Time Series Model and Forecast of CAD/USD FX Price with Interest Rate, Inflation, Trade Balance and Unemployment Rate

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Abstract

This paper aims to discover and explain the dynamics and interactions between FX Price and different macroeconomic variables. By the market efficiency hypothesis, long-run arbitrage opportunities should not appear in the long run for any financial instruments in spot, forward or futures market. So, these variables under a macroeconomic system should have stability (i.e. the modulus of all eigenvalues/roots of the reverse characteristic polynomial is less than 1). We will be testing these relationships within the paper and hope to draw inferential conclusions and forecasts accordingly.

1 Introduction

This paper is an empirical study on the relationship of exchange rate between USD and CAD in the spot market and the macroeconomic variables including interest rate, inflation rate, trade balance and unemployment rate. The relationship between exchange rate, interest rate, inflation rate and trade balance has been discussed in multiple ways in the last couple of years. Al - Masbhi and Du (2006), Tafa (2015), Deka and Dube (2021) have shown that the variables listed explain a considerable degree of both short and long term dynamics of how FX Prices (CAD/USD) trends and fluctuates. Here, we hypothesizes that the mid-to-long term dynamics can be captured by inflation rate differential, interest rate differential, trade balance differential, and unemployment rate differential such that we can construct a vector of simultaneous equations with each variable defined by a selection of lags of each variable respectively. This paper will also investigate the causal relationship between each variable and see if unit root processes (e.g. inflation, interest rates, FX Price, etc.) cointegrates in the long run, as suggested by Ghalayini (2014). We will run through these tests and construct an appropriate vector time series model based on the statistical results.

2 Literature Review

Interest rate and inflation rate are arguably the two biggest macroeconomic factor that has an impact on the movement of exchange rate. Al - Masbhi and Du (2006) built a model to explain the exchange rate between the Yemeni Rial (YER) against the US Dollar (USD) from 1998 to 2008, including interest rate, inflation rate, GDP growth and current account (CA) balance. The authors used a VECM model to predict the long-run relationship and short-run dynamics between variables.

In the end, the coefficients for the error correction terms are significant such that there exists long term correlation between variables, in a sense that GDP growth and current trade balance negatively affects exchange rate. Conversely, exchange rate shares a positive relation with interest rate and inflation rate. Tafa (2015) showed that interest rate can have two-way relationship with exchange rate. She built 2 separate simple regression models using monthly data from 2002 to 2014, to analyze how interest rate on deposits in Albania affects the exchange rates between the Albanian Lek (ALL) and both the US Dollar (USD) and the Euro (EUR). The results indicate that an increase in interest rate would increase USD/ALL but decrease EUR/ALL. While all the findings are statistically significant, the fact that the author does not control for other macroeconomic factors and the impact of the 2008 financial crisis makes us highly doubt the paper's accuracy. Hacker, Karlsson and Mansson (2012) provides an empirical study suggests that time horizon has an influence on the relationship between exchange rate and interest rate. The authors used data in Serbia in roughly 10 years and reggressed interest rate on exchange rate then found that the negative relationship between the difference of interest rate between 2 countries and exchange rate appears to be negative in short term, but this effect gradually gets smaller and eventually becomes positive after the 1 year juncture. This result is consistent with multiple exchange rate (SEK/US, SEK/JPY, SEK/EUR, SEK/CHF, SEK/KRW) and both monthly and quarterly data. Recently, a long run relationship between inflation rate and exchange rate was found by Deka and Dube (2021) in a way that an increase in inflation level would make exchange rate depreciate, accounting for renewable energy. The study was done by using ARDL (Autoregressive distributed lag) model and yearly data from 1990 to 2019. On the other hand, it is essential to explore the correlation between exchange rates and another key macroeconomic indicator: unemployment. Adzugbele, Eze, Morba and Nwokocha (2020) analyzed this relationship using the ADRL (Autoregressive distributed lag) and found that a rise in real exchange rate would increase the unemployment rate. Theories and past empirical studies show that the effect of exchange rate on unemployment rate is more direct than the other way around. Therefore, we are interested in verifying for the opposite (ie. How unemployment affects exchange rate)

3 Data

The paper analyzes US/CAD Forex prices, interest rate differential, inflation differential, trade balance percentage change differential and unemployment rate differential over a time frame of 10 years (January 2013 - December 2023). We choose this specific period of time because we want to make sure that the 2008 financial crisis does not make any negative impact on our results. All differential variables are constructed as US's variable - Canada's variable. 1 year T-bills rate is used to proxy for interest rate due to its sensitivity to monetary policy changes. We calculate percentage change in CPI for both countries which eventually will give us the inflation rate. Countries' trade balance are in terms of their currency (eg. the unit of Canada's trade balance is CAD), therefore it is not sufficient to compute the difference of two variables that have different currency. We normalize trade balance by calculating their percentage change then taking the difference to address this problem. The final dataset consists of 132 observations. Table 1 and Table 2 breaks down the characteristics of the variables.

4 Empirical Framework

4.1 Time Series Models

The main econometric framework in this paper is a Vector Autoregressive Model (VAR). The logical flow of our statistical tests are as follow:

Firstly, we run the Augmented Dickey Fuller Test to see if any of the 5 variables are unit root processes. If so, we will run the Engle-Granger Test among those unit root variables to see if there exist a bivariate cointegration relationship. If there exists a pairwise cointegration, we will proceed with Vector Error Correction Model (VECM) as our main model. Otherwise, if none of the unit root processes are cointegrated, we will construct a full-rank Vector Autoregressive Model (VAR) with stationary variables and n-differenced unit root variables such that the n-th difference is $I(0)$. Our model selection criteria are based on information criterion minimization, serial correlation test, Autoregressive Conditional Heteroskedasticity (ARCH) test, Normality test, and structural break/stability test. Additionally, we run Granger Causality Test and Impulse Response Analysis to check for causal relationships and contemporaneous shock effects. Here, we lay out the major time series models, estimation methods, and model selection procedures used in this paper.

4.1.1 Vector Autoregressive Model (VAR)

The VAR model with p lags generally denoted as VAR(p) is represented by the following mathematical model:

$$y_t = v + A_1 y_{t-1} + \cdots A_p y_{t-p} + u_t \tag{1}$$

where y_t in a $K \times 1$ vector of K endogenous time series variables of interest. v is a $K \times 1$ vector of intercept constants permitting $E(y_t) \neq 0$, and u_t is a $K \times 1$ vector of white noise (i.e. $u_t \overset{\text{iid}}{\sim} (0, \Sigma_u)$). A_j is the $K \times K$ coefficient matrix of the j-th lag.

For example, in using the variables of the paper to construct a VAR model, it will be represented in matrix form as follows:

$$\begin{bmatrix} \Delta\text{FX}_t \\ \Delta\text{Int.Diff}_t \\ \text{Inf.Diff}_t \\ \text{Trade.Diff}_t \\ \Delta\text{Unemp.Diff}_t \end{bmatrix} = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{bmatrix} + A_1 \begin{bmatrix} \Delta\text{Fx}_{t-1} \\ \Delta\text{Int.Diff}_{t-1} \\ \text{Inf.Diff}_{t-1} \\ \text{Trade.Diff}_{t-1} \\ \Delta\text{Unemp.Diff}_{t-1} \end{bmatrix} + \cdots + A_p \begin{bmatrix} \Delta\text{Fx}_{t-p} \\ \Delta\text{Int.Diff}_{t-p} \\ \text{Inf.Diff}_{t-p} \\ \text{Trade.Diff}_{t-p} \\ \Delta\text{Unemp.Diff}_{t-p} \end{bmatrix} + \begin{bmatrix} u_{1t} \\ u_{2t} \\ u_{3t} \\ u_{4t} \\ u_{5t} \end{bmatrix} \tag{2}$$

Notice that we include the first-differenced variables for FX, Int.Diff and Unemp.Diff instead of the undifferenced variables at time t. One underlying assumption of a standard VAR model is that all variables have to be stationary, and hence we will explain the reason behind this when we reach Augmented Dickey Fuller's Test. Note that we use Yule Walker's Equations in finding the estimators for autocovariance/autocorrelation at lag k (denoted by $\Gamma_y(k)$ and $\rho_y(k)$ respectively) and suppose that VAR(p) model is stable. We estimate the coefficient matrices using maximum likelihood method.

4.1.2 Seasonal Autoregressive Integrated Moving Average (SARIMA) Model

The autoregressive integrated moving average model with seasonality (ARIMA(p,d,q)(P,D,Q)[s]) using backshift operator is defined as follow:

$$\begin{aligned} &\underbrace{(1-\phi_1 B-\phi_2 B^2-\cdots-\phi_p B^p)}_{\text{non-seasonal AR(p) poly}}\underbrace{(1-B)^d}_{\text{d-th diff}}\underbrace{(1-\Phi_1 B^s-\Phi_2 B^{2s}-\cdots-\Phi_P B^{Ps})}_{\text{seasonal AR(P) poly}}\underbrace{(1-B^s)}_{\text{D-th diff}}^{D} Y_t^* \\ &\qquad = \underbrace{(1-\theta_1 B-\theta_2 B^2-\cdots-\theta_q B^q)}_{\text{non-seasonal MA(q) poly}}\underbrace{(1+\Theta_1 B^s+\Theta_2 B^{2s}+\cdots+\Theta_Q B^{Qs})}_{\text{seasonal MA(Q) poly}}\epsilon_t \end{aligned} \tag{3}$$

Note that Y_t^* is a de-meaned stocastic variable, i.e. $Y_t^* = Y_t - E(Y_t) = Y_t - \mu$, and the backshift operator B is defined as $B^p Y_t = Y_{t-p}$. Since we can denote any ARIMA/AR/ARMA/MA models under the SARIMA family with the correct indices, this paper takes the liberty in generalizing all these models under SARIMA. For example, we are able to fit an ARIMA(1,1,1) model to FX Price in the train-validation-test section, which is equivalent to a SARIMA(1,1,1)(0,0,0)[0] model. We will explain how each variables are fit to their respective autoregressive models under the SARIMA family later on in this paper.

4.2 Estimation Strategy and Model Selection Criteria

The paper is divided into two main sections:

1. ARIMA/SARIMA inferences of each macroeconomic variable

2. Construction of VAR model

The reason why we try to fit ARIMA/SARIMA models before aggregating our analysis is that this will enable us to understand the behaviour of each variable, such as stationarity, seasonality, trend, etc. and provide insights in lag order identification, and potentially capturing short-term or long-term dynamics of each variable which can improve model robustness and forecasting accuracy.

4.2.1 Sample ACF and PACF

In fitting the appropriate model, we can use the ACF and PACF to make an educated guess of the lag order of each variable may follow. Since we have finite number of observations (132 monthly), we used the sample ACF to predict MA order and the sample PACF to predict the AR order. Note that both estimators are unbiased and consistent. The sample autocovariance function is defined as follow:

$$\hat{\gamma}(h) = \frac{1}{n}\sum_{t=1}^{n-h}(x_{t+h} - \bar{x})(x_t - \bar{x}) \tag{4}$$

with $\hat{\gamma}(-h) = \hat{\gamma}(h)\ \forall h \in \mathbb{N}$, $h = 0, 1, \ldots, n-1$ where n is the number of observations we have (in this case 132) and h is the constant difference in time indices.

The sample autocorrelation function at lag h is defined as a fraction of the autocovariance at lag h over the autocovariance at lag 0 (which is also the variance of the time series variable Y_t:

$$\hat{\rho}(h) = \frac{\hat{\gamma}(h)}{\hat{\gamma}(0)} \equiv \frac{\hat{\gamma}_{t,s}}{\sqrt{\hat{\gamma}_{t,t}\hat{\gamma}_{s,s}}} \tag{5}$$

Note that writing the ACF in terms of the time indices difference $h = |t - s|$ for any $t, s \in \mathbb{R}$ implies the process is stationary. Otherwise, a more prudent expression will be $\hat{corr}(Y_t, Y_{t-h})$. One important application of the ACF is to provide insights on the lag order of the MA part by counting the number of lags protruding the 95% confidence band.

On the contrary, the partial autocorrelation function (PACF) provides insights on the lag order in the AR part. Suppose we want to find the PACF for a de-meaned stationary time series x_{t+h}, denote $\hat{x}_{t+h} = \hat{\beta}_1 x_{t+h-1} + \hat{\beta}_2 x_{t+h-2} + \cdots + \hat{\beta}_{h-1} x_{t+1}$, i.e. we perform a regression on all the variables between x_{t+h} and x_t. Then the PACF at lag h for any $h \geq 2$ is defined as follow:

$$\phi_{hh} = corr(x_{t+h} - \hat{x}_{t+h}, x_t - \hat{x}_t) \tag{6}$$

Note that the ACF and PACF graphs are generated by these equations.

4.2.2 Information Criterion

In the ARIMA model selection for each variable in the first section, we mainly use the **Akaike's Information Criterion** (AIC) to determine the model which minimizes information loss. The formula of AIC is as follow:

$$\text{AIC} = -2log(\mathcal{L}) + 2k \tag{7}$$

where $\mathcal{L}$ is maximum likelihood function of the joint probability distribution of observing the time series variables, e.g. FX Price and its estimators. Since a larger likelihood ratio under the negative log function will yield a smaller number, the function intuitively is minimizing information loss by maximizing the joint observance of the data and estimators. Note the 2k is a penalizing factor inversely proportional to the model complexity where k is the number of parameters in the stochastic process.

