STA305 Data Analysis

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1 Terminology

GPU: Graphics Processing Unit. The piece of computer hardware used to do lots of mathematical calculations and renders graphics which are displayed on your computer screens.
CPU: Central Processing Unit. The main piece of computer hardware is responsible for coordinating everything on your machine.
Frequency: The amount of operations a CPU/GPU does within a second.
Frame Rate: The number of different images that are displayed on the computer screen within a second.

2 Methodology

In our experiment, we test different CPU frequencies and GPU types on three levels of games. We are aiming to find out whether GPU types and CPU frequencies have an impact on video games' frame rate.

GPU Types: GTX 1060 (produced in 2016) and RTX 3060 Ti (Produced in 2020)

CPU types: 3.2GHz, 3.7GHz, 4.2GHz

Games types: Low-End(GTA 5), Mid-End(Forza Horzion 5), High-End(Cyberpunk 2077)

We run each game based on its built-in benchmark. The benchmark system would run a couple of in-game scenarios and record the average frame rate. It is easy for us to compare between different adjustments we chose. We chose games as our experimental units as different games have different recommended system requirements, which is useful for us to categorize them and block them into distinct groups to experiment precisely. We've selected the top gamer-rated games from each level as players nowadays consider not only the content of the game, but also the graphics and optimization.

The control design is used in our experiment. For each test we are using the same internet and the same monitor. For the CPU and GPU, we also choose the same manufacturer to minimize our errors. We make sure the only difference in our experiment is the type of GPU, the rest remains the same. We then run each game based on our treatments using the built-in benchmark system.

We run the game in the order of Low-End, Mid-End and finally High-End, with frequency from 3.2GHz, 3.7GHz and then 4.2GHz. We first run on the old GPU GTX1060 and then we turn the computer off for an hour before testing RTX3060 Ti. Since we are running each treatment on all the games, this is a within-subjects design.

3 Analysis

3.1 Assumptions for linear model

To observe whether our data satisfies normality and equal variance assumptions, we used the Shapiro-Wilk test and Bartlett's test separately here.

Normality assumption:

H_0 : Data is normally distributed, H_1 : Data is not normally distributed

Equal Variance assumption:

H_0 : All treatments have equal variance, H_1 : All treatments do not have equal variance.

```
        Shapiro-Wilk normality test

data:  residuals(res.aov)
W = 0.85332, p-value = 5.557e-08
```

```
        Bartlett test of homogeneity of variances

data:  Average_FPS by interaction(CPU_Frequency, GPU_Types)
Bartlett's K-squared = 33.269, df = 5, p-value = 3.327e-06
```

Since the outcome of each test is strictly less than 0.05, we reject our H_0 here and conclude that none of these assumptions are satisfied for our data.

3.2 Sample Size

We calculated our sample size using the power of the two-way ANOVA test. We set our power to be 0.95, and our significance level to be 0.05. We have 6 groups and the effect size was set to 0.5, our sample size was calculated as 14.1435, rounded up to 15. The output is shown below:

```
     Balanced one-way analysis of variance power calculation

              k = 6
              n = 14.14353
              f = 0.5
      sig.level = 0.05
          power = 0.95

NOTE: n is number in each group
```

3.3 ANOVA Table

We use Two-way ANOVA to interpret our data.

$$Y_{ijk} = \mu + \alpha_j + \beta_k + (\alpha\beta)_{jk} + \epsilon_{ijk}$$

where

$$i = 1, ..., n_{jk}; j = 1, ..., a; k = 1, ..., b$$

α_j represents CPU frequencies and β represents GPU types.
Our Null hypothesis is that neither CPU frequency nor GPU types have effect on video games' frame rate. But first, we need to check the interaction effects.
Interaction effects:
$H_0 : (\alpha\beta)_{jk} = 0$ for all j,k; H_1 : at least one $(\alpha\beta)_{jk} \neq 0$

$$SSA = \sum_{j=1}^{3}\sum_{k=1}^{2} n_{jk}(\bar{Y}_{Aj} - \bar{Y}_T)^2$$

$$SSB = \sum_{k=1}^{2}\sum_{j=1}^{3} n_{jk}(\bar{Y}_{Bk} - \bar{Y}_T)^2$$

$$SSAB = \sum_{j=1}^{3}\sum_{k=1}^{2} n_{jk}(\bar{Y}_{jk} - \bar{Y}_{Aj} - \bar{Y}_{Bk} + \bar{Y}_T)^2$$

