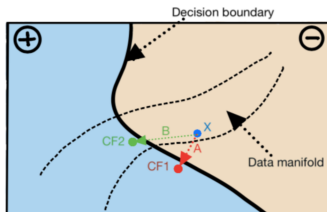


Interpretable Machine Learning

Counterfactual Explanations

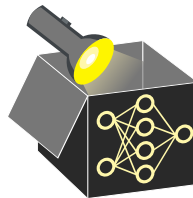
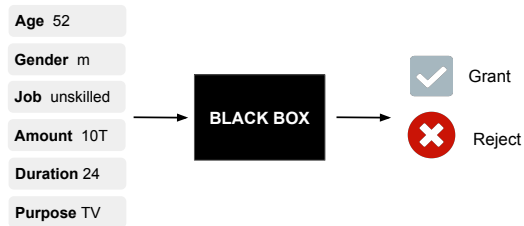


Learning goals

- Understand the motivation behind CEs
- See the mathematical foundation of CEs

EXAMPLE: CREDIT RISK APPLICATION

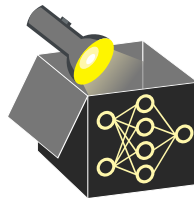
- x : customer and credit information
- y : grant or reject credit



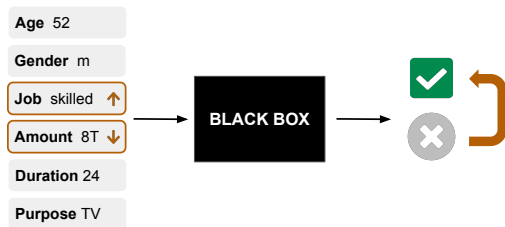
Questions:

- Why was the credit rejected?
- Is it a fair decision?
- **How should x be changed so that the credit is accepted?**

EXAMPLE: CREDIT RISK APPLICATION



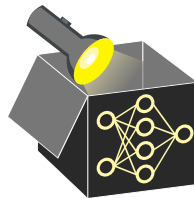
Counterfactual Explanations provide answers in the form of "What-If"-scenarios.



"If the person was more skilled and the credit amount had been reduced to \$8.000, the credit would have been granted."

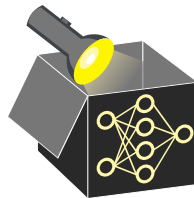
COUNTERFACTUAL EXPLANATIONS: MAIN IDEA

- Counterfactual explanations == counterfactuals == CEs

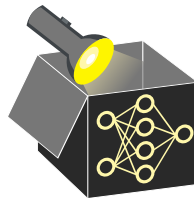


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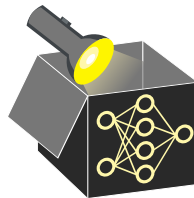


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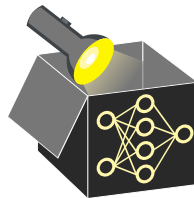
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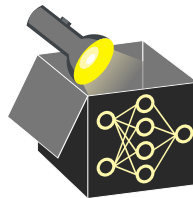


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- The targeted audience of CEs are often end-users

AIMS & ROLES

CEs can serve various purposes; the user can decide what to learn from them. For example:

"If the person had been **one year older** and the **credit amount had been increased** to \$12.000, the credit would have been granted."



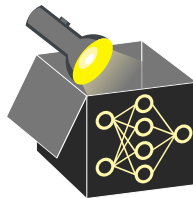
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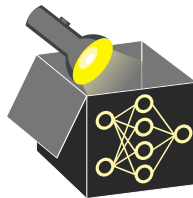
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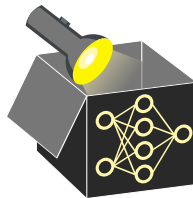
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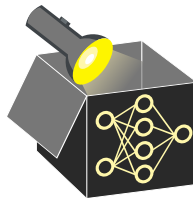
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- **Detect model biases:**

There is a bug, an increase in amount should not increase approval rates.

