_pmelt(T)	Melting pressure [bar] vs temperature [K], $T = 0.0009 - 31$ K
he3_pm	Melting curve minimum pressure, $29.3113$ [bar]
he3_tm	Melting curve minimum temperature, 0.31524 [K]
he3_pa	Superfluid trans. at melting curve, pressure, $34.3380$ [bar]
he3_ta	Superfluid trans. at melting curve, temp., $2.491~[\mathrm{mK}]$
he3_pb	A-B trans. at melting curve, pressure, 34.3580 [bar]
he3_tb	A-B trans. at melting curve, temp., 1.932 $[mK]$
he3_ps	Neel transition at melting curve, pressure, $34.3905$ [bar]
he3_ts	Neel transition at melting curve, temp., $0.9291~[\mathrm{mK}]$
he3_tc(P)	Superfluid transition temperature [mK] vs pressure [bar], $P=0-34.358~\mathrm{bar}$
he3_tab(P)	A-B transition temperature [mK] vs pressure [bar], $P=0-34.3609~{\rm bar,~below~21.22~bar~is~equal~to~he3\_tc}$
he3_tabn	A-B-Normal point temperature, 2.2311 $[mK]$
he3_pabn	A-B-Normal point pressure, 21.22 [bar]
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PLTS-2000 temperature scale:

he3_pmelt_plts(T)	PLTS-2000 melting presure [bar] vs temperature [K].
he3_pa_plts	Superfluid trans. at melting curve, pressure, $34.3407$ [bar]
he3_ta_plts	Superfluid trans. at melting curve, temp., $2.444~[\mathrm{mK}]$
he3_pb_plts	A-B trans. at melting curve, pressure, $34.3609$ [bar]
he3_tb_plts	A-B trans. at melting curve, temp., 1.896 [mK]
he3_ps_plts	Neel transition at melting curve, pressure, $34.3934~[\mathrm{bar}]$
he3_ts_plts	Neel transition at melting curve, temp., 0.902 [mK]



#### Fermi-liquid parameters

Fermi-liquid parameters according to Wheatley-75 table. Values are calculated using experimental data for molar volume, speed of sound, magnetic temperature  $T^*$ , collected by Wheatley-75 and heat capacity  $(\gamma_f = C/RT)$  measured by Graywall-86. Argument is pressure, [bar] in the range 0 – 34.4. See more information in the source file he3\_fermi.f.

Measured values:

he3\_vm(P) Molar volume 
$$v_m$$
, [cm<sup>3</sup>/mol]

he3\_c1(P) First sound velocity, 
$$c_1$$
, [cm/s]

he3\_gammaf(P) R-gas constant 
$$\gamma_f = C/RT$$
,  $[1/(\text{K mol})]$ 

he3\_tmag(P) Magnetic temperature 
$$T^*$$
, [K]

Derived values:

he3\_rho(P) Density 
$$\rho = \mu/v_m$$
, [g/cm<sup>3</sup>]

he3\_2n0(P) 
$$2N(0) = \frac{\gamma_f}{v_m} \frac{3N_A}{k_B \pi^2}, [1/(\text{erg cm}^3)]$$

$$\mbox{he3\_pf(P)} \qquad p_F = h \left( \frac{3}{8\pi} \; \frac{N_A}{v_m} \right)^{1/3}, \, [\mbox{g cm/s}] \label{eq:pf_P}$$

$$\label{eq:mass_mass_mass_mass_mass} \text{he3.meff(P)} \quad m^\star = \frac{h^3}{8\pi} \; \frac{2N(0)}{p_F}, \, [\text{g}]$$

he3\_mm(P) 
$$m^*/m_3$$

he3\_vf(P) 
$$v_F = p_F/m^\star, \, [\mathrm{cm/s}]$$

$$\label{eq:chin} \mbox{he3\_chin(P)} \ \chi_N = 2N(0) \frac{(\gamma \hbar)^2}{4(1+F_0^a)}.$$

he3\_f0s(P) 
$$F_0^s = 3 \ m^{\star} m_3 \ c_1^2/p_F^2 - 1$$

he3\_f1s(P) 
$$F_1^s = 3(m^*/m_3 - 1)$$

he3\_f0a(P) 
$$F_0^a = Z_0/4 = 3k_B T^{\star} m^{\star}/p_F^2 - 1$$

he3\_f1a(P) 
$$F_1^c$$

he3\_a(P) Average atomic spacing, 
$$a = (v_m/N_A)^{1/3}$$
, Å

he3\_gdk(P) Average dipolar coupling enegy, 
$$g_d/k_B = \frac{2\pi\gamma^2\hbar^2}{3v_mk_B}$$
, [K]

he3\_tfeff(P) Effective Fermi temperature, 
$$T_{F_{eff}} = \frac{\pi^2}{2\gamma_f}$$
, [K]

#### TODO:

Recalculate  $F_0^a$   $F_1^a$  from experimental data (Now there are some extra not-consistent data)! (see Ramm-1970, Halperin-92 ...). Add  $F_2^a$  (zero or from Halperin-95). Check  $g_d$ .

