Homework 4

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```
library(foreign)
hsb <- read.dta('data/hsbdemo.dta')
```

1. Cross-tabulate the variables ses and prog.

(a) (half a point) Which program was chosen by the largest fraction of students with high socio-economic status?

(b) (half a point) How many percent of students with low socio-economic status selected the general program?

(c) (half a point) In the academic program are there more students with middle socioeconomic status than students with high socio-economic status?

(d) (half a point) What is the least-frequent combination of the two variables?

2. You continue with your analysis of the relationship between ses and prog.

(a) (half a point) Draw a mosaicplot visualising the contingency table of program choice and socio-economic status.

(b) (1.5 points) Are students with low ses less likely (as measured in odds) to choose the academic program than students with higher socio-economic status? Calculate the odds ratios for choosing the academic program comparing students with low ses to students with middle ses and to students with high ses. [hint: use the command loddsratio in the package vcd. First, aggregate the variable prog into a binary variable indicating whether the student has chosen an academic program yes or no.]

3. Now, you assess the relationship between prog and ses using the χ^2-statistic.

(a) (1 point) Calculate the χ^2-test to assess the relationship between ses and prog. Is the relationship statistically significant?

(b) (1 point) Calculate the expected frequencies under the assumption that socio-economic status has no effect on program choice. For which cells are expected frequencies higher than the observed ones?

4. In the following, perform the last analysis separately for female and male students.

(a) (half a point) Calculate the χ^2-test to assess the relationship between ses and prog.

(b) (half a point) Calculate the expected frequencies under the assumption that socio-econmic status has no effect on program choice. For which cells are expected frequencies higher than the observed ones?

(c) (half a point) Do the results differ for the two sexes?

(d) (half a point) Visualise the relationships using mosaicplots. Get any differences between females and males in relation to socio-economic status and program choice visible in the plots?

5. Create a multinomial logistic regression model using prog as dependent variable and the following predictors: female, ses, schtype, read, write, math, science, honors, awards. [hint: use the function multinom in hte package nnet.]

```r
library(nnet)
mod_1 <- multinom(prog ~ . - id - socst - cid,
                  data = hsb)
```

```
# weights:  36 (22 variable)
initial  value 219.722458
iter  10 value 184.828226
iter  20 value 157.499339
final  value 157.443537
converged
```

```r
summary(mod_1)
```

```
Call:
multinom(formula = prog ~ . - id - socst - cid, data = hsb)

Coefficients:
          (Intercept) femalefemale sesmiddle   seshigh schtypprivate
academic   -5.498379   -0.1522507 0.3751645 1.0802062     0.5372474
vocation    4.163618    0.2804404 1.3235922 0.7959664    -1.2758046
                 read       write        math     science honorsenrolled
academic  0.052955458  0.06429399  0.10006643 -0.10199019      0.5743597
vocation -0.009609139 -0.02166161 -0.02995725 -0.03800852      1.9991952
             awards
academic -0.2703505
vocation -0.3525181

Std. Errors:
          (Intercept) femalefemale sesmiddle   seshigh schtypprivate
academic    2.347466    0.4533749 0.5061641 0.5772063     0.5550686
vocation    2.455693    0.5179411 0.5388177 0.7084789     0.8817585
               read      write       math    science honorsenrolled
academic 0.02919946 0.04927104 0.03480492 0.03090596      0.8644712
vocation 0.03322165 0.05054331 0.03756352 0.03224286      1.1038196
            awards
academic 0.2936491
vocation 0.3856715

Residual Deviance: 314.8871
AIC: 358.8871
```

(a) (half a point) How large is the AIC score for this model?

The AIC score is 358.8871.

```r
AIC(mod_1)
```

```
[1] 358.8871
```

(b) (1.5 points) The default output does not include p-values. Compute p-values based on the Wald-test statistics and determine the coefficients that are statistically significantly different from zero!

As the table below shows, the variables *academic:(Intercept)*, *academic:math*, *academic:science* and *vocation:sesmiddle* are significant under significance level of 5%.

