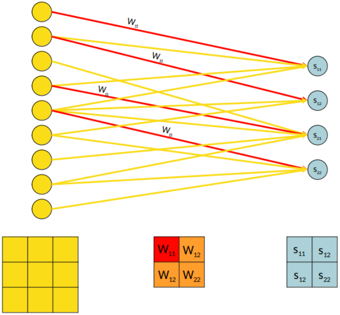
Deep Learning

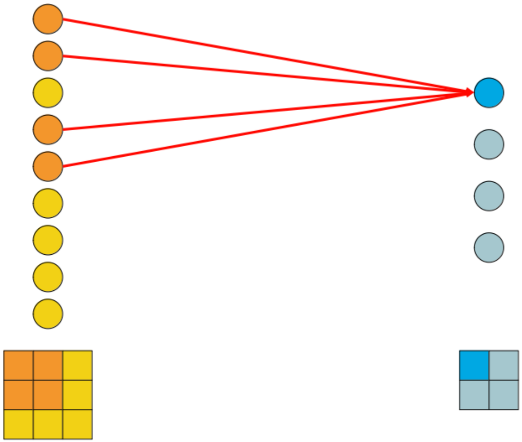
Properties of Convolution



Learning goals

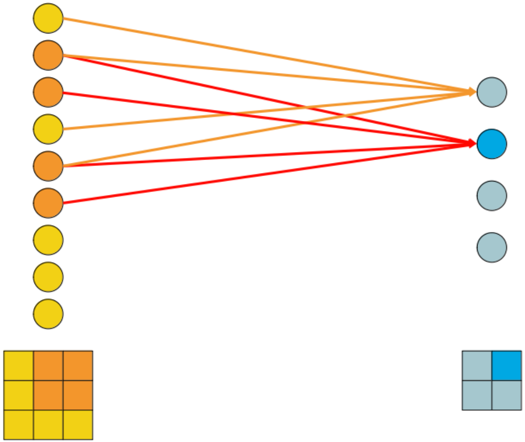
- Sparse Interactions
- Parameter Sharing
- Equivariance to Translation

SPARSE INTERACTIONS



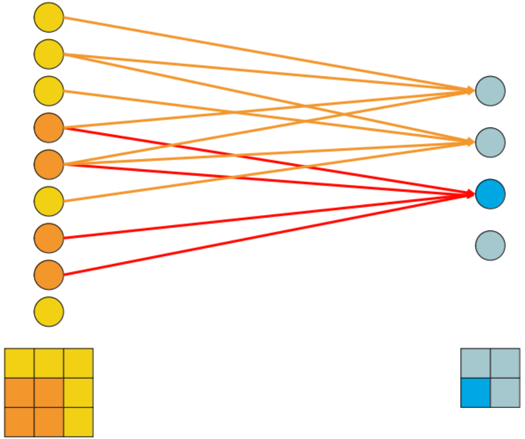
- We want to use the “neuron-wise” representation of our CNN.
- Moving the filter to the first spatial location yields the first entry of the feature map which is composed of these four connections.

SPARSE INTERACTIONS



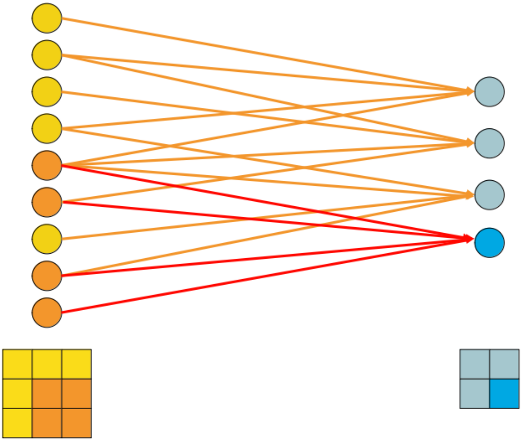
- Similarly...

SPARSE INTERACTIONS



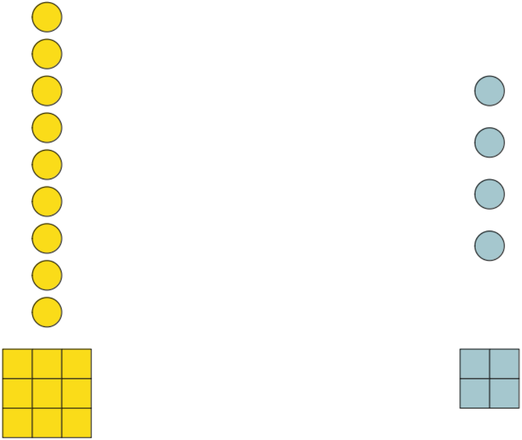
- Similarly...

SPARSE INTERACTIONS



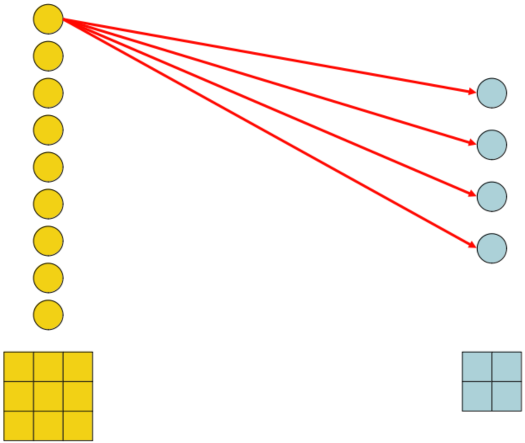
- Similarly...

SPARSE INTERACTIONS



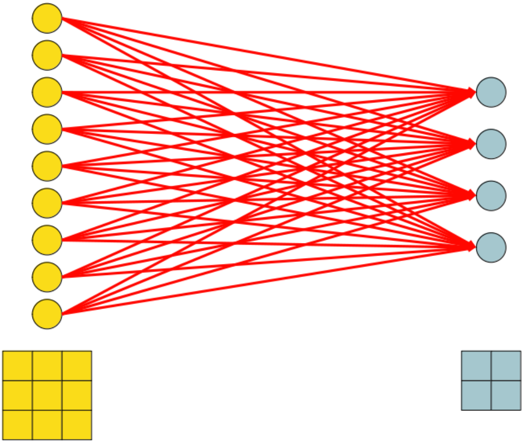
- Assume we would replicate the architecture with a dense net.

SPARSE INTERACTIONS



- Each input neuron is connected with each hidden layer neuron.

SPARSE INTERACTIONