Homework 4

Shun-Lung Chang, Dilip Hiremath

```r
library(ggplot2)
```

```r
load('data/creditcard.Rdata')
```

1. From your previous analysis you know that HomeOwnership is an important predictor for getting a credit card approved. You hence limit your analysis in this homework to homeowning applicants only. Create a subset of the data just including the homeowners.

```r
creditcard <- creditcard[creditcard$HomeOwner == 1, ]
```

(a) (half a point) For this subset, compute the median income.

```r
median(creditcard$Income)
```

```
[1] 3.75
```

(b) (half a point) For this subset, compute the standard deviation of income.

```r
sd(creditcard$Income)
```

```
[1] 5.566508
```

(c) (half a point) How many applicants are in this subset?

```r
nrow(creditcard)
```

```
[1] 295
```

(d) (half a point) How many of the applicants in the subset are married?

```r
sum(creditcard$MaritalStatus == 1)
```

```
[1] 228
```

(e) (half a point) How many applicants in the subset own a home?

All the applicants own a home in this subset of the data, hence the number is 295.

2. On a typical working day, your team is able to process 120 credit card applications. To simulate this situation you draw a random sample of size 120 from the data subset generated in Question 1. (In order to make the results reproducible, use set.seed(201803) prior to drawing the sample.) Based on this sample of size 120, you want to test the null hypothesis that the mean income in the population is equal to 4750 USD. [hint: use the command t.test to perform a one-sample t-test to answer this question.]

```r
set.seed(201803)
sample_creditcard <- creditcard[sample(nrow(creditcard), 120), ]
```

(a) (1 point) Based on the result obtained, do you conclude to reject the null hypothesis of the true population mean being equal to 4750 USD?

```r
t.test(sample_creditcard$Income, mu = 4.75)
```

```

    One Sample t-test

data:  sample_creditcard$Income
t = 2.1488, df = 119, p-value = 0.03367
alternative hypothesis: true mean is not equal to 4.75
95 percent confidence interval:
 4.832375 6.765792
sample estimates:
mean of x
 5.799083
```

(b) (half a point) How large is the test-statistic?

```r
t.test(sample_creditcard$Income, mu = 4.75)$statistic
```

```
       t
2.148828
```

(c) (half a point) How large is the corresponding p-value?

```r
t.test(sample_creditcard$Income, mu = 4.75)$p.value
```

```
[1] 0.03367372
```

(d) (half a point) Does the 95%-confidence interval contain the score 4.75?

```r
t.test(sample_creditcard$Income, mu = 4.75)$conf.int
```

```
[1] 4.832375 6.765792
attr(,"conf.level")
[1] 0.95
```

3. To check whether R actually computes the right thing, you decide to double check.

(a) (1 point) You first compute the mean and standard deviation of income in your sample and report these numbers.

```
mean(sample_creditcard$Income)
```

```
[1] 5.799083
```

```
sd(sample_creditcard$Income)
```

```
[1] 5.348094
```

(b) (half a point) Next you compute the standard error of the mean by dividing the standard deviation of your sample by the square root of the sample size.

```
sde_mean <- sd(sample_creditcard$Income) / sqrt(nrow(sample_creditcard))
sde_mean
```

```
[1] 0.488212
```

(c) (1 point) Finally, you compute the test statistic t which is the ratio of the difference between sample mean and hypothetical value and the standard error of the mean.

```
t_value <- (mean(sample_creditcard$Income) - 4.75) / sde_mean
t_value
```

```
[1] 2.148828
```

4. Now, you compare the empirical results with the corresponding theoretical distribution.

(a) (1 point) Compute the 2.5% quantile and the 97.5% quantile of the t-distribution with 119 degrees of freedom. Does the test statistic fall inside this range?

```
qt(c(0.025,0.975), df = 119)
```

```
[1] -1.9801  1.9801
```

(b) (1.5 points) Compute the probability that a random variable that follows a t-distribution with 119 degrees of freedom takes on values that are in absolute values larger than the observed test-statistic i.e. P $(T \geq |2.1488|)$.

```
pt(-abs(t_value), df = 119) * 2
```

```
[1] 0.03367372
```

5. Now, you simulate a full years work of your team, by drawing a total of 220 samples of size 120 from the income variable in the credit card data set.

```
sample_size <- 120
no_of_samples <- 220
sample <- matrix(0, nrow = sample_size, ncol = no_of_samples)
for (i in 1:no_of_samples) {
    index <- sample(length(creditcard$Income), size = sample_size,replace = FALSE)
    sample[, i] <- creditcard[index, "Income"]
}
```

