

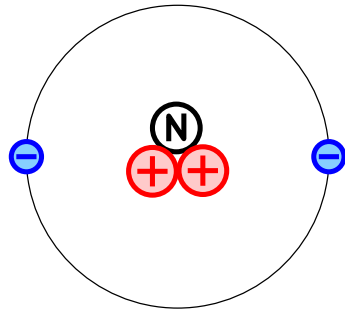
^3He

Superfluidity, collective modes, spin waves

Vladislav Zavjalov

Helium: two stable isotopes

^3He

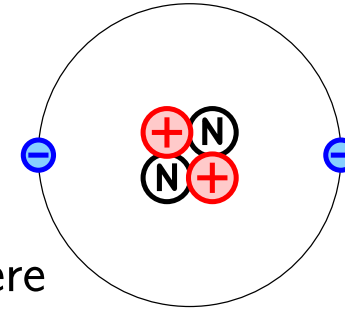


2 protons
1 neutron
2 electrons

1:1'000'000 ratio in atmosphere

Spin: $1/2$, Fermi particle

^4He



2 protons
2 neutrons
2 electrons

Spin: 0, Bose particle

Inert atoms with symmetric electron shells.

Weak interactions between atoms, small atom mass.

Liquid down to absolute zero temperature (at small pressures).

^3He :

Superfluid transition at ~ 1 mK

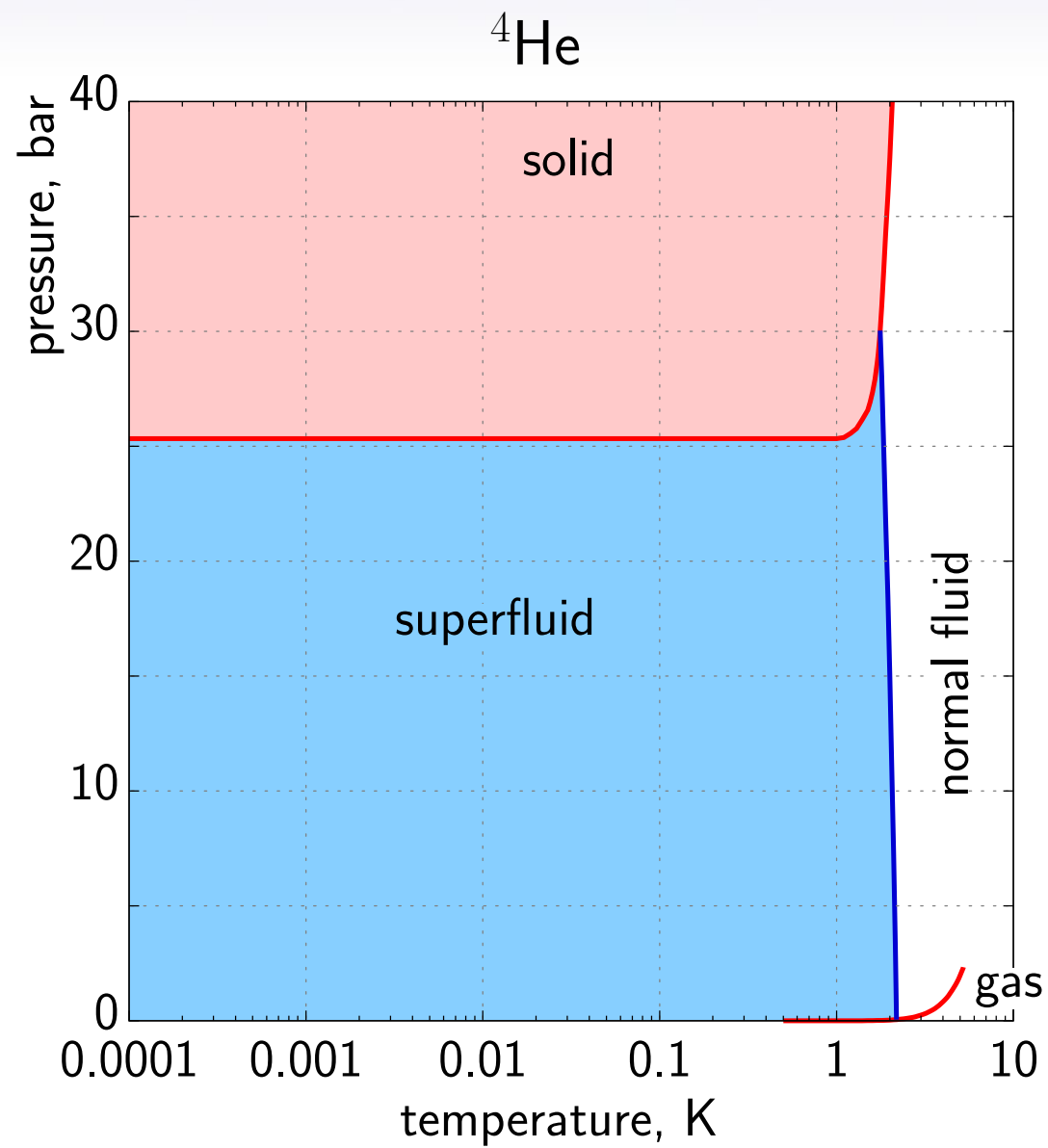
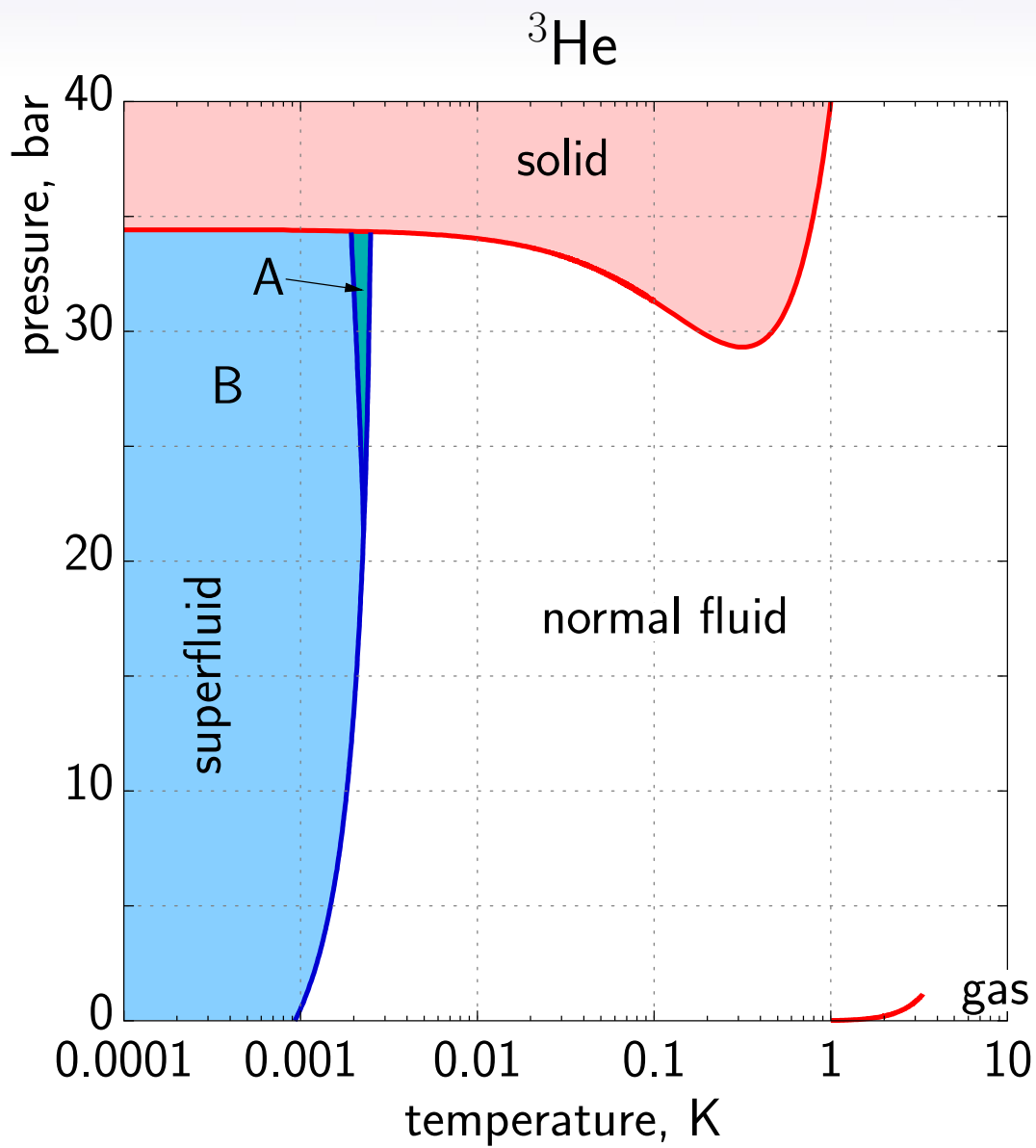
Cooper pairing with $L = 1$ and $S = 1$.

