Homework 3

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```
load('data/bankmarketing.Rdata')
```

1. Create a logistic regression model to predict the efficiency of the marketing campaign using y as dependent variable and no predictor, just the intercept.

```
mod_1 <- glm(y ~ 1, data = bankmarketing, family = binomial(link = 'logit'))
summary(mod_1)
```

```
Call:
glm(formula = y ~ 1, family = binomial(link = "logit"), data = bankmarketing)

Deviance Residuals:
    Min       1Q   Median       3Q      Max
-0.4949  -0.4949  -0.4949  -0.4949   2.0788

Coefficients:
            Estimate Std. Error z value Pr(>|z|)
(Intercept) -2.03830    0.04658  -43.76   <2e-16

(Dispersion parameter for binomial family taken to be 1)

    Null deviance: 3231  on 4520  degrees of freedom
Residual deviance: 3231  on 4520  degrees of freedom
AIC: 3233

Number of Fisher Scoring iterations: 4
```

(a) (half a point) How large is the AIC score for this model?

The AIC score is 3233.

```
AIC(mod_1)
```

```
[1] 3233
```

(b) (half a point) How large is the BIC score for this model?

The BIC score is 3239.417.

```
BIC(mod_1)
```

```
[1] 3239.417
```

(c) (1 point) Compute the log odds for the mean response and compare it to the coefficient estimate for the intercept in this model [hint: Be careful about the internal numeric coding of the variable y in the data set].

The log odds is computed as follows and the value is -2.0383, which is same as the intercept in the model.

```
log(sum(bankmarketing$y == 'yes') / sum(bankmarketing$y == 'no'))
```

```
[1] -2.0383
```

2. Add duration as a predictor to the model.

```
mod_2 <- glm(y ~ duration, data = bankmarketing, family = binomial(link = 'logit'))
summary(mod_2)
```

```
Call:
glm(formula = y ~ duration, family = binomial(link = "logit"),
    data = bankmarketing)

Deviance Residuals:
    Min       1Q   Median       3Q      Max
-3.8683  -0.4303  -0.3548  -0.3106   2.5264

Coefficients:
             Estimate Std. Error z value Pr(>|z|)
(Intercept) -3.2559346  0.0845767  -38.50   <2e-16
duration     0.0035496  0.0001714   20.71   <2e-16

(Dispersion parameter for binomial family taken to be 1)

    Null deviance: 3231.0  on 4520  degrees of freedom
Residual deviance: 2701.8  on 4519  degrees of freedom
AIC: 2705.8

Number of Fisher Scoring iterations: 5
```

(a) (half a point) In comparison to the naive model, by how much has the AIC changed?

The AIC score drops by 527.2476 after *duration* is added to the model.

```
AIC(mod_2) - AIC(mod_1)
```

```
[1] -527.2476
```

(b) (half a point) Is last contact duration (duration) a significant predictor in this model for modeling the probability of subscribing to a term deposit?

The small p-value implys that the variable *duration* is significant.

(c) (half a point) With growing duration of the last contact are clients less likely or are they more likely to subscribe to a term deposit?

The positive coefficient suggests that customer with larger duration will be more likely to have a term deposit.

(d) (half a point) According to the second model, what is the estimated probability of subscribing to a term deposit for a client who immediately terminated the last contact (i.e. duration equals 0)?

The probability is 0.037 if a client with 0 duration.

```
predict(mod_2, data.frame(duration = 0), type = 'response')
```

```
         1
0.03711422
```

3. Using the second model with duration as predictor,

(a) (1 point) compute the halfway point, i.e the last contact duration at which the estimated probability of subscribing to a term deposit equals 0.5.

```
d = unname(-mod_2$coefficients[1] / mod_2$coefficients[2])
d
```

```
[1] 917.2802
```

```
predict(mod_2, data.frame(duration = d), type = 'response')
```

```
  1
0.5
```

(b) (1 point) Compute the slope of the tangent to the regression curve at the halfway point.

```
unname(mod_2$coefficients[2]) * 0.5 * (1 - 0.5)
```

```
[1] 0.0008873882
```

4. Compute a logistic regression model for subscribing to a term deposit using age, marital and duration as predictors.