Pressure-dependent values:

P, bar	0	3	6	9	12	15	18	21	24	27	30	33
$T_c$ , mK	0.929	1.290	1.560	1.769	1.934	2.067	2.177	2.267	2.339	2.395	2.438	2.474
$T_{AB}$ , mK	_	_	_	_	_	_	_	_	2.217	2.137	2.056	1.969
$v_m,  \mathrm{cm}^3/\mathrm{mol}$	36.84	33.95	32.03	30.71	29.71	28.89	28.18	27.55	27.01	26.56	26.17	25.75
$c_1,  \mathrm{m/s}$	182.9	227.4	259.9	285.8	307.9	327.3	344.8	360.6	375.2	389.2	402.9	416.1
$\gamma_f$ , 1/(K mol)	2.784	2.981	3.158	3.321	3.475	3.625	3.772	3.918	4.065	4.211	4.355	4.496
$T^{\star}$ , mK	358.8	305.8	276.2	255.7	238.7	224.0	212.4	204.0	197.8	191.9	184.9	178.4
$\rho$ , g/cm <sup>3</sup>	0.082	0.089	0.094	0.098	0.102	0.104	0.107	0.109	0.112	0.114	0.115	0.117
$2N(0), 10^{38}$	1.002	1.164	1.307	1.434	1.551	1.663	1.774	1.885	1.995	2.102	2.207	2.315
$p_F, 10^{-20} \text{ g cm/s}$	8.280	8.509	8.675	8.798	8.896	8.979	9.054	9.122	9.182	9.234	9.280	9.330
$m^{\star}, 10^{-23} \text{ g}$	1.401	1.584	1.744	1.887	2.018	2.144	2.269	2.392	2.515	2.635	2.753	2.872
$m^{\star}/m_3$	2.797	3.162	3.482	3.767	4.030	4.282	4.530	4.777	5.022	5.261	5.497	5.735
$v_F,\mathrm{m/s}$	59.11	53.73	49.74	46.64	44.08	41.87	39.91	38.13	36.51	35.05	33.71	32.49
$\chi_N, 10^{-9}$	38.1	48.5	56.9	64.1	71.0	77.8	84.1	89.6	94.2	98.8	104.1	109.7
$F_0^s$	9.27	15.99	22.52	28.91	35.32	41.81	48.43	55.17	62.10	69.32	76.95	84.82
$F_1^s$	5.39	6.49	7.45	8.30	9.09	9.85	10.59	11.33	12.07	12.78	13.49	14.21
$F_0^a$	-0.70	-0.72	-0.73	-0.74	-0.75	-0.75	-0.76	-0.76	-0.76	-0.75	-0.76	-0.76
$F_1^a$	-0.55	-0.73	-0.80	-0.85	-0.90	-0.95	-0.99	-1.00	-0.99	-0.99	-0.99	-1.00
a, Å	3.940	3.834	3.761	3.708	3.668	3.634	3.604	3.577	3.553	3.533	3.516	3.497
$g_d/k_B,\mu\mathrm{K}$	114.5	124.3	131.7	137.4	142.0	146.0	149.7	153.1	156.2	158.8	161.2	163.9
$T_{F_{eff}}$ , K	1.773	1.656	1.563	1.486	1.420	1.361	1.308	1.260	1.214	1.172	1.133	1.098
< $W$ $>$	83.4	104.1	116.0	123.7	130.7	137.2	141.6	142.4	140.6	139.0	140.3	141.8
$<\!W_I\!>$	10.92	14.68	16.37	17.32	18.23	19.08	19.48	19.19	18.41	17.73	17.63	17.60
$<\!\!W_D\!\!>$	24.89	29.75	32.62	34.42	35.98	37.41	38.38	38.56	38.18	37.88	38.23	38.60
$<\!W_L\!>$	-11.42	-8.14	-6.53	-4.70	-2.28	0.29	2.38	3.62	4.10	4.25	4.53	5.19
$\gamma_0$	0.131	0.141	0.141	0.140	0.139	0.139	0.138	0.135	0.131	0.127	0.126	0.124
$\delta_0$	0.298	0.286	0.281	0.278	0.275	0.273	0.271	0.271	0.272	0.272	0.272	0.272
$w_0$	1.211	1.192	1.187	1.185	1.182	1.180	1.179	1.181	1.184	1.187	1.189	1.189
$\lambda_1^a$	-0.137	-0.078	-0.056	-0.038	-0.017	0.002	0.017	0.025	0.029	0.031	0.032	0.037
$\tau_N(0,T_c),\mu\mathrm{s}$	0.609	0.237	0.137	0.095	0.072	0.057	0.048	0.043	0.039	0.036	0.034	0.031
$\Delta_{WC+}(T=0)/T_c$	1.771	1.795	1.813	1.825	1.834	1.843	1.850	1.856	1.861	1.866	1.870	1.875
$\chi_B(T=0)/\chi_N$	0.378	0.357	0.346	0.340	0.335	0.331	0.328	0.327	0.328	0.329	0.329	0.328
$\nu_B(T=0), kHz$	125.3	161.9	188.0	208.5	225.7	241.2	255.7	269.8	283.5	296.6	308.7	320.5
$c_{\perp}(T=0), cm/s$	2190	1901	1724	1594	1485	1393	1315	1253	1202	1157	1111	1068
$c_{\parallel}(T=0), cm/s$	2509	2172	1967	1817	1692	1585	1497	1426	1368	1316	1264	1215
P, bar	0	3	6	9	12	15	18	21	24	27	30	33