^4He :

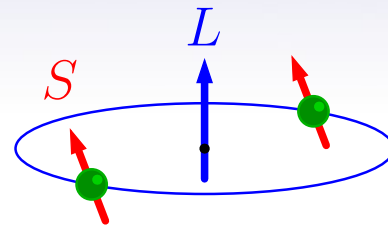
Superfluid transition at ~ 2 K

Bose condensation of atoms.

Helium: Phase diagram

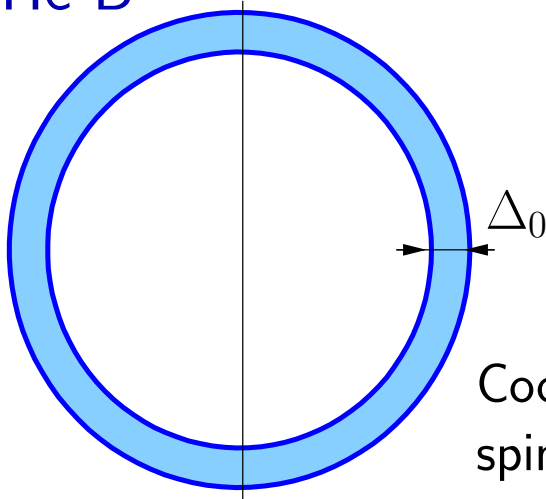


Superfluid ^3He



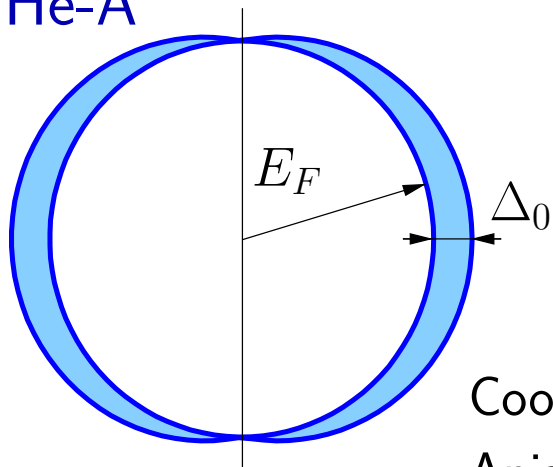
Cooper pair
 $S = 1, L = 1$

$^3\text{He-B}$

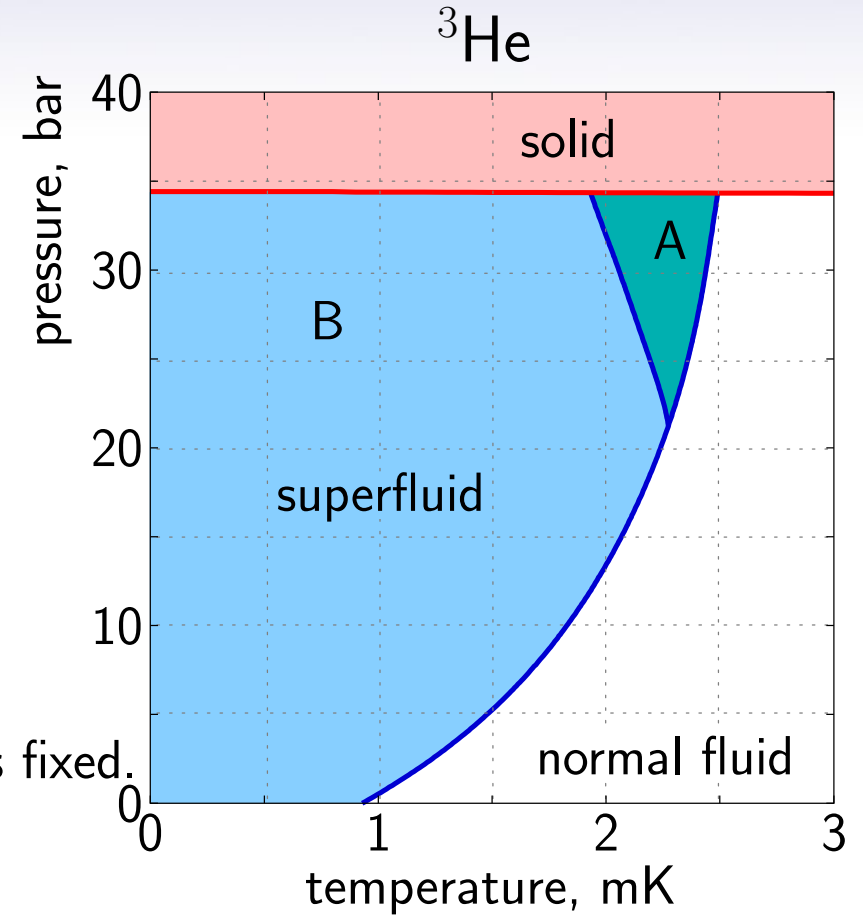


Cooper pairs with all possible spin projections $S_z = 0, \pm 1$.
Mutual orientation of S and L is fixed.
Isotropic energy gap.

$^3\text{He-A}$



Cooper pairs with $S_z = \pm 1$.
Anisotropic energy gap.



Superfluid ^3He :

- complicated theory
- pure system, good agreement with experiments
- macroscopic system

Phase transitions: 1st order

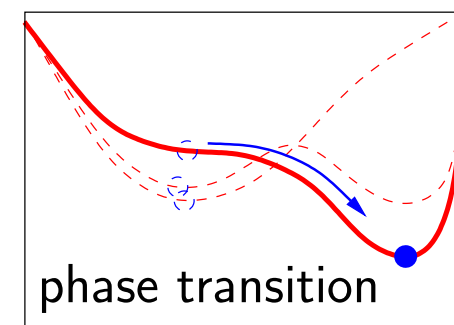