From our data, we calculated that $MSE = 1216.7$, $\bar{Y}_T = 86.4$, $\bar{Y}_{A1} = 82.2435$, $\bar{Y}_{A2} = 87.1747$, $\bar{Y}_{A3} = 89.7823$, $\bar{Y}_{B1} = 67.801$ and $\bar{Y}_{B2} = 104.9993$. Further more, we have $\bar{Y}_{11} = 65.1275$, $\bar{Y}_{12} = 99.3595$, $\bar{Y}_{21} = 68.2589$, $\bar{Y}_{22} = 106.0905$, $\bar{Y}_{31} = 70.0166$ and $\bar{Y}_{32} = 109.5479$ Bringing into the formulas, we can calculate SSAB, the Interaction Sum of Squares.

$$SSAB = 15 * ((65.1275 - 82.2435 - 67.801 + 86.4)^2$$
$$+(99.3595 - 82.2435 - 104.9993 + 86.4)^2 + (68.2589$$
$$-87.1747 - 67.801 + 86.4)^2 + (106.0905 - 87.1747-$$
$$104.9993 + 86.4)^2 + (70.0166 - 89.7823 - 67.801+$$
$$86.4)^2 + (109.5479 - 89.7823 - 104.9993 + 86.4)^2)$$
$$= 109.824$$
$$MSAB = \frac{SSAB}{(a-1)(b-1)} = \frac{109.824}{2 * 1} = 54.912$$
$$F - statistic = \frac{MSAB}{MSE} = \frac{54.912}{1216.7} = 0.0451$$

From F table, $F_{0.05,2,84} \approx 3.09$. Because $0.0451 < 3.09$, we conclude that the model does not have an interaction effect.

According to the Two-way ANOVA table, we also can clearly see that the p-value of the F-statistic is 0.9559, much greater than $\alpha = 0.05$. We got the same

```
> # Two way ANOVA
> model1 = lm(fps ~ cpu*gpu, data = data) # interaction model
> anova(model1)
Analysis of Variance Table

Response: fps
          Df Sum Sq Mean Sq F value    Pr(>F)    
cpu        2    879   439.7  0.3614    0.6978    
gpu        1  31134 31133.6 25.5890 2.447e-06 ***
cpu:gpu    2    110    54.9  0.0451    0.9559    
Residuals 84 102201  1216.7                      
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

numbers by hand calculation as well. Thus, we fail to reject our null hypothesis that the model has an interaction effect.

Since the model does not have an interaction effect, we can now use the addition model for our experiment.
However, we should check the assumptions for this model. Independence should hold based on how the experiment was designed. The only effect other hardware would have on the system is when they are in the system, which they will not be since the hardware is swapped out for different groups. The system needs to be shut down when swapping hardware, so there will be no leftover issues with memory, and leaving the computer off for an hour between testing GPUs attempts to prevent heat from affecting performance.

Main effect:

Factor A:

$H_0 : \alpha_j = 0$ for all j, H_1 : at least one $\alpha_j \neq 0$

Factor B:

$H_0 : \beta_k = 0$ for all j, H_1 : at least one $\beta_k \neq 0$

Based on the data we listed above, we can calculate our SSA, the Factor A Sum of Squares.

$$SSA = 15*((82.2435-86.4)^2+(87.1747-86.4)^2+(89.7823-86.4)^2)*2 = 879.4982$$

We can also calculate our SSB, Factor B Sum of Squares as below:

$$SSB = 15*((67.801-86.4)^2+(104.9993-86.4)^2)*3 = 31133.5543$$

Our MSA can be calculated as:

$$MSA = \frac{SSA}{(a-1)} = \frac{879.4982}{2} = 439.7491$$

and our MSB is:

$$MSB = \frac{SSB}{(b-1)} = \frac{31133.5543}{1} = 31133.5543$$

our MSE is 1189.7, and now we can calculate our F-statistics.
F for Factor A is:

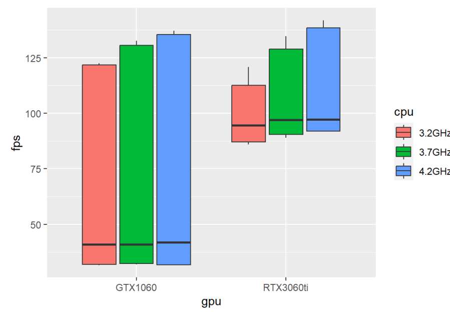
$$F = \frac{MSA}{MSE} = \frac{439.7491}{1189.7} = 0.3696$$

F for Factor B is:

$$F = \frac{MSB}{MSE} = \frac{31133.5543}{1189.7} = 26.1692$$

According to the ANOVA table, we can also conclude that CPU has no effect on frame rate, since CPU p-value $= 0.6921 > \alpha = 0.05$, we fail to reject $\alpha_j = 0$ Also, because p-value for GPU $= 1.876 * 10^-6 < \alpha = 0.05$, we reject that $\beta_k = 0$. We are confident to say that GPU types have a major impact on video games frame rate. In addition, we also use the box plot to support our claims.

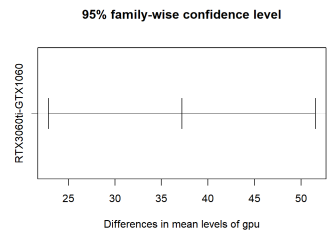
```
Analysis of Variance Table