In the VAR model selection section, we will use multiple information criteria to determine the optimal model. Including Alkaike's Information Criterion, the additional criteria used are **Schwarz Bayesian (SC(m)/BIC)**, **Hannan-Quinn (HQ(m))**, and **Final Prediction Error (FPE(n))**. These information criteria are very similar to AIC, just differs in terms of model fit and complexity penalization. However, note that a VAR model is doing analysis on $\mathbb{R}^m$ instead of $\mathbb{R}^1$, so the formulas for the information criteria for VAR(m) will differ slightly:

$$\text{AIC}(m) = log|\hat{\Sigma}_u(m)| + \frac{2mK^2}{n} \tag{8}$$

where m is the order/number of lags of the VAR model, $\hat{\Sigma}_u(m)$ is the exact likelihood estimator for the white noise covariance matrix, K is the dimension of the time series, and n is the sample size.

For order m Schwarz Bayesian Information Criterion (BIC) in VAR, which in R is denoted as SC(m), the formula is as follow:

$$\text{SC(m)} = log|\hat{\Sigma}_u(m)| + \frac{log(n)}{n}mK^2 \tag{9}$$

where the $\hat{\Sigma}_u(m)$, n, m and K share the same definition as above.

For order m Final Prediction Error (FPE) Information Criterion, the formula is as follow:

$$\text{FPE(m)} = \left[\frac{n + Km + 1}{n - Km - 1}\right]^K det\hat{\Sigma}_u(m) \tag{10}$$

where the $\hat{\Sigma}_u(m)$, n, m and K share the same definition as above.

For order m Hannaqn-Quinn (HQ) Information Criterion, the formula is as follow:

$$\text{HQ(m)} = log|\hat{\Sigma}_u(m)| + \frac{2log(log(n))}{n} mK^2 \tag{11}$$

where the $\hat{\Sigma}_u(m)$, n, m and K share the same definition as above.

Note that in R, instead of reporting the actual information criterion at order m, it reports the difference between the order m criterion and the minimum criterion of some order k ($\exists k \in m$).

4.2.3 Data Partition (Train-Test Split)

On a high level, this is a machine learning technique. For FX Price, we observed potential seasonality and trend, i.e. $Y_t = \hat{S}_t + \hat{T}_t + \hat{R}_t$, i.e. Y_t is composed of seasonality, trend and residual/remainder components respectively. Hence, initially we split the 132 observations into two data sets: training set (120 obs) and testing set (12 obs). We use the 120 observations to train 3 models and compare their 12-period forecast against with the testing set. We chose the model that minimizes root mean square error (RMSE) where

$$\text{RMSE} = \sqrt{\frac{1}{n} \sum_{i=1}^{n} (y_i - \hat{y}_i)^2} \tag{12}$$

Eventually, we will use the selected model from this method to perform the actual forecast. Note that we also perform a three-way split to the data set, i.e. train-validation-test split which hypothetically should give a better fit to the actual model.

4.2.4 Augmented Dickey Fuller Test (ADF Test)

In constructing ARIMA and VAR models, it is essential to see if the variables are unit root processes are not. Hence, the ADF Test comes into play here. Note that if there exists a rough linear decay (i.e. not a sharp decay) in the ACF graph, it often implies that the variable is not stationary. Consider the following AR(1) model to illustrate ADF Test:

$$Y_t = \alpha Y_{t-1} + X_t \Rightarrow \Delta Y_t = (\alpha - 1) Y_{t-1} + X_t \tag{13}$$

where X_t is an independent AR(k) process. If we rename $\alpha - 1$ as another parameter Φ to be estimated, i.e. $Y_t = \Phi Y_{t-1} + X_t$. Then our unit root test will be H_0 as $\Phi = 0$ being non-stationary, and H_1 as $\Phi < 0$ being stationary.

4.2.5 Engle-Granger Cointegration Test

Another essential factor we should consider in constructing our VAR model is to see if the unit root processes cointegrates. If there exists cointegration, then a standard VAR model will not be the optimal model. Otherwise, VAR will be a good model in capturing short-to-medium term effects. Note that Engle-Granger Test is a two-step testing process. We firstly regress two unit root processes against each other, in our case, we will have $3P2$ total number of pairwise cointegrated relationships to test. Note that the order of regression should not matter as the both results should be consistent, however, for added robustness, we conduct regressions in both directions in our paper.

The first step of this test is to regress a unit root process Y_t against another unit root process X_t, and predict the residuals from the linear combination, i.e. $Y_t - \hat{\beta} X_t = \hat{\epsilon}_t$.

The second step is to perform the Dickey Fuller Test on the estimated residuals. If the residuals are stationary, i.e. $I(0)$, then this will imply a long-run cointegrated relationship. Otherwise, if the residuals are not stationary, then X_t and Y_t are not cointegrated.

4.2.6 Granger Causality Test

Note the Granger Causality Test is used to determine if one stochastic variable granger causes another stochastic variable. The definition of Granger Causality here is that if one variable is able to minimize the prediction error of the other variable in $t+h$ periods for any $h \geq 1$. In mathematical notations, it is defined as follow:

$$\Sigma_z(h|\Omega_t) < \Sigma_z(h|\Omega_t \backslash \{x_s | s \leq t\}) \tag{14}$$

for at least one $h = 1, 2, \ldots$. In short, the inequality is saying the prediction covariance matrix of h innovations ahead conditioning on all relevant information set Ω_t which includes the information of the variable of interest x_t and its lags should be strictly less than the prediction covariance matrix of z_t of h innovations ahead conditioning on the information subset Ω_t without x_t and all its lags, if and only if x_t granger-causes z_t. For example, if FX price minimizes the prediction error/variance of trade balance percentage change differential, then we say FX price granger-causes trade balance differential.

5 Results

5.1 Foreign Exchange Price

In Figure 1, we plot the time series for the monthly average FX closing price. From first glance, the FX price seems to have a positive upward trend implying non-stationarity. Hence, we run an Augmented Dickey Fuller's (ADF) Test (Table 3) to see whether it is a unit root process. The obtained p-value is $0.44 > 0.05$. Hence, we fail to reject H_0 at 5% significance level, suggesting significant evidence for FX price to be non-stationary (An additional ADF Test is run (Table 4) and shows that FX price is I(1)). This is also supported by the autocorrelation function (ACF) graph (Figure 2) which shows the lags for the moving average (MA) exhibits a slow linear decay. On the other hand, the partial autocorrelation (PACF) graph (Figure 3) indicates only 1 lag lying outside the 95% confidence band, suggesting FX price should hypothetically follow at least an ARIMA(1,1,0) process.

After the in-sample diagnostic test using train-validate-test split method (created 4 samples using the ARIMA diagnostic plot), we notice that SARIMA(1,1,1)(0,0,1)[12] should be the best model as it minimizes both average mean absolute percentage error (AVG MAPE) and average root mean square error (AVG RMSE) (Table 6). Using maximum likelihood method, the model with estimated parameters is as follow:

$$(1-0.0522B)(1-B)\mathrm{FX}_t = (1+0.2175B)(1+0.0721B^{12})\epsilon_t \tag{15}$$

However, visually from Figure 5, model 4 (ARIMA(2,1,2) with drift) seems to have a closer in-sample forecast. We will factor in this issue in creating our VAR model to balance the model complexity and goodness of fit. Nonetheless, using SARIMA(1,1,1)(0,0,1)[12], we create a time series plot with both 80% and 95% prediction intervals for the next 12 periods (Figure 6).

5.2 Interest Rate Differential

In Figure 7, we plot the time series for the monthly interest rate differential. We then run an ADF test (Table 3) which shows that interest rate differential follows a unit root process (P-value $= 0.62 > 0.05$). An additional ADF test is performed (Table 4) on the first-difference of interest rate differential and indicates that it is an I(1) process. Figure 8 and Figure 9 showed the ACF and PACF graph, we can see from the ACF graph, the MA lags show a slow linear decay and oscillates around the confidence band. Using R command "auto.arima", we determine that interest rate differential follows an ARIMA(0,1,1) process, i.e. MA(1) process with 1 differencing. Here, the model using backshift operators with parameter estimates is as follow:

$$(1 - B)\text{Int.Diff}_t = 0.2104\epsilon_t \tag{16}$$

From Figure 10, we can see that the inverse root of the characteristic polynomial of the MA process lies within the unit circle (real part on x-axis and imaginary part on y-axis), which implies the modulus of the root is outside the unit circle, hence invertibility is guaranteed (i.e. we are able to rewrite the MA process as an AR process of infinite order due to the invertibility of the characteristic polynomial). From the residual plot (Figure 11), we can see that the residuals of the MA process seems to be fairly Normally-distributed. In Figure 12, we again create a time series plot with both 80% and 95% prediction intervals for the next 12 periods.

5.3 Inflation Differential

Figure 13 shows the time series for the monthly inflation differential. We then run an ADF test (Table 3) which shows inflation differential is stationary (P-value $= 0.01 < 0.05$).Figure 14 and Figure 15 shows the ACF and PACF graph, we can see that there is only one lag protruding out the confidence band in the ACF graph and only the third lag in the PACF is outside the confidence band. With the help of R, we identify that inflation differential is approximately a white noise process:

$$\text{Inf.Diff}_t \approx \epsilon_t \tag{17}$$

One reason for this to occur is that taking percentage change in CPI can have removed the AR lag or integration order CPI itself exhibits. This could potentially be a limitation of the data and we will discuss more about this in the conclusion part of how this could be done differently.
Note that a time series forecast plot for the next 12 periods is also included (Figure 17).

5.4 Trade Balance Percentage Change Differential

In Figure 18, we plot the time series for the monthly trade balance percentage (pct) change differential. We then run an ADF test (Table 3) which shows trade balance percentage change differential is stationary (P-value = 0.01 < 0.05).Figure 19 and Figure 20 shows the ACF and PACF graph, we can see that there is only one lag protruding out the confidence band in the ACF graph and all the first 20 lags in the PACF are within the confidence band. With R, we identify that trade balance percentage change differential is approximately a white noise process:

$$\text{trade}_t \approx \epsilon_t \tag{18}$$

Again, a reason this occurring may well be because of the nature of the data being percentage change, which by definition is taking the first difference between adjacent data points.
Note that a time series forecast plot for the next 12 periods is also included (Figure 22).

5.5 Unemployment Rate Differential

In Figure 23, we plot the time series for the monthly unemployment rate differential. We then run an ADF test (Table 3) which shows unemployment rate differential to be a unit root process (P-value = 0.29 > 0.05). An additional ADF test is performed (Table 4) on the first-difference of unemployment rate differential and showed that it is an I(1) process. Figure 24 and Figure 25 showed the ACF and PACF graph, we can see from the ACF graph, the MA lags are showing a slow linear decay, whereas the PACF graph suggests unemployment rate differential should have 3 lags in the AR part. Using R, we determine that unemployment rate differential follows an ARIMA(2,1,0) process. Here, the model using backshift operators with parameter estimates is as follow:

$$(1 + 0.7926B + 0.4897B^2)(1 - B)\text{Unemp}_t = \epsilon_t \tag{19}$$

From Figure 26, we can see that both inverse roots of the characteristic polynomial of the AR process lies within the unit circle, which implies the modulus of both roots are outside the unit circle, hence stationarity is guaranteed. From the residual plot (Figure 27), we can see that the residuals seems to be fairly Normally-distributed. In Figure 28, we again create a time series plot with both 80% and 95% prediction intervals for the next 12 periods. One thing to note is that the reason of the prediction interval getting wider when we progress to the future is because the prediction error is increasing (the error is bound to increase when we try to make inferences further into the future with limited data, i.e. data up to time t only).

5.6 Engle Granger Cointegration Tests

From sections 6.1 - 6.5, we identify 3 unit root processes, i.e. FX Price (FX), Interest Rate Differential (Int.Diff), and Unemployment Differential (Unemp.Diff). We ran $6P2 = 6$ pairwise cointegration tests and discover that none of the processes are cointegrated at 5% significance level.

5.7 Vector Autoregressive Model (VAR)

We take first difference of FX, Interest Rate Differential and Unemployment Rate Differential to meet the core assumptions of the VAR model. From Table 4, we can see that FX, Interest Rate Differential, and Inflation Differential are all I(1). Hence, in constructing the VAR model, we take the first difference of these 3 variables and construct a VAR model with a coefficient matrix of dimension of 131×131. From the VAR(m) Information Criteria Table (Table 7), we can see that HQ(m) and SC(m) suggest a VAR(1) model; whereas AIC(m) and FPE(m) suggest a VAR(5) model. Note that after running both models, VAR(1) has a higher AIC (1634.02) but a lower BIC (1720.05); whereas VAR(5) has a lower AIC (1579.39) but a higher BIC (1948.11). Since both models perform similar in the robustness check, so we proceed to select the VAR(1) model due to lower complexity.