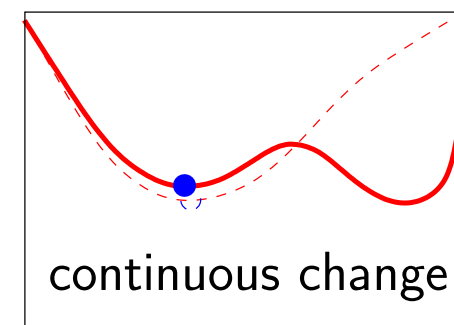
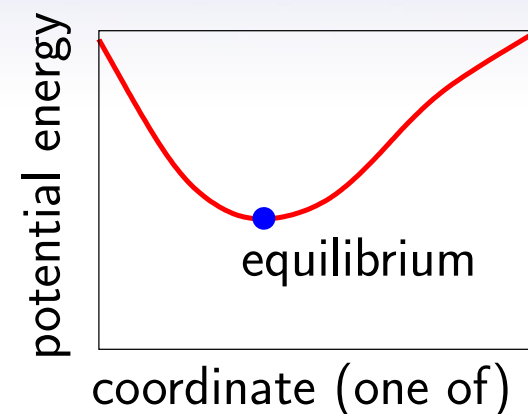
Phase transition: sudden jump of a system to a new state during continuous change of some external condition (temperature, pressure, magnetic field...).

A system state can be represented by a point in a multi-dimensional coordinate space.

Potential energy depends on coordinates, it continuously changes following external conditions.

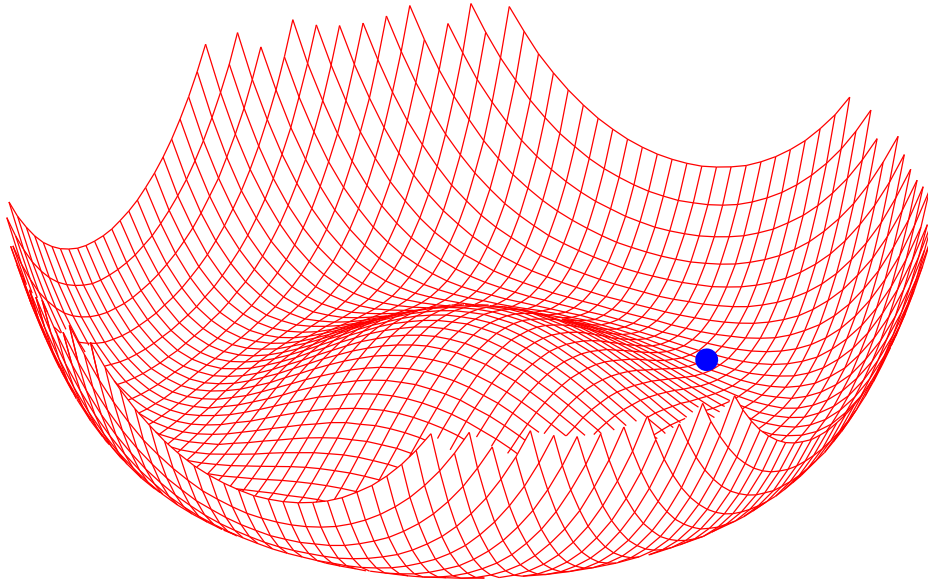
First order phase transition:

- All properties of the system have a jump.
- Hysteresis: backward transition at a different point.
- Latent heat of the transition, singularity in heat capacity (extra kinetic energy has to be removed or added to thermalize the system).

