Homework 1

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```r
load("data/bank.Rdata") # read the data into R
```

1. To start with, you compute the naive model for current salary (SALNOW) as the dependent variable.

(a) Calculate the model and specify the model equation.

```r
mod <- lm(SALNOW ~ 1, data = bank)
summary(mod)
```

```
Call:
lm(formula = SALNOW ~ 1, data = bank)

Residuals:
   Min     1Q Median     3Q    Max 
 -7468  -4168  -2218   1007  40232 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)  13767.8      313.7   43.88   <2e-16

Residual standard error: 6830 on 473 degrees of freedom
```

As the summary shows, the model equation is $\widehat{Salary}_i = 13767.8$.

(b) Compute the residual sum of squares for this model, i.e. compute the sum of the squared residuals.

```r
deviance(mod)
```

```
[1] 22066639270
```

The sum of the squared residuals is 22066639270.

(c) Compute the residual standard error for this model, i.e. compute the square root of the residual sum of squares divided by n - 1, where n is the sample size.

```r
sqrt(deviance(mod)/(nrow(bank) - 1))
```

```
[1] 6830.265
```

The residual standard error is 6830.26.

2. As second step, you compute a simple linear regression model for current salary (SALNOW) as the dependent variable using education level (EDLEVEL) as a predictor.

(a) Calculate the model and specify the model equation.

```
mod_2 <- lm(SALNOW ~ EDLEVEL, data = bank)
summary(mod_2)
```

```
Call:
lm(formula = SALNOW ~ EDLEVEL, data = bank)

Residuals:
   Min     1Q Median     3Q    Max 
 -8627  -3284  -1001   2351  31617 

Coefficients:
            Estimate Std. Error t value      Pr(>|t|)
(Intercept) -7332.47    1128.76  -6.496 0.00000000021
EDLEVEL      1563.96      81.82  19.115       < 2e-16

Residual standard error: 5133 on 472 degrees of freedom
Multiple R-squared:  0.4363,    Adjusted R-squared:  0.4351 
F-statistic: 365.4 on 1 and 472 DF,  p-value: < 2.2e-16
```

The model equation is $\widehat{Salary_i} = -7332.47 + 1563.96 * EDLEVEL_i$.

(b) Compute the residual sum of squares for this model, i.e. compute the sum of the squared residuals.

```
deviance(mod_2)
```

```
[1] 12438124428
```

The residual sum of squares is 12438124428.

(c) Compute the residual standard error for this model, i.e. compute the square root of the residual sum of squares divided by n - 2, where n is the sample size.

```
sqrt(deviance(mod_2)/(nrow(bank) - 2))
```

```
[1] 5133.416
```

The residual standard error is 5133.42.

3. In a third model, you add gender (SEX) as an additional predictor to education level.

(a) Calculate the model and specify the model equation.

```r
mod_3 <- lm(SALNOW ~ EDLEVEL + SEX, data = bank)
summary(mod_3)
```

```
Call:
lm(formula = SALNOW ~ EDLEVEL + SEX, data = bank)

Residuals:
    Min      1Q  Median      3Q     Max
-9263.0 -3077.3  -783.3  2054.7 31223.6

Coefficients:
            Estimate Std. Error t value        Pr(>|t|)
(Intercept) -6369.78    1084.52  -5.873 0.0000000080779
EDLEVEL      1356.67      83.44  16.259          < 2e-16
SEXMale      3369.38     482.81   6.979 0.0000000000102

Residual standard error: 4892 on 471 degrees of freedom
Multiple R-squared:  0.4892,    Adjusted R-squared:  0.487
F-statistic: 225.5 on 2 and 471 DF,  p-value: < 2.2e-16
```

The model equation is $\widehat{Salary}_i = -6369.78 + 1356.67 * EDLEVEL_i + 3369.38 * SEX(ifMale)_i$.

(b) Compute the residual sum of squares for this model, i.e. compute the sum of the squared residuals.

```r
deviance(mod_3)
```

```
[1] 11272531174
```

The residual sum of squares is 11272531174.

(c) Compute the residual standard error for this model, i.e. compute the square root of the residual sum of squares divided by n - 3, where n is the sample size.

