Homework 5

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```
library(magrittr)
library(dplyr)
library(ggplot2)
library(car)
```

1. First of all, load the data frame Wage from the library ISLR. You start out with a close look at wage differences between the two health levels.

```
data(Wage, package = "ISLR")
```

(a) (1.5 points) Compute mean and standard deviation of wage for each health level separately. Summarize the result in an English sentence.

```
wage_stats <- Wage %>%
    group_by(health) %>%
    summarise(mean_wage = mean(wage),
              sd_wage = sd(wage),
              counts = n())
wage_stats
```

```
# A tibble: 2 x 4
          health mean_wage  sd_wage counts
          <fctr>     <dbl>    <dbl>  <int>
1      1. <=Good  101.6613 35.18500    858
2 2. >=Very Good  115.7262 43.43896   2142
```

(b) (1 point) Compute the standard errors for the mean wages in the two groups.

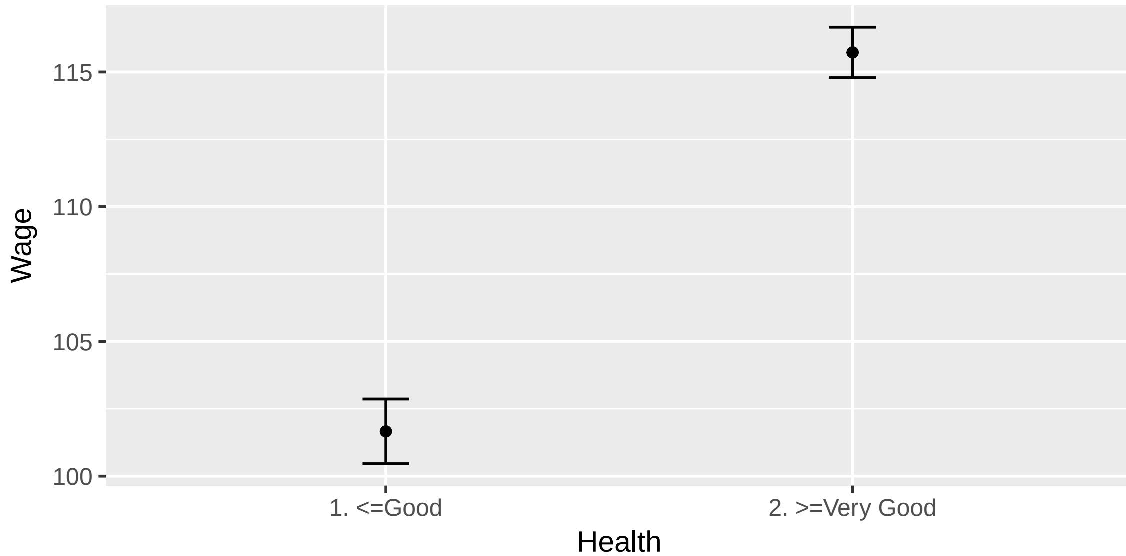
```
wage_stats$se_wage <- wage_stats$sd_wage / sqrt(wage_stats$counts)
wage_stats
```

```
# A tibble: 2 x 5
          health mean_wage  sd_wage counts   se_wage
          <fctr>     <dbl>    <dbl>  <int>     <dbl>
1      1. <=Good  101.6613 35.18500    858 1.2011960
2 2. >=Very Good  115.7262 43.43896   2142 0.9385766
```

2. (2.5 points) Create a plot showing the mean wages for the two groups and corresponding error bars, i.e. add lines of length one standard error of the mean to both sides of the mean.

```
ggplot(wage_stats, aes(x = health, y = mean_wage, group = 1)) +
    geom_errorbar(aes(x = health, ymin = mean_wage - se_wage, ymax = mean_wage + se_wage),
```

```
                  width = 0.1) +
    geom_point() +
    labs(x = 'Health' , y = 'Wage')
```



3. (2.5 points) Using an appropriate statistical procedure, test whether average wage is the same for workers with health level "1. at most Good" and workers with health level "2. at least Very Good". Formulate the null and alternative hypothesis and report the results in an English sentence refering to the relevant numbers.

$H_0 : Average\ wage_{level_1} = Average\ wage_{level_2}$

$H_1 : Average\ wage_{level_1} \neq Average\ wage_{level_2}$

