^{3}He
Superfluidity, collective modes, spin waves

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Lancaster
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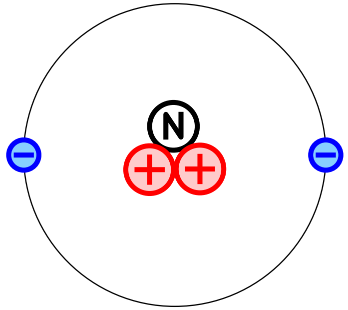
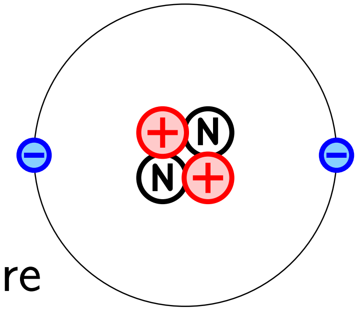
Helium: two stable isotopes

^{3}He



2 protons
1 neutron
2 electrons

^{4}He



2 protons
2 neutrons
2 electrons

1:1'000'000 ratio in atmosphere

Spin: 1/2, Fermi particle

Spin: 0, Bose particle

Inert atoms with symmetric electron shells.
Weak interactions between atoms, small atom mass.

Liquid down to absolute zero temperature (at small pressures).

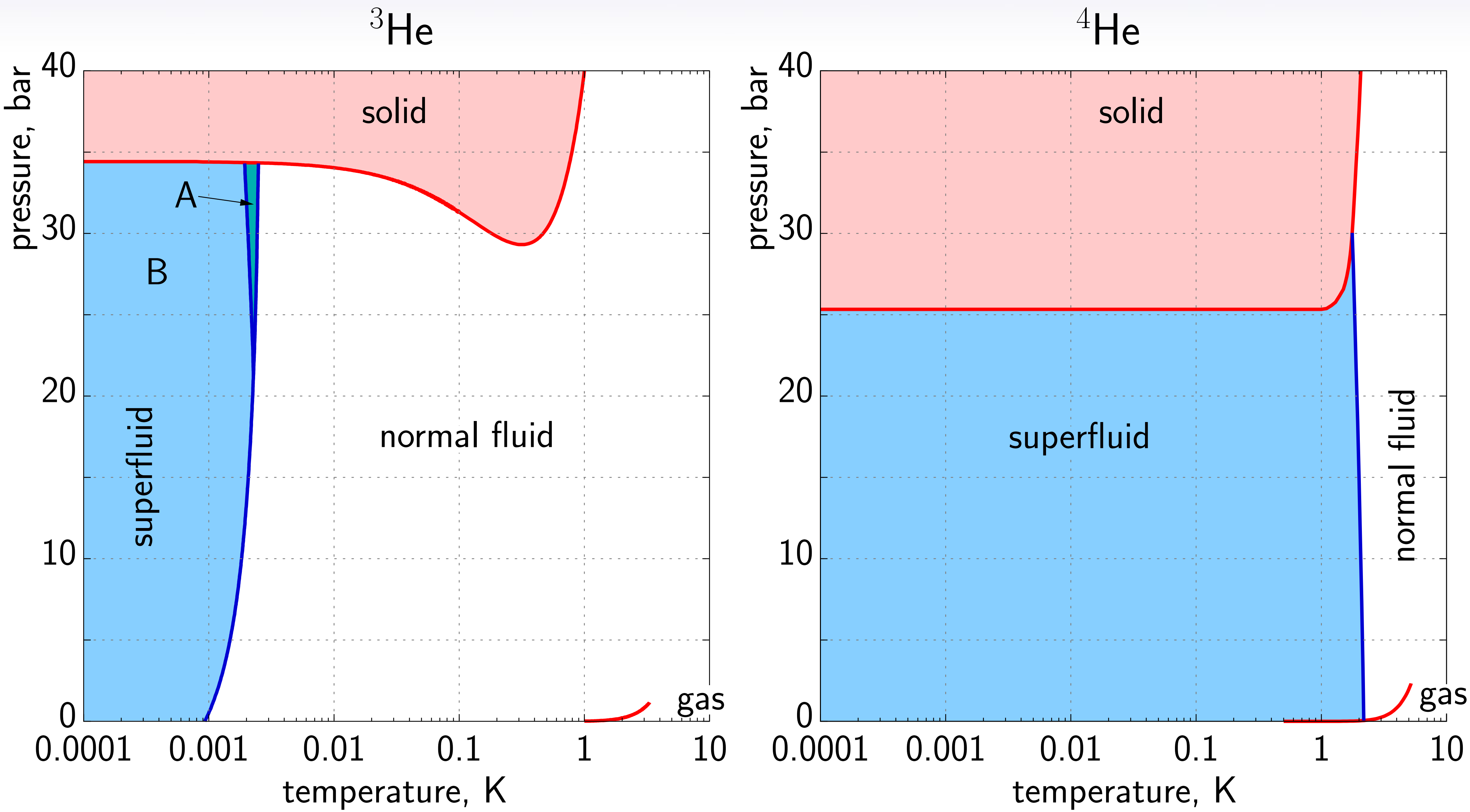
^{3}He:

Superfluid transition at ~ 1 mK
Cooper pairing with $L = 1$ and $S = 1$.

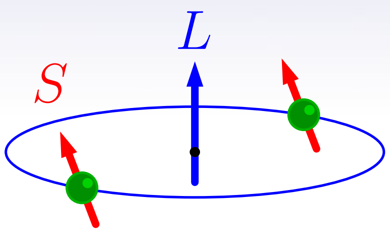
^{4}He:

Superfluid transition at ~ 2 K
Bose condensation of atoms.