```
mod_3 <- glm(y ~ age + marital + duration, data = bankmarketing,
             family = binomial(link = 'logit'))
summary(mod_3)
```

```
Call:
glm(formula = y ~ age + marital + duration, family = binomial(link = "logit"),
    data = bankmarketing)

Deviance Residuals:
    Min       1Q   Median       3Q      Max
```

```
-3.9035  -0.4360  -0.3542  -0.2943   2.6029

Coefficients:
                 Estimate Std. Error z value   Pr(>|z|)
(Intercept)     -3.818323   0.211710 -18.036    < 2e-16
age              0.022285   0.005068   4.397 0.00001095
maritaldivorced -0.230097   0.177829  -1.294      0.196
maritalmarried  -0.568236   0.127543  -4.455 0.00000838
duration         0.003534   0.000172  20.548    < 2e-16

(Dispersion parameter for binomial family taken to be 1)

    Null deviance: 3231.0  on 4520  degrees of freedom
Residual deviance: 2670.8  on 4516  degrees of freedom
AIC: 2680.8

Number of Fisher Scoring iterations: 5
```

(a) (1 point) Calculate the predictive probability of subscribing to a term deposit for a married client at the mean values of the numeric predictors.

```
predict(mod_3, data.frame(age = mean(bankmarketing$age),
                          marital = 'married',
                          duration = mean(bankmarketing$duration)),
        type = 'response')
```

```
         1
0.07335345
```

(b) In the above model, calculate the effect on the probability of subscribing when keeping all other predictors constant and

i. (half a point) changing duration from the mean score to 300 seconds;

```
predict(mod_3, data.frame(age = mean(bankmarketing$age),
                          marital = 'married',
                          duration = 300),
        type = 'response')
```

```
         1
0.08249455
```

ii. (half a point) changing age from the mean score to one standard deviation above the mean score.

```
predict(mod_3, data.frame(age = mean(bankmarketing$age) +
                              sd(bankmarketing$age),
                          marital = 'married',
                          duration = mean(bankmarketing$duration)),
        type = 'response')
```

```
         1
0.09107385
```

5. (2 points) Compute a logistic regression model for the decision to subscribe to a term deposit using duration, campaign, and the interaction between the two. How do you interpret the regression coefficients? Are these interpretations meaningful? Give reasons for your answer!

```
mod_4 <- glm(y ~ duration + campaign + duration * campaign,
             data = bankmarketing,
             family = binomial(link = 'logit'))
summary(mod_4)
```

```
Call:
glm(formula = y ~ duration + campaign + duration * campaign,
    family = binomial(link = "logit"), data = bankmarketing)

Deviance Residuals:
    Min       1Q   Median       3Q      Max
-3.7907  -0.4464  -0.3683  -0.2786   2.7341

Coefficients:
                    Estimate  Std. Error z value    Pr(>|z|)
(Intercept)       -2.61917477  0.14105246 -18.569     < 2e-16
duration           0.00277087  0.00026692  10.381     < 2e-16
campaign          -0.27602084  0.05590684  -4.937 0.000000793
duration:campaign  0.00032618  0.00009033   3.611    0.000305

(Dispersion parameter for binomial family taken to be 1)

    Null deviance: 3231.0  on 4520  degrees of freedom
Residual deviance: 2664.1  on 4517  degrees of freedom
AIC: 2672.1

Number of Fisher Scoring iterations: 6
```

6. (2 points) Center the variables duration and campaign and re-build the model built in question 5. How do you interpret the regression coefficients? Are these interpretations meaningful? Give reasons for your answer! Draw the effects plot for this model and interpret!