```r
sqrt(deviance(mod_3)/(nrow(bank) - 3))
```

```
[1] 4892.156
```

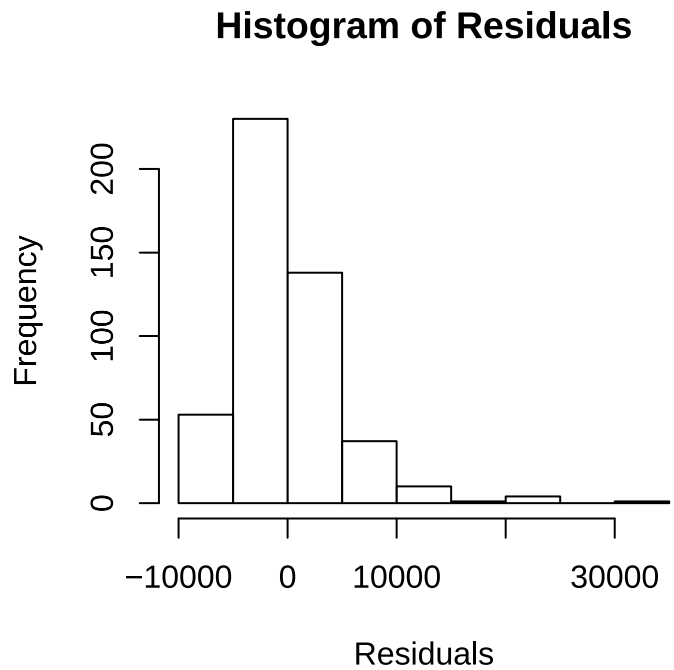
The residual standard error is 4892.16.

4. You continue with the last model using education level and gender as predictors and investigate the residuals in more detail.

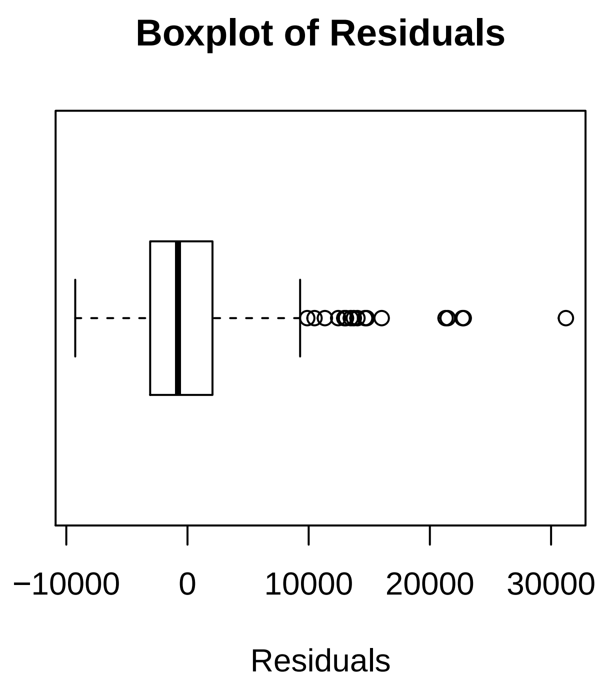
(a) Draw a histogram, a boxplot, a density plot, and a Q-Q-plot to assess normality of the residuals. Give a brief summary report on these plots!