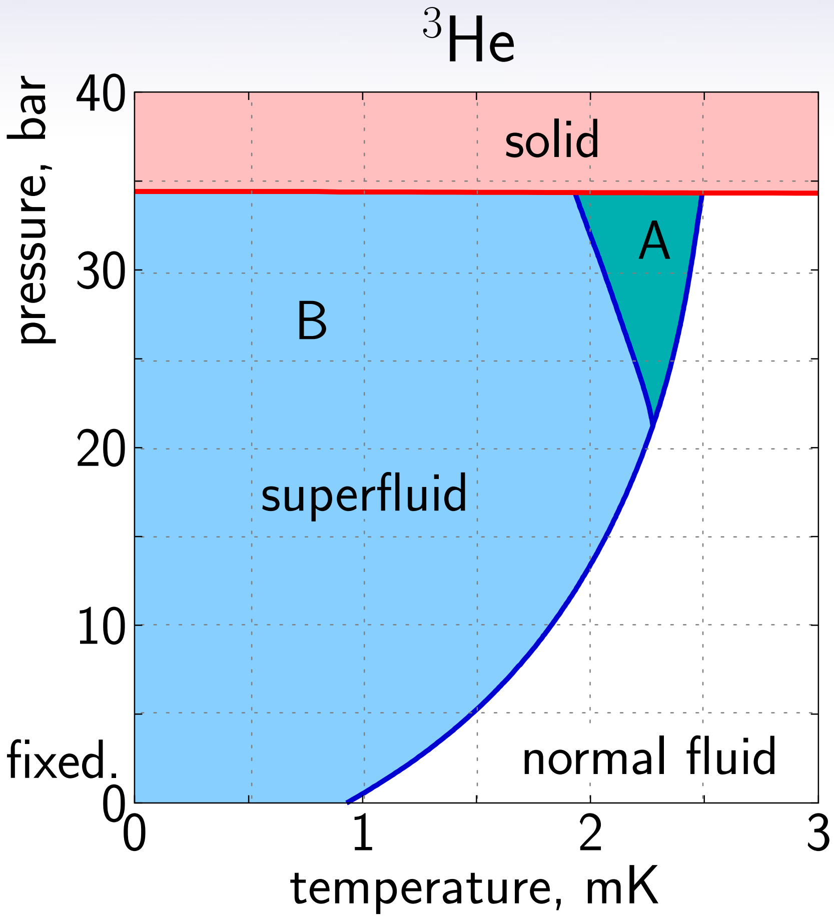
Helium: Phase diaglam



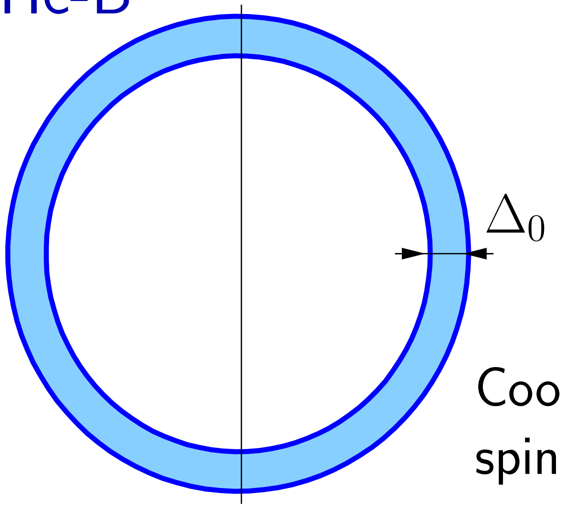
Superfluid ^{3}He



Cooper pair
$S = 1,\ L = 1$

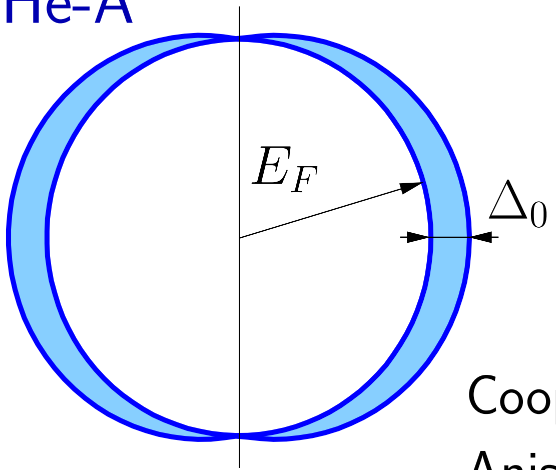


^{3}He-B



Cooper pairs with all possible spin projections $S_z = 0, \pm 1$.
Mutial orientation of S and L is fixed.
Isotropic energy gap.

^{3}He-A



Cooper pairs with $S_z = \pm 1$.
Anisotropic energy gap.