Response: fps
          Df Sum Sq Mean Sq F value    Pr(>F)
cpu        2    879   439.7  0.3696    0.6921
gpu        1  31134 31133.6 26.1702 1.876e-06 ***
Residuals 86 102311  1189.7
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
```



It is obvious that there's a huge difference between different GPU types. But when we are testing the same GPU with distinct CPU frequencies, the disparity is small.

3.4 Mean CI

To be able to see where the difference is caused within the groups, a Tukey's pairwise comparison of means was used. Here we used TukeyHSD for FPS on CPU Frequency and FPS on GPU type. From the first plot, where the comparison was made for FPS on CPU Frequency, none of the confidence intervals listed in this graph contains 0, we hereby conclude that they fail to reject the null hypothesis, or in other words, the CPU frequency does not effect the video games' average frame rate. In the second plot, where the comparison was made for FPS on GPU type. The confidence interval listed on this graph does not contain 0, and we conclude it rejects its null hypothesis, that the GPU type does affect video games' average frame rates.



4 Appendix

```
#Read Data
data <- read.table("C:/Users/huang/Desktop/University/
2023_Second_Session/STA305/Project/STA305_Clean_New.csv",
                    sep=",", header=TRUE)
cpu <- factor(data$CPU_Frequency)
gpu <- factor(data$GPU_Types)
fps <- data$Average_FPS

#Check Necessary Assumptions
res.aov <- aov(Average_FPS ~ CPU_Frequency + GPU_Types,
               data = data)
summary(res.aov)
shapiro.test(residuals(res.aov))
plot(res.aov, 1)
plot(res.aov, 2)
df = read.csv("STA305_Clean.csv")
df = within(df,{group<-paste(GPU_Types, CPU_Frequency)})
bartlett.test(Average_FPS ~ group, data=df)

#Two way ANOVA
model1 = lm(fps ~ cpu*gpu, data = data) # interaction model
model2 = lm(fps ~ cpu+gpu, data = data) # additive model
summary(model1)
summary(model2)
anova(model1)
anova(model2)

#Graphs for Linear Model
boxplot(fps ~ x, data = data1, xlab = 'treatments')
ggplot(data = data, aes(x = gpu, y = fps, fill = cpu)) + geom_boxplot()
ggplot(data = data, aes(x = cpu, y = fps, fill = gpu)) + geom_boxplot()

#Means
data1 <- within(data, {x <- paste(cpu, gpu)})
```

```
mean_cpu = with(data1, tapply(fps, cpu, mean))
mean_gpu = with(data1, tapply(fps, gpu, mean))
mean_cell = with(data1, tapply(fps, list(cpu, gpu), mean))
mean_grand = mean(fps)


#Mean CI
tukeyCIs1 <- TukeyHSD(aov(lm(fps ~ cpu, data = data)),
factor = 3, conf.level = 0.95)
tukeyCIs2 <- TukeyHSD(aov(lm(fps ~ gpu, data = data)),
factor = 2, conf.level = 0.95)
plot(tukeyCIs1)
plot(tukeyCIs2)


#Sample size
sample_size <- pwr.anova.test(k = 6, n = NULL, f = 0.5,
sig.level = 0.05, power = 0.95)
```