Referring to Table 8, the results of the VAR(5) model are shown. Notice that a VAR(5) model says that we are including the data of the 5 variables of the previous 5 months to explain the changes in the response variable.

With FX Price (first-differenced) as the response variable, we can see that only the coefficients of the FX Price at lag 1 and lag 5, Interest Rate Differential at lag 2, Trade Balance Differential at lag 2 are significant at 10% level. The interpretation of the results of each explanatory variable are as follow:

1. We can see that an increase in 1 unit of lag 1 FX Price is associated with 0.2 units increase in the current FX Price. And for lag 5 FX Price, we can see that an increase in 1 unit is associated with 0.24 units increase in the current FX Price. This implies that there is a lag dependence of previous FX price on the current FX price, which also partially resonates with our findings in the ARIMA model we fitted in Section 5.1, as FX Price defined by itself follows a SARIMA(1,1,1)(0,0,1)[12].

2. For every unit increase in the inflation rate differential at lag 2, FX Price (CAD/USD) decreases by 0.045 units. This is saying that if the gap between the US and Canada risk-free interest rate increases, then the exchange rate will have a negative lagged response by 2 months. One reason to this is that when the US for example has a higher risk-free interest rate than Canada, then intuitively more investors will seek to invest in US assets as they then will have a higher expected return. Hence, the capital influx into US relative to Canada increases such that the demand for USD increases, which pushes the exchange rate of CAD/USD down.

3. The Trade Balance Differential at lag 2 is also significant at 10% level. However, the coefficient shown is smaller than 0.0001 (2.467e-05 in R). This indicates that a 1 unit increase in lagged variable of Trade Balance Differential of the 5th month is associated with approximately 0.000025 units increase in current FX Price.

With Interest Rate Differential (first-differenced) as the response variable, we can see that only the coefficient of Trade Balance Differential at lag 5 is at 10% significance level. The interpretation of the results are as follow:

1. For every unit increase in Trade Balance Differential 5 months ago is associated with 0.0001 decrease in Interest Rate Differential. One reason can be because when US trade balance percentage change is greater than the Canada trade balance percentage change (or Canada trade balance percentage change is smaller), then this implies that the demand for CAD relative to USD becomes smaller, then the Canadian central bank may relax its interest rate to attract foreign investment, whereas the US central bank may raise their interest rate to

put the breaks on inflationary momentum. Note that the impact of trade to have a latent effect, hence the lag variable of trade from 5 months ago associating with changes of Interest Rate Differential will intuitively and statistically be reasonable.

Here, with Inflation Rate Differential as the response variable, we can see that Unemployment Rate Differential at lag 1, and Inflation Rate Differential at lag 2 are significant at 10% level. While Inflation Rate Differential at lag 1, and FX Price at lag 3 are significant at 5% level. Most importantly, the Unemployment Rate Differential at lag 2, Inflation Rate Differential at lag 3, FX Price at lag 4, Unemployment Rate Differential at lag 4, and Unemployment Rate Differential at lag 5 are significant at 5% level. The interpretation of the results are as follow:

1. We can see that 1 unit increase of FX Price 3 months ago is associated with 91.99 units increase in Inflation Rate Differential. While 1 unit increase of FX Price at 4 months ago is associated with 157.36 units increase in Inflation Rate Differential.

2. We can see that 1 unit increase of Inflation Rate Differential 1 month ago is associated with 0.18 units decrease in the current Inflation Rate Differential; 1 unit increase of Inflation Rate Differential 2 months ago is associated with 0.12 units decrease in current Inflation Rate Differential; and 1 unit increase in Inflation Rate Differential 3 months ago is associated with 0.26 units decrease in current Inflation Rate Differential.

3. 1 unit increase of Unemployment Rate Differential 1 month ago is associated with 11.97 units increase in Inflation Rate Differential at present. 1 unit increase of Unemployment Rate Differential 2 months ago is associated with 32.60 units increase of current Inflation Rate Differential. 1 unit increase of Unemployment Rate Differential 3 months ago is associated with 68.70 units increase of current Inflation Rate Differential. 1 unit increase of Unemployment Rate Differential 4 months ago is associated with 67.44 units increase of current Inflation Rate Differential. 1 unit increase of Unemployment Rate Differential 5 months ago is associated with 46.19 units increase of current Inflation Rate Differential.

For Trade Balance Differential as the response variable, we can see that the coefficient of FX Price at lag 1 is significant at 10% level. While the coefficients of FX Price at lag 4, Interest Rate

Differential at lag 4, Inflation Rate Differential at lag 4, FX Price at lag 5, and Interest Rate Differential at lag 5 are significant at 5% significance level.

1. For the lags of FX Price, we can see that 1 unit increase in FX Price 1 month ago is associated with 1329.15 units decrease in the present Trade Balance Differential. 1 unit increase in FX Price 4 months ago is associated with 1694.48 units decrease in the present Trade Balance Differential. 1 unit increase in FX Price 5 months ago is associated with 2067.53 units decrease in the present Trade Balance Differential.

2. For the lags of interest rate, we can see that 1 unit increase in Interest Rate Differential 4 months ago is associated with 352.56 units increase in present Trade Balance Differential, while 1 unit increase in Interest Rate Differential 5 months ago is associated with 382.83 units decrease in present Trade Balance Differential.

3. For the lags of inflation rate differential, we can see that 1 unit increase in inflation rate differential 4 months ago is associated with 2.57 units decrease in present trade balance differential.

For Unemployment Rate Differential as the response variable, we can see that only the coefficient of Trade Balance Differential at lag 2 is significant at 10% level, while Unemployment Rate Differential at lag 1 and lag 2 are both significant at 1% level. Below are the interpretations of the results:

1. For the lags of Trade Balance Differential, we can see that 1 unit increase in trade balance differential 2 months ago is associated with 0.0002 units increase in present Unemployment Rate Differential.

2. For the lags of Unemployment Rate Differential, we can see that 1 units increase in Unemployment Rate Differential 1 month ago is associated with 0.84 units decrease in present Unemployment Rate Differential, while 1 units increase in Unemployment Rate Differential 2 months ago is associated with 0.59 units decrease in present Unemployment Rate Differential.

5.8 Robustness Check

In Table 9, we can see that our VAR(5) model passes the Serial Correlation test at lag 17 (p-value $= 0.06 > 0.05$, i.e. we fail to reject $H_0 \Rightarrow$ there is no autocorrelation of the residuals at lag 17).

Note that we use a VAR(5) instead of a VAR(17) as the main model is due to VAR(17) having a very high BIC while having similar performances in the following robustness tests results.

The VAR(5) model passes the Autoregressive Conditional Heteroskedasticity (ARCH) test at 5 lags as the p-value = $0.25 > 0.05$. Hence, we fail to reject H_0 implying that the variance of the error term of our time series process is constant over time and does not have serial dependence over past error terms.

In our Normality Test, the p-value is 2.2e-16 < 0.05. Hence, we reject the H_0 meaning that our residuals are not Normally-distributed.

In Figure 30, we can see that all 5 variables, i.e FX Price, Inflation Differential, Interest Rate Differential, Trade Balance Differential, and Unemployment Rate Differential are stable under the cumulative sum test. This implies that there is evidence that the data does not contain significant structural breaks.

5.9 Granger Causality Test

Finally, we construct a F-test with 10 lags to test for granger causality between variables. Here we only consider stationary variables (i.e. we will take first difference for variables those are I(1) process). Table 10 shows that there are 2 pairs that have causation at 10% significant level, 1 at 5% and 1 at 1% level. In particular, we can be sure at 10% first difference of FX Price causes Inflation Rate Differential and Inflation Rate Differential causes Trade Balance Differential. While Trade Balance Differential causes first difference of Interest Rate Differential at 5%. The most significant causation relation we have is between first difference of Unemployment Rate Differential and Inflation Rate, in a sense that first difference of Unemployment Rate Differential causes Inflation Rate at 1% significant level.

6 Conclusion

From the ARIMA models we constructed for each variable, we can see that we can explain each variable reasonably well with ARIMA or SARIMA models with 1 to 2 lags of their own variable respectively. However, these models do provide statistical/inferential knowledge in constructing the macroeconomic VAR(5) model.

From our VAR(5) model, robustness test, and Granger Causality Test results, we can see that VAR(5) is a reasonable model to explain the short-to-mid term dynamics of FX Price (CAD/USD) and the relationship of FX Price with other macroeconomic variables like Interest Rate Differential, Inflation Differential, Trade Balance Differential and Unemployment Differential. Inflation Rate Differential is the most defined variable we can find, in a way that it has a very strong positive relationship with unemployment. This finding remains consistent and statistically significant across both the Granger Causality and VAR model analyses. Additionally, there is a notable positive association between FX Price and inflation. Recent Unemployment Rate Differential demonstrates a pronounced self-connection. Moreover, we uncover a two-way relationship between Trade Differential and both FX price and Interest Rate Differential in the past, indicative of an unpredictable market dynamic between Canada and the US.

7 References

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8 Tables

Statistic	N	Mean	Median	St. Dev.	Min	Max
FX	132	1.26	1.30	0.10	0.99	1.42
Int.Diff	132	-0.14	-0.08	0.45	-1.00	0.74
Inf.Diff	132	-0.05	0.00	11.17	-78.46	56.25
Trade.Diff	132	10.00	1.00	136.02	-1055.14	855.53
Unemp.Diff	132	-0.38	-0.42	0.19	-1.00	0.30

Table 1: Summary Statistics

	FX	Int.Diff	Inf.diff	Trade.Diff	Unemp.Diff
FX	1	0.821	0.154	-0.095	-0.652
Int.Diff	0.821	1	0.070	-0.014	-0.631
Inf.diff	0.154	0.070	1	0.022	-0.032
Trade.Diff	-0.095	-0.014	0.022	1	-0.026
Unemp.Diff	-0.652	-0.631	-0.032	-0.026	1

Table 2: Correlation Matrix

ADF Test	Variable	Test Statistics	P-Value	Lag order
1	FX	-2.33	0.44	5
2	Int.Diff	-1.90	0.62	5
3	Inf.Diff	-5.23	0.01	5
4	Trade.Diff	-4.958	0.01	5
5	Unemp.Diff	-2.68	0.29	5

Table 3: ADF Test Results

ADF Test	Variable	Statistics	P-Value	Lag order
1	ΔFX	-4.27	0.01	5
2	ΔInt.Diff	-4.27	0.01	5
3	ΔUnemp.Diff	-5.58	0.01	5

Table 4: ADF Test Results for I(1) variable

Co-integration	Relationship	Statistics	P-Value	Lag order
1	FX - Int.Diff	-2.69	0.28	5
2	Int.Diff - FX	-2.34	0.43	5
3	FX - Unemp.Diff	-2.79	0.24	5
4	Unemp.Diff - FX	-3.26	0.08	5
5	Int.Diff - Unemp.Diff	-2.26	0.47	5
6	Unemp.Diff - Int.Diff	-3.29	0.08	5

Table 5: Co-integration Test Results

Model id	Model	Avg MAPE	AVG RMSE
1	SARIMA(1,1,1)(0,0,1)[12]	0.0208	0.0324
2	ARIMA(1,1,1)	0.0212	0.0329
3	ARIMA(2,1,2)	0.0230	0.0339
4	ARIMA(2,1,2)(drift)	0.0230	0.0339

Table 6: Fx Price Train-Validate-Test Split Method

	1	2	3	4	5	6
AIC(m)	-1.457	-1.499	-1.424	-1.387	-1.624∗	-1.565
HQ(m)	-1.182∗	-0.994	-0.689	-0.422	-0.429	-0.140
SC(m)	-0.779∗	-0.255	0.386	0.988	1.318	1.942
FPE(m)	0.233	0.224	0.242	0.254	0.203∗	0.220