Superfluid ^{3}He:
- complicated theory
- pure system, good agreement with experiments
- macroscopic system
- no applications

Phase transitions: 1st order

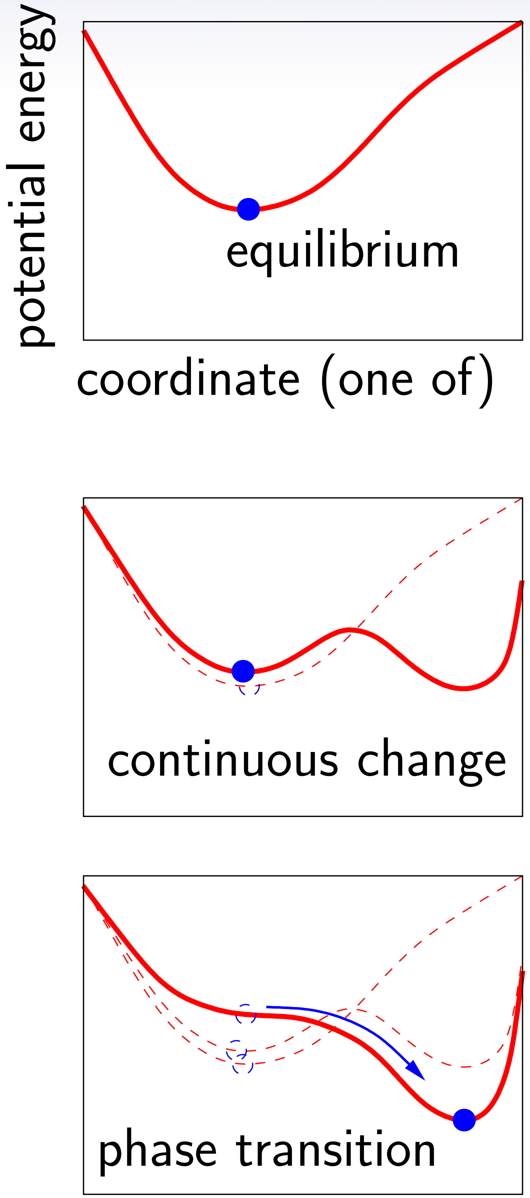
Phase transition: sudden jump of a system to a new state during continuous change of some external condition (temperature, pressure, magnetic field...).

A system state can be represented by a point in a multi-dimensional coordinate space.

Potential energy depends on coordinates, it continuously changes following external conditions.

First order phase transition:

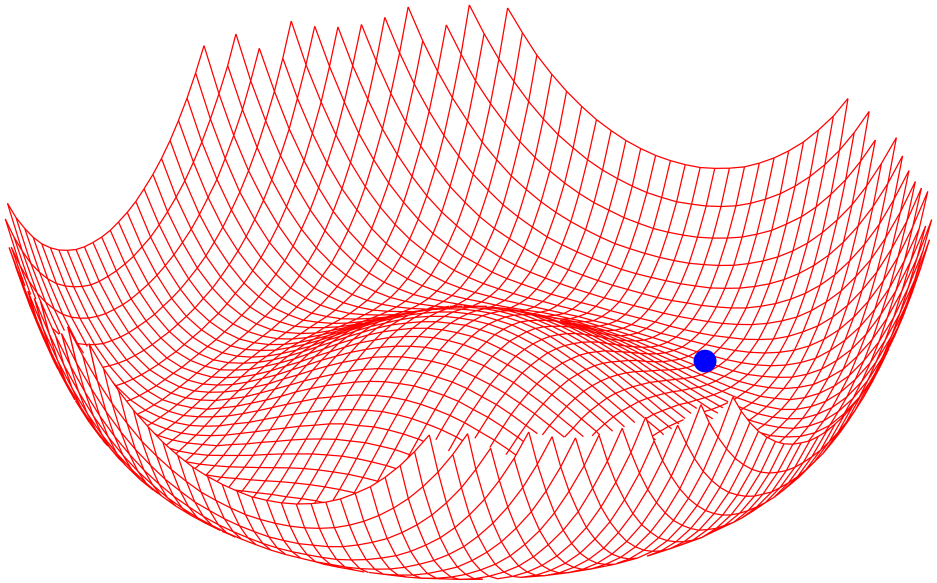
- All propertis of the system have a jump.
- Hysteresis: backward transition at a different point.
- Latent heat of the transiton, singularity in heat capacity (extra kynetic energy has to be removed or added to thermalize the system).

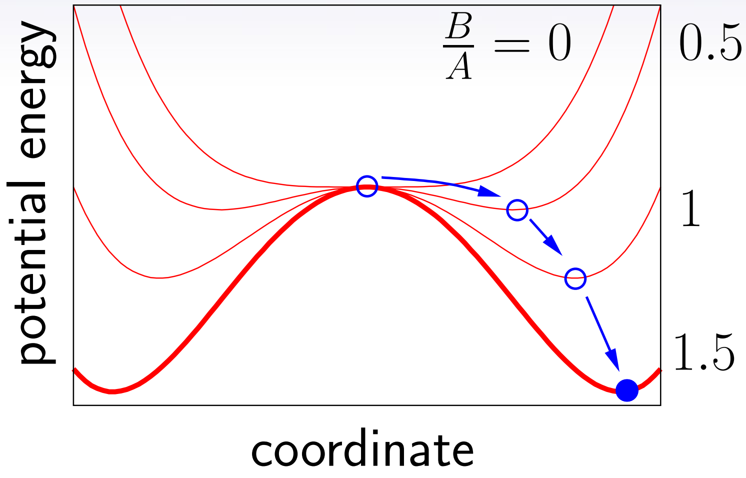


Phase transitions: 2nd order

"Mexican-hat", or "wine-bottle" potential.