```r
res <- resid(mod_3)
```

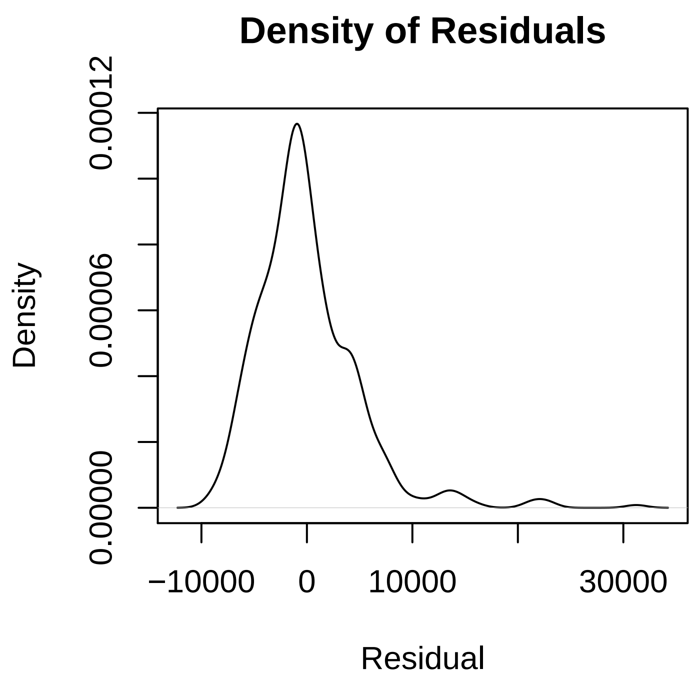
```r
hist(res, main = 'Histogram of Residuals', xlab = 'Residuals')
```



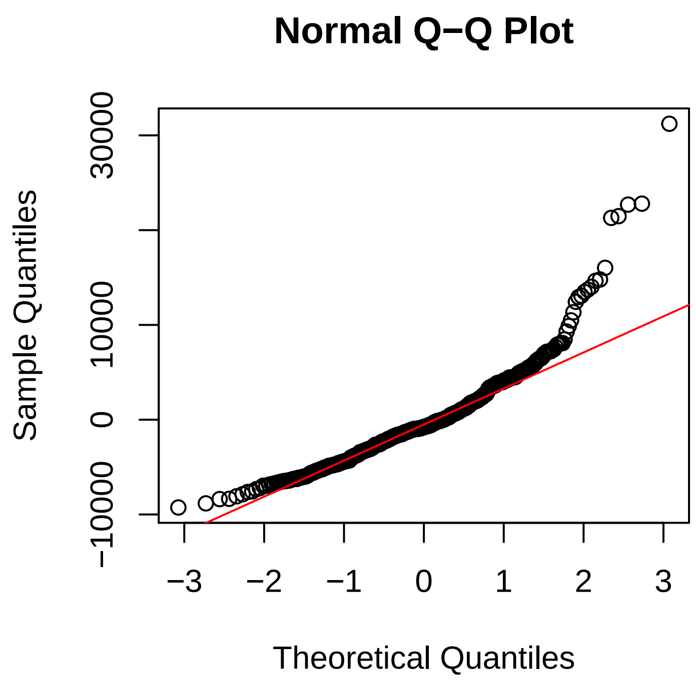

```r
boxplot(res, main = 'Boxplot of Residuals', xlab = 'Residuals', horizontal = TRUE)
```




```r
plot(density(res), main = "Density of Residuals", xlab = "Residual")
```




```r
qqnorm(res)
qqline(res, col = "red")
```



The distribution is right-skewed and the points in the Q-Q plot do not all lie on the theoretical line, and hence the residuals do not follow a normally distribution.

(b) Use the Kolmogorov-Smirnov-Test to check whether the residuals follow a normal distribution.

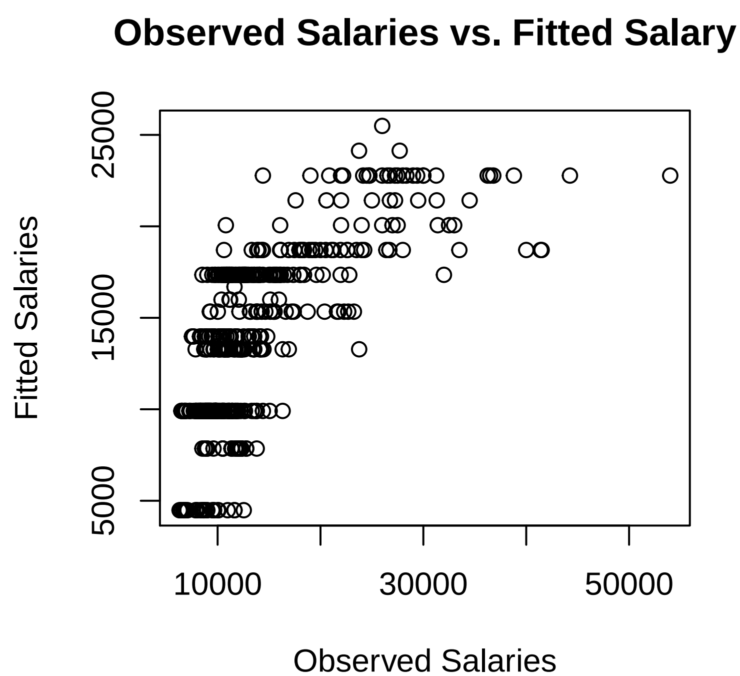
```r
ks.test(res, "pnorm")
```

```
Warning in ks.test(res, "pnorm"): ties should not be present for the
Kolmogorov-Smirnov test


    One-sample Kolmogorov-Smirnov test

data:  res
D = 0.59705, p-value < 2.2e-16
alternative hypothesis: two-sided
```

The p-value is small enough to reject the null hypothesis in Kolmogorov-Smirnov test, thus the residuals are not normally distributed.

5. Plot observed salaries against the ones predicted by the above model (use either the command *fitted* or the stored scores in *modelname$fitted.values* to obtain the fitted scores). Compute the Pearson correlation coefficient between observed and fitted salaries. How can you check your result using results from the regression table?

```r
plot(bank$SALNOW, mod_3$fitted.values,
     main = "Observed Salaries vs. Fitted Salary",
     xlab = "Observed Salaries",
     ylab = "Fitted Salaries")
```



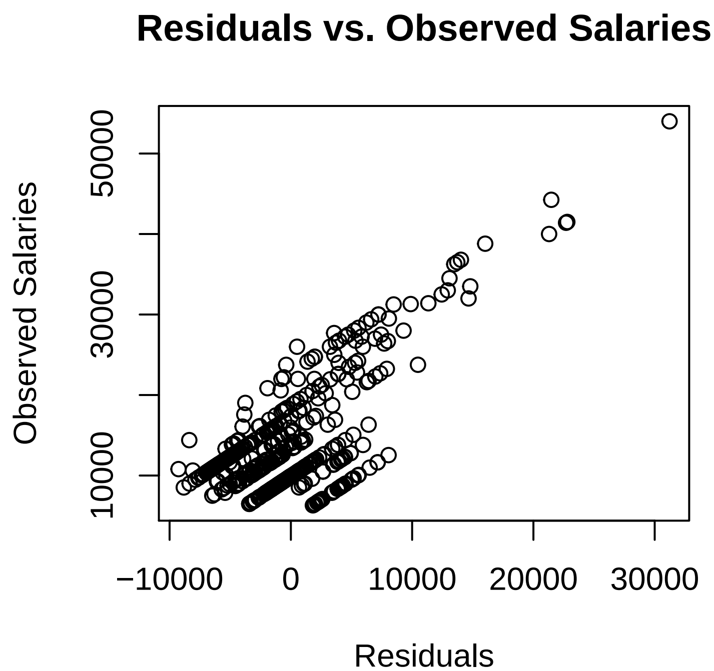

```r
cor(bank$SALNOW, mod_3$fitted.values)
```

```
[1] 0.6993994
```

The Pearson correlation coefficient is around 0.699. Besides, the square of the correlation coefficient (0.489) is same as the R-squared reported in Task 3(a).

6. Plot the residuals against the observed salaries. Does the plot look similar to what you had expected? Compute the Pearson correlation coefficient and comment on it!

```
plot(res, bank$SALNOW,
     main = "Residuals vs. Observed Salaries",
     xlab = "Residuals",
     ylab = "Observed Salaries")
```



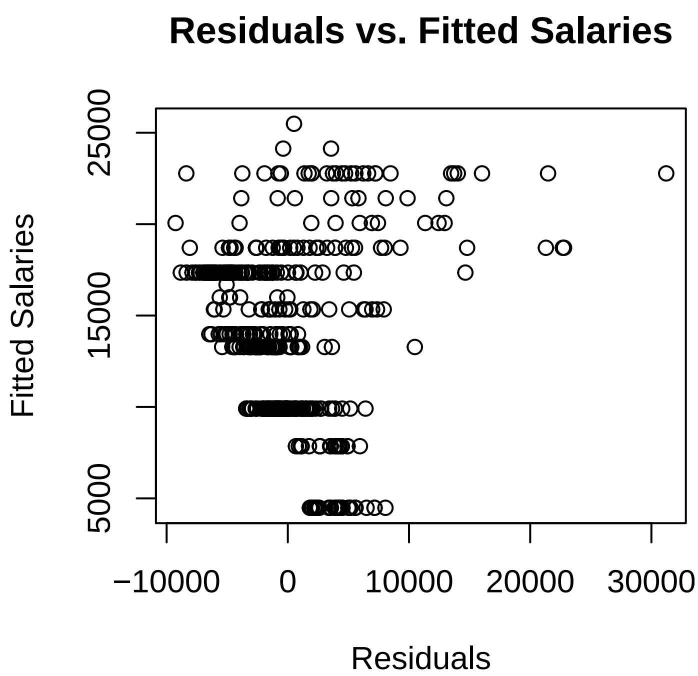

```
cor(res, bank$SALNOW)
```

```
[1] 0.714731
```

The Pearson correlation coefficient is 0.71, indicating observed salaries are positively correlated with residuals. The reason lies in the fact that regression models with few predictors tend to predict average values of the response variable. Therefore, the actual salaries deviated greatly from the mean led to larger absolute residuals.

7. Plot the residuals against the fitted salaries. Does the plot look similar to what you had expected? Compute the Pearson correlation coefficient and comment on it!

```
plot(res, mod_3$fitted.values,
     main = "Residuals vs. Fitted Salaries",
     xlab = "Residuals",
     ylab = "Fitted Salaries")
```




```
cor(res, mod_3$fitted.values)
```

```
[1] 1.491875e-17
```

The infinitesimal coefficient indicates residuals have no linear relationship with fitted values. Thus we can say how fitted values vary does not affect the residuals linearly.

8. In the next analysis step, you want to look at the relationship between the current salary (SALNOW) and all available predictors except ID.

```
mod_4 <- lm(SALNOW ~ . - ID, data = bank)
summary(mod_4)
```

```
Call:
lm(formula = SALNOW ~ . - ID, data = bank)

Residuals:
     Min       1Q   Median       3Q      Max
-10080.8  -1222.3   -250.1    986.8  18680.1

Coefficients:
                                Estimate  Std. Error t value Pr(>|t|)
(Intercept)                  -2377.95062  1475.56464  -1.612 0.107745
SALBEG                           1.41350     0.09043  15.630  < 2e-16
SEXMale                        573.73592   326.74078   1.756 0.079765
TIME                            58.59362    12.80883   4.574 6.15e-06
AGE                            -31.52532    20.29194  -1.554 0.120970
EDLEVEL                        181.58295    64.34962   2.822 0.004982
WORK                           -66.77509    27.82811  -2.400 0.016812
JOBCATCollegeTrainee          4967.29912   593.55330   8.369 7.05e-16
JOBCATExempt                  2792.67798   788.65372   3.541 0.000439
```

```
JOBCATMBATrainee                   4022.51936  1366.88277   2.943 0.003417
JOBCATOffice                       -190.35313   338.43790  -0.562 0.574086
JOBCATSecurity                     2517.02042   651.88793   3.861 0.000129
JOBCATTechnical                    4142.34777  1615.99866   2.563 0.010684
MINORITYMinority                   -391.18092   315.47220  -1.240 0.215613

Residual standard error: 2701 on 460 degrees of freedom
Multiple R-squared:  0.8479,    Adjusted R-squared:  0.8436
F-statistic: 197.3 on 13 and 460 DF,  p-value: < 2.2e-16
```

(a) Which variables are significant at the 5% level?

As the table above shows, significant variables are `SALBEG`, `TIME`, `EDLEVEL`, `WORK`, `JOBCAT (except for Office)`.

(b) How much variability in salaries is explained by this model?

The R-squared shows that 84.79 % of the variability is explained.

(c) Is there evidence for discrimination?

If we consider discrimination with respect to gender, age and ethnic group size, those variables are not statistically significant to conclude that discrimination exists.

9. Remove AGE from the previous model.