Table 7: VAR(m) Information Criteria

	Dependent variable:				
	y				
	FX	Int.Diff	Inf.Diff	Trade.Diff	Unemp.Diff
FX.l1	0.20*	0.54	62.58	−1,329.15*	0.40
	(0.11)	(0.55)	(42.53)	(740.03)	(0.70)
Int.Diff.l1	0.03	0.16	−3.82	100.69	−0.09
	(0.02)	(0.11)	(8.60)	(149.68)	(0.14)
Inf.Diff.2.132..l1	−0.0001	−0.001	−0.18**	−1.75	−0.0003
	(0.0002)	(0.001)	(0.08)	(1.44)	(0.001)
Trade.Diff.2.132..l1	0.0000	0.0000	−0.01	0.05	−0.0000
	(0.0000)	(0.0001)	(0.01)	(0.10)	(0.0001)
Unemp.Diff.l1	−0.03	−0.02	11.97*	120.94	−0.84***
	(0.02)	(0.08)	(6.41)	(111.50)	(0.11)
FX.l2	0.03	0.57	50.28	351.53	0.37
	(0.11)	(0.57)	(43.75)	(761.25)	(0.72)
Int.Diff.l2	−0.04*	−0.12	−7.28	69.64	0.05
	(0.02)	(0.11)	(8.74)	(152.11)	(0.14)
Inf.Diff.2.132..l2	−0.0000	0.0003	−0.12*	−1.76	0.0005
	(0.0002)	(0.001)	(0.07)	(1.26)	(0.001)
Trade.Diff.2.132..l2	0.0000*	0.0001	0.0002	−0.02	0.0002*
	(0.0000)	(0.0001)	(0.01)	(0.09)	(0.0001)
Unemp.Diff.l2	−0.02	0.01	32.60***	89.76	−0.59***
	(0.02)	(0.11)	(8.35)	(145.27)	(0.14)
FX.l3	−0.15	−0.07	91.99**	894.07	−0.84
	(0.11)	(0.55)	(42.15)	(733.50)	(0.70)
Int.Diff.l3	0.01	−0.02	−5.55	−248.49	−0.01
	(0.02)	(0.12)	(8.98)	(156.30)	(0.15)
Inf.Diff.2.132..l3	−0.0002	−0.001	−0.26***	1.36	0.0000
	(0.0002)	(0.001)	(0.07)	(1.18)	(0.001)
Trade.Diff.2.132..l3	0.0000	0.0000	0.001	−0.06	−0.0001
	(0.0000)	(0.0001)	(0.01)	(0.10)	(0.0001)
Unemp.Diff.l3	−0.02	0.08	68.70***	169.23	−0.17
	(0.02)	(0.12)	(9.28)	(161.56)	(0.15)
FX.l4	0.03	−0.17	157.36***	−1,694.48**	0.88
	(0.11)	(0.55)	(42.63)	(741.70)	(0.71)
Int.Diff.l4	−0.02	0.05	−14.46	352.56**	0.01
	(0.02)	(0.12)	(8.96)	(155.96)	(0.15)
Inf.Diff.2.132..l4	−0.0001	−0.0003	−0.10	−2.57**	−0.001
	(0.0002)	(0.001)	(0.07)	(1.27)	(0.001)
Trade.Diff.2.132..l4	−0.0000	0.0000	−0.0004	0.02	−0.0000
	(0.0000)	(0.0001)	(0.01)	(0.10)	(0.0001)
Unemp.Diff.l4	0.01	0.18	67.44***	248.68	−0.10
	(0.02)	(0.11)	(8.81)	(153.36)	(0.15)
FX.l5	0.24*	0.51	30.07	2,067.53**	−0.68
	(0.12)	(0.60)	(46.22)	(804.22)	(0.77)
Int.Diff.l5	−0.01	0.01	10.52	−382.83**	−0.06
	(0.02)	(0.12)	(9.22)	(160.43)	(0.15)
Inf.Diff.2.132..l5	−0.0000	0.0004	−0.08	0.83	−0.001
	(0.0002)	(0.001)	(0.07)	(1.19)	(0.001)
Trade.Diff.2.132..l5	−0.0000	−0.0001*	−0.002	−0.05	0.0001
	(0.0000)	(0.0001)	(0.01)	(0.10)	(0.0001)
Unemp.Diff.l5	−0.01	0.12	46.19***	155.36	0.002
	(0.02)	(0.09)	(6.79)	(118.13)	(0.11)
const	0.001	0.01	0.43	13.87	−0.01
	(0.002)	(0.01)	(0.76)	(13.22)	(0.01)
Observations	126	126	126	126	126
R^2	0.28	0.16	0.58	0.19	0.51
Adjusted R^2	0.10	−0.05	0.47	−0.01	0.39
Residual Std. Error (df = 100)	0.02	0.10	7.99	138.99	0.13
F Statistic (df = 25; 100)	1.54*	0.78	5.51***	0.94	4.21***
Note:					*p<0.1; **p<0.05; ***p<0.01

Table 8: VAR(5) Model Result

Test Name	Test Statistic	Df	P-value
Serial Correlation Test	$\chi^2 = 339.31$	300	0.06
Arch Test	$\chi^2 = 1156.20$	1125	0.25
Normality Test	$\chi^2 = 4247.20$	5	2.2e-16

Table 9: Robustness Check Results for VAR(5)

F-Test	Null Hypothesis	F-Statistics	P-Value	Degree of freedom
1	ΔInt.Diff $\nRightarrow$ ΔFX	0.89	0.55	-10
2	ΔFX $\nRightarrow$ ΔInt.Diff	0.48	0.90	-10
3	Inf.Diff $\nRightarrow$ ΔFX	0.79	0.63	-10
4	ΔFX $\nRightarrow$ Inf.Diff	1.72	0.09*	-10
5	Trade.Diff $\nRightarrow$ ΔFX	0.98	0.47	-10
6	ΔFX $\nRightarrow$ Trade.Diff	1.40	0.19	-10
7	Inf.Diff $\nRightarrow$ ΔInt.Diff	0.19	1.00	-10
8	ΔInt.Diff $\nRightarrow$ Inf.Diff	0.74	0.69	-10
9	Trade.Diff $\nRightarrow$ ΔInt.Diff	2.52	0.01**	-10
10	ΔInt.Diff $\nRightarrow$ Trade.Diff	1.01	0.44	-10
11	Trade.Diff $\nRightarrow$ Inf.Diff	0.16	1.00	-10
12	Inf.Diff $\nRightarrow$ Trade.Diff	2.19	0.02*	-10
13	ΔUnemp.Diff $\nRightarrow$ ΔFX	0.85	0.58	-10
14	ΔFX $\nRightarrow$ ΔUnemp.Diff	1.28	0.25	-10
15	ΔUnemp.Diff $\nRightarrow$ ΔInt.Diff	0.49	0.89	-10
16	ΔInt.Diff $\nRightarrow$ ΔUnemp.Diff	1.56	0.13	-10
17	Inf.Diff $\nRightarrow$ ΔUnemp.Diff	0.47	0.90	-10
18	ΔUnemp.Diff $\nRightarrow$ Inf.Diff	11.64	0.00***	-10
19	Trade.Diff $\nRightarrow$ ΔUnemp.Diff	0.89	0.55	-10
20	ΔUnemp.Diff $\nRightarrow$ Trade.Diff	0.52	0.87	-10
Note:			*p<0.1; **p<0.05; ***p<0.01	