#### Energy gap

BCS energy gap + trivial strong coupling correction + some values derived from energy gap. See more information in the source file he3\_gap.f.

he3\_bcsgap(ttc) BCS gap for 3He-B in  $T_c$  units,  $\Delta_{\rm BCS}/k_BT_c$ 

he3\_bcsgap\_fast(ttc) Einzel approximation for BCS gap (0.5% accuracy, 70 times faster)

(Einzel-1991, f.68)

he3\_trivgap(ttc, P) Trivial strong-coupling correction to the BCS gap

he3\_todogap(ttc, P) Gap based on Todoschenko's measurements: linear interpolation in den-

sity between BCS value at zero bar and measured value 1.99 at melting

pressure + temperature behaviour as in he3\_trivgap

he3\_gap(ttc, P) Wrapper which is used everywhere in the lib (trivgap by default)

he3\_yosida(ttc, gap, n) Yosida functions  $Y_n(T/T_c, \Delta) = \int_{-\infty}^{\infty} \left(\frac{\xi_k}{E_k}\right)^n \phi_k \ d\xi_k$ 

Low temperature limit:  $Y_n(T,\Delta) = 2\Gamma\left(\frac{n+1}{2}\right)\left(\frac{T}{\Delta}\right)^{\frac{n-1}{2}}\exp\left(-\frac{\Delta}{T}\right)$ 

Note: type of n parameter in fortran should be real\*8 (this was done for standard function handling). If n is integer, result is unpredictable!

he3\_yosida\_par(ttc, gap)  $Y_q^{\parallel}(T/T_c,\Delta)$ 

he3\_yosida\_perp(ttc, gap)  $Y_q^\perp(T/T_c,\Delta)$ 

he3\_z3(ttc, gap)

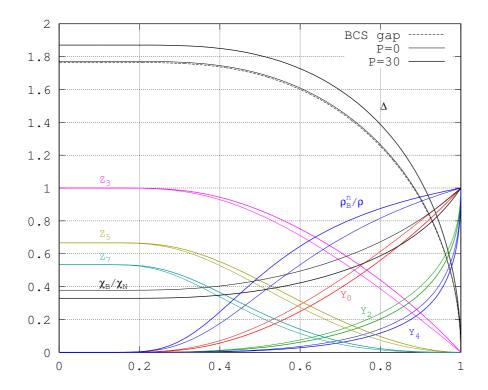
he3\_z5(ttc, gap)

he3\_z7(ttc, gap)

he3\_lambda(ttc, gap)

he3\_rho\_nb(ttc, p) B-phase normal component density:  $\frac{\rho_B^n}{\rho_N} = \frac{(3+F_1^s)Y_0}{3+F_1^sY_0}$ 

he3\_chi\_b(ttc, p) B-phase susceptibility:  $\frac{\chi_B}{\chi_N} = \frac{(1+F_0^a)(2+Y_0)}{3+F_0^a(2+Y_0)}$ 



### Dipole energy and Legget frequency

Values are given according to Thuneberg's paper JLTP 122, p657 (2001). Temperature independent  $g_d$ parameter is restored from experimental data for  $\Omega_B$ , measured in transverse and longitudinal NMR. See more information in the source file he3\_dipole.f.