Ginzburg-Landau model: $A\,r^4 - B\,r^2$

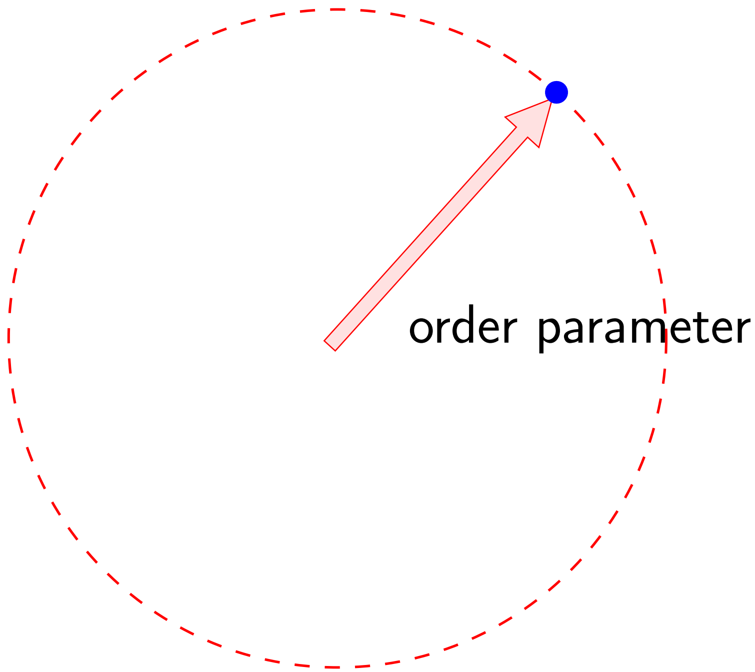




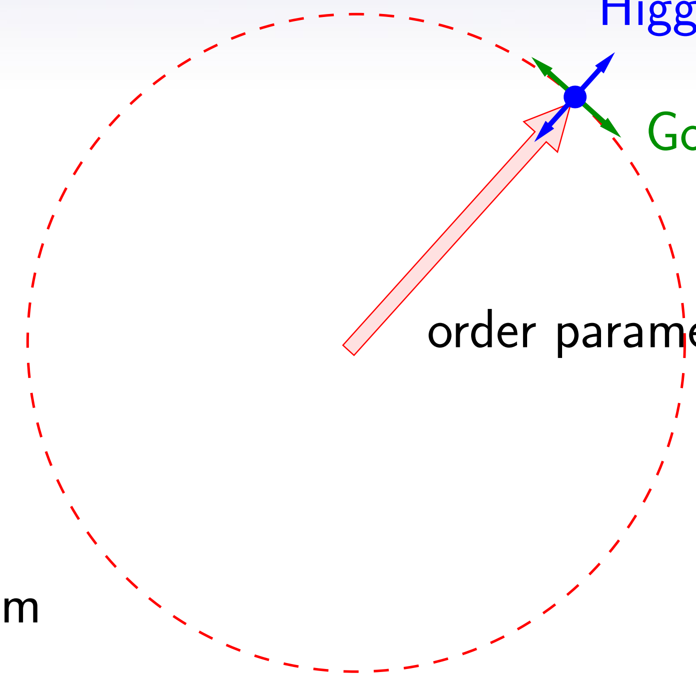
- System state changes continuously, but a new degree of freedom appears.
- No hysteresis.
- Jump in heat capacity.

- Spontaneous symmetry breaking

- Order parameter (phase and amplitude).



2nd order phase transition, collective modes



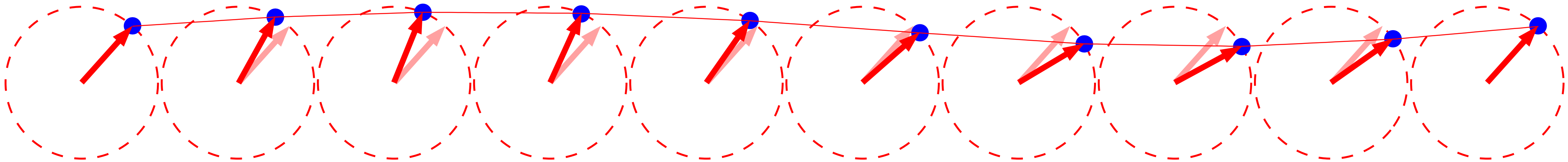
Order parameter space: 2D plane
2 degrees of freedom,
2 oscillation modes.

Degenerate space: a circle where energy is at minimum
1 degree of freedom,
1 Goldstone mode - motion of the order parameter phase

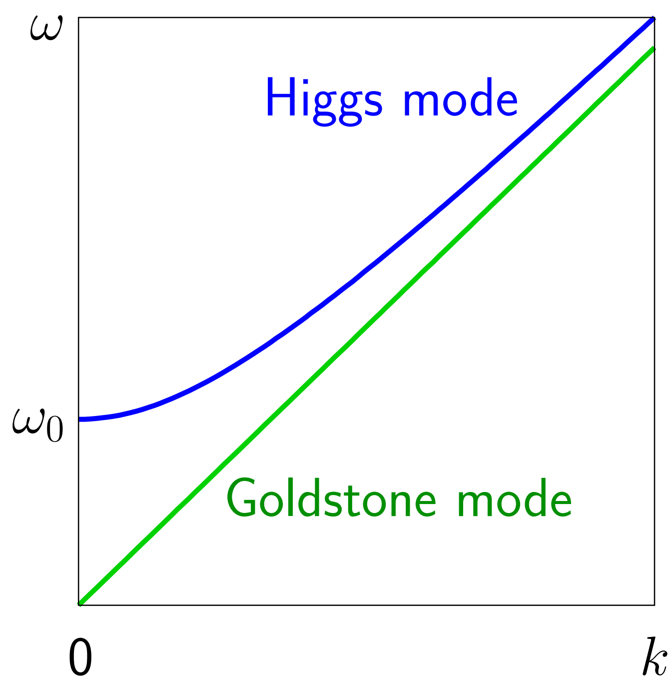
Higgs mode - motion of the order parameter amplitude.
non-zero frequency ω_0.