(a) (1 point) Compute the median income for each sample. Report the median of the sample medians as well as the interquartile range of the sample medians.

```
sample_medians <- apply(sample, 2, median, na.rm = TRUE)
median(sample_medians)
```

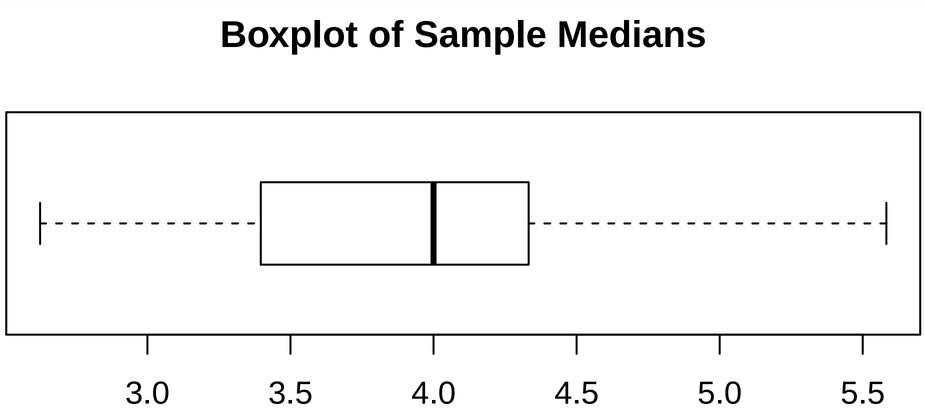
```
[1] 4
```

```
IQR(sample_medians)
```

```
[1] 0.925625
```

(b) (1 point) Draw a boxplot of the sample medians. Based on this plot comment on the sampling distribution of the median income!

```
boxplot(sample_medians, horizontal = TRUE, main = 'Boxplot of Sample Medians')
```



(c) (half a point) Compute the 0.025-quantile and the 0.975-quantile of your sampling distribution of the median income.

```
quantile(sample_medians, probs = c(0.025, 0.975))
```

```
 2.5% 97.5%
   3     5
```

6. Using the data obtained in Question 5 compute the following:

(a) (1 point) Compute the mean income for each sample. Report the mean of the sample means as well as the standard deviation of the sample means.

```
sample_means <- apply(sample, 2, mean, na.rm = TRUE)
mean(sample_means)
```

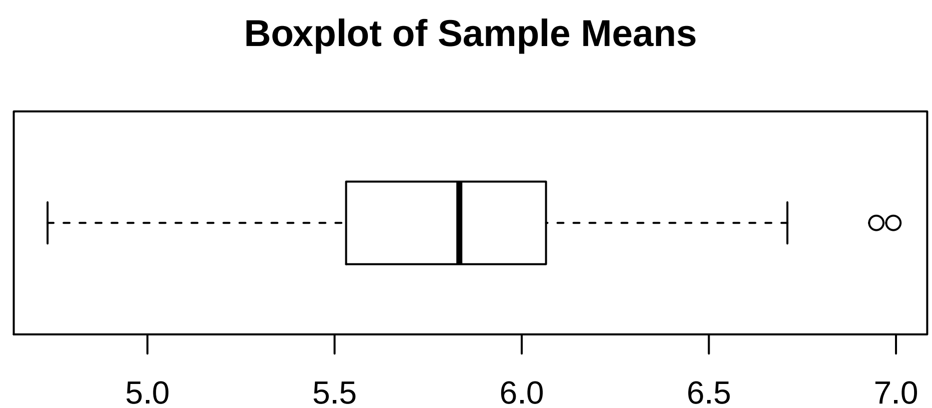
```
[1] 5.802627
```

```
sd(sample_means)
```

```
[1] 0.3820323
```

(b) (1 point) Draw a boxplot of the sample means. Based on this plot comment on the sampling distribution of the mean income!

```
boxplot(sample_means, horizontal = TRUE, main = 'Boxplot of Sample Means')
```



(c) (half a point) Compute the 0.025-quantile and the 0.975-quantile of your sampling distribution of the mean income.

```
quantile(sample_means, probs = c(0.025, 0.975))
```

```
    2.5%    97.5% 
5.008343 6.456191 
```