```
# Assume that population variances of the two classes are not equal
t.test(Wage$wage ~ Wage$health, var.equal = FALSE)
```

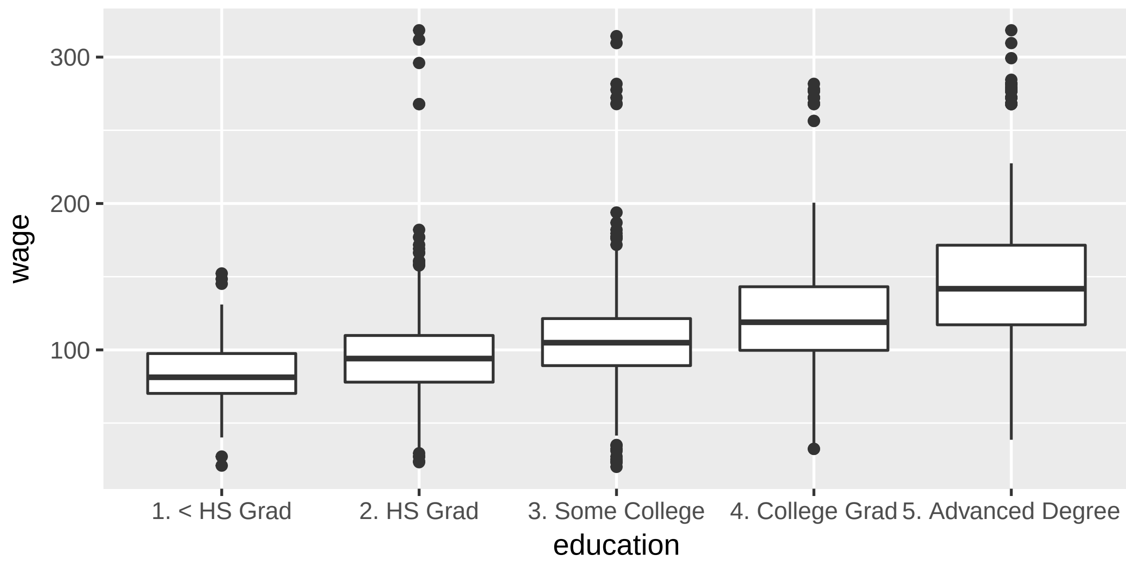
```

    Welch Two Sample t-test

data:  Wage$wage by Wage$health
t = -9.2265, df = 1934.3, p-value < 2.2e-16
alternative hypothesis: true difference in means is not equal to 0
95 percent confidence interval:
 -17.05452 -11.07524
sample estimates:
     mean in group 1. <=Good mean in group 2. >=Very Good
                    101.6613                     115.7262
```

4. Plot a box plot of the workers raw wage (variable wage using the education level (variable education) as grouping.

```
ggplot(Wage) +
    geom_boxplot(aes(x = education, y = wage))
```



(a) (half a point) Are half of the wages for workers who have less than a high school degree below the first quartile of the wage for workers with some college degree?

(b) (half a point) Do half of the workers with a HS degree have higher wages than three quarters of the high school dropouts?

(c) (half a point) The minimum wage of workers with advanced degree is larger than the median wage of high school dropouts?

(d) (half a point) The interquartile range differs substantially between all groups.

(e) (half a point) Spread as measured by the length of the whiskers differs substantially between all groups.

5. You want to assess the wage difference between educational groups. Before you run the appropriate statistical test, you check some of the assumptions for ANOVA. In particular, you assess homoscedasticity.

(a) (1.5 points) Looking at the boxplot in Question 4. Does homoscedasticity hold for the five groups? Give reasons for your answer!

(b) (1 point) Select a suitable variance test to check on this. Does the test confirm homoscedasticity?

```
leveneTest(Wage$wage, Wage$education)
```

```
Levene's Test for Homogeneity of Variance (center = median)
        Df F value    Pr(>F)
group    4  50.021 < 2.2e-16
      2995
```

6. Now, you assess the wage difference between educational groups using a statistical test.

(a) (1 point) Using an appropriate statistical test check whether wages are equal across education groups. Report the result in a complete English sentence including the relevant numbers!