Waves

We can have a Goldstone wave with non-zero frequency:



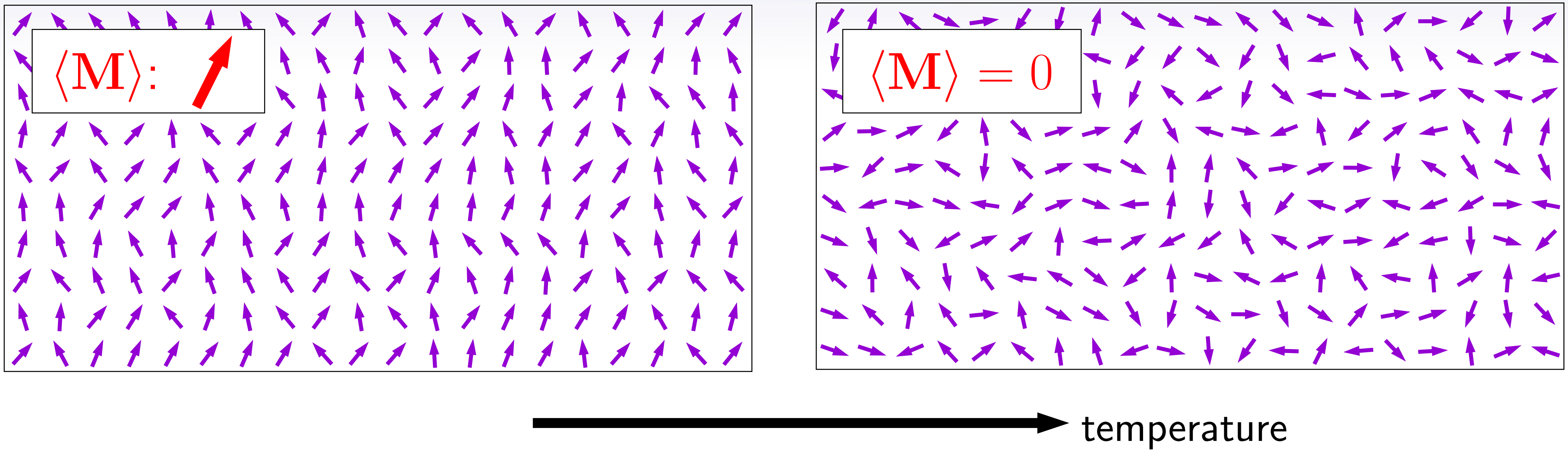
Wave length L, wave vector $k = 2\pi/L$
$\omega = k\,c$ where c is speed of the wave



For Higgs mode minimum frequency is ω_0
$\omega = \sqrt{\omega_0^2 + (c\,k)^2}$

Relativstic spectrum with non-zero rest mass.

Simple example: ferromagnet



Order parameter - magnetization $\langle \mathrm{M} \rangle$

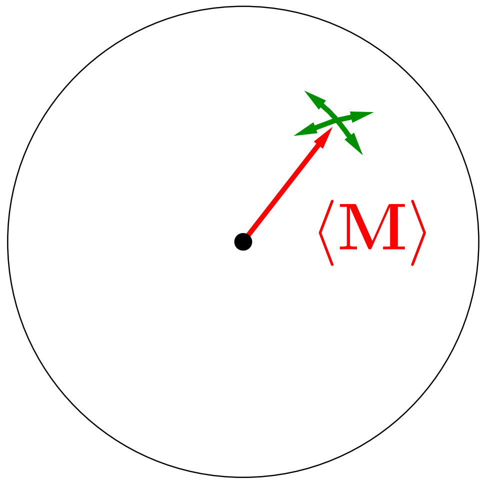
Order parameter space: 3D $\rightarrow$ three modes.

Degenerate space: a sphere $\rightarrow$ two Goldstone modes.

Higgs mode: oscillation of magnetization amplitude

Goldstone modes: rotation of magnetization, spin waves

(2 modes, can be splitted by magnetic field)



Order parameter in superfluid helium

^{4}He (and superconductors):

Order parameter: $A = \Delta\, e^{i\varphi}$ (wave function of Bose condensate, a complex number)

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^{3}He:

Cooper pairs have $S = 1$ and $L = 1$, the condensate state is a linear combination of states with $L_z = -1, 0, +1$ and $S_z = -1, 0, 1$

Order parameter: $A_{\mu j}$, a 3x3 complex matrix.

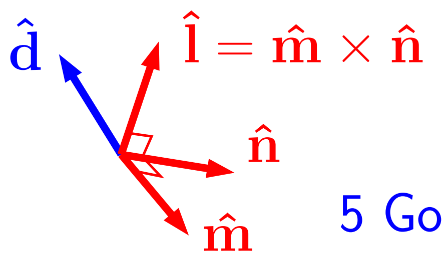
18 degrees of freedom!

Symmetry breaks in different ways in A- and B-phases:

^{3}He-A

$$A_{\mu j} = \Delta_0\, \hat{\mathbf{d}}_\mu (\hat{\mathbf{m}}_j + i\hat{\mathbf{n}}_j)$$

$\hat{\mathbf{m}}$ and $\hat{\mathbf{n}}$ - orthogonal unit vectors

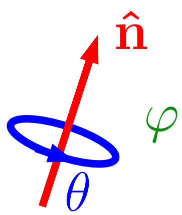


5 Goldstone modes

^{3}He-B

$$A_{\mu j} = \Delta_0\, R_{\mu j}(\hat{\mathbf{n}}, \theta)\, e^{i\varphi}$$

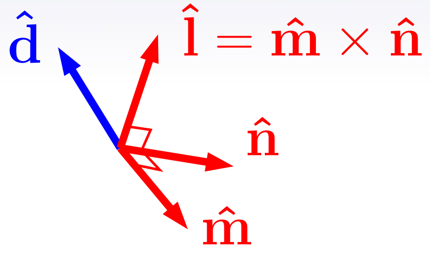
$R_{\mu j}$ - rotation matrix with axis $\hat{\mathbf{n}}$ and angle θ



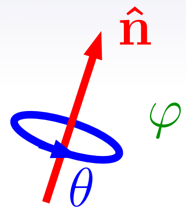
4 Goldstone modes

Collective modes in superfluid helium-3

^{3}He-A



^{3}He-B



Goldstone modes (5):

- sound: rotation of $\hat{\mathbf{n}}$ and $\hat{\mathbf{m}}$ around $\hat{\mathbf{l}}$
- 2 spin waves: motion of $\hat{\mathbf{d}}$
- 2 orbital waves: motion of $\hat{\mathbf{l}}$

Higgs modes (13):

- 6 Clapping modes
- 4 Flapping (spin-orbit) modes
- 1 Pseudo-sound mode
- 2 Pseudo-spin modes

Goldstone modes (4):

- sound: motion of φ
- 3 spin wave modes: motion of $\hat{\mathbf{n}}$ and θ

Higgs modes (14):

- 4 Pair-breaking modes
- 5 Real squashing modes
- 5 Imaginary squashing modes

Finate temperature:

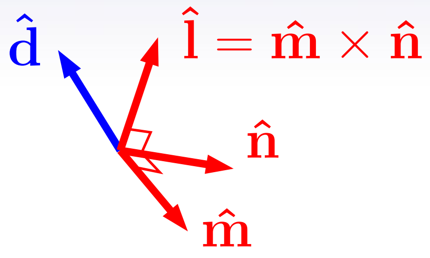
- first and second sound

Non-hyrodynamic regime:

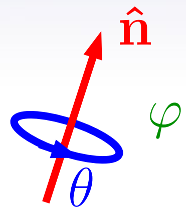
- zero sound
- collisionless spin waves

Collective modes in superfluid helium-3

^{3}He-A



^{3}He-B



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Spin waves in ^{3}He-B

Rotation of the order parameter changes total spin of the system.
We can write Hamiltonian eqations using spin S and $R_{\mu j}$ as coordinates.

Energy of spin in magnetic field:

$$F_M = -(\gamma \mathbf{S} \cdot \mathbf{H}) + \frac{\gamma^2 \mathbf{S}^2}{2\chi} \qquad \text{(minimum at } \gamma\mathbf{S} = \chi\mathbf{H}\text{)}$$

Energy of weak spin-orbit interaction in Cooper pairs:

$$F_{SO} = \frac{\chi \Omega_B^2}{15\gamma^2} \sum_{k,j} (R_{jj}R_{kk} + R_{jk}R_{kj}) = \frac{\chi \Omega_B^2}{15\gamma^2} \frac{1}{2} (4\cos\theta + 1)^2$$

(minimum at $\theta \approx 104^\circ$)

Gradient energy:

$$F_\nabla = \langle \text{some long expression} \rangle$$

Leggett equations, texure and spin waves

Leggett equations:

$$\begin{aligned} \dot{S}_a &= [\mathbf{S} \times \gamma \mathbf{H}]_a + T_a(R), \\ \dot{R}_{aj} &= e_{abc} R_{cj} \left(\frac{\gamma^2}{\chi_B} \mathbf{S} - \gamma \mathbf{H} \right)_b, \end{aligned}$$

Gradient energy

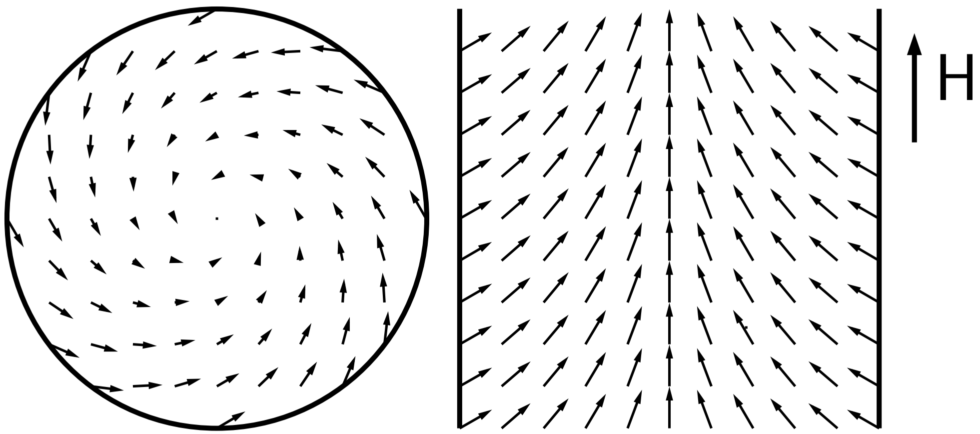
Spin-orbit interaction, Ω_B

Interaction with walls

Equilibrium distribution of $R(\hat{\mathbf{n}}, \vartheta)$ – texture.

Motion of $R(\hat{\mathbf{n}}, \vartheta)$ – spin waves.

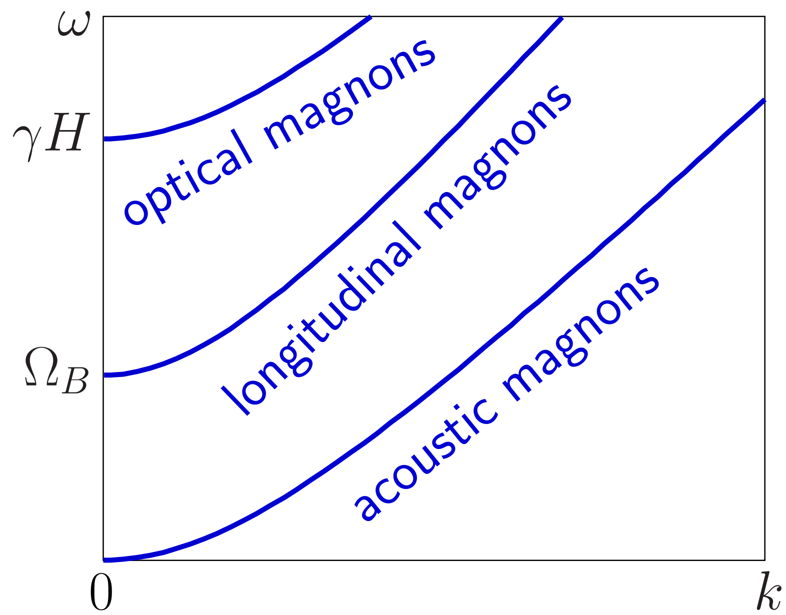
Flare-out texture in a cylindrical cell



$\vartheta = 104°$

(Leggett angle, minimum of spin-orbit interaction)

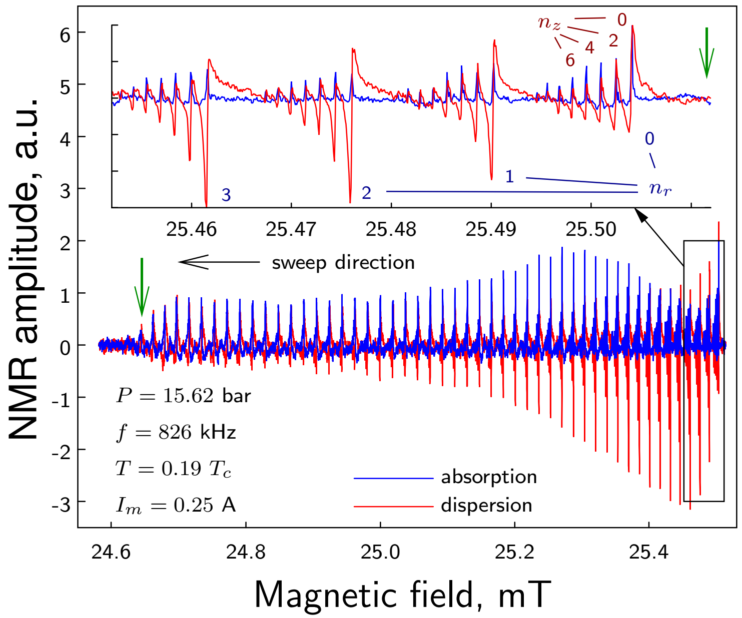
Linear NMR, $\Omega_B \ll \gamma H$, $\hat{\mathbf{n}} \parallel \mathbf{H}$