Table 10: Granger Causality Test Results

9 Figures

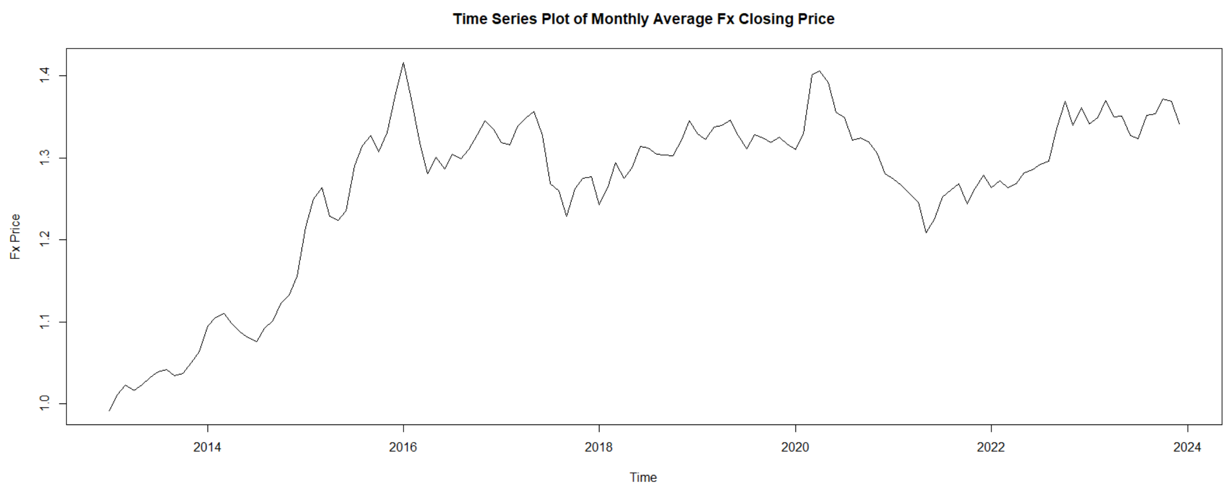

Figure 1: Time Series Plot of Average FX Closing Price

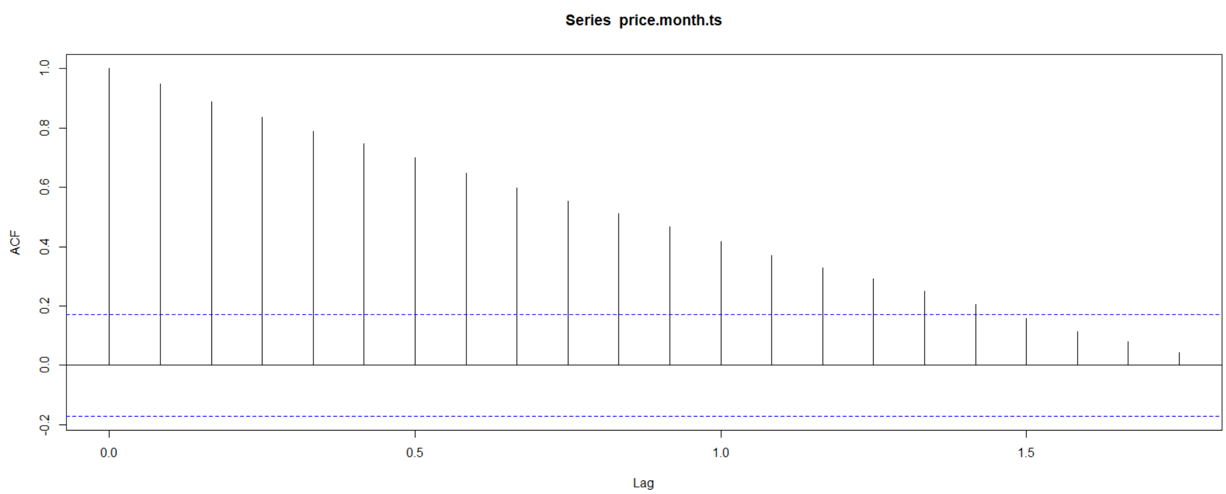

Figure 2: ACF Graph of Monthly FX Price

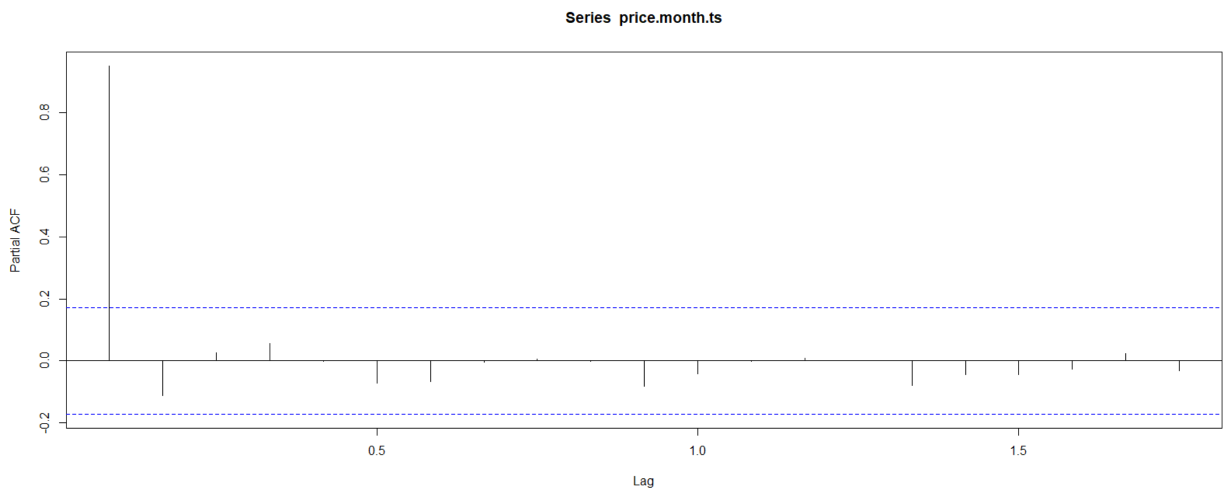

Figure 3: ACF Graph of FX Price

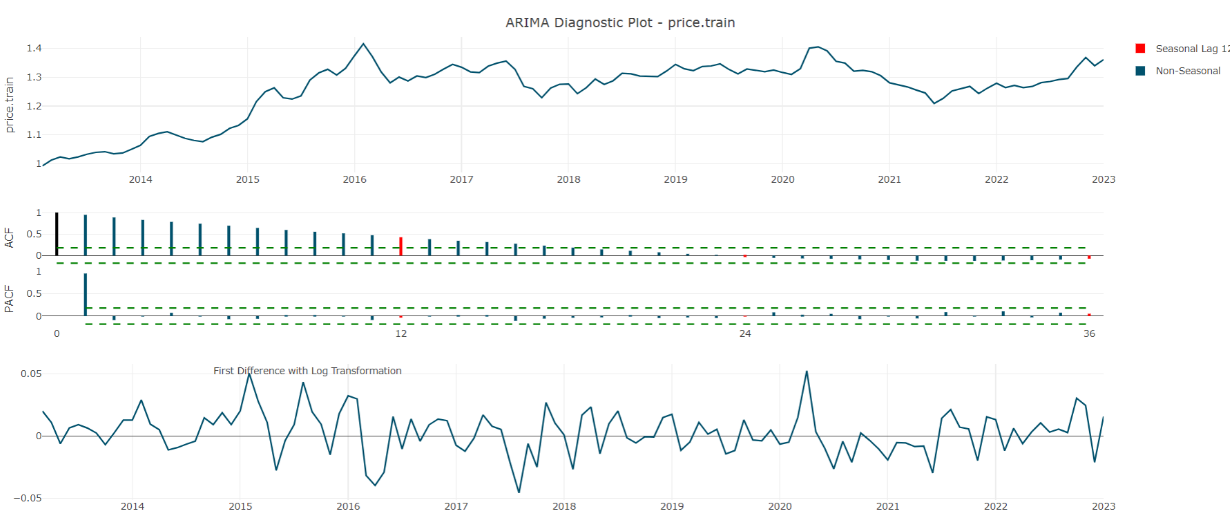

Figure 4: ARIMA Diagnostic Plot of FX Price

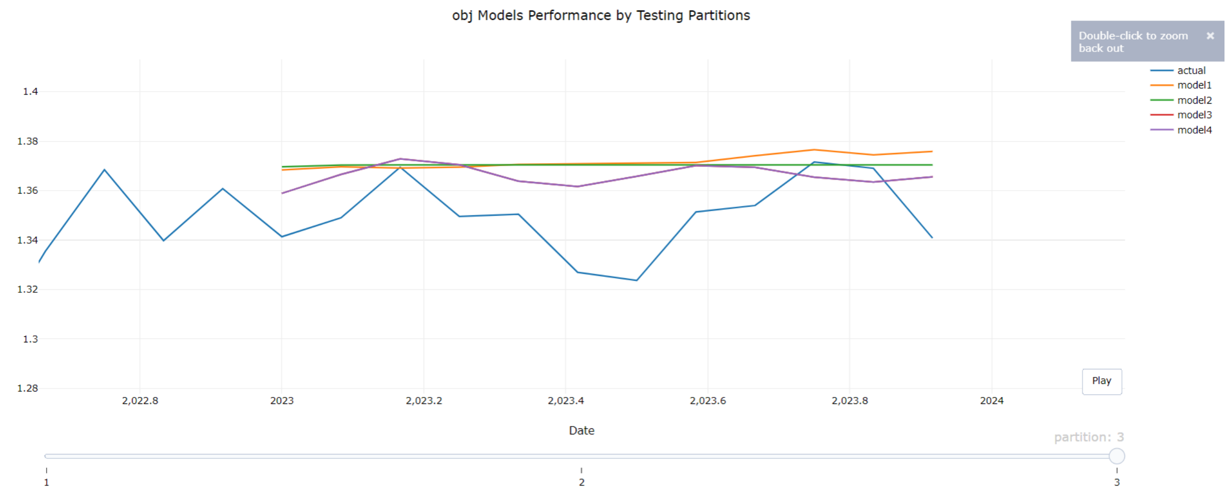

Figure 5: Train-Validate-Test Patition of FX Price

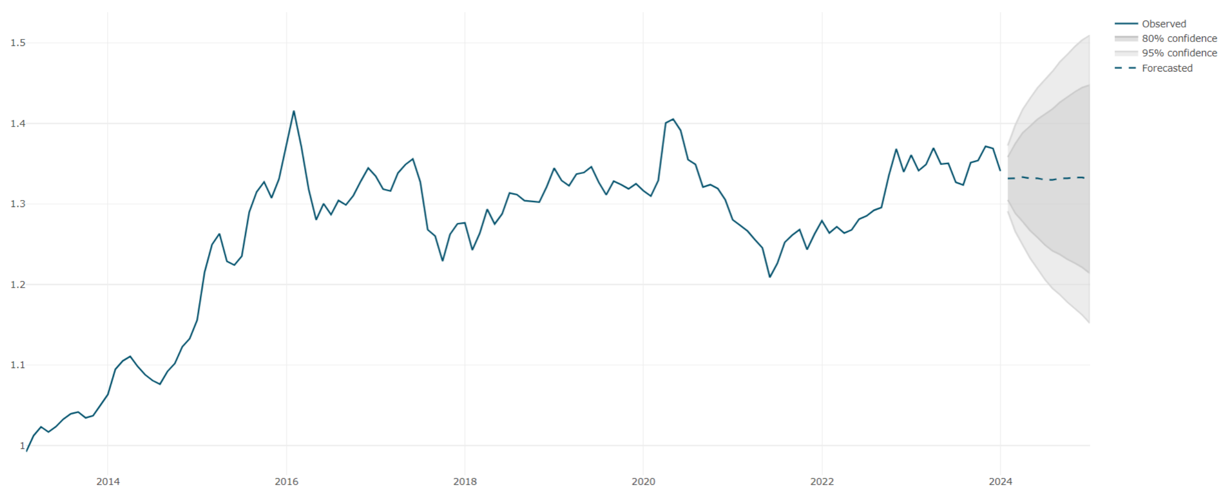

Figure 6: Time Series Plot with Forecast of FX Price

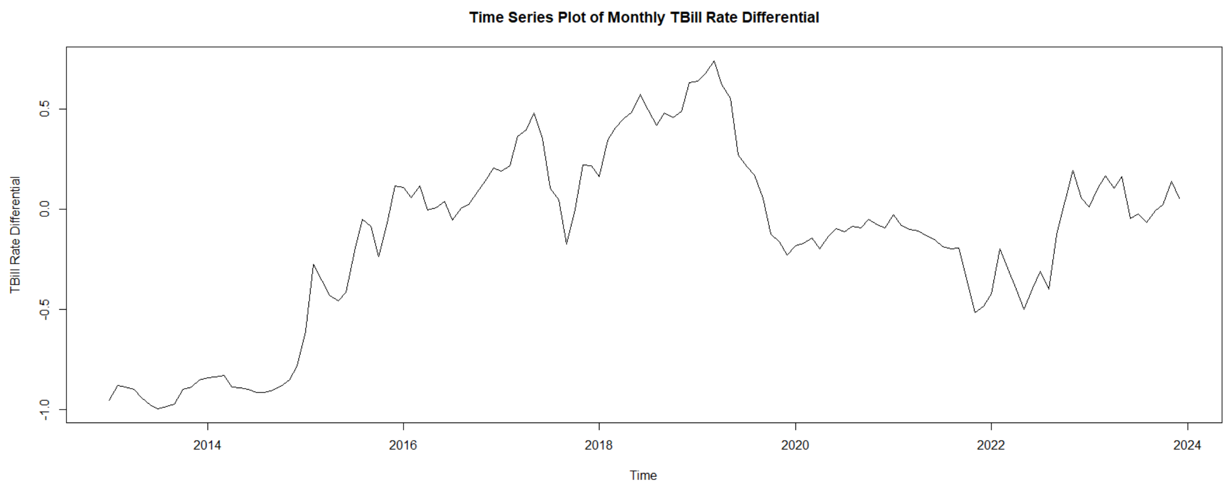

Figure 7: Time Series Plot of Interest Rate Differential

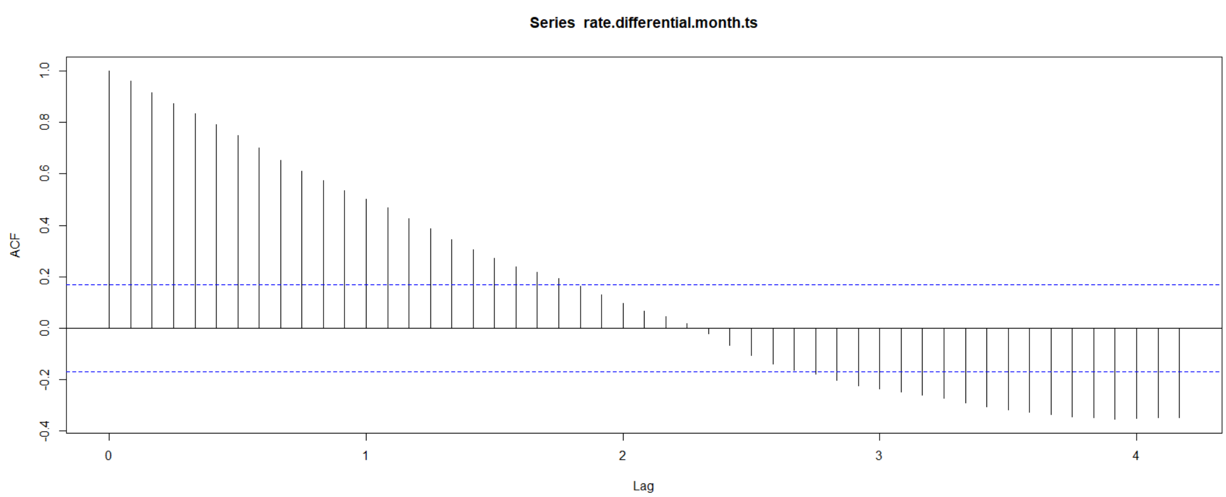

Figure 8: ACF Graph of Interest Rate Differential

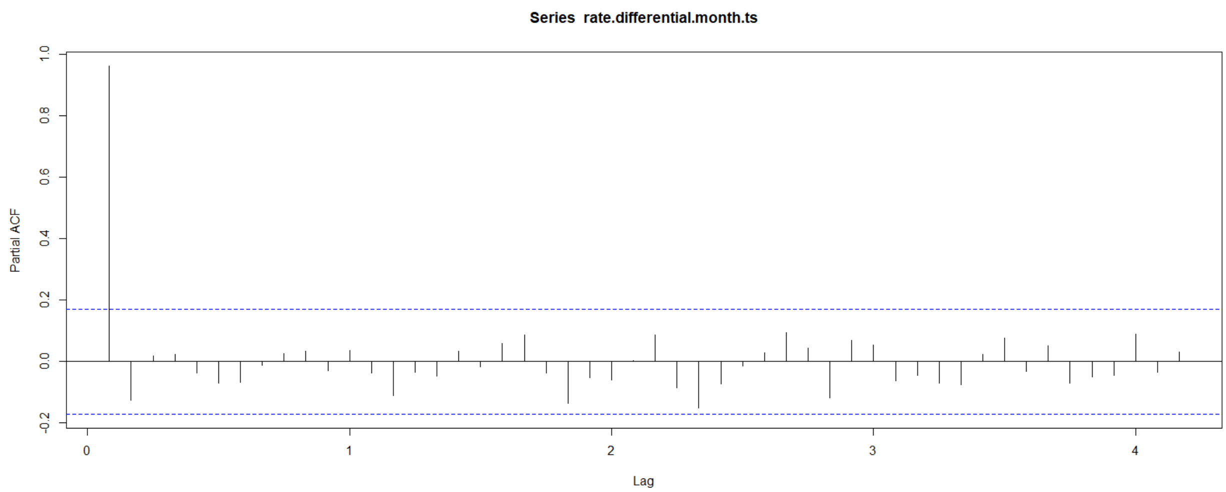

Figure 9: PACF Graph of Interest Rate Differential

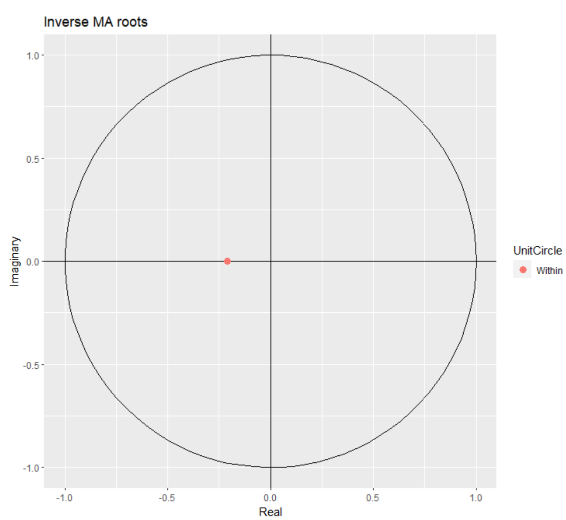

Figure 10: Inverse Characteristic Polynomial Root Graph of Interest Rate Differential