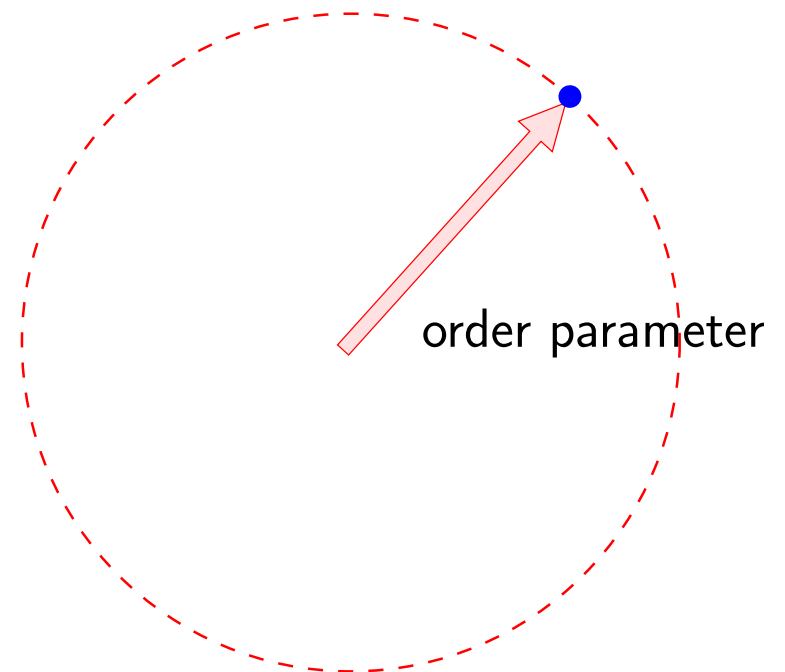
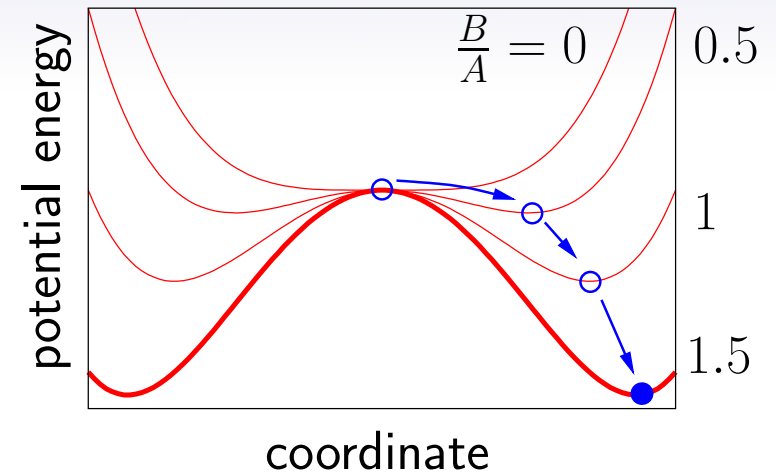


“Mexican-hat”, or “wine-bottle” potential.

Ginzburg-Landau model: $A r^4 - B r^2$



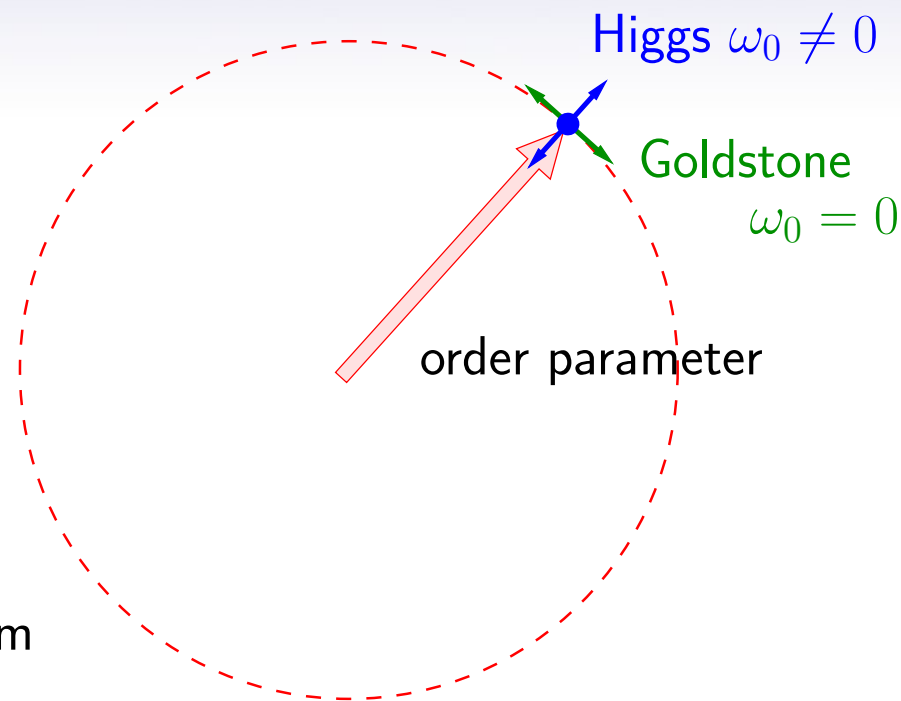
- System state changes continuously, but a new degree of freedom appears.
- No hysteresis.
- Jump in heat capacity.
- Spontaneous symmetry breaking
- Order parameter (phase and amplitude).



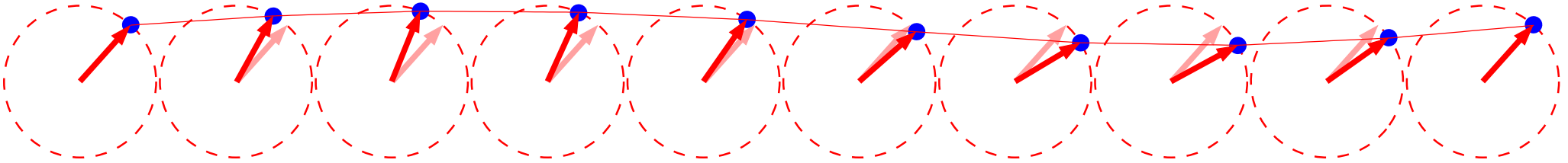
Order parameter space: 2D plane
2 degrees of freedom,
2 oscillation modes.

