Homework 5

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```
# import packages
library(vcd)
library(RcmdrMisc) # stepwise()
library(tidyr) # spread()
```

1. (2 points) Construct the two-way table for gender and whether admitted. Find the odds ratio for admission of males vs. females and interpret. For which gender is the probability of admission higher?

The table below shows the numbers of admitted students with respect to genders, and the odds ratio for admission of males vs. females is 1.84108. Given that this ratio is greater than 1, male students are more likely to get admission than the females ones.

```
UCBAdmissions_m <- margin.table(UCBAdmissions, c(2, 1))
UCBAdmissions_m

Admit
Gender Admitted Rejected
Male 1198 1493
Female 557 1278

oddsratio(UCBAdmissions_m, log = FALSE)

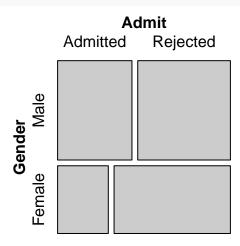
odds ratios for Gender and Admit

[1] 1.84108</pre>
```

2. Draw a mosaic plot for the data aggregated over departments.

The mosaic plot for the aggregated data is shown below.

mosaic(UCBAdmissions_m)



3. (3 points) Fit a saturated log-linear model to the frequencies using A and G as predictors. Calculate the odds ratio for admission of males vs. females. Display the predicted frequencies in a table and compare them to the observed frequencies displayed in Question 1.

The log-linear model with frequency as the response and A and G as predictors is shown below. The odds ratio for admission of males vs. females can be obtained by taking exponential of the coefficient of *AdmitRejected:GenderFemale*, and the odd ratio is 1.84108, which is same as the ratio in task 1.

```
berkeley <- data.frame(UCBAdmissions)</pre>
mod_1 <- glm(Freq ~ Admit * Gender,</pre>
             family = 'poisson', data = berkeley)
summary(mod_1)
Call:
glm(formula = Freq ~ Admit * Gender, family = "poisson", data = berkeley)
Deviance Residuals:
                1Q
                      Median
    Min
                                     3Q
                                              Max
-18.9074
                                          18.4285
          -7.8908
                     -0.1399
                                 5.6871
Coefficients:
                            Estimate Std. Error z value
                                                             Pr(>|z|)
(Intercept)
                             5.29665
                                        0.02889 183.329
                                                              < 2e-16
AdmitRejected
                                                  5.675 0.0000000138
                             0.22013
                                        0.03879
GenderFemale
                            -0.76584
                                        0.05128 -14.933
                                                              < 2e-16
AdmitRejected:GenderFemale 0.61035
                                        0.06389
                                                  9.553
                                                              < 2e-16
(Dispersion parameter for poisson family taken to be 1)
    Null deviance: 2650.1 on 23 degrees of freedom
Residual deviance: 2163.7 on 20 degrees of freedom
AIC: 2330.8
Number of Fisher Scoring iterations: 5
exp(unname(mod_1$coefficients[4]))
[1] 1.84108
The predicted frequency table is same as the observed frequency table in task 1.
mod_table <- xtabs(mod_1$fitted.values ~ berkeley$Admit + berkeley$Gender)</pre>
mod table
              berkeley$Gender
berkeley$Admit Male Female
      Admitted 1198
                       557
      Rejected 1493
                      1278
```

4. (2 points) Fit an independence log-linear model to the frequencies using A, D and G as predictors. Display the predicted frequencies in a table and compare them to the observed frequencies. Calculate the estimated odds ratios for admission of males vs. females for each department.

The requested model is shown as follows. In the predicted frequency table, the ratio of the accepted to the rejected, regardless of gender, is always the same (0.6333454) in each department. The reason is that this ratio can be simplied to total number of accepted students dividing by the total number of rejected students. Given the identical ratios, the estimated odds ratios are all equal to 1.