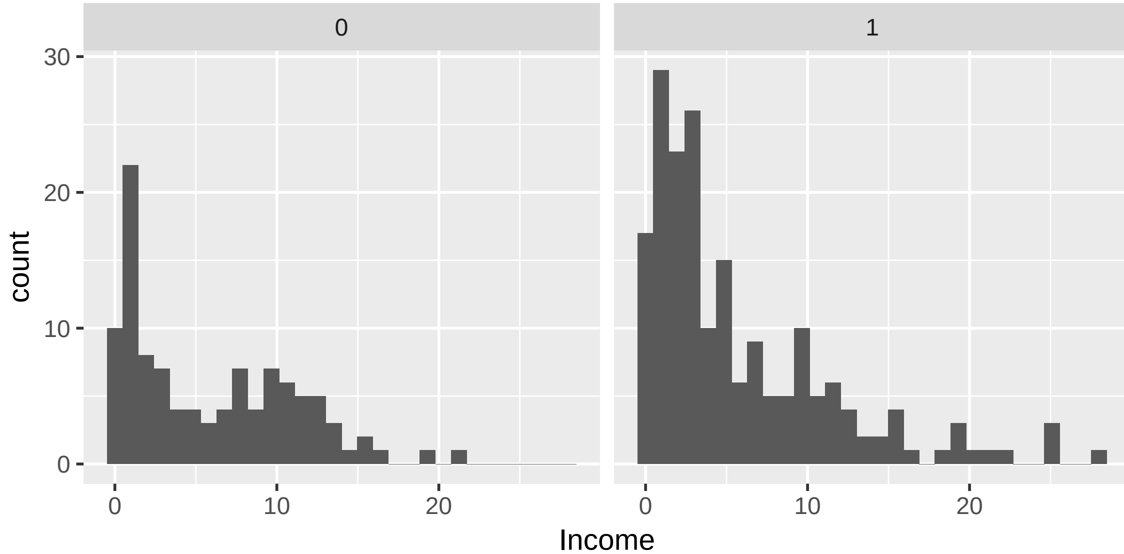
7. (2.5 points) Coming back to your subset data of homeowners (see Question 1), you want to investigate if there is a gender bias in income. For that, briefly describe in plain English the distributions of income, separately for males and females (variable Gender). Use relevant numerical summaries as well as one graphical representation for each of the two distributions.

```
by(creditcard$Income, creditcard$Gender, summary,
   quantiles = c(0,.25,.5,.75,1))
```

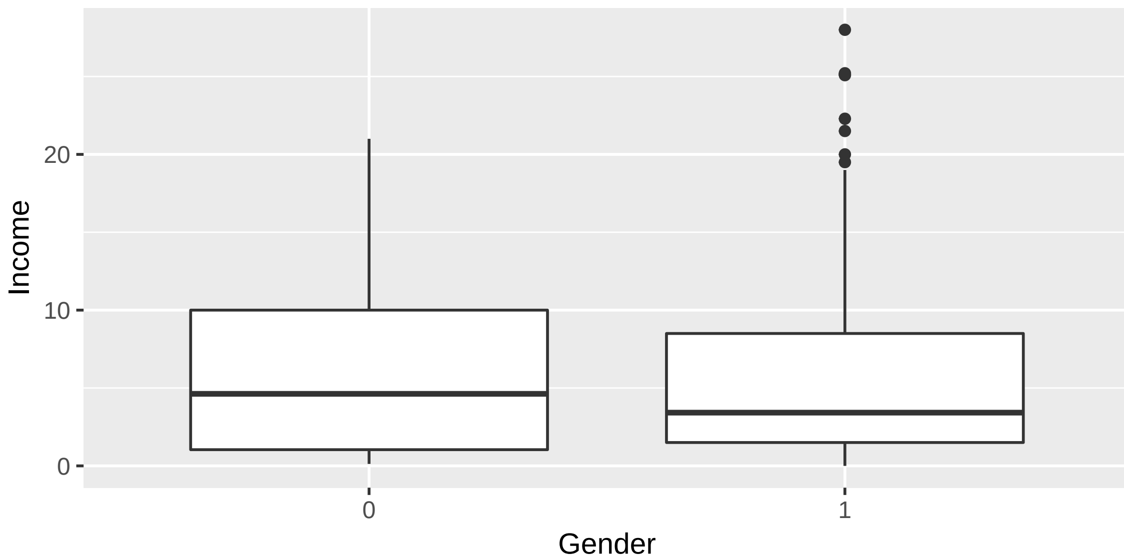
```
creditcard$Gender: 0
   Min. 1st Qu.  Median    Mean 3rd Qu.    Max. 
  0.125   1.040   4.625   5.914  10.000  21.000 
```

```
--------------------------------------------------------
creditcard$Gender: 1
   Min. 1st Qu.  Median    Mean 3rd Qu.    Max. 
  0.000   1.500   3.417   5.683   8.500  28.000 
```

```
ggplot(creditcard) +
    geom_histogram(aes(x = Income)) +
    facet_grid(~ Gender)
```




```
ggplot(creditcard) +
    geom_boxplot(aes(x = Gender, y = Income))
```