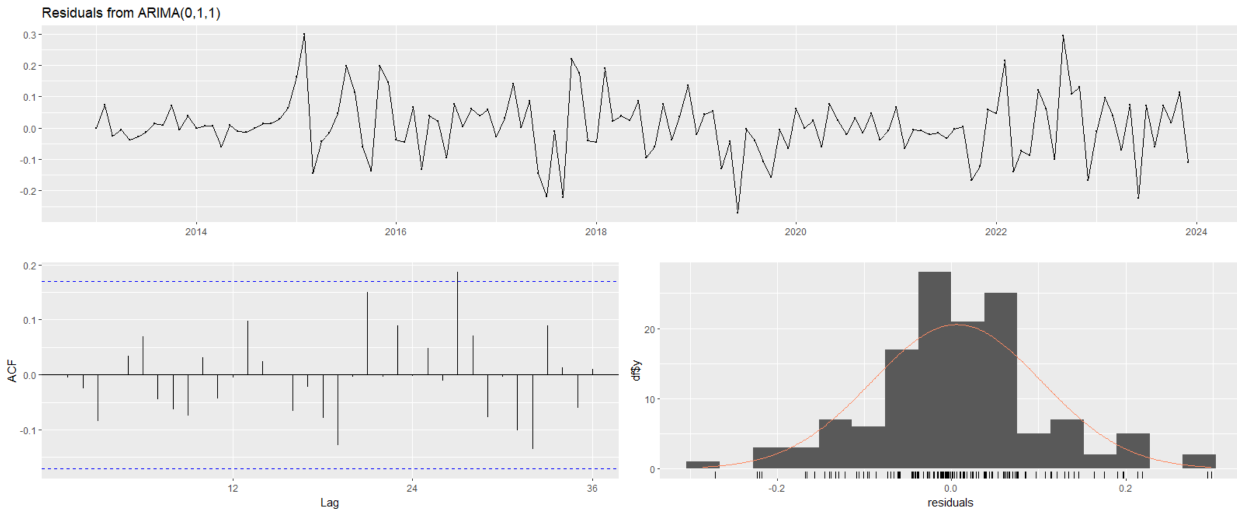

Figure 11: Residual Plot of Interest Rate Differential

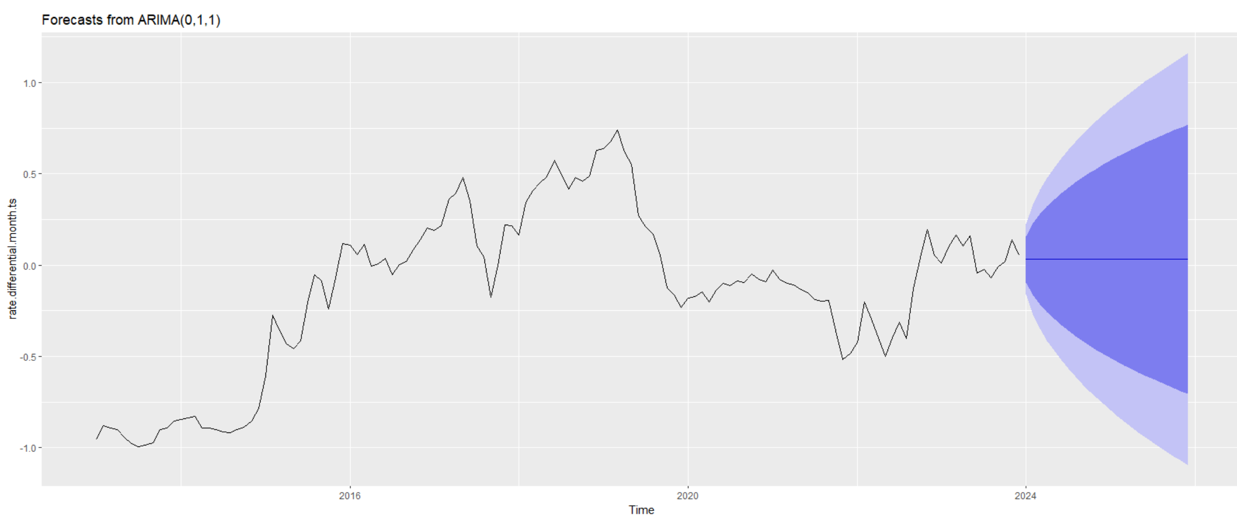

Figure 12: Time Series Plot with Forecast of Interest Rate Differential

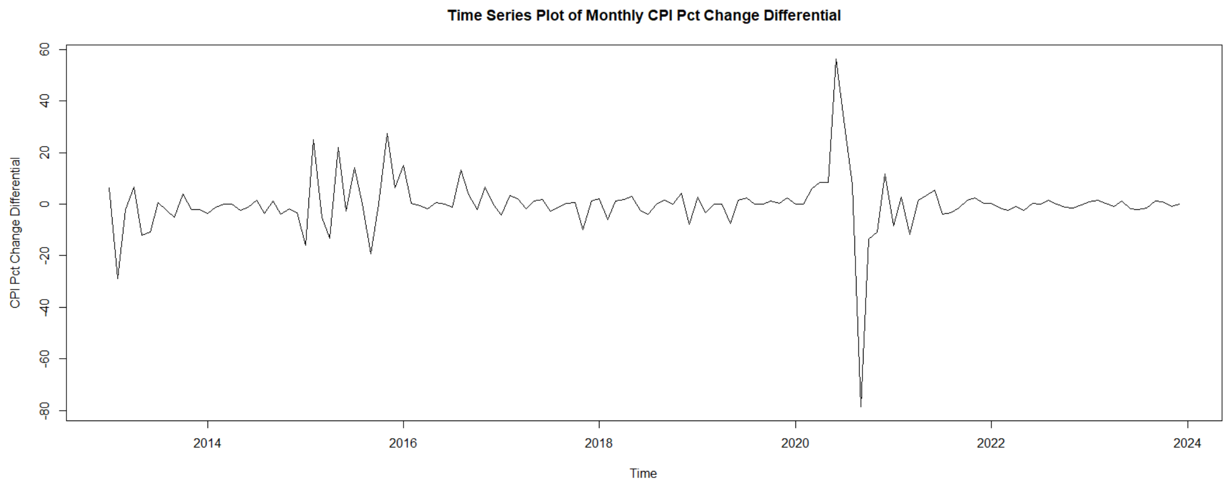

Figure 13: Time Series Plot of Inflation Differential

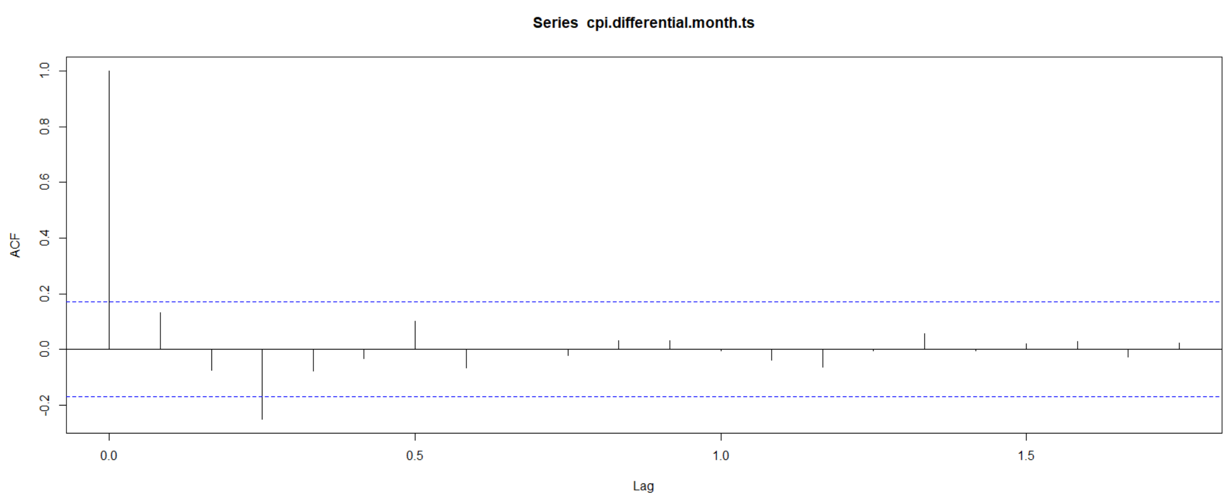

Figure 14: ACF Graph of Inflation Differential

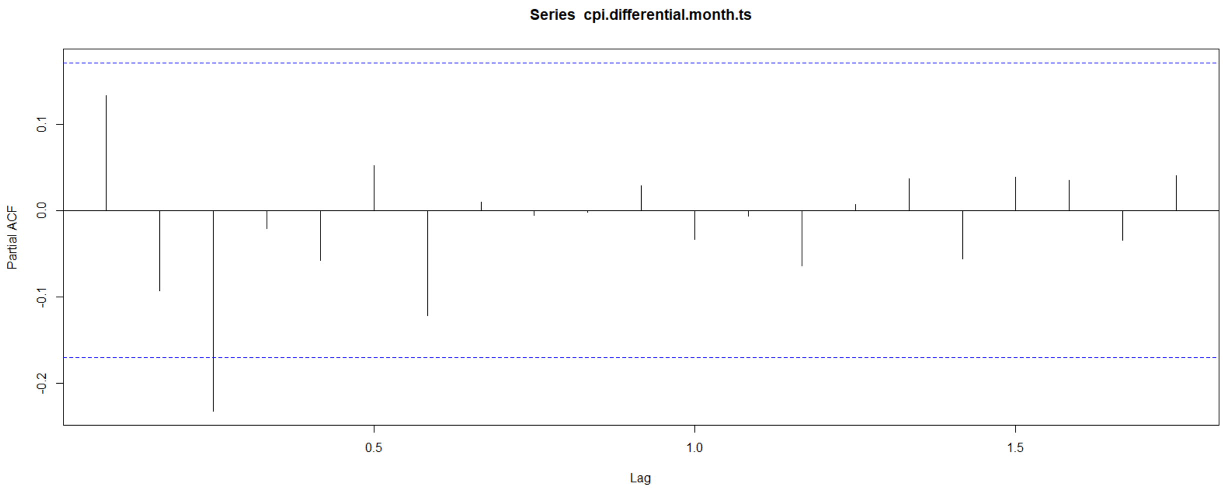

Figure 15: PACF Graph of Inflation Differential

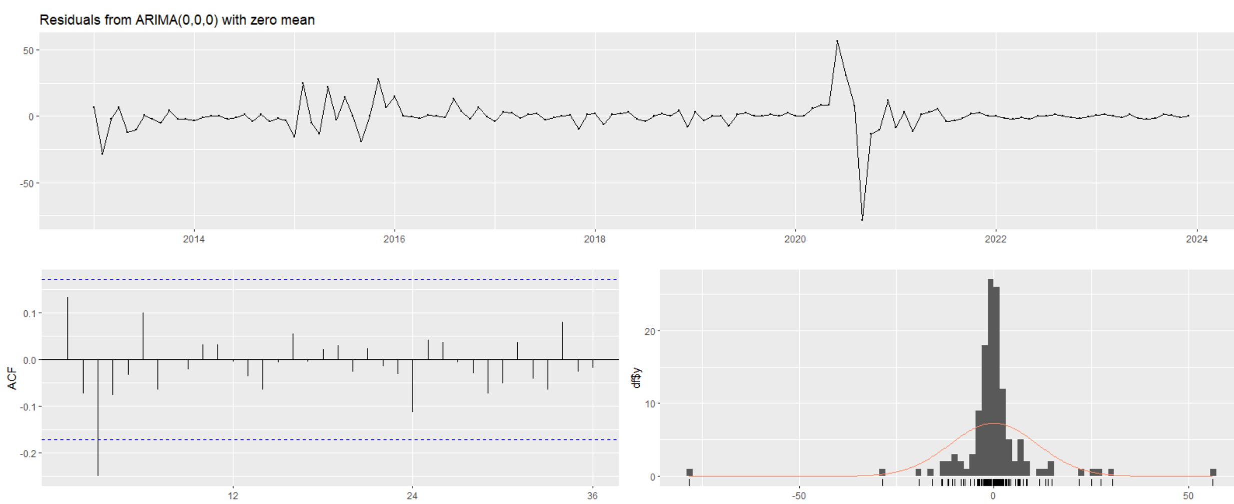

Figure 16: Residual Plot of Inflation Differential

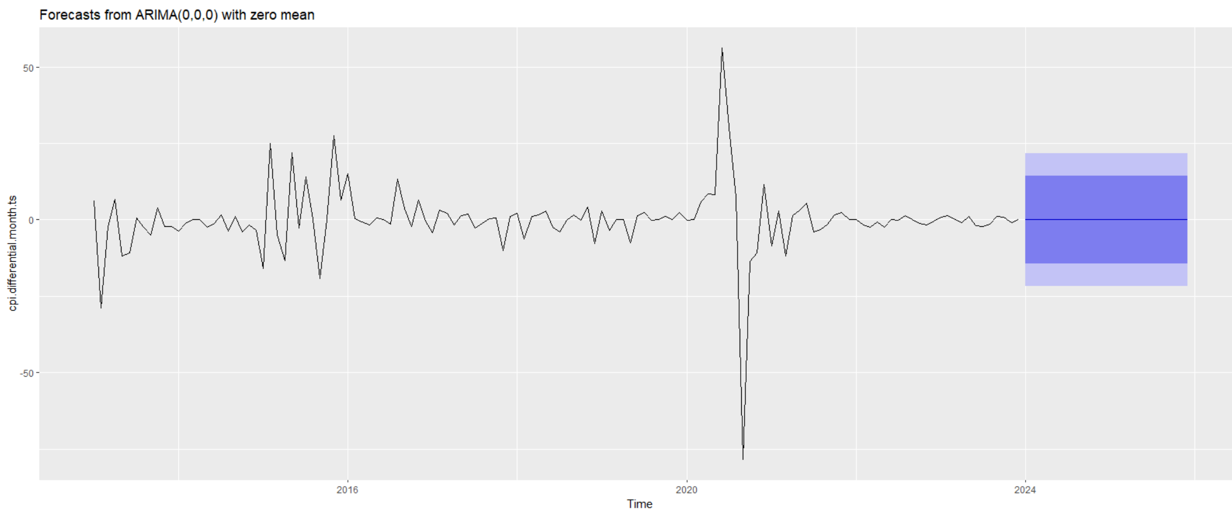

Figure 17: Time Series Plot with Forecast of Inflation Differential