Dipolar energy:

$$F_D = \lambda_D \int (R_{ii}R_{jj} + R_{ij}R_{ji}) d^3r = 4\lambda_D \int \cos\theta (1 + 2\cos\theta) d^3r$$
$$\lambda_D = \Delta^2 g_d, \qquad \Omega_B = \gamma \sqrt{15\lambda_D/\chi_B}$$

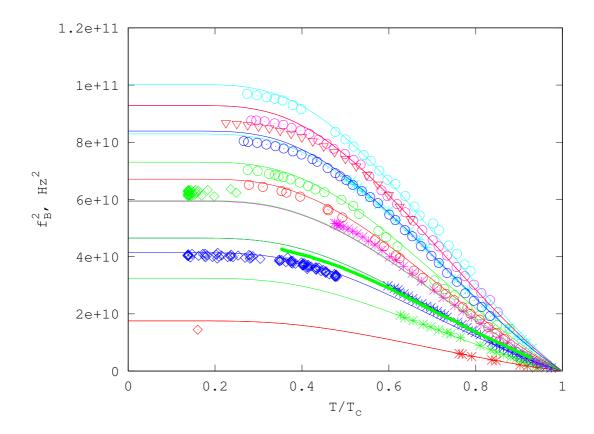
Note: In old papers (Leggett, ...)  $g_D/5$  is used instead of  $\lambda_G$ .

he3\_gd(p) 
$$g_d$$
,  $[1/({\rm erg} \ {\rm cm}^3)]$ 

he3\_ld(p) 
$$\lambda_D = \Delta^2 g_d, \, [\mathrm{erg/cm^3}]$$

he3\_nu\_b(ttc, p) B-phase Leggett frequency  $\nu_B = \frac{\gamma}{2\pi} \sqrt{15\Delta^2 g_d/\chi_B}$ , Hz

he3\_nu\_b1(ttc, p) Less accurate formula without using 
$$g_d$$
, Hz 
$$\nu_B = \frac{1}{2\pi} \sqrt{\frac{3\pi}{2\chi}} \; \frac{\gamma^2 \hbar}{2} \; N(0) \; \Delta \log \frac{e_f}{\Delta}$$



# Gradient energy and spin wave velocity

Gradient energy coefficients and derived values, including spin wave velocities. See more information in the source file he3\_grad.f.

$$F_{\nabla} = \frac{1}{2} \Delta^2 \left[ K_1(\nabla_j R_{ak})(\nabla_j R_{ak}) + K_2(\nabla_j R_{ak})(\nabla_k R_{aj}) + K_3(\nabla_j R_{aj})(\nabla_k R_{ak}) \right]$$

he3\_grad\_K0(ttc, p)  $K_1 = K_2 = K_3$  without fermi-liquid corrections, see VW7.23m

he3\_grad\_c(ttc, p) c and  $\delta$  parameters calculated with fermi-liquid corrections (Cross-1975), he3\_grad\_delta(ttc, p) see VW7.25. These values are used to calculate all other things:

$$c = -\frac{\rho_s}{10} \frac{3 + F_1^a}{3 + F_1^s} \frac{1}{1 + F_1^a (5 - 3\rho_s/\rho)/15}, \qquad \delta = \frac{F_1^a \rho_s/\rho}{3 + F_1^s (1 - \rho_s/\rho)}$$

 $\label{eq:K12} \begin{array}{ll} \text{he3\_grad\_K12(ttc, p)} & K_1 = K_2 = -\frac{2}{\Delta^2} \left(\frac{\hbar}{2m}\right)^2 c, \qquad K_3 = -\frac{2}{\Delta^2} \left(\frac{\hbar}{2m}\right)^2 (1+\delta) c \\ \text{he3\_grad\_K3(ttc, p)} & \\ \end{array}$ 

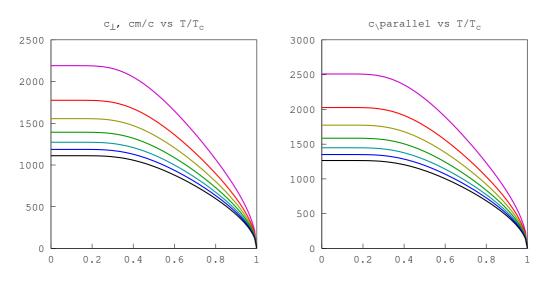
he3\_grad\_K(ttc, p)  $K=2K_1+K_2+K_3, \qquad K'=K_2+K_3 \label{eq:K}$  he3\_grad\_Kp(ttc, p)

he3\_grad\_lg1(ttc, p) Thunebergs  $\lambda_{G1}$ ,  $\lambda_{G2}$  and  $\lambda_{SG}^b$ :
he3\_grad\_lg2(ttc, p)
he3\_grad\_lsgb(ttc, p)  $\lambda_{G1} = \frac{1}{2}\Delta^2(K_2 + K_3), \qquad \lambda_{G2} = \frac{1}{2}\Delta^2K_1, \qquad \lambda_{SG}^b = \Delta^2K_2$ 

he3\_cpar(ttc, p) velocity of transverse spin waves parallel and perpendicular to the l direction [cm/s]:

 $c_{\parallel}^2 = \frac{\gamma^2 \Delta^2}{\chi_B} K, \qquad c_{\perp}^2 = \frac{\gamma^2 \Delta^2}{\chi_B} (K - K'/2), \qquad c_{\parallel}^2 / c_{\perp}^2 \approx 4/3$ 

he3\_clpar(ttc, p) same for longitudinal waves [cm/s]:  $C_{\parallel}^2 = \frac{\gamma^2 \Delta^2}{\chi_B} (K - K'), \qquad C_{\perp}^2 = \frac{\gamma^2 \Delta^2}{\chi_B} K, \qquad C_{\parallel}^2 / C_{\perp}^2 \approx 1/2$ 



#### Other textural parameters

Values are given according to Thuneberg's paper JLTP 122, p657 (2001). High order Fermi liquid parameters  $(F_2^a, F_3^a)$  are set to 0. See more information in the source file he3\_text.f.