Degenerate space: a circle where energy is at minimum
1 degree of freedom,
1 Goldstone mode - motion of the order parameter phase

Higgs mode - motion of the order parameter amplitude.
non-zero frequency ω_0 .



We can have a Goldstone wave with non-zero frequency:



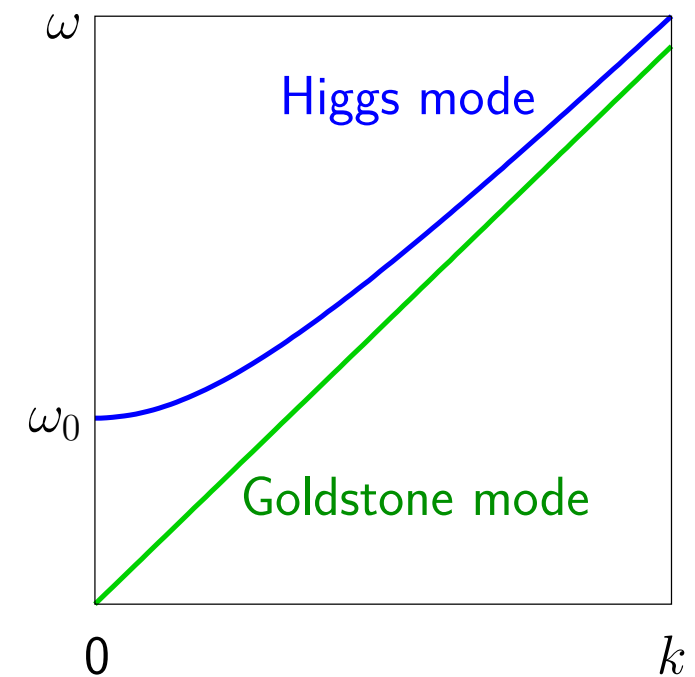
Wave length L , wave vector $k = 2\pi/L$

$\omega = kc$ where c is speed of the wave

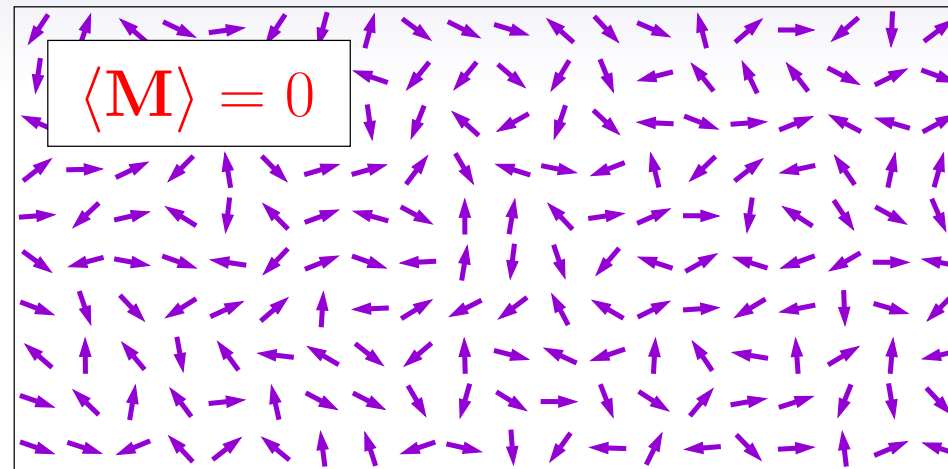
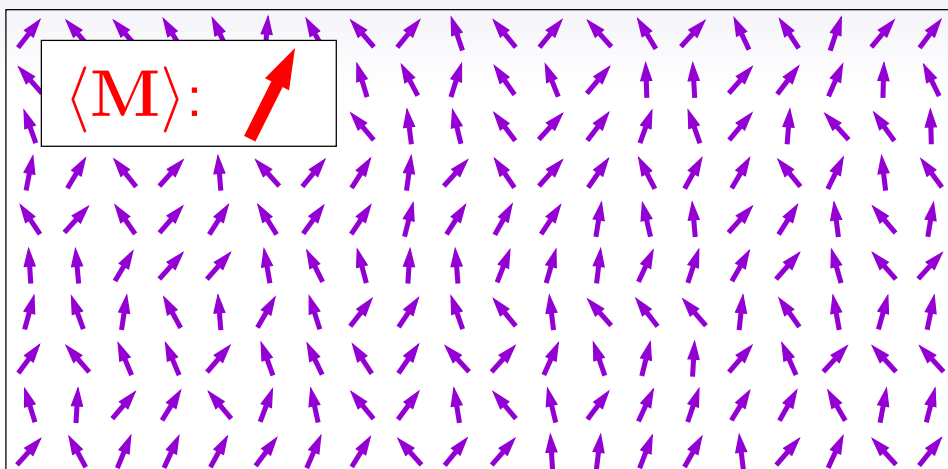
For Higgs mode minimum frequency is ω_0

$$\omega = \sqrt{\omega_0^2 + (ck)^2}$$

Relativistic spectrum with non-zero rest mass.



Simple example: ferromagnet



—————→ temperature

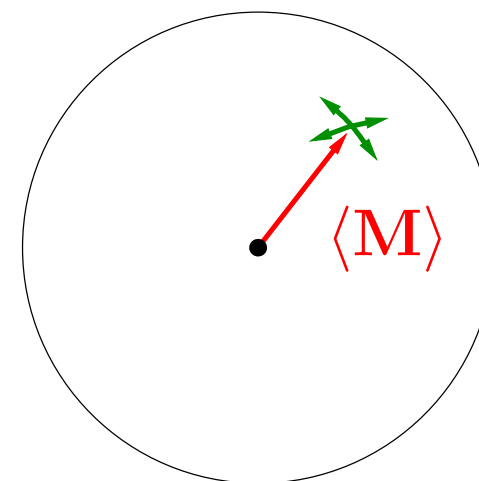
Order parameter - magnetization $\langle \mathbf{M} \rangle$

Order parameter space: 3D \rightarrow three modes.