```r
mod_5 <- lm(SALNOW ~ . - ID - AGE, data = bank)
summary(mod_5)
```

```
Call:
lm(formula = SALNOW ~ . - ID - AGE, data = bank)

Residuals:
    Min      1Q  Median      3Q     Max 
-9988.2 -1274.7  -274.2  1002.5 18545.5 

Coefficients:
                                   Estimate Std. Error t value Pr(>|t|)
(Intercept)                      -3402.0567  1322.1600  -2.573  0.01039
SALBEG                               1.4078     0.0905  15.556  < 2e-16
SEXMale                            733.4405   310.6239   2.361  0.01863
TIME                                57.1926    12.7966   4.469 9.89e-06
EDLEVEL                            190.2858    64.2035   2.964  0.00320
WORK                               -99.8780    17.9271  -5.571 4.30e-08
JOBCATCollegeTrainee              4996.0276   594.1741   8.408 5.23e-16
JOBCATExempt                      2816.4936   789.7128   3.566  0.00040
JOBCATMBATrainee                  4040.8572  1368.9259   2.952  0.00332
JOBCATOffice                        -7.7030   317.8477  -0.024  0.98068
JOBCATSecurity                    2616.9634   649.6998   4.028 6.58e-05
JOBCATTechnical                   4148.0839  1618.4702   2.563  0.01069
```

```
MINORITYMinority                    -383.6564   315.9183  -1.214  0.22521

Residual standard error: 2705 on 461 degrees of freedom
Multiple R-squared:  0.8471,    Adjusted R-squared:  0.8431
F-statistic: 212.9 on 12 and 461 DF,  p-value: < 2.2e-16
```

(a) Which variables are now significant at the 5% level?

The significant variables are `SALBEG`, `SEX (if male)`, `TIME`, `WORK`, `JOBCAT (except for Office)`.

(b) How much variability in salaries is explained by this model?

The R-squared suggests that the model explains 84.71% of the variability.

(c) Is there evidence for discrimination?

In this model, we can see that SEX (if male) became a significant variable with a positive coefficient. Therefore, without considering age, women may suffer from discrimination.

10. Compare all models that you have built in this home work assignment using the anova function. Briefly summarize your findings.

From the anova tables below, we can see that the sum of the squared residuals decreases as the number of predictors increases, since the sum of the squared residuals is the variance of the response variable that cannot be explained by the predictors. And more predictors allow the model to explain more variance of the response variable. Also, the larger the variance a predictor explains, the more likely the predictor would be statistically significant.

```r
anova(mod)
```

```
Analysis of Variance Table

Response: SALNOW
           Df      Sum Sq  Mean Sq F value Pr(>F)
Residuals 473 22066639270 46652514
```

```r
anova(mod_2)
```

```
Analysis of Variance Table

Response: SALNOW
           Df      Sum Sq    Mean Sq F value    Pr(>F)
EDLEVEL     1  9628514842 9628514842  365.38 < 2.2e-16
Residuals 472 12438124428   26351959
```

```r
anova(mod_3)
```

```
Analysis of Variance Table

Response: SALNOW
           Df      Sum Sq    Mean Sq F value           Pr(>F)
EDLEVEL     1  9628514842 9628514842 402.308        < 2.2e-16
SEX         1  1165593254 1165593254  48.702 0.00000000001016
```

```
Residuals 471 11272531174   23933187
```

```r
anova(mod_4)
```

```
Analysis of Variance Table

Response: SALNOW
           Df      Sum Sq     Mean Sq   F value             Pr(>F)
SALBEG      1 17092967800 17092967800 2342.7693          < 2.2e-16
SEX         1    64224764    64224764    8.8027           0.003165
TIME        1   208781551   208781551   28.6157 0.0000001394186974
AGE         1   427757745   427757745   58.6287 0.0000000000001132
EDLEVEL     1   133653116   133653116   18.3186 0.0000227529764503
WORK        1    42296045    42296045    5.7971           0.016445
JOBCAT      6   729555926   121592654   16.6655          < 2.2e-16
MINORITY    1    11218146    11218146    1.5376           0.215613
Residuals 460  3356184177     7296053
```

```r
anova(mod_5)
```

```
Analysis of Variance Table

Response: SALNOW
           Df      Sum Sq     Mean Sq   F value           Pr(>F)
SALBEG      1 17092967800 17092967800 2335.6072        < 2.2e-16
SEX         1    64224764    64224764    8.7758          0.00321
TIME        1   208781551   208781551   28.5282 0.00000014533853
EDLEVEL     1   330141836   330141836   45.1110 0.00000000005497
WORK        1   258483194   258483194   35.3195 0.00000000552480
JOBCAT      6   727452651   121242108   16.5667        < 2.2e-16
MINORITY    1    10793270    10793270    1.4748          0.22521
Residuals 461  3373794205     7318426
```