```
# compute p-values by definition
# z <- summary(mod_1, Wald.ratios = TRUE)$Wald.ratios
# p <- (1 - pnorm(abs(z))) * 2
# p

# compute p-values by package function
library(AER)
coeftest(mod_1)[, 4] < 0.05
```

```
   academic:(Intercept)   academic:femalefemale      academic:sesmiddle
                   TRUE                   FALSE                   FALSE
       academic:seshigh  academic:schtypprivate           academic:read
                  FALSE                   FALSE                   FALSE
         academic:write           academic:math        academic:science
                  FALSE                    TRUE                    TRUE
academic:honorsenrolled         academic:awards    vocation:(Intercept)
                  FALSE                   FALSE                   FALSE
  vocation:femalefemale      vocation:sesmiddle        vocation:seshigh
                  FALSE                    TRUE                   FALSE
 vocation:schtypprivate           vocation:read          vocation:write
                  FALSE                   FALSE                   FALSE
          vocation:math        vocation:science vocation:honorsenrolled
                  FALSE                   FALSE                   FALSE
        vocation:awards
                  FALSE
```

6. Using the model from the previous question and the backward strategy with criterion AIC for variable selection, determine the significant coefficients in the resulting model.

```
mod_2 <- step(mod_1, direction = 'backward')
```

```
summary(mod_2)
```

```
Call:
multinom(formula = prog ~ ses + schtyp + read + math + science,
    data = hsb)

Coefficients:
         (Intercept) sesmiddle   seshigh schtypprivate        read
academic   -3.745688  0.323115 1.0358034      0.608257  0.05912408
vocation    3.907946  1.183126 0.7014962     -1.408038 -0.01218565
                math     science
academic  0.10745053 -0.09076914
vocation -0.03078266 -0.04770648

Std. Errors:
```

```
          (Intercept) sesmiddle   seshigh schtypprivate       read
academic    1.401302 0.4876102 0.5648570     0.5484718 0.02807034
vocation    1.564001 0.5201513 0.6880376     0.8662165 0.03127526
               math    science
academic 0.03270710 0.02859475
vocation 0.03535214 0.03005018

Residual Deviance: 322.9919
AIC: 350.9919
```

(a) (1 point) Which predictors are included in the resulting model?

According to the table above, the variables *ses*, *schtyp*, *read*, *math* and *science* are included in the final model.

(b) (half a point) What is the BIC score of the resulting model?

The BIC score is 397.1684.

```
BIC(mod_2)
```

```
[1] 397.1684
```

(c) (half a point) What is the log-likelihood score of this model?

The log-likelihood score is -161.496.

```
logLik(mod_2)
```

```
'log Lik.' -161.496 (df=14)
```

7. (2 points) Using the final model that resulted in Question 6 predict the probabilities for the three program types for the combination of all factor levels and the average score of numeric predictors in the model.

The table below indicates academic is the most possible outcome in all combinations. But the students in the private school are more likely to take an academic program than that in the public school.

```
d <- expand.grid(ses = c('low', 'middle', 'high'),
                 schtyp = c('private', 'public'),
                 read = mean(hsb$read),
                 math = mean(hsb$math),
                 science = mean(hsb$science))
```

```
pred <- predict(mod_2, newdata = d, type = 'probs', se = TRUE)
cbind(d, pred)
```

```
     ses  schtyp  read   math science   general  academic   vocation
1    low private 52.23 52.645   51.85 0.2801551 0.6897408 0.03010409
2 middle private 52.23 52.645   51.85 0.2104441 0.7157334 0.07382253
3   high private 52.23 52.645   51.85 0.1226533 0.8507663 0.02658044
4    low  public 52.23 52.645   51.85 0.3597989 0.4821528 0.15804831
5 middle  public 52.23 52.645   51.85 0.2333606 0.4319957 0.33464371
6   high  public 52.23 52.645   51.85 0.1766363 0.6668812 0.15648254
```

8. (2 points) Again using the final model that resulted in Question 6, we now want to investigate the specific dependency on the math score. Generate new data such that you have for each combination of factor levels a total of 51 math scores running from 30 to 80 in increments of one. The other numeric predictors enter again with their mean score into the prediction. Compute the predictions and average them for each level of socio-economic status.

The table below shows the average probabilities of taking different programs with respect to different socio-economic levels. As can be seen from the table, taking an academic program is the most probable in three socio-economic levels. Furthermore, people with a high socio-economic status is more likely to have an academic program.

```
d <- expand.grid(ses = c('low', 'middle', 'high'),
                 schtyp = c('private', 'public'),
                 read = mean(hsb$read),
                 math = 30:80,
                 science = mean(hsb$science))

pred <-  predict(mod_2, newdata = d, type = 'probs', se = TRUE)

bind_d <- cbind(d, pred)
aggregate(bind_d[, 6:8], by = list(bind_d$ses), FUN = 'mean')
```

```
  Group.1   general  academic  vocation
1     low 0.2978392 0.5956783 0.1064825
2  middle 0.1993257 0.5887810 0.2118934
3    high 0.1629723 0.7138563 0.1231714
```