Degenerate space: a sphere \rightarrow two Goldstone modes.

Higgs mode: oscillation of magnetization amplitude

Goldstone modes: rotation of magnetization, spin waves
(2 modes, can be splitted by magnetic field)



⁴He (and superconductors):

Order parameter: $A = \Delta e^{i\varphi}$ (wave function of Bose condensate, a complex number)

³He:

Cooper pairs have $S = 1$ and $L = 1$, the condensate state is a linear combination of states with $L_z = -1, 0, +1$ and $S_z = -1, 0, 1$

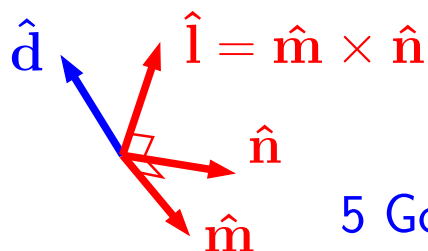
Order parameter: $A_{\mu j}$, a 3x3 complex matrix. 18 degrees of freedom!

Symmetry breaks in different ways in A- and B-phases:

³He-A

$$A_{\mu j} = \Delta_0 \hat{\mathbf{d}}_\mu (\hat{\mathbf{m}}_j + i \hat{\mathbf{n}}_j)$$

$\hat{\mathbf{m}}$ and $\hat{\mathbf{n}}$ - orthogonal unit vectors

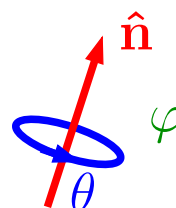


5 Goldstone modes

³He-B

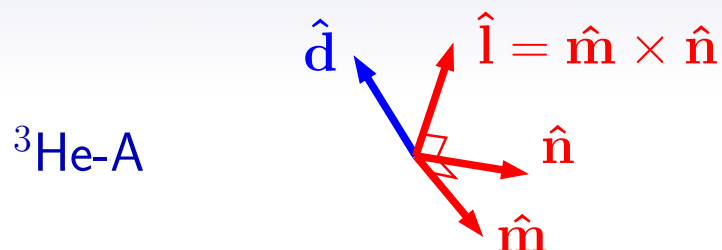
$$A_{\mu j} = \Delta_0 R_{\mu j}(\hat{\mathbf{n}}, \theta) e^{i\varphi}$$

$R_{\mu j}$ - rotation matrix with axis $\hat{\mathbf{n}}$ and angle θ



4 Goldstone modes

Collective modes in superfluid helium-3



Goldstone modes (5):

- sound: rotation of $\hat{\mathbf{n}}$ and $\hat{\mathbf{m}}$ around $\hat{\mathbf{l}}$
- 2 spin waves: motion of $\hat{\mathbf{d}}$
- 2 orbital waves: motion of $\hat{\mathbf{l}}$

Higgs modes (13):

- 6 Clapping modes
- 4 Flapping (spin-orbit) modes
- 1 Pseudo-sound mode
- 2 Pseudo-spin modes

Goldstone modes (4):

- sound: motion of φ
- 3 spin wave modes: motion of $\hat{\mathbf{n}}$ and θ

Higgs modes (14):

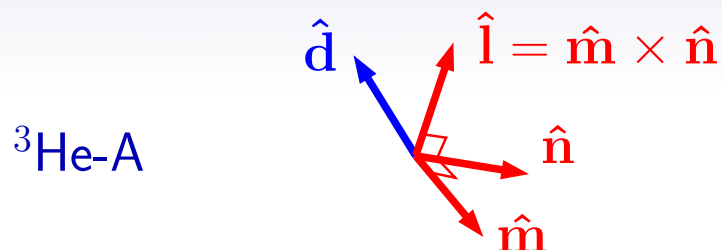
- 4 Pair-breaking modes
- 5 Real squashing modes
- 5 Imaginary squashing modes

Finite temperature:

- first and second sound

Non-hydrodynamic regime:

- zero sound
- collisionless spin waves



Goldstone modes (5):

- sound: rotation of $\hat{\mathbf{n}}$ and $\hat{\mathbf{m}}$ around $\hat{\mathbf{l}}$
- 2 spin waves: motion of $\hat{\mathbf{d}}$
- 2 orbital waves: motion of $\hat{\mathbf{l}}$

Higgs modes (13):

- 6 Clapping modes
- 4 Flapping (spin-orbit) modes
- 1 Pseudo-sound mode
- 2 Pseudo-spin modes

Goldstone modes (4):

- sound: motion of φ
- 3 spin wave modes: motion of $\hat{\mathbf{n}}$ and θ

Higgs modes (14):

- 4 Pair-breaking modes
- 5 Real squashing modes
- 5 Imaginary squashing modes

Finite temperature:

- first and second sound

Non-hydrodynamic regime:

- zero sound
- collisionless spin waves

Rotation of the order parameter changes total spin of the system.

We can write Hamiltonian equations using spin S and $R_{\mu j}$ as coordinates.

Energy of spin in magnetic field:

$$F_M = -(\gamma \mathbf{S} \cdot \mathbf{H}) + \frac{\gamma^2 \mathbf{S}^2}{2\chi} \quad (\text{minimum at } \gamma \mathbf{S} = \chi \mathbf{H})$$

Energy of weak spin-orbit interaction in Cooper pairs:

$$F_{SO} = \frac{\chi \Omega_B^2}{15\gamma^2} \frac{1}{2} (4 \cos \theta + 1)^2 \quad (\text{minimum at } \theta \approx 104^\circ)$$

Gradient energy:

$$F_{\nabla} = \langle \text{some long expression} \rangle$$

Leggett equations, texture and spin waves

Leggett equations:

$$\begin{aligned}\dot{S}_a &= [\mathbf{S} \times \gamma \mathbf{H}]_a + T_a(R), \\ \dot{R}_{aj} &= e_{abc} R_{cj} \left(\frac{\gamma^2}{\chi_B} \mathbf{S} - \gamma \mathbf{H} \right)_b,\end{aligned}$$