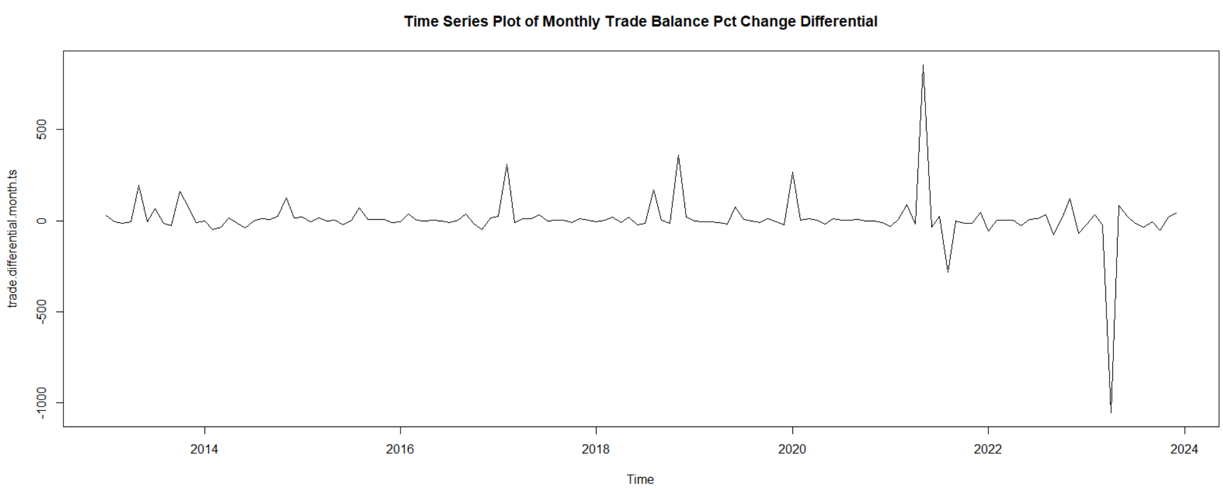

Figure 18: Time Series Plot of Trade Balance Differential

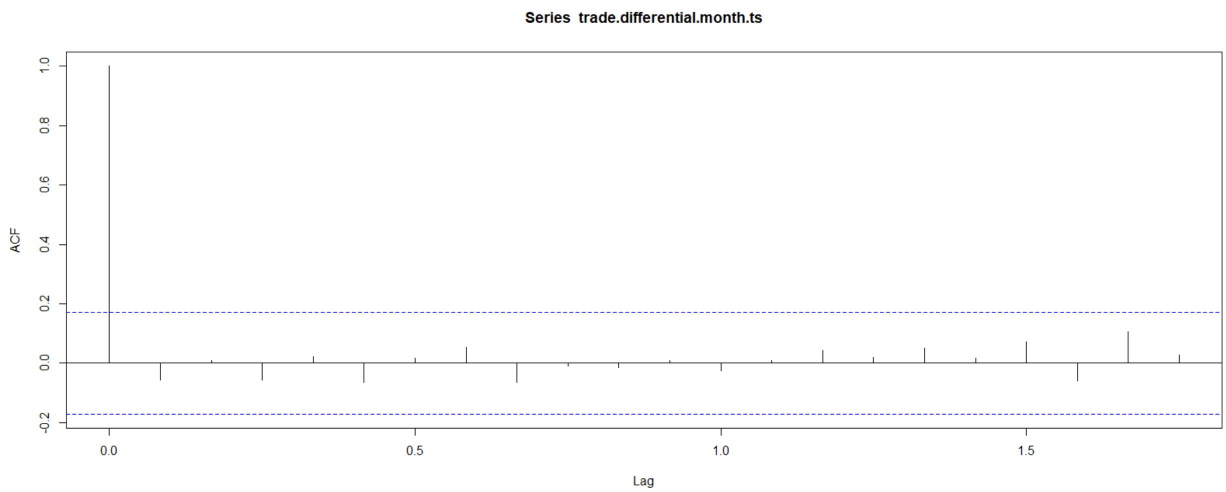

Figure 19: ACF Graph of Trade Balance Differential

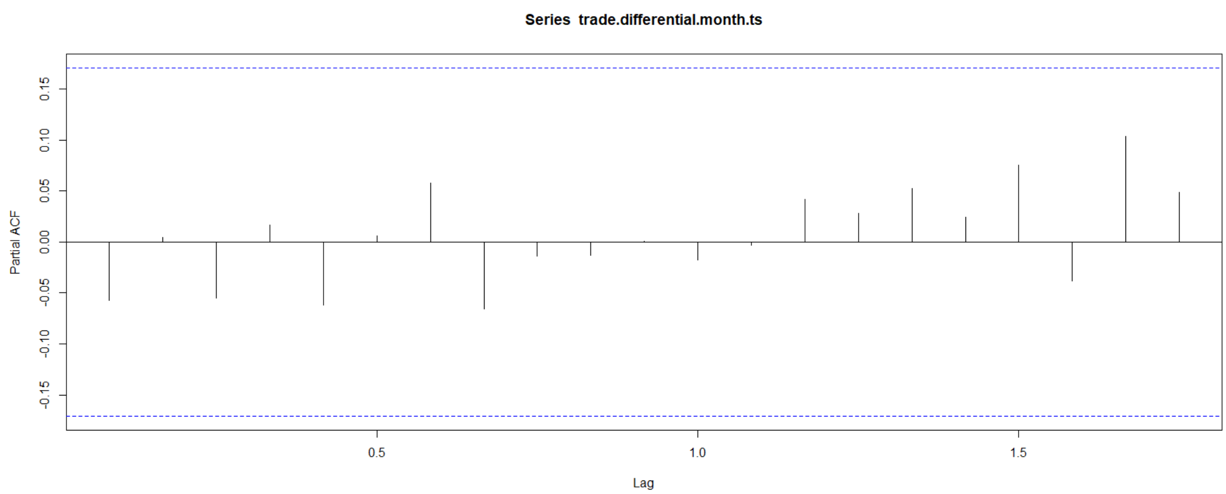

Figure 20: PACF Graph of Trade Balance Differential

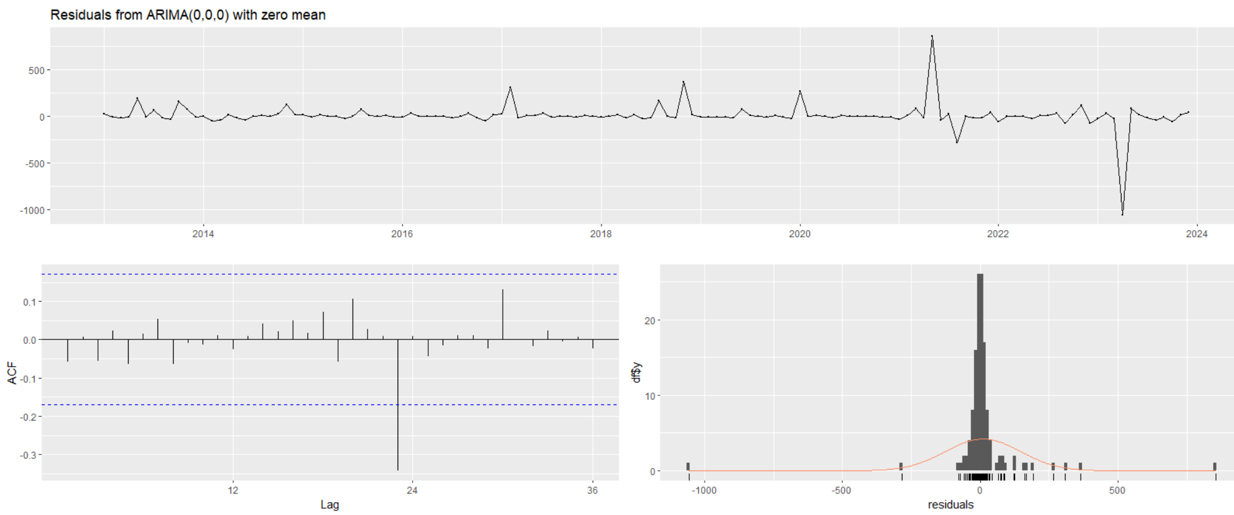

Figure 21: Residual Plot of Trade Balance Differential

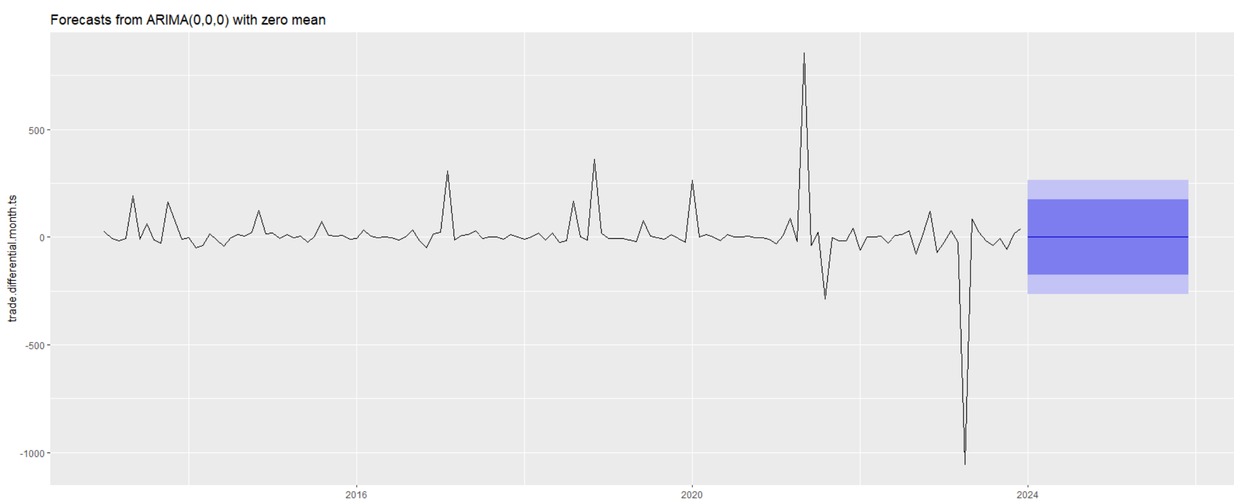

Figure 22: Time Series Plot with Forecast of Trade Balance Differential

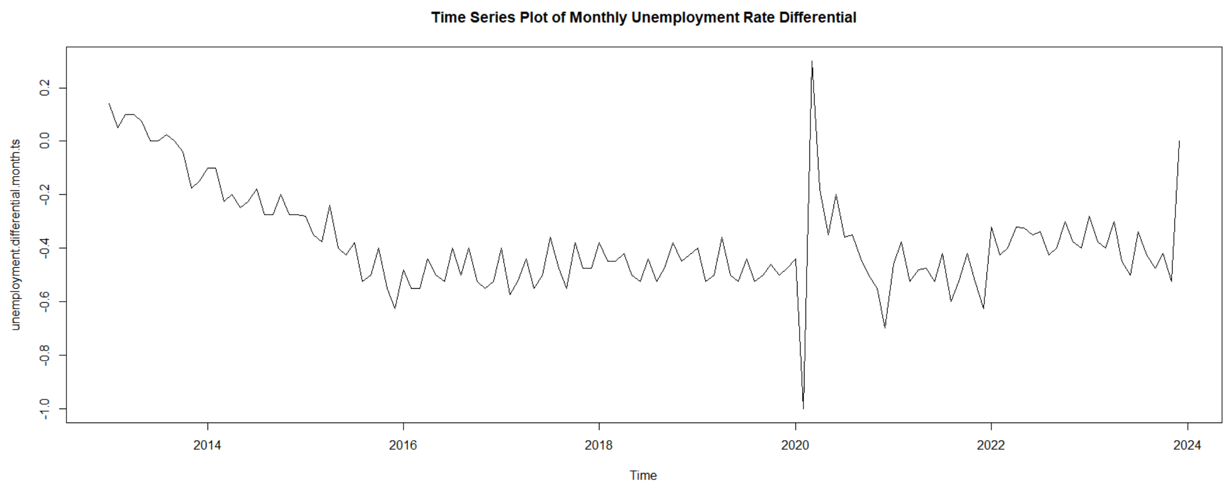

Figure 23: Time Series Plot of Unemployment Rate Differential

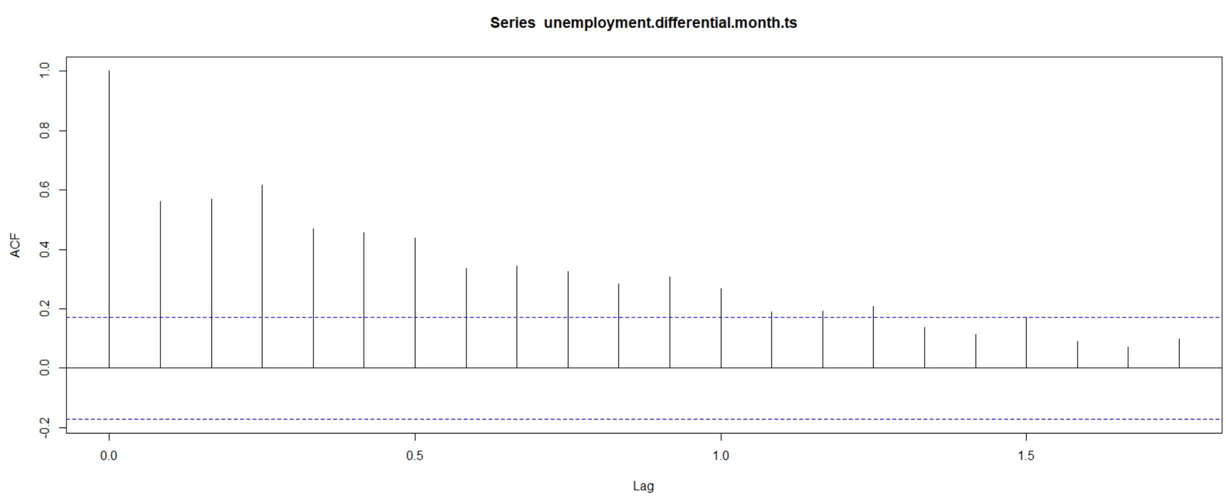