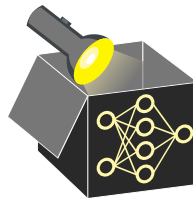


PHILOSOPHICAL BASIS

Counterfactuals have a long-standing tradition in analytic philosophy

⇒ According to ▶ Lewis (1973), a **counterfactual conditional** is a statement of the form:

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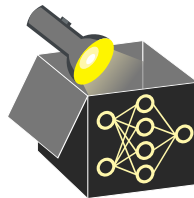
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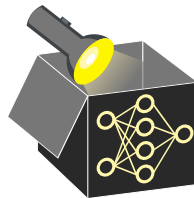
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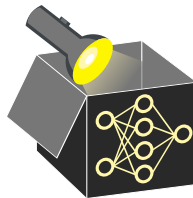
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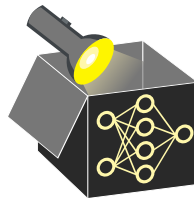
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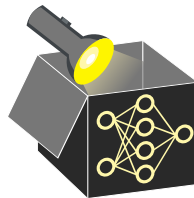
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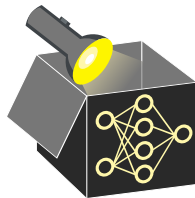
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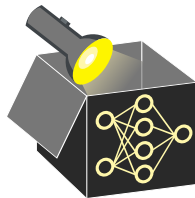
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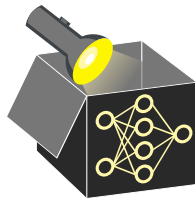
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- CEs are often contrastive, i.e., they explain a decision by referring to an alternative outcome
 - ↪ e.g., if the loan applicant was 30 instead of 60 years old, the approved loan would have been over \$100.000 instead of \$40.000



MATHEMATICAL PERSPECTIVE

Terminology:

- \mathbf{x} : original/factual datapoint whose prediction we want to explain
- $y' \subset \mathbb{R}^g$: desired prediction ($y' = 1000$ or $y' = \text{"grant credit"}$) or interval ($y' = [1000, \infty[$)



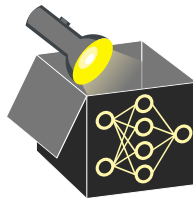
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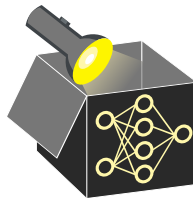
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Reformulate these two objectives (denoted by o_1 and o_2) as optimization problem:

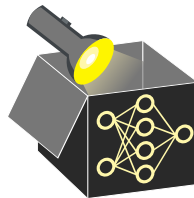
$$\arg \min_{\mathbf{x}'} \lambda_1 o_p(\hat{f}(\mathbf{x}'), y') + \lambda_2 o_f(\mathbf{x}', \mathbf{x})$$

- λ_1 and λ_2 balance the two objectives
- Choice of o_p (distance on prediction space) and of o_f (distance on feature space) is crucial

MATHEMATICAL PERSPECTIVE

► Dandl et al. (2020)

- Regression: o_p could be the L_1 -distance $o_p(\hat{f}(\mathbf{x}'), y') = |\hat{f}(\mathbf{x}') - y'|$
- Classification: L_1 -distance for scores and 0-1 Loss for labels, e.g.,
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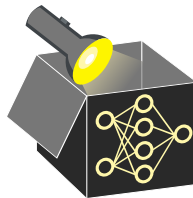
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 $o_p(\hat{f}(\mathbf{x}'), y') = \mathcal{I}_{\{\hat{f}(\mathbf{x}') \neq y'\}}$
- o_f could be the Gower distance (suitable for mixed feature space):

$$o_f(\mathbf{x}', \mathbf{x}) = d_G(\mathbf{x}', \mathbf{x}) = \frac{1}{p} \sum_{j=1}^p \delta_G(x'_j, x_j) \in [0, 1]$$

The value of δ_G depends on the feature type (numerical or categorical):

$$\delta_G(x'_j, x_j) = \begin{cases} \frac{1}{\hat{R}_j} |x'_j - x_j| & \text{if } x_j \text{ is numerical} \\ \mathcal{I}_{\{x'_j \neq x_j\}} & \text{if } x_j \text{ is categorical} \end{cases}$$

with \hat{R}_j as the value range of feature j in the training dataset (to ensure that $\delta_G(x'_j, x_j) \in [0, 1]$)



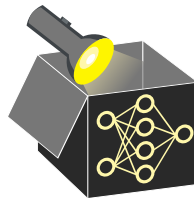
FURTHER OBJECTIVES

Additional constraints can improve the explanation quality of the corresponding CEs

~> popular constraints include sparsity and plausibility

Sparsity:

- End-users often prefer short over long explanations
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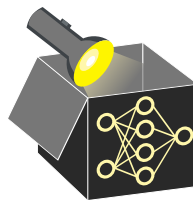
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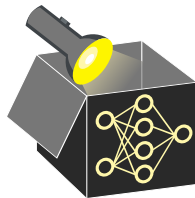
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- Independently from o_f , sparsity in the changes can be additionally considered by another objective that counts the number of changed features via the L_0 -norm:

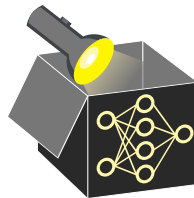
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FURTHER OBJECTIVES

Plausibility:

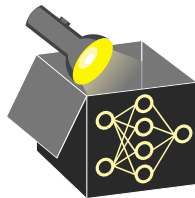
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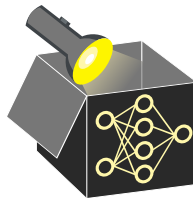
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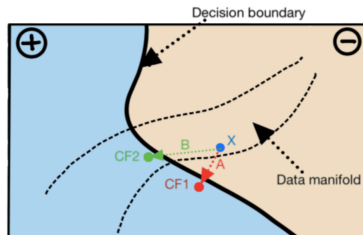
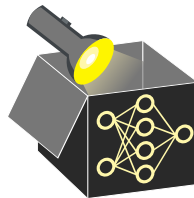
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~> Proxy: ensure that \mathbf{x}' is close to training data \mathbf{X}



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Example from [Verma et al. \(2020\)](#)

- Two possible paths for x , originally classified to \ominus
- Two valid CEs in class \oplus : **CF1** and **CF2**
- **Path A** for **CF1** is shorter
- **Path B** for **CF2** is longer but adheres to data manifold

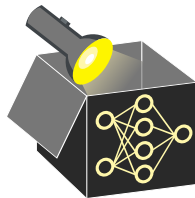
FURTHER OBJECTIVES

To ensure plausibility, o_4 could, e.g., be the Gower distance of \mathbf{x}' to its nearest data point of the training dataset which we denote $\mathbf{x}^{[1]}$:

$$o_4(\mathbf{x}', \mathbf{X}) = d_G(\mathbf{x}', \mathbf{x}^{[1]}) = \frac{1}{p} \sum_{j=1}^p \delta_G(x'_j, x_j^{[1]})$$

We can extend the previous optimization problem by adding o_s (for sparsity) and o_4 (for plausibility):

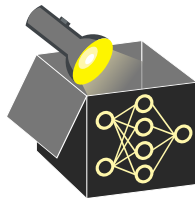
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REMARKS: THE RASHOMON EFFECT

Issue (Rashomon effect):

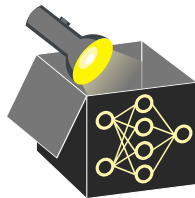
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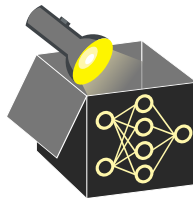
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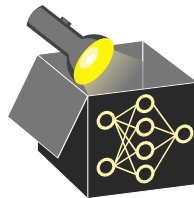
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Note:

- As the model is generally non-linear, inconsistent and diverse CEs can arise e.g. suggesting either an increase or decrease in credit duration (confuses the explainee)
- How to deal with the Rashomon effect is considered an open problem in IML

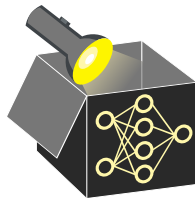
REMARKS: MODEL OR REAL-WORLD

- Most CEs provide explanations of model predictions, but CEs might appear to explain the real-world for end-users
~> Transfer of model explanations to explain real-world is generally not permitted

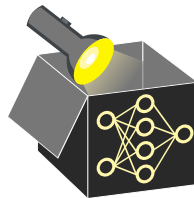


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 - ~> Transfer of model explanations to explain real-world is generally not permitted
- Consider a CE that proposes to increase the feature age by 5 to obtain the loan
 - ~> a loan applicant takes this information and applies 5 years later for the loan

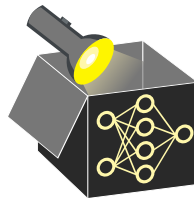


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- Also, the bank's algorithm might change and previous CEs are not applicable anymore