Bulk terms:

$$F_{DH} = -a \int_{V} (\mathbf{n} \cdot \mathbf{H})^{2} d^{3}r$$

$$F_{DV} = -\lambda_{DV} \int_{V} [\mathbf{n} \cdot (\mathbf{v_{s}} - \mathbf{v_{n}})]^{2} d^{3}r \ll F_{HV} \text{ at } H \approx 200 \text{ Oe}$$

$$F_{HV} = -\lambda_{HV} \int_{V} [\mathbf{H} \cdot R \cdot (\mathbf{v_{s}} - \mathbf{v_{n}})]^{2} d^{3}r$$

Surface terms:

$$F_{SH} = -d \int_{S} [\mathbf{H} \cdot R \cdot \mathbf{s}]^{2} d^{2}r$$

$$F_{SG} = -\lambda_{SG} \int_{S} s_{j} R_{\alpha j} \frac{\partial R_{\alpha i}}{\partial r_{i}} d^{2}r$$

Vortex term:

$$F_{LH} = \frac{\lambda_{LH}}{2\Omega} \int_{L} |\omega_s| [\mathbf{H} \cdot R \cdot \mathbf{l}]^2 d^3r$$

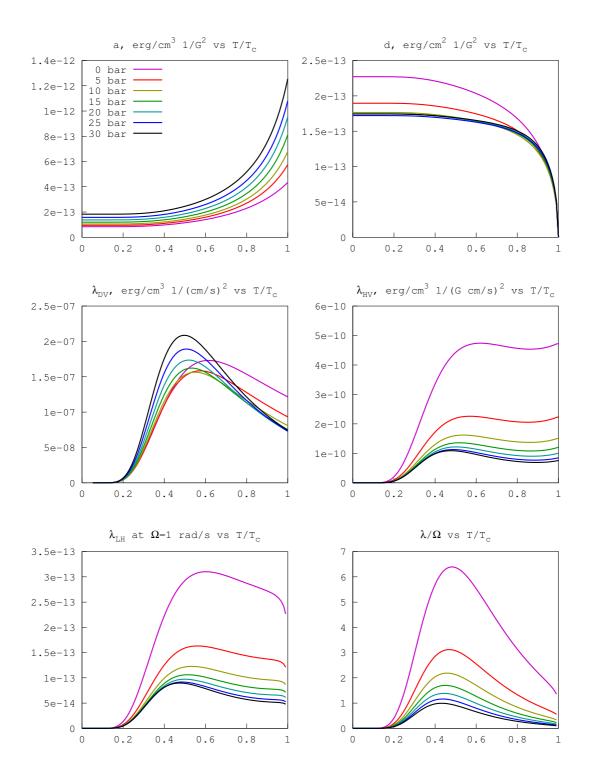
he3\_text\_a(ttc, p) 
$$a = \frac{5g_d}{2} \left[ \frac{\hbar \gamma/2}{1 + F_0^a(2 + Y_0)/3} \right]^2 \left( 5 - 3\frac{Z_3}{Z_5} \right), [\text{erg/cm}^3 \text{ G}^{-2}]$$
 he3\_text\_d(ttc, p) 
$$d, [\text{erg/(cm}^2 \text{ G}^2)] - \text{GL extrapolation!}$$
 he3\_text\_ldv(ttc, p) 
$$\lambda_{DV} = 5g_d \left( \frac{m^* v_F}{1 + F_1^s Y_0/3} \right)^2 \left( 1 - \frac{3Z_5}{2Z_3} \right), [\text{erg/cm}^3 \text{ (cm/s)}^{-2}]$$
 he3\_text\_lhv(ttc, p) 
$$\lambda_{HV} = \frac{\rho}{\Delta^2} \frac{1 + F_1^s/3}{(1 + F_1^s Y_0/3)^2} \left( \frac{\hbar \gamma/2}{1 + F_0^a(2 + Y_0)/3} \right)^2 \left( Z_3 - \frac{9}{10} Z_5 + \frac{9}{10} \frac{Z_5^2}{Z_3} - \frac{3}{2} Z_7 \right)$$
 he3\_text\_lh(ttc, p, omega) 
$$\lambda_{LH} = \frac{\hbar}{2m} \Omega \lambda_{HV} \left( \ln \frac{R}{r} - \frac{3}{4} \right), [\text{erg/(cm}^3 \text{ G}^2 \text{ s})] - \text{counterflow part only!}$$