Gradient energy

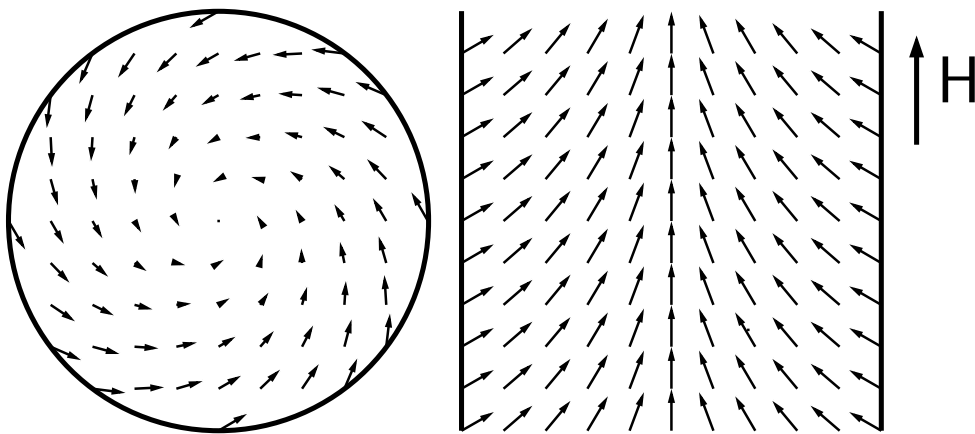
Spin-orbit interaction, Ω_B

Interaction with walls

Equilibrium distribution of $R(\hat{\mathbf{n}}, \vartheta)$ – texture.

Motion of $R(\hat{\mathbf{n}}, \vartheta)$ – spin waves.

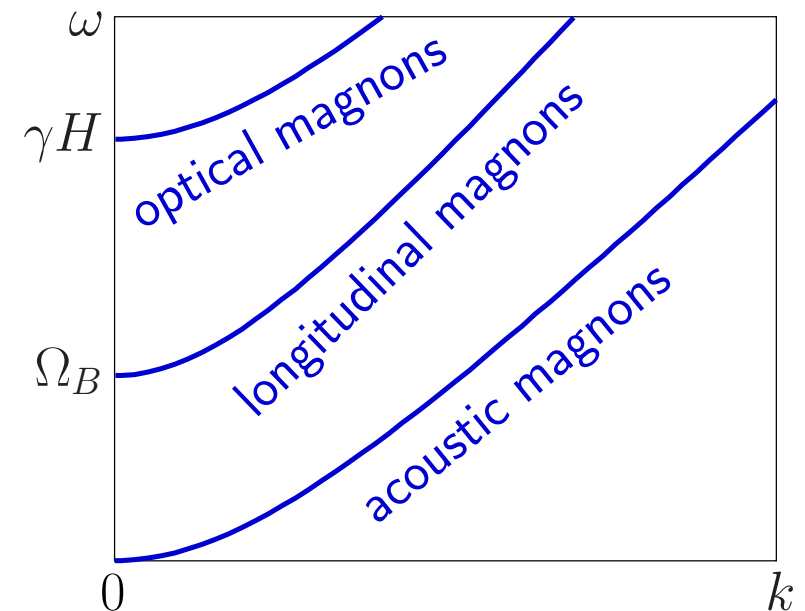
Flare-out texture in a cylindrical cell



$$\vartheta = 104^\circ$$

(Leggett angle, minimum of spin-orbit interaction)

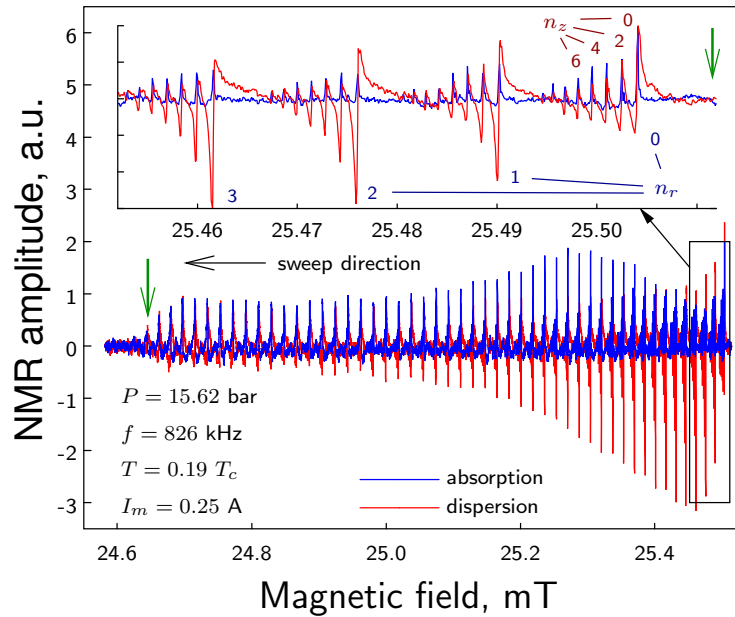
Linear NMR, $\Omega_B \ll \gamma H$, $\hat{\mathbf{n}} \parallel \mathbf{H}$