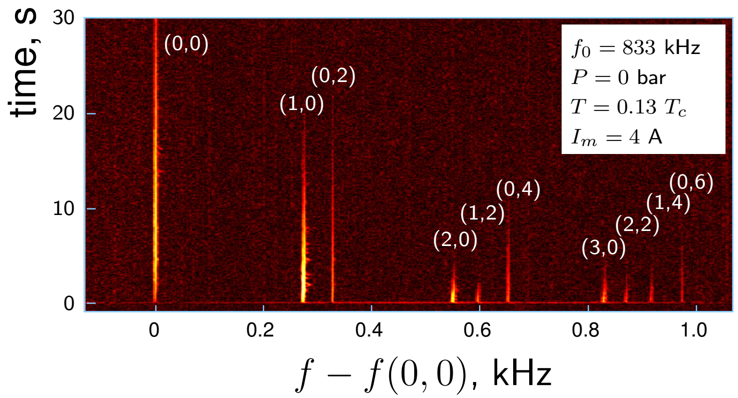
Trap for magnons

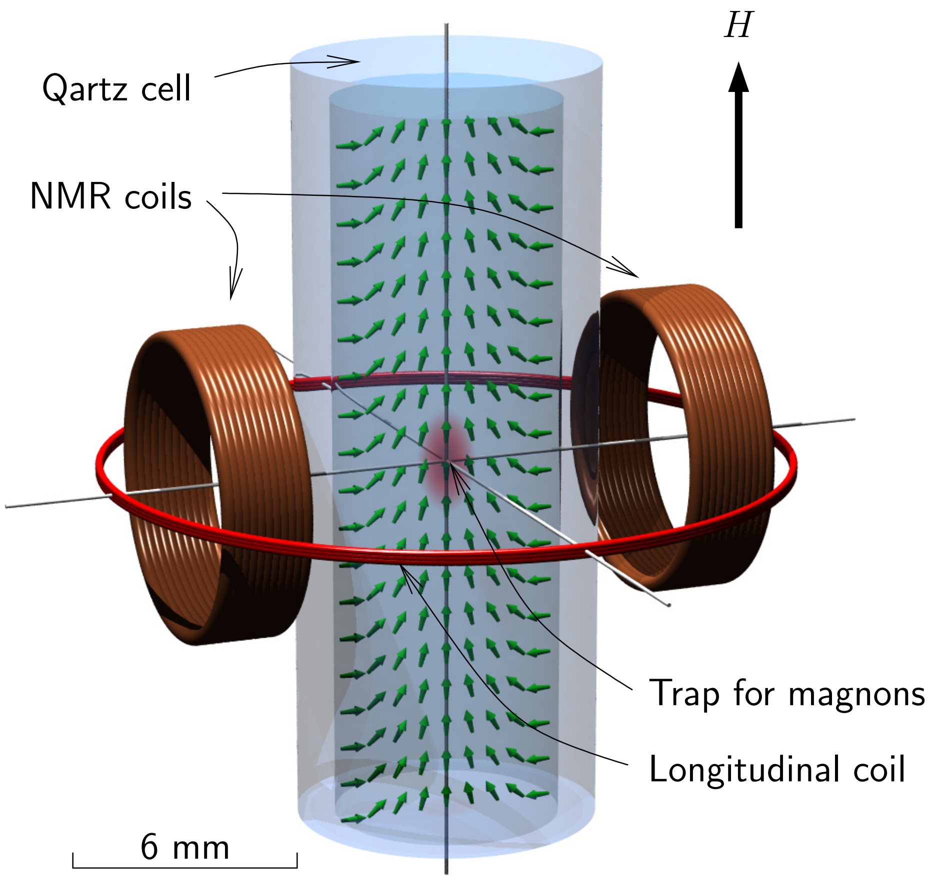
Experments in Helsinki 2012-2015

continuous pumping



pulsed pumping

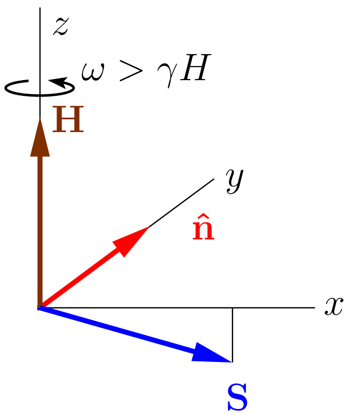




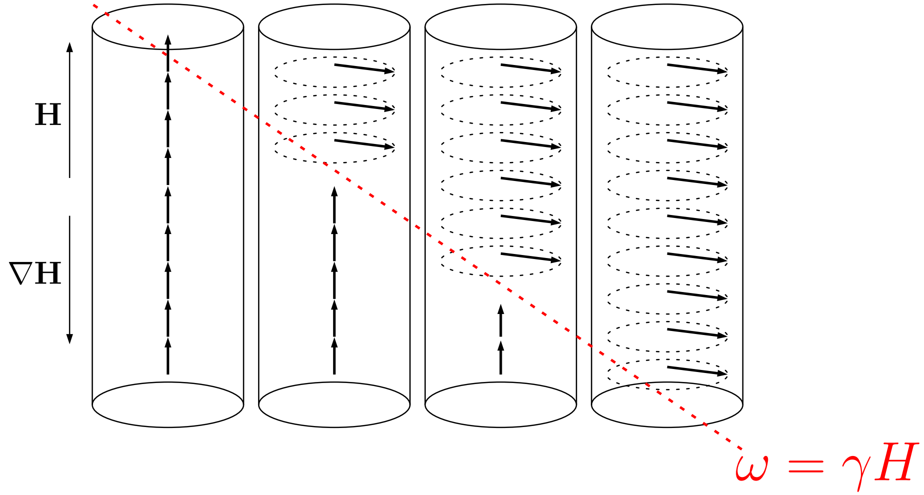
Thermometry, interacton with free surface, with vortices, Studying properties of ^{3}He.

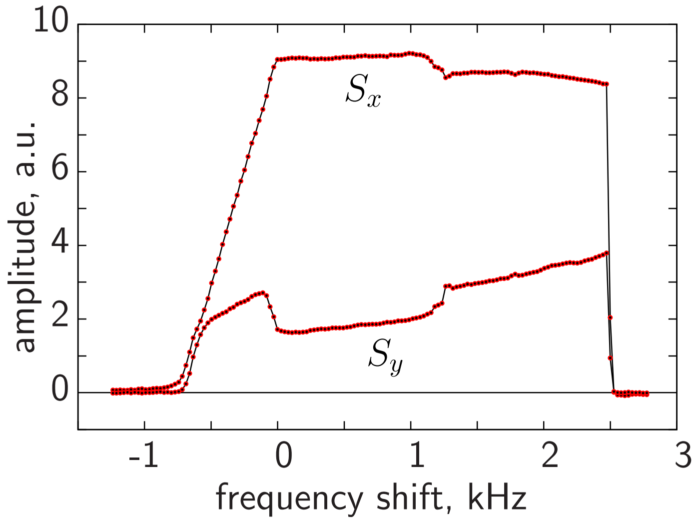
Homogeneously precessing domain (HPD)

HPD



creation of HPD in CW NMR





Experiment with two HPDs (Moscow, 1987)