Fourier Series and Sound: A Loving Relationship

Sound!

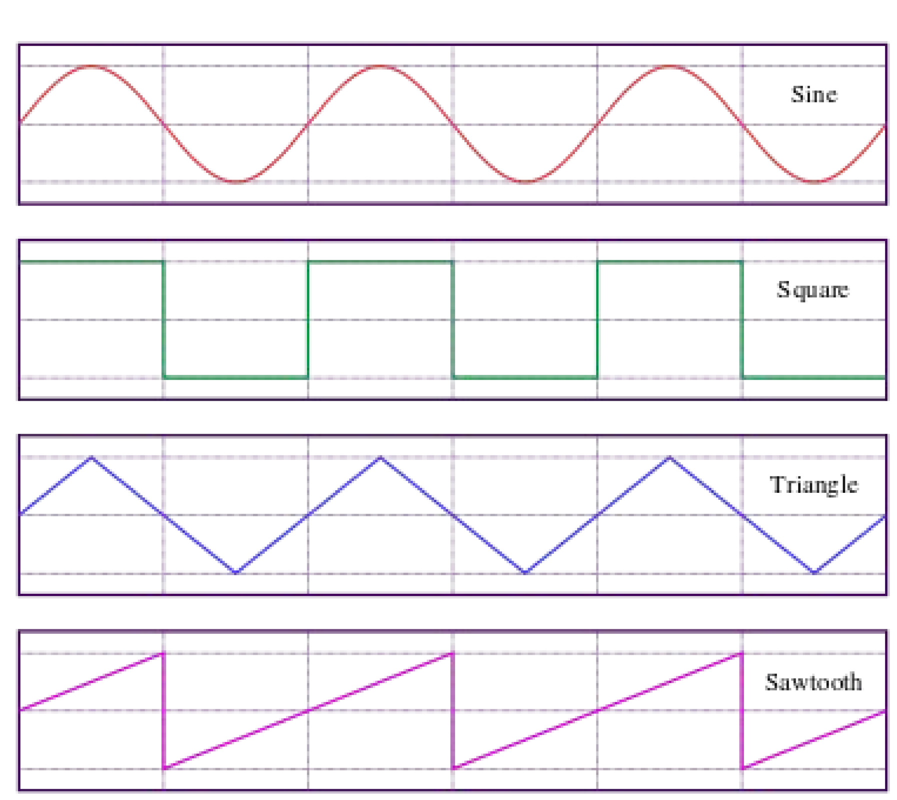
- A disruption, or vibration, that travels through a medium and transfer energy.
- A longitudinal wave! The oscillations are parallel to the direction of the transfer of energy. Similar to a spring oscillation, it consists of compressions and rarefactions.
  - Imagine a long line of angry New Yorkers pushing each other back and forth.
- Sound needs a medium, unlike light, it can not travel in a vacuum.
  - The speed of sound varies and is dependent on the medium. The speed of sound in dry air at 20° Celsius is about 343m/s.
- Every sound wave has an amplitude and a frequency and can be represented by sine and cosine functions.
- Sound waves can add together to produce new sounds.
  - Equal frequencies will resonate. Frequencies with whole number ratios are harmonics.

Fourier Series!

- A decomposition of a function, usually periodic, into sine and cosine functions of varying amplitudes and frequencies.
- Doesn’t care about differentiability! The function just has to be continuous.
  - Obiously far superior to other series. Can a Taylor series approximate a non-differentiable function? I don’t think so!
- Represented as a summation of sine and cosine functions of varying amplitudes (coefficients) and frequencies (the n-th harmonic of a fundamental frequency).
- The n-th coefficient can be found by integrating the product of the n-th harmonic sine or cosine and the orginal function over the period of the function.
- Any continuous function can be approximated. The accuracy increases as the number of terms approaches infinity.

Fourier Series and Sound!

- Both have amplitudes, frequencies, n-th harmonics. They’re the perfect couple.
- Any sound can be synthesized by a fourier series.
- The ear, with tiny follicles that resonate to different frequencies and add them together, is like a tiny Fourier Series machine.
- Unusual waves, like the sawtooth, square, and triangle waves of classic video game music can be synthesized and heard.



Fourier series of a

Square wave: $f(x) = \sum_{n=1}^{\infty} \frac{4 sin(2\pi nx - \pi x)}{2\pi n - \pi}$

Triangle wave: $f(x) = \sum_{n=1}^{\infty} \frac{8(-1)^{(n-1)} sin(2\pi nx - \pi x)}{(\pi n)^2}$

Sawtooth wave: $f(x) = \sum_{n=1}^{\infty} \frac{2(-1)^{n+1} sin(\pi nx)}{\pi n}$