```
c_duration <- scale(bankmarketing$duration, scale = FALSE)
c_campaign <- scale(bankmarketing$campaign, scale = FALSE)

mod_5 <- glm(bankmarketing$y ~ c_duration + c_campaign + c_duration * c_campaign,
             family = binomial(link = 'logit'))
summary(mod_5)
```

```
Call:
glm(formula = bankmarketing$y ~ c_duration + c_campaign + c_duration *
    c_campaign, family = binomial(link = "logit"))

Deviance Residuals:
```

```
    Min       1Q   Median       3Q      Max
-3.7907  -0.4464  -0.3683  -0.2786   2.7341

Coefficients:
                        Estimate  Std. Error z value    Pr(>|z|)
(Intercept)           -2.41834174  0.06419453 -37.672     < 2e-16
c_duration             0.00368210  0.00018080  20.366     < 2e-16
c_campaign            -0.18992117  0.03835754  -4.951 0.000000737
c_duration:c_campaign  0.00032618  0.00009033   3.611    0.000305

(Dispersion parameter for binomial family taken to be 1)

    Null deviance: 3231.0  on 4520  degrees of freedom
Residual deviance: 2664.1  on 4517  degrees of freedom
AIC: 2672.1

Number of Fisher Scoring iterations: 6
```

7. (2 points) Create a logistic regression model using y as dependent variable and all available predictors, except day and month. Which predictors are significant? Do the estimated coefficients make common sense to you?

```
mod_6 <- glm(y ~ . - day - month, data = bankmarketing,
             family = binomial(link = 'logit'))
summary(mod_6)
```

```
Call:
glm(formula = y ~ . - day - month, family = binomial(link = "logit"),
    data = bankmarketing)

Deviance Residuals:
    Min       1Q   Median       3Q      Max
-4.0779  -0.4067  -0.2785  -0.1705   3.0457

Coefficients:
                     Estimate  Std. Error z value       Pr(>|z|)
(Intercept)        -2.58481120  0.49297812  -5.243 0.000000157766
age                 0.00051541  0.00687649   0.075       0.940252
jobblue-collar     -0.45713374  0.23660305  -1.932       0.053351
jobentrepreneur    -0.46289495  0.37353608  -1.239       0.215262
jobhousemaid       -0.32234424  0.39517541  -0.816       0.414672
jobmanagement      -0.11365818  0.23572561  -0.482       0.629690
jobretired          0.58415128  0.29855600   1.957       0.050396
jobself-employed   -0.24456557  0.34018308  -0.719       0.472188
jobservices        -0.23224216  0.26610659  -0.873       0.382804
jobstudent          0.51174726  0.36139116   1.416       0.156761
jobtechnician      -0.26279749  0.22354603  -1.176       0.239761
jobunemployed      -0.69747294  0.41220863  -1.692       0.090639
jobunknown          0.46787823  0.54422750   0.860       0.389948
maritaldivorced     0.19815965  0.19766206   1.003       0.316094
maritalmarried     -0.23249919  0.14359896  -1.619       0.105429
```

```
educationsecondary  0.07954357  0.19583232   0.406       0.684609
educationtertiary   0.36044330  0.22596527   1.595       0.110684
educationunknown   -0.34886940  0.33917035  -1.029       0.303669
defaultyes          0.46738306  0.42123528   1.110       0.267192
balance             0.00000330  0.00001756   0.188       0.850967
housingyes         -0.47768772  0.12435061  -3.841       0.000122
loanyes            -0.76137581  0.19372118  -3.930 0.000084851850
contacttelephone   -0.03948148  0.22245467  -0.177       0.859131
contactunknown     -1.09294023  0.17621602  -6.202 0.000000000557
duration            0.00404244  0.00019582  20.644        < 2e-16
campaign           -0.07430257  0.02700051  -2.752       0.005925
pdays              -0.00014347  0.00093454  -0.154       0.877991
previous           -0.00473025  0.03806701  -0.124       0.901109
poutcomesuccess     2.40156770  0.26266846   9.143        < 2e-16
poutcomeother       0.46819766  0.25950506   1.804       0.071201
poutcomeunknown    -0.28114500  0.30276181  -0.929       0.353096

(Dispersion parameter for binomial family taken to be 1)

    Null deviance: 3231.0  on 4520  degrees of freedom
Residual deviance: 2277.2  on 4490  degrees of freedom
AIC: 2339.2

Number of Fisher Scoring iterations: 6
```

8. (2 points) Starting with the model of Question 7 use the automatic backward/forward selection method to derive a suitable model. Report the significant predictors and the AIC score of the resulting model.