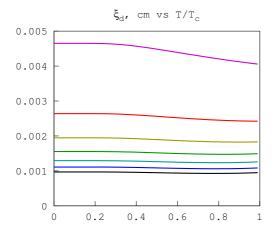
he3\_text\_lo(ttc, p, omega) lambda/omega used in the texture library:  $\frac{\lambda}{\Omega} = \frac{5\lambda_{LH}}{2a\Omega}$ ,  $(rad/s)^{-1}$ 

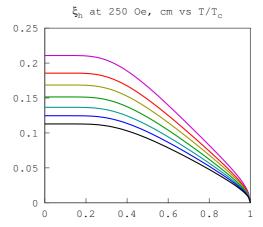
he3\_text\_xih(ttc, p, h) Magnetic length  $\xi_H = \sqrt{65\lambda_{G2}/(8aH^2)}$ 

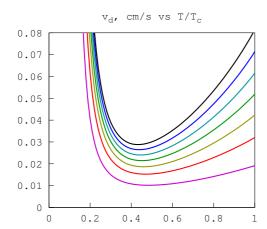
he3\_text\_xid(ttc, p) Dipolar length (according to Hakonen-1989)  $\xi_D = \sqrt{13\lambda_{G2}/12\lambda_D}$ . Note: in Thuneberg's paper  $\xi_D = \sqrt{\lambda_{G2}/\lambda_D}$  is used.

he3\_text\_vd(ttc, p) Dipolar velocity (according to Thuneberg-2001)  $v_D = \sqrt{2a/5\lambda_{HV}}$ 









#### Transport properties in the normal phase

See more information in the source file he3\_transp\_n.f.

Crossections and scattering factors:

he3\_scatt\_w(P) Scattering crossection < W > (Einzel-1978, f.82)

 $< W_I >$  (Einzel-1978, f.82) he3\_scatt\_wi(P)

 $< W_D >$  (Einzel-1978, f.82) he3\_scatt\_wd(P)

he3\_scatt\_w1(P)  $< W_L > (Einzel-1978, f.71)$ 

he3\_scatt\_g0(P) Scattering factor  $\gamma_0 = \langle W_I \rangle / \langle W \rangle$  (Einzel-1978, f.66)

Scattering factor  $\delta_0 = \langle W_D \rangle / \langle W \rangle$  (Einzel-1978, f.67) he3\_scatt\_d0(P)

he3\_scatt\_w0(P)  $w_0 = 1 - 3/2\gamma_0 + \delta_0$  (Einzel-1978, f.79)

he3\_scatt\_l1a(P) Scattering factor  $\lambda_1^a$  used in spin diffusion transport time. Can be neglected.

(Einzel-1978, f.74, p.350).

Normal state quasiparticle lifetime at the Fermi level, s. he3\_tau\_n0(ttc,p)

 $\tau_N(0,T) = \tau_N(0,T_c) \left(\frac{T_c}{T}\right)^2 = \frac{32E_F\hbar}{\langle W \rangle (\pi k_B T)^2}$ (Fingel 1991, 1992)

(Einzel-1991, p.325)

he3\_tau\_n\_av(ttc,p) Thermal average of normal state quasiparticle lifetime, s.

 $\bar{\tau}_N = \left\langle \frac{\tau_N(0, T)}{1 + (\xi_k / \pi T)^2} \right\rangle = \frac{3}{4} \tau_N(0, T)$ (Einzel-1991, f.4-5)

Spin diffusion transport time for a normal Fermi-liquid, s he3\_tau\_nd(ttc,p)

 $\tau_{ND} = \tau_1 = \bar{\tau}_N / (1 - \lambda_1^a)$  (Einzel-1991, p.328)

Hydrodynamic spin diffusion in normal liquid,  $D_0$ , cm<sup>2</sup>/s he3\_diffn\_hydr(ttc, p)

(Einzel JLTP84 (1991) f.23)

he3\_diffn\_perp(ttc, p, nu0) Perpendicular component of spin diffusion, reduced because of the

Leggett-Rice effect,  $D_N^{\perp}$ , cm<sup>2</sup>/s (Einzel JLTP84 (1991) f.22)

TODO: Temperature is measured in  $T_c$  units which is strange for the normal phase. Maybe it is better to use K of  $T_F$  units here.

#### Transport properties in the B phase

Values for he3\_tau0, he3\_tau\_av, he3\_diff\_\* are extended to  $T > T_c$  region using normal phase functions. See more information in the source file he3\_transp\_b.f and in doc\_tech/spin\_diff.pdf.