```
wage_education <- aov(wage ~ education, data = Wage)
anova(wage_education)
```

```
Analysis of Variance Table

Response: wage
            Df  Sum Sq Mean Sq F value    Pr(>F)
education    4 1226364  306591  229.81 < 2.2e-16
Residuals 2995 3995721    1334
```

(b) (1 point) From the ANOVA table derive the total sum of squares for wages and compare this result with the variance of wage when multiplied by 2999.

```
sum(anova(wage_education)[, 2])
```

```
[1] 5222086
```

```
var(Wage$wage) * 2999
```

```
[1] 5222086
```

(c) (half a point) Which proportion of total variation in wages is due to the group differences in education?

```
anova(wage_education)[1, 2] / sum(anova(wage_education)[, 2])
```

```
[1] 0.2348419
```

7. Having found an overall difference, you now want to use a post-hoc test with Holm correction, to asses which marital status groups do actually differ significantly in wages?

```
pairwise.t.test(Wage$wage, Wage$education, p.adjust.method = "holm")
```

```

    Pairwise comparisons using t tests with pooled SD

data:  Wage$wage and Wage$education

                   1. < HS Grad 2. HS Grad 3. Some College 4. College Grad
2. HS Grad         3.7e-06      -          -               -
3. Some College    < 2e-16      2.3e-10    -               -
4. College Grad    < 2e-16      < 2e-16    3.5e-16         -
5. Advanced Degree < 2e-16      < 2e-16    < 2e-16         < 2e-16
```

```
P value adjustment method: holm
```

(a) (2 points) According to the post hoc test which groups differ significantly?

(b) (half a point) According to the post hoc test which groups do not differ significantly?

8. You now investigate the relationship between wage and the two predictors education and health status.

(a) (1 point) First, calculate a main effects model only. Give a verbal summary of the model result!

```
wage_edu_heal <- aov(wage ~ education + health, data = Wage)
anova(wage_edu_heal)
```

```
Analysis of Variance Table

Response: wage
            Df  Sum Sq Mean Sq F value      Pr(>F)
education    4 1226364  306591 231.248   < 2.2e-16
health       1   26239   26239  19.791 0.000008956
Residuals 2994 3969483    1326
```

(b) (1 point) Second, calculate a model with interaction. Give a verbal summary of the model result!

```
wage_edu_heal_inter <- aov(wage ~ education + health + education:health, data = Wage)
anova(wage_edu_heal_inter)
```

```
Analysis of Variance Table

Response: wage
                   Df  Sum Sq Mean Sq  F value      Pr(>F)
education           4 1226364  306591 231.3193   < 2.2e-16
health              1   26239   26239  19.7967 0.000008928
education:health    4    6530    1632   1.2316      0.2952
Residuals        2990 3962953    1325
```

(c) (half a point) Using the TukeyHSD post-hoc tests, which education levels do actually differ significantly in wages?

```
TukeyHSD(wage_edu_heal)
```

```
  Tukey multiple comparisons of means
    95% family-wise confidence level

Fit: aov(formula = wage ~ education + health, data = Wage)

$education
                                      diff        lwr      upr     p adj
2. HS Grad-1. < HS Grad           11.67894   4.821323 18.53655 0.0000343
```

```
3. Some College-1. < HS Grad       23.65115 16.436562 30.86574 0.0000000
4. College Grad-1. < HS Grad       40.32349 33.162914 47.48407 0.0000000
5. Advanced Degree-1. < HS Grad    66.81336 59.064785 74.56194 0.0000000
3. Some College-2. HS Grad         11.97221  6.935590 17.00884 0.0000000
4. College Grad-2. HS Grad         28.64456 23.685608 33.60351 0.0000000
5. Advanced Degree-2. HS Grad      55.13443 49.358811 60.91004 0.0000000
4. College Grad-3. Some College    16.67234 11.230411 22.11427 0.0000000
5. Advanced Degree-3. Some College 43.16221 36.966958 49.35746 0.0000000
5. Advanced Degree-4. College Grad 26.48987 20.357598 32.62214 0.0000000

$health
                             diff      lwr      upr     p adj
2. >=Very Good-1. <=Good 6.443038 3.558529 9.327548 0.0000123
```