Figure 24: ACF Graph of Unemployment Rate Differential

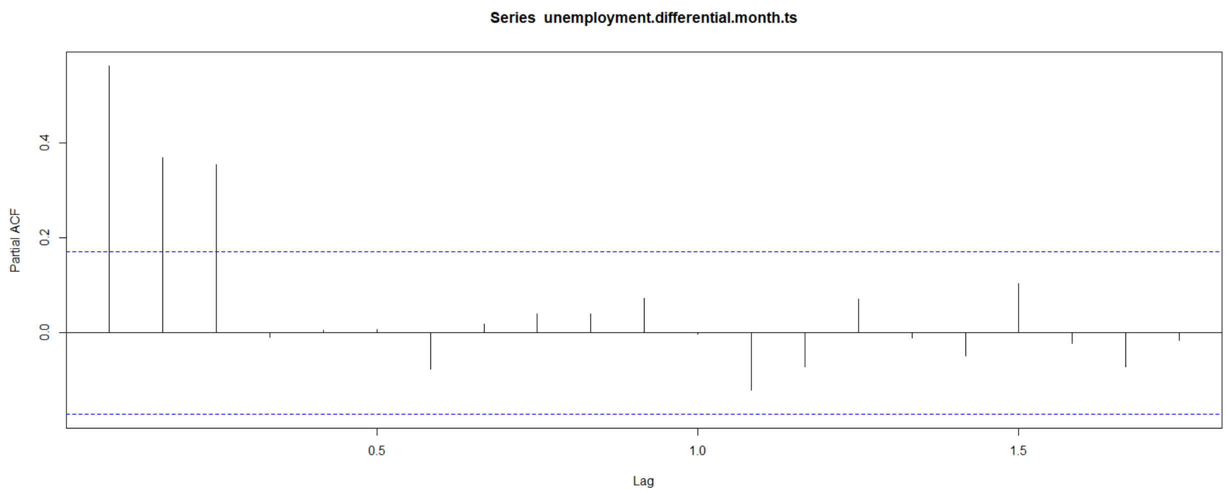

Figure 25: PACF Graph of Unemployment Rate Differential

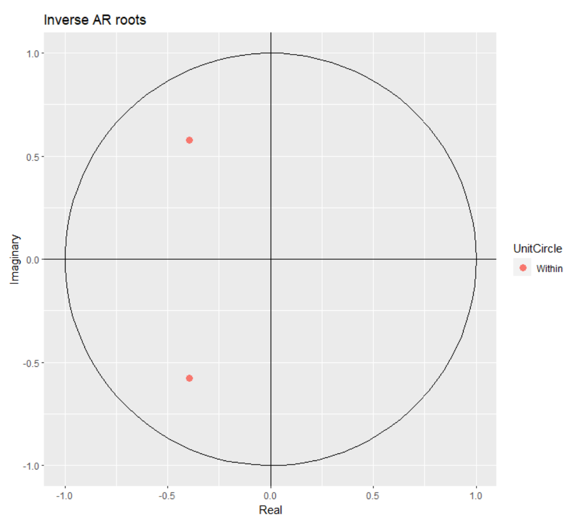

Figure 26: Inverse Characteristic Polynomial Root Graph of Unemployment Rate Differential

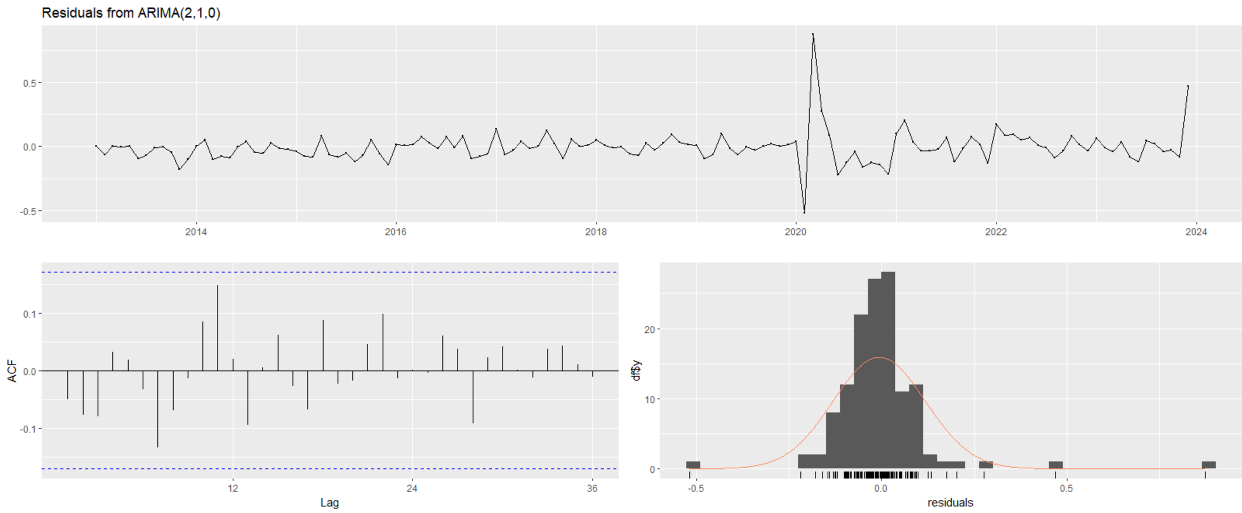

Figure 27: Residual Plot of Unemployment Rate Differential

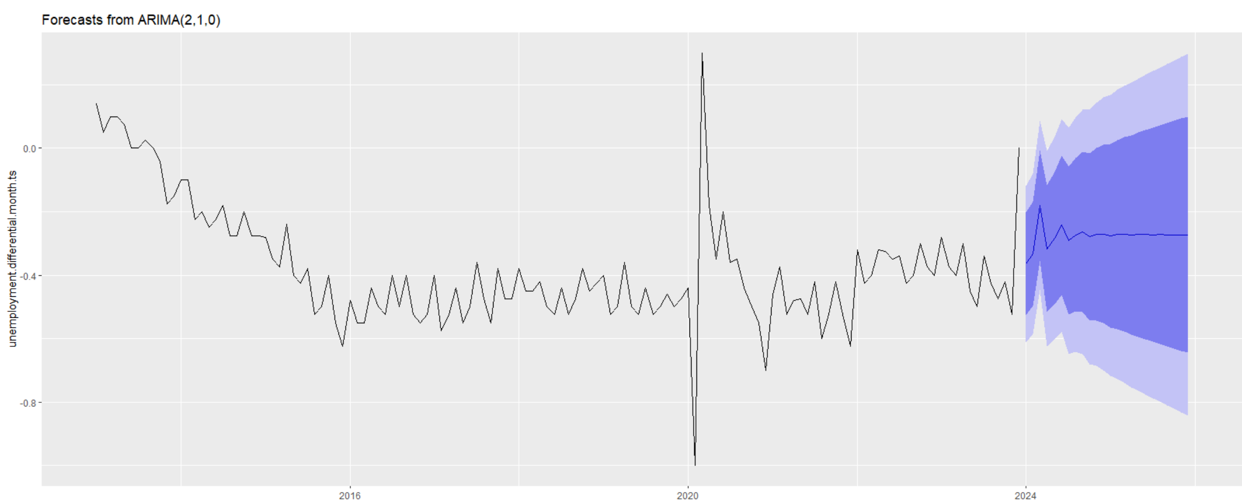

Figure 28: Time Series Plot with Forecast of Unemployment Rate Differential

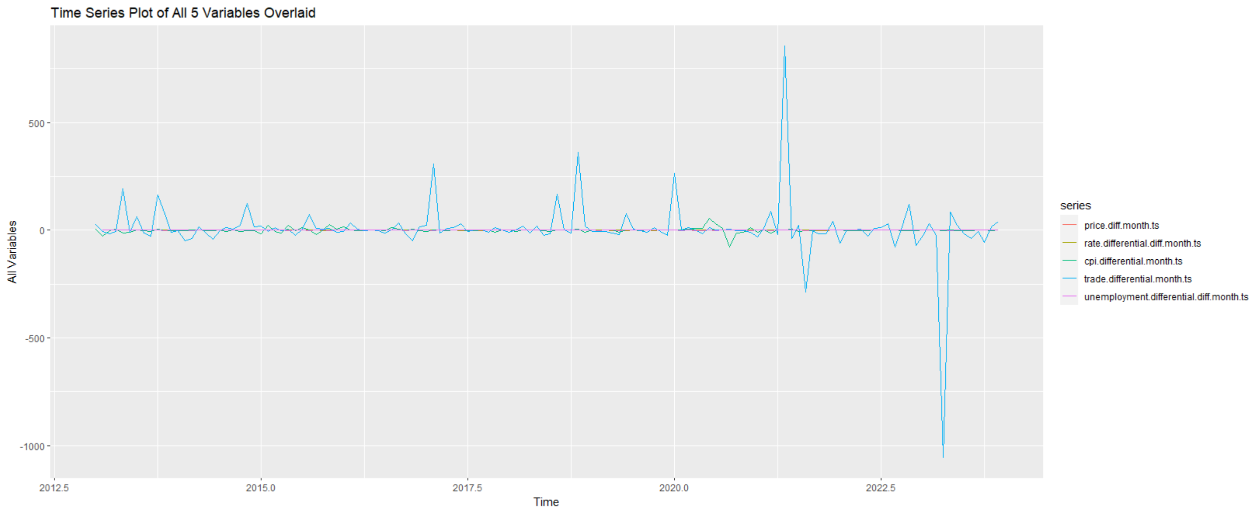

Figure 29: Time Series Plot of all 5 Variables Overlaid

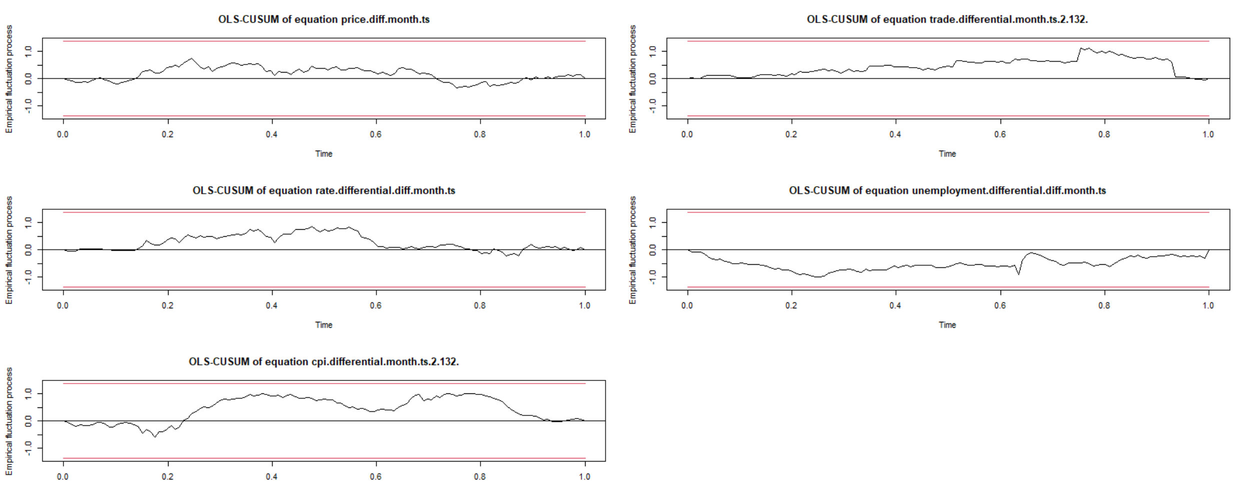

Figure 30: Stability Test of all 5 Variables