```
mod_2 <- glm(Freq ~ Admit + Gender + Dept,</pre>
            family = 'poisson',
            data = berkeley)
summary(mod_2)
Call:
glm(formula = Freq ~ Admit + Gender + Dept, family = "poisson",
   data = berkeley)
Deviance Residuals:
   Min
         10 Median
                              30
                                       Max
                 -1.008
                            4.734
-18.170 -7.719
                                    17.153
Coefficients:
             Estimate Std. Error z value
                                             Pr(>|z|)
              5.37111 0.03964 135.498
                                              < 2e-16
(Intercept)
AdmitRejected 0.45674
                         0.03051 14.972
                                              < 2e-16
GenderFemale -0.38287
                         0.03027 -12.647
                                              < 2e-16
DeptB
             -0.46679
                         0.05274 -8.852
                                              < 2e-16
DeptC
                         0.04649 -0.349
             -0.01621
                                             0.727355
DeptD
             -0.16384 0.04832 -3.391
                                             0.000696
             -0.46850
                         0.05276 -8.879
DeptE
                                              < 2e-16
             -0.26752
                         0.04972 -5.380 0.0000000744
DeptF
(Dispersion parameter for poisson family taken to be 1)
   Null deviance: 2650.1 on 23 degrees of freedom
Residual deviance: 2097.7 on 16 degrees of freedom
AIC: 2272.7
Number of Fisher Scoring iterations: 5
# predicted values (mod 2)
mod_2_table <- xtabs(mod_2$fitted.values ~ berkeley$Gender +</pre>
                        berkeley$Admit + berkeley$Dept)
# observed values
obs_table <- xtabs(berkeley$Freq ~ berkeley$Gender +</pre>
                        berkeley$Admit + berkeley$Dept)
structable(obs_table)
```

berkeley\$Admit Admitted Rejected

berkeley\$Gender berkeley\$Dept

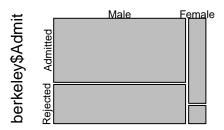
```
Male
                Α
                                                   512
                                                            313
                В
                                                   353
                                                            207
                С
                                                   120
                                                            205
                D
                                                   138
                                                            279
                Ε
                                                    53
                                                            138
                F
                                                    22
                                                            351
Female
                                                    89
                                                             19
                Α
                В
                                                    17
                                                              8
                C
                                                   202
                                                            391
                D
                                                            244
                                                   131
                Ε
                                                    94
                                                            299
                F
                                                    24
                                                            317
structable(mod_2_table)
                              berkeley$Admit Admitted Rejected
berkeley$Gender berkeley$Dept
Male
                Α
                                              215.10146 339.62744
                В
                                              134.87069 212.94968
                С
                                              211.64324 334.16719
                D
                                              182.59417 288.30110
                Ε
                                              134.64014 212.58566
                F
                                              164.61141 259.90781
Female
                Α
                                              146.67825 231.59285
                В
                                               91.96868 145.21095
                С
                                              144.32008 227.86949
                D
                                              124.51144 196.59328
                Е
                                               91.81147 144.96272
                                              112.24895 177.23182
# odds ratio for observed values
(obs_table[1, 1, ]/obs_table[1, 2, ])/(obs_table[2, 1, ]/obs_table[2, 2, ])
                                       D
0.3492120 0.8025007 1.1330596 0.9212838 1.2216312 0.8278727
# odds ratio for predicted values
(mod_2_table[1, 1, ]/mod_2_table[1, 2, ])/(mod_2_table[2, 1, ]/mod_2_table[2, 2, ])
ABCDEF
1 1 1 1 1 1
```

5. Draw mosaic plots of admission versus gender for each department separately.

The following plots demonstrate admission versus gender for each department separately.

