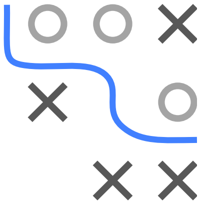
# COST-SENSITIVE LEARNING: IN A NUTSHELL



- Cost-sensitive learning:
  - Classical learning: data sets are balanced, and all errors have equal costs
  - We now assume given, unequal cost
  - And try to minimize them in expectation
- Applications:
  - Medicine — Misdiagnosing as healthy vs. having a disease
  - (Extreme) Weather prediction — Incorrectly predicting that no hurricane occurs
  - Credit granting — Lending to a risky client vs. not lending to a trustworthy client.

| | | Truth | |
|---|---|---|---|
| | | Default | Pays Back |
| Pred. | Default | 0 | 10 |
| | Pays Back | 1000 | 0 |

- In these examples, **the costs of a false negative is much higher than the costs of a false positive**.
- In some applications, the costs are **unknown** $\rightsquigarrow$ need to be specified by experts, or be learnt.

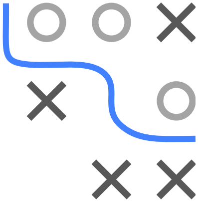
# COST MATRIX

- Input: cost matrix **C**

| | | True Class $y$ | | | |
|---|---|---|---|---|---|
| | | 1 | 2 | $\ldots$ | $g$ |
| **Classification** | 1 | $C(1,1)$ | $C(1,2)$ | $\ldots$ | $C(1,g)$ |
| | 2 | $C(2,1)$ | $C(2,2)$ | $\ldots$ | $C(2,g)$ |
| $\hat{y}$ | | | | | |
| | $\vdots$ | $\vdots$ | $\vdots$ | $\ldots$ | $\vdots$ |
| | $g$ | $C(g,1)$ | $C(g,2)$ | $\ldots$ | $C(g,g)$ |

- $C(j,k)$ is the cost of classifying class $k$ as $j$,
- 0-1-loss would simply be: $C(j,k) = \mathbb{1}_{[j \neq k]}$
- **C** designed by experts with domain knowledge
  1. Too low costs: not enough change in model, still costly errors
  2. Too high costs: might never predict costly classes

# COST MATRIX FOR IMBALANCED LEARNING



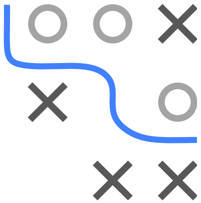
- Common heuristic for imbalanced data sets:
  - $C(j,k) = \frac{n_j}{n_k}$ with $n_k \ll n_j$,
    misclassifying a minority class $k$ as a majority class $j$
  - $C(j,k) = 1$ with $n_j \ll n_k$,
    misclassifying a majority class $k$ as a minority class $j$
  - 0 for a correct classification

- Imbalanced binary classification:

| | True class | |
|---|---|---|
| | $y = 1$ | $y = -1$ |
| Pred. $\hat{y}$ = 1 | 0 | 1 |
| class $\hat{y}$ = -1 | $\frac{n_-}{n_+}$ | 0 |

- So: much higher costs for FNs

# MINIMUM EXPECTED COST PRINCIPLE



- Suppose we have:
  - a cost matrix $\mathbf{C}$
  - knowledge of the true posterior $p(\cdot \mid \mathbf{x})$
- Predict class j with smallest expected costs when marginalizing over true classes:

$$\mathbb{E}_{K \sim p(\cdot \mid \mathbf{x})}(C(j, K)) = \sum_{k=1}^{g} p(k \mid \mathbf{x}) C(j, k)$$

- If we trust we trust a probabilistic classifier, we can convert its scores to labels:

$$h(\mathbf{x}) := \underset{j=1,\dots,g}{\arg\min} \sum_{k=1}^{g} \pi_k(\mathbf{x}) C(j, k).$$

- Can be better to take a less probable class (Elkan et. al. 2001)

# OPTIMAL THRESHOLD FOR BINARY CASE

- Optimal decisions do not change if
  - **C** is multiplied by positive constant
  - **C** is added with constant shift
- Scale and shift **C** to get simpler **C**′:

| | | True class | |
|---|---|---|---|
| | | $y = 1$ | $y = -1$ |
| Pred. | $\hat{y}$ = 1 | $C'(1,1)$ | 1 |
| class | $\hat{y}$ = -1 | $C'(-1,1)$ | 0 |

where

  - $C'(-1,1) = \frac{C(-1,1)-C(-1,-1)}{C(1,-1)-C(-1,-1)}$
  - $C'(1,1) = \frac{C(1,1)-C(-1,-1)}{C(1,-1)-C(-1,-1)}$
- We predict **x** as class 1 if

$$\mathbb{E}_{K\sim p(\cdot\,\mid\,\mathbf{x})}(C'(1,K)) \leq \mathbb{E}_{K\sim p(\cdot\,\mid\,\mathbf{x})}(C'(-1,K))$$