Collision integral for Bogoliubov quasiparticles:

```
he3_coll_int(xi, ttc, gap, g0, d0) Full temperature range approximation
(Einzel, Wolfle, Hirschfeld, JLTP80 (1990), p.66)

he3_coll_int_lt(xi, ttc, gap, g0, d0) Collision integral for low temp (good for < 0.7T<sub>c</sub>)
(Einzel, JLTP84 (1991), f.76)

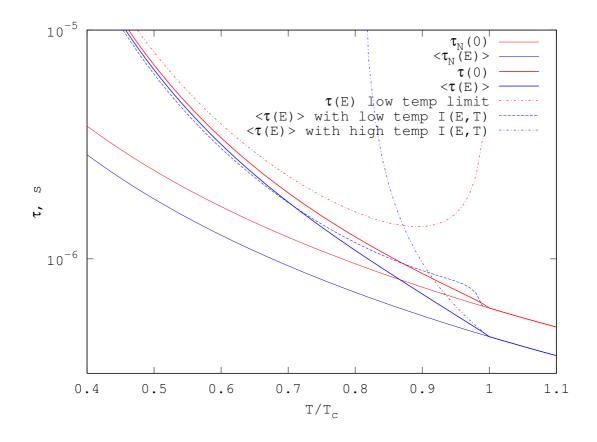
he3_coll_int_ht(xi, ttc, gap, g0, d0) Collision integral for high temp
(Einzel, JLTP84 (1991), f.75)
```

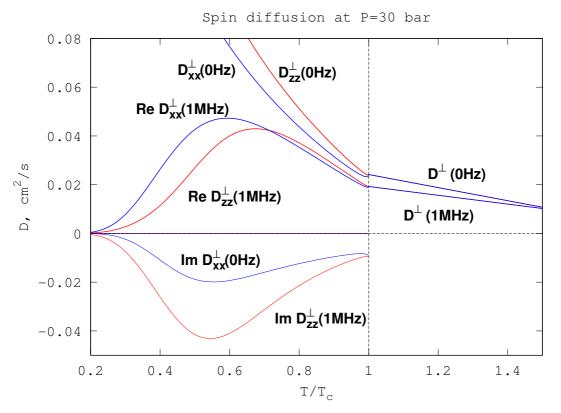
Bogoliubov quasiparticles lifetime and mean free path:

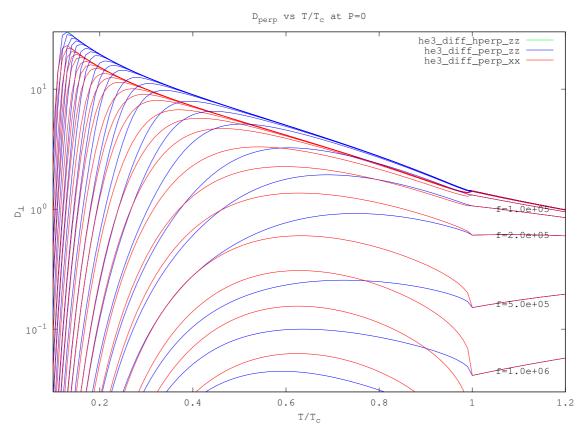
```
Lifetime at the Fermi level, s. 1/\tau(0) = \frac{I(0,T)}{\tau_N(0,T)},
he3_tau0(ttc, p)
                                (Einzel-1991, f.74,76. Einzel-1990 f.A1)
                                1/\tau, T \to 0 limit (does not depend on energy). (Einzel-1978, f.79)
he3_tau0lt(ttc, p)
                               Thermal average lifetime, s. 1/\bar{\tau} = \frac{1}{Y_0} \int_{-\infty}^{\infty} \frac{\phi_k \ d\xi}{\tau}
he3_tau_av(ttc, p)
                                (Einzel-1991, f.77)
he3_fpathv(ttc, p)
                               Mean free path, cm
                                (Einzel JLTP32 (1978) f.84)
                               Spin diff. transport time, s. \tau_D^{\perp,\parallel} = \bar{\tau}/(1-\lambda_1^a Y^{\perp,\parallel})
he3_tau_dperp(ttc, p)
he3_tau_dpar(ttc, p)
                                (Einzel-1991, f.90,96)
   Spin diffusion:
                                              Spin diffusion in hydrodynamic limit (\omega_L \tau \ll 1), D_{zz}^{\perp,\parallel}(\omega_L = 0),
he3_diff_hperp_zz(ttc, p)
he3_diff_hpar_zz(ttc, p)
                                              cm<sup>2</sup>/s. According to Einzel's paper (see f.105) nonhydrodynamic
                                              effects do not affect D^{\parallel}. In Mukharsky paper they do.
                                              (Einzel JLTP84 (1991) f.102)
he3_diff_perp_xx(ttc, p, nu0)
                                              Components of spin diffusion tenzor, cm<sup>2</sup>/s
he3_diff_perp_zz(ttc, p, nu0)
                                              (Bunkov et al, 1990 f.3; Einzel, 1991, f.108;
he3_diff_perp_xx_im(ttc, p, nu0)
                                              Markelov, Mukharsky, 1992, f.7,8)
he3_diff_perp_zz_im(ttc, p, nu0)
he3_diff_par_xx(ttc, p, nu0)
he3_diff_par_zz(ttc, p, nu0)
```