```
mosaicplot(obs_table[1:2,1:2,1], main = "Dept. A")
```

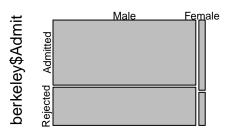
Dept. A



berkeley\$Gender

mosaicplot(obs_table[1:2,1:2,2], main = "Dept. B")

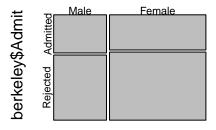
Dept. B



berkeley\$Gender

mosaicplot(obs_table[1:2,1:2,3], main = "Dept. C")

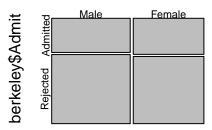
Dept. C



berkeley\$Gender

mosaicplot(obs_table[1:2,1:2,4], main = "Dept. D")

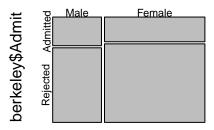
Dept. D



berkeley\$Gender

mosaicplot(obs_table[1:2,1:2,5], main = "Dept. E")

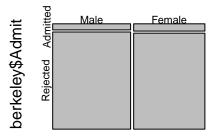
Dept. E



berkeley\$Gender

mosaicplot(obs_table[1:2,1:2,6], main = "Dept. F")

Dept. F



berkeley\$Gender

6. (3 points) Fit a log-linear model to the frequencies using A, D and G as predictors that includes all two-way interactions but not the three-way interaction. Display the predicted frequencies in a table and compare them to the predicted frequencies displayed in Question 4 as well as the observed frequencies. Calculate the estimated odds ratio for admission of males vs. females in this model.

The requested model and the predicted frequencies are shown below. We can see the ratios of the accepted to the rejected differ accross departments and genders, since the interactions were included in this model. Besides, the predicted frequencies are more closer to the observed frequencies than that of the model in task

4. The estimated odds ratio for admission of males vs. females was also computed, and the value is 1.84108, which is the same as the odds ratio in task 1.

Call:

```
glm(formula = Freq ~ Admit + Gender + Dept + Admit * Dept + Gender *
    Dept + Admit * Gender, family = "poisson", data = berkeley)
```

Deviance Residuals:

1	2	3	4	5	6	7
-0.75481	0.99471	1.96454	-3.15768	-0.03402	0.04449	0.15709
8	9	10	11	12	13	14
-0.22034	1.01273	-0.73839	-0.74367	0.54896	0.06760	-0.04741
15	16	17	18	19	20	21
-0.06911	0.05080	1.05578	-0.61236	-0.73617	0.42678	-0.20117
22	23	24				
0.05113	0.19803	-0.05370				

Coefficients:

	Estimate	Std. Error	z value	Pr(> z)
(Intercept)	6.27150	0.04271	146.855	< 2e-16
AdmitRejected	-0.58205	0.06899	-8.436	< 2e-16
GenderFemale	-1.99859	0.10593	-18.866	< 2e-16
DeptB	-0.40322	0.06784	-5.944	0.0000000278
DeptC	-1.57790	0.08949	-17.632	< 2e-16
DeptD	-1.35000	0.08526	-15.834	< 2e-16
DeptE	-2.44982	0.11755	-20.840	< 2e-16
DeptF	-3.13787	0.16174	-19.401	< 2e-16
AdmitRejected:DeptB	0.04340	0.10984	0.395	0.693
AdmitRejected:DeptC	1.26260	0.10663	11.841	< 2e-16
AdmitRejected:DeptD	1.29461	0.10582	12.234	< 2e-16
AdmitRejected:DeptE	1.73931	0.12611	13.792	< 2e-16
AdmitRejected:DeptF	3.30648	0.16998	19.452	< 2e-16
GenderFemale:DeptB	-1.07482	0.22861	-4.701	0.00000258269
GenderFemale:DeptC	2.66513	0.12609	21.137	< 2e-16
GenderFemale:DeptD	1.95832	0.12734	15.379	< 2e-16
GenderFemale:DeptE	2.79519	0.13925	20.073	< 2e-16
GenderFemale:DeptF	2.00232	0.13571	14.754	< 2e-16
AdmitRejected:GenderFemale	-0.09987	0.08085	-1.235	0.217

(Dispersion parameter for poisson family taken to be 1)

Null deviance: 2650.095 on 23 degrees of freedom Residual deviance: 20.204 on 5 degrees of freedom

AIC: 217.26

Number of Fisher Scoring iterations: 4