```
mod_7 <- step(mod_6, direction = 'both')
```

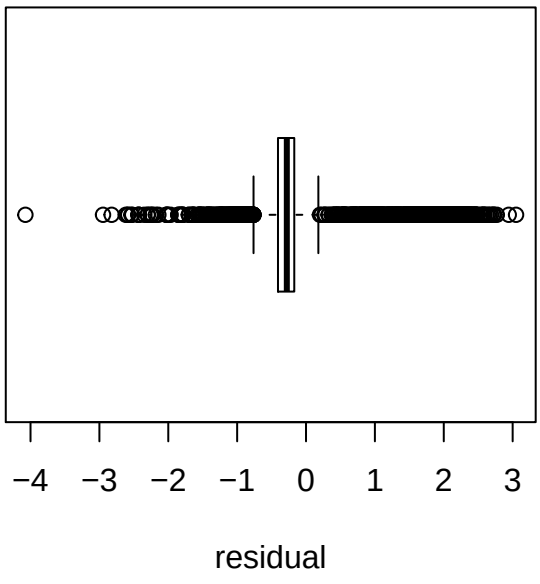
```
summary(mod_7)
```

```
Call:
glm(formula = y ~ job + marital + education + housing + loan +
    contact + duration + campaign + poutcome, family = binomial(link = "logit"),
    data = bankmarketing)

Deviance Residuals:
    Min       1Q   Median       3Q      Max
-4.0745  -0.4072  -0.2788  -0.1704   3.0496

Coefficients:
                   Estimate Std. Error z value Pr(>|z|)
(Intercept)       -2.6139848  0.3255815  -8.029 9.85e-16
jobblue-collar    -0.4494496  0.2363066  -1.902  0.05717
jobentrepreneur   -0.4300067  0.3706045  -1.160  0.24593
jobhousemaid      -0.3032897  0.3920882  -0.774  0.43921
jobmanagement     -0.1039637  0.2354379  -0.442  0.65880
jobretired         0.6054857  0.2691559   2.250  0.02448
jobself-employed  -0.2343157  0.3400443  -0.689  0.49078
jobservices       -0.2283782  0.2659796  -0.859  0.39054
```

```
jobstudent          0.5139966  0.3550967   1.447  0.14776
jobtechnician      -0.2509853  0.2231478  -1.125  0.26070
jobunemployed      -0.6949974  0.4126048  -1.684  0.09210
jobunknown          0.4676864  0.5433718   0.861  0.38940
maritaldivorced     0.2094087  0.1861087   1.125  0.26051
maritalmarried     -0.2297354  0.1329582  -1.728  0.08401
educationsecondary  0.0846584  0.1943381   0.436  0.66311
educationtertiary   0.3604550  0.2236398   1.612  0.10701
educationunknown   -0.3377539  0.3378899  -1.000  0.31751
housingyes         -0.4801920  0.1217720  -3.943 8.03e-05
loanyes            -0.7579438  0.1933619  -3.920 8.86e-05
contacttelephone   -0.0411671  0.2194125  -0.188  0.85117
contactunknown     -1.0960326  0.1761025  -6.224 4.85e-10
duration            0.0040386  0.0001956  20.650  < 2e-16
campaign           -0.0749988  0.0269881  -2.779  0.00545
poutcomesuccess     2.4100962  0.2559920   9.415  < 2e-16
poutcomeother       0.4772387  0.2573995   1.854  0.06373
poutcomeunknown    -0.2275282  0.1732319  -1.313  0.18904

(Dispersion parameter for binomial family taken to be 1)

    Null deviance: 3231.0  on 4520  degrees of freedom
Residual deviance: 2278.4  on 4495  degrees of freedom
AIC: 2330.4

Number of Fisher Scoring iterations: 6
```

9. (2 points) Draw a box plot of the residuals and look for extreme outliers. Remove the outlier and re-run the model you have obtained in Question 8. Which changes in the model are to be noted?

```r
res <- residuals(mod_7)
boxplot(res, horizontal = TRUE,
        main = 'Boxplot of Residuals',
        xlab = 'residual')
```


Boxplot of Residuals
−4
−3
−2
−1
0
1
2
3
residual