8. Again using the subset data for homeowners, you want to see whether the difference in means is large in comparison to the spread of the data.

(a) (1 points) Calculate the means ($\bar{x}_{inc.f}$, $\bar{x}_{inc.m}$) and the standard deviations ($s_{inc.f}$, $s_{inc.m}$) of income separately. Now calculate the test-statistic of the independent-samples t-test.

```
inc_x_bar <- tapply(creditcard$Income, creditcard$Gender,
                    mean, na.rm = TRUE)
inc_sd <- tapply(creditcard$Income, creditcard$Gender,
                    sd, na.rm = TRUE)
```

```
n1 <- nrow(creditcard[creditcard$Gender == 0 & !is.na(creditcard$Income), ])

n2 <- nrow(creditcard[creditcard$Gender == 1 & !is.na(creditcard$Income), ])

t_sd <- ((n1 - 1) * inc_sd[1] ^ 2 + (n2 - 1) * inc_sd[2] ^ 2) / (n1 + n2 - 2)

t_value <- (inc_x_bar[1] - inc_x_bar[2]) /
    sqrt(t_sd * (1 / n1 + 1 / n2))
t_value <- unname(t_value)
t_value
```

```
[1] 0.340173
```

(b) (1 point) Using a t-distribution with n1 + n2 - 2 degrees of freedom, calculate the probability of a t-distributed random variable being larger than or equal the above calculated t-statistic score.

```
p_larger <- pt(abs(t_value), df = n1 + n2 - 2, lower.tail = FALSE)
p_larger
```

```
[1] 0.366985
```

(c) (half a point) Based on the results so far, compute the probability under the null hypothesis to obtain a result for the test statistic that is as extreme as the one we have obtained.

```
2 * pt(abs(t_value), df = n1 + n2 - 2, lower.tail = FALSE)
```

```
[1] 0.73397
```

9. (2.5 points) Use the function t-test to check with an independent samples t-test whether income significantly differs between males and females in your subset of homeowners. Assume equal variances for the two groups. State the statistical null hypothesis to be tested as well as the alternative hypothesis.

Take a look at the output and compare it with your results above.

```
f_income <- creditcard[creditcard$Gender == 0 & !is.na(creditcard$Income), 'Income']
m_income <- creditcard[creditcard$Gender == 1 & !is.na(creditcard$Income), 'Income']

t.test(f_income, m_income, mu = 0, var.equal = TRUE)
```

```
	Two Sample t-test

data:  f_income and m_income
t = 0.34017, df = 293, p-value = 0.734
alternative hypothesis: true difference in means is not equal to 0
95 percent confidence interval:
 -1.103595  1.564813
sample estimates:
mean of x mean of y
 5.913714  5.683105
```

10. (2.5 points) Visualise the previous results. Draw a plot for the pdf of the t-distribution with the adequate number of degrees of freedom for the test statistic t. Color the areas under the pdf for all values smaller than t and larger than t.

```r
d <- data.frame(xt = seq(-5, 5, by = 0.01))
d$dt <- dt(d$xt, df = n1 + n2 - 2)

ggplot(d) +
    geom_path(aes(xt, dt)) +
    geom_linerange(data = d[d$xt > qt(p_larger, n1 + n2 - 2, lower.tail = FALSE), ],
                   aes(xt, ymin = 0, ymax = dt),
                   color = "red") +
    geom_linerange(data = d[d$xt < qt(p_larger, n1 + n2 - 2), ],
                   aes(xt, ymin = 0, ymax = dt),
                   color = "red") +
    theme_bw()
```