```
D
                                               137.207390 279.792610
                Ε
                                                45.680810 145.319190
                F
                                                22.957096 350.042904
Female
                Α
                                                71.730081 36.269919
                В
                                                16.360491
                                                            8.639509
                C
                                               212.754724 380.245276
                D
                                               131.792610 243.207390
                Ε
                                               101.319190 291.680810
                F
                                                23.042904 317.957096
```

```
 (sum(mod_3_table[1, 1, ])/sum(mod_3_table[1, 2, ]))/(sum(mod_3_table[2, 1, ])/sum(mod_3_table[2, 2, ])) / (sum(mod_3_table[2, 2, ])) / (sum(mod_3_table[2, 2, ])/sum(mod_3_table[2, 2, ])/sum(mod_3_table[2, 2, ]) / (sum(mod_3_table[2, 2, ])/sum(mod_3_table[2, 2, ])) / (sum(mod_3_table[2, 2, ])/sum(mod_3_table[2, 2, ])) / (sum(mod_3_table[2, 2, ])/sum(mod_3_table[2, 2, ])/sum(mod_3_table[2, 2, ]) / (sum(mod_3_table[2, 2, ])/sum(mod_3_table
```

[1] 1.84108

predicted values (mod_3)

7. (2 points) Starting with the model in Question 6, use the stepwise method with the backward/forward option and BIC as criterion. Does this result in a simpler model? Interpret this model in plain English.

The model obtained by stepwise method is shown below, and it omitted the interaction term between Admit and Gender. In a three-main-effect model, if we have already known that the interaction between A and B is independent of that between B and C (denoted as (AB, BC)) and the interaction between B and C is independent of that between A and C, which is (BC, AC), we can safely conclude that the interaction between A and B is independent of that between A and C ((AB, AC)). Therefore, the redundant interaction term was omitted.

```
mod_4 <- stepwise(mod_3, direction = 'backward/forward', criterion = 'BIC')
summary(mod_4)</pre>
```

```
Call:
```

```
glm(formula = Freq ~ Admit + Gender + Dept + Admit:Dept + Gender:Dept,
    family = "poisson", data = berkeley)
```

Deviance Residuals:

```
Min 1Q Median 3Q Max
-3.4776 -0.4144 0.0098 0.3089 2.2321
```

Coefficients:

	Estimate	Std. Error	z value	Pr(> z)
(Intercept)	6.27557	0.04248	147.744	< 2e-16
AdmitRejected	-0.59346	0.06838	-8.679	< 2e-16
GenderFemale	-2.03325	0.10233	-19.870	< 2e-16
DeptB	-0.40575	0.06770	-5.993	0.00000000206

```
DeptC
                   -1.53939
                               0.08305 -18.536
                                                      < 2e-16
DeptD
                                                      < 2e-16
                   -1.32234
                               0.08159 -16.207
DeptE
                   -2.40277
                               0.11014 -21.816
                                                      < 2e-16
DeptF
                   -3.09624
                               0.15756 -19.652
                                                      < 2e-16
AdmitRejected:DeptB 0.05059
                               0.10968
                                         0.461
                                                        0.645
AdmitRejected:DeptC 1.20915
                               0.09726 12.432
                                                      < 2e-16
AdmitRejected:DeptD 1.25833
                               0.10152 12.395
                                                      < 2e-16
AdmitRejected:DeptE
                                                      < 2e-16
                   1.68296
                               0.11733 14.343
AdmitRejected:DeptF 3.26911
                               0.16707 19.567
                                                      < 2e-16
GenderFemale:DeptB -1.07581
                               0.22860 -4.706 0.00000252480
GenderFemale:DeptC
                    2.63462
                                0.12343 21.345
                                                     < 2e-16
                                                      < 2e-16
GenderFemale:DeptD
                    1.92709
                               0.12464 15.461
GenderFemale:DeptE
                    2.75479
                               0.13510 20.391
                                                      < 2e-16
GenderFemale:DeptF
                    1.94356
                               0.12683 15.325
                                                     < 2e-16
(Dispersion parameter for poisson family taken to be 1)
   Null deviance: 2650.095
                            on 23
                                   degrees of freedom
Residual deviance:
                    21.736
                                   degrees of freedom
                            on 6
AIC: 216.8
```

Number of Fisher Scoring iterations: 4

8. (2 points) Calculate the saturated model using all three predictors and compare this model to the one obtained in Question 7 using AIC, BIC and the deviance criterion. Give a verbal summary of your findings.

The saturated model is shown below. A decrease can be seen in AIC and BIC scores and deviance criterion if we compare the model to the one in task 7. Furthermore, the predicted frequency table is the same as the observed table, since all the interaction terms were all taken into account in this model.

```
Call:
glm(formula = Freq ~ Admit * Gender * Dept, family = "poisson",
   data = berkeley)
Deviance Residuals:
 [24] 0
Coefficients:
                             Estimate Std. Error z value Pr(>|z|)
(Intercept)
                              6.23832
                                       0.04419 141.157 < 2e-16
AdmitRejected
                             -0.49212
                                        0.07175 -6.859 6.94e-12
GenderFemale
                             -1.74969
                                        0.11484 -15.235 < 2e-16
DeptB
                             -0.37186
                                        0.06918 -5.375 7.65e-08
DeptC
                             -1.45083
                                        0.10142 -14.305 < 2e-16
DeptD
                                        0.09591 -13.669 < 2e-16
                             -1.31107
```

```
DeptE
                                -2.26803
                                            0.14430 -15.718 < 2e-16
                                -3.14728
DeptF
                                            0.21773 -14.455 < 2e-16
                                            0.26271 -4.005 6.21e-05
AdmitRejected:GenderFemale
                                -1.05208
                                            0.11319 -0.368 0.71304
AdmitRejected:DeptB
                                -0.04163
AdmitRejected:DeptC
                                 1.02764
                                            0.13550 7.584 3.34e-14
AdmitRejected:DeptD
                                            0.12641 9.462 < 2e-16
                                 1.19608
AdmitRejected:DeptE
                                            0.17681 8.196 2.49e-16
                                 1.44908
                                            0.23120 14.109 < 2e-16
AdmitRejected:DeptF
                                 3.26187
GenderFemale:DeptB
                                -1.28357
                                            0.27358 -4.692 2.71e-06
GenderFemale:DeptC
                                 2.27046
                                            0.16270 13.954 < 2e-16
GenderFemale:DeptD
                                 1.69763
                                            0.16754 10.133 < 2e-16
                                            0.20663 11.241 < 2e-16
GenderFemale:DeptE
                                 2.32269
GenderFemale:DeptF
                                 1.83670
                                            0.31672
                                                    5.799 6.66e-09
                                            0.51039 1.630 0.10306
AdmitRejected:GenderFemale:DeptB 0.83205
AdmitRejected:GenderFemale:DeptC 1.17700
                                            0.29956
                                                      3.929 8.53e-05
AdmitRejected:GenderFemale:DeptD 0.97009
                                            0.30262
                                                      3.206 0.00135
                                                      3.791 0.00015
AdmitRejected:GenderFemale:DeptE 1.25226
                                            0.33032
AdmitRejected:GenderFemale:DeptF 0.86318
                                            0.40267
                                                      2.144 0.03206
(Dispersion parameter for poisson family taken to be 1)
   Null deviance: 2.6501e+03 on 23 degrees of freedom
Residual deviance: 1.1191e-13 on 0 degrees of freedom
AIC: 207.06
Number of Fisher Scoring iterations: 3
# compare AIC scores
AIC(mod_4)
[1] 216.7952
AIC(mod_5)
[1] 207.0597
# compare BIC scores
BIC(mod_4)
[1] 238.0002
BIC(mod_5)
[1] 235.333
# Compute DIC = -2 * log(likelihood)
-2 * logLik(mod_4)
'log Lik.' 180.7952 (df=18)
-2 * logLik(mod_5)
'log Lik.' 159.0597 (df=24)
# predicted values (mod_5)
mod_5_table <- xtabs(mod_5$fitted.values ~ berkeley$Gender +</pre>
                        berkeley$Admit + berkeley$Dept)
structable(mod_5_table)
```

		berkeley\$Admit	Admitted	Rejected
berkeley\$Gender	berkeley\$Dept	·		J
Male	A		512	313
	В		353	207
	C		120	205
	D		138	279
	E		53	138
	F		22	351
Female	A		89	19
	В		17	8
	C		202	391
	D		131	244
	E		94	299
	F		24	317

9. To run a logistic regression model with aggregated data it is best to create a data frame that comprises two frequency variables: one for the numbers of admitted students, and one for the numbers of rejected students.

Treating cbind(A,R) as response and D and G as qualitative predictors, fit the logit model having main effects only.