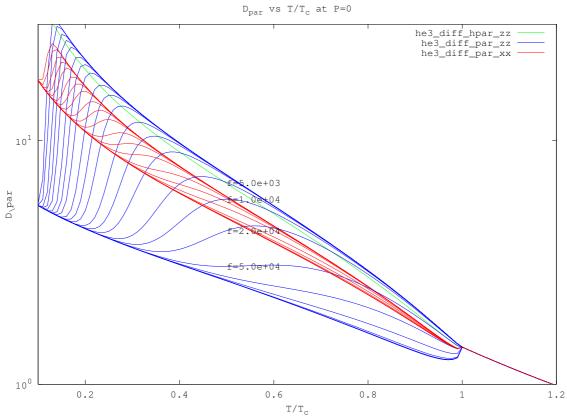
Problem with  $D^{\parallel}$ . According to Einzel-1991 (see f.105) nonhydrodynamic effects do not affect  $D^{\parallel}$  and you should use he3\_diff\_hpar\_zz for any frequency. In Markelov-Mukharsky paper the result for  $D^{\parallel}$  is completely different (it is used in he3\_diff\_par\_xx, \_zz).  $D^{\perp}$  is the same in all papers. TODO:

Exchange coupling strength is in the  $F_1^a=0$  limit. (small difference from normal phase values) How does he3\_diff\_perp\_??\_im extends to normal phase?









#### Other

he3\_xigl(ttc, p) Extrapolated GL coherence length, cm he3\_vneq(ttc, p, omega, r) Equilibrium vortex number

#### Normal liquid beyond zero temperature limit

he3\_cv\_n(t, p) Normal phase heat capacity  $C_v$ , Greywall-1983 TODO:  $V_m$ 

### ROTA specific functions and constants

rota\_c\_ns(t, i) Nuclear stage heat capacity [J/K] vs T[K] and I[A]  $C_{ns} = 9.66 \cdot 10^{-5} [J/T^2] (0.113 [T/A] I/T)^2$ 

rota\_fork\_cal(w, p, n) Calibration of fork  $n, T/T_c$ , vs fork width w [Hz] and pressure P [bar]

 $w = a_n \exp\left(-\frac{\Delta_{\text{WC+}}(P, T/T_c)}{T/T_c}\right),$  $a = \alpha_n \ p_F^4(P), \text{ where } \alpha_n \text{ is a geometrical factor}$ 

Fork K, calibration 30.4.2010, 29 bar, a = 11700N=2: Fork E, calibration 30.4.2010, 29 bar, a = 17543

rota\_rcell cell radius

field/current in nmrA solenoid [G/A] rota\_nmra field/current in nmrB solenoid [G/A] rota\_nmrb

Q value of the nmrA spectrometer vs frequency rota\_nmra\_q(f0)

frequencies of nmrA spectrometer,kHz for n=1..8 (use real\*8 n!) rota\_nmra\_f(n)

rota\_hmina\_r effective radius of the HminA coil [cm]

field/current in the center of HminA coil [G/A] rota\_hmina

quadratic radial term of the HminA field, [G/A/cm<sup>2</sup>] rota\_hmina\_mr

effective HminA coil current caused by NMR field distortion, divided by rota\_hmina\_i0

NMR freq [G/Hz]

radiation damping constant for the nmrA spectrometer rota\_rrda

# Q-balls in the zero temperature limit

qball_nu_b(P)	Leggett frequency (measurements, same as he3_nu_b(0,p) ) [Hz]
qball_cpar(P)	cpar (measured), $[cm/s]$
qball_cper(P)	cper (measured), [cm/s]
qball_dbetan(P, f0)	Derivative of the textural angle beta_N in the center of the cell (rota-specific, measured), $[{\rm rad/cm}]$
qball_fz0(P,f0,Imin)	1/2 of distance between visible axial levels (rota-specific, measured), [Hz]
qball_fr0(P,f0,Imin)	$1/2$ of distance between visible radial levels (rota-specific, measured), $[\mathrm{Hz}]$
qball_az0(P,f0,Imin)	$\mathbf{z}$ size of the magnon condensate (rota-specific, measured), [cm]
qball_ar0(P,f0,Imin)	r size of the magnon condensate (rota-specific, measured), $[\mathrm{cm}]$
qball_trd(P,f0,fr,fz)	$\tau_{RD}$ for the magnon condensate with given radial and axial frequencies (rota-specific, measured) [s]
qball_trd0(P,f0,imin)	$\tau_{RD}$ for the magnon condensate (no rotation, rota-specific, measured) [s]