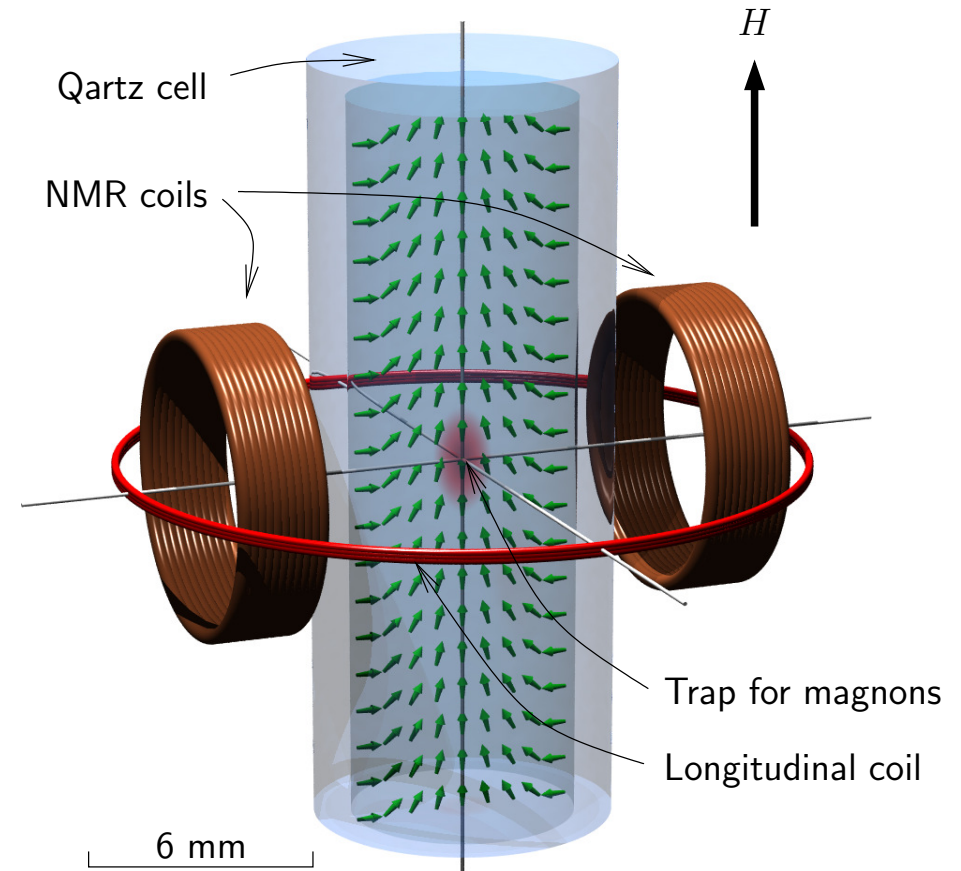
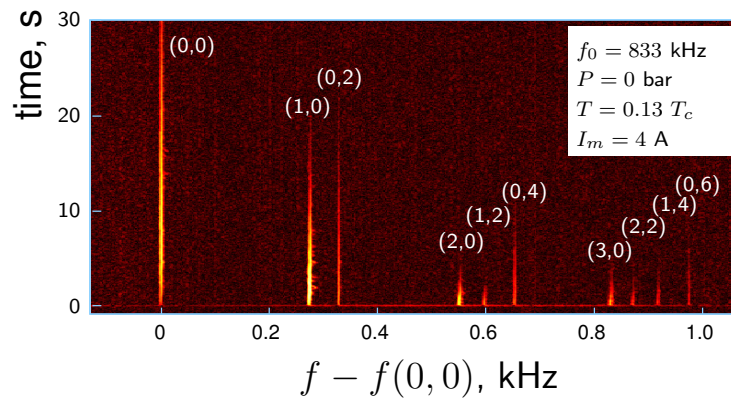
Trap for magnons

Experiments in Helsinki 2012-2015

continuous pumping



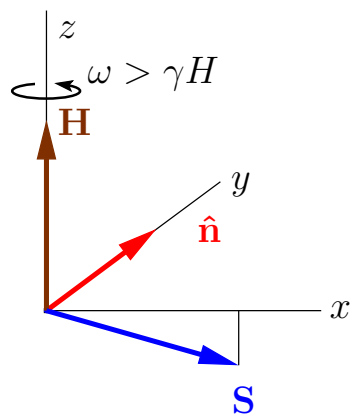
pulsed pumping



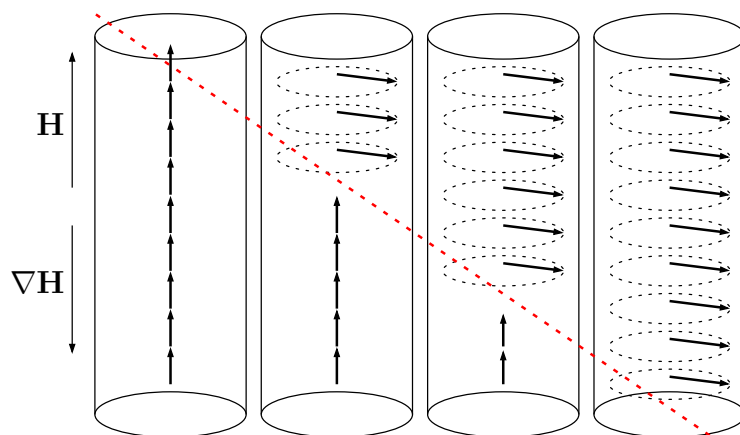
Thermometry, interaction with free surface, with vortices,
 Studying properties of ^3He .

Homogeneously precessing domain (HPD)

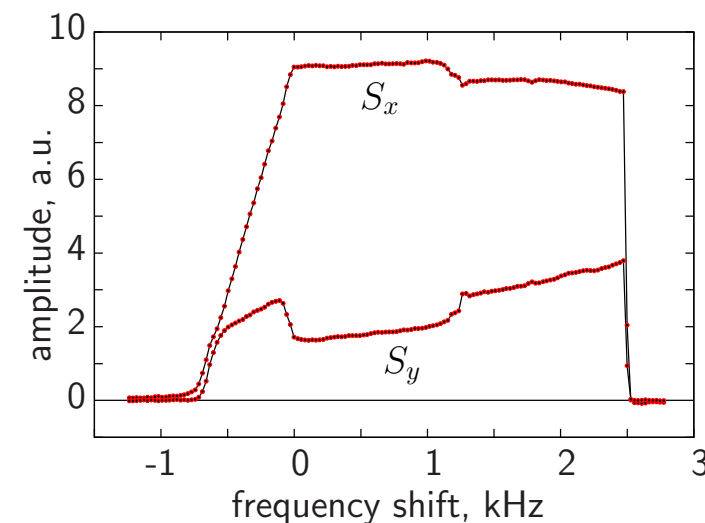
HPD



creation of HPD in CW NMR



$$\omega = \gamma H$$



Experiment with two HPDs (Moscow, 1987)