```
berkeley_t <- spread(berkeley, Admit, Freq)</pre>
mod_6 <- glm(cbind(Rejected, Admitted) ~ Gender * Dept,</pre>
            family = binomial(link = "logit"),
            data = berkeley_t)
summary(mod_6)
Call:
glm(formula = cbind(Rejected, Admitted) ~ Gender * Dept, family = binomial(link = "logit"),
   data = berkeley t)
Deviance Residuals:
[1] 0 0 0 0 0 0 0 0 0 0 0 0
Coefficients:
                  Estimate Std. Error z value Pr(>|z|)
(Intercept)
                             0.07175 -6.859 6.94e-12
                  -0.49212
GenderFemale
                  -1.05208
                              0.26271 -4.005 6.21e-05
DeptB
                  -0.04163
                              0.11319 -0.368 0.71304
DeptC
                  1.02764
                              0.13550 7.584 3.34e-14
                   1.19608
                              0.12641 9.462 < 2e-16
DeptD
```

```
DeptE
                   1.44908
                              0.17681
                                       8.196 2.49e-16
                   3.26187
                              0.23120 14.109 < 2e-16
DeptF
                                       1.630 0.10306
GenderFemale:DeptB 0.83205
                              0.51039
GenderFemale:DeptC
                   1.17700
                              0.29956
                                        3.929 8.53e-05
GenderFemale:DeptD 0.97009
                              0.30262
                                        3.206 0.00135
GenderFemale:DeptE 1.25226
                              0.33032
                                        3.791 0.00015
GenderFemale:DeptF 0.86318
                              0.40267
                                        2.144 0.03206
```

(Dispersion parameter for binomial family taken to be 1)

```
Null deviance: 877.0564132197750951 on 11 degrees of freedom Residual deviance: 0.0000000000001088 on 0 degrees of freedom
```

AIC: 92.94

Number of Fisher Scoring iterations: 3

0.9212838

(a) (1.5 points) Report the prediction equation. Interpret the coefficients of D, G and the interaction in this equation.

The prediction equation and the coefficients of each term are shown above. If we add the coefficient of *GenderFemale* and the coefficients of interaction terms between *Dept*, and *Gender*, the results are exactly the logarithmic odds ratios for admission of males vs. females in each department. And the exponential of the results are the observed odd ratios in task 4.

0.8278727

(b) (half a point) To which log-linear model is this model equivalent?

1.2216312

GenderFemale:DeptD GenderFemale:DeptE GenderFemale:DeptF

The model is equivalent to the saturated model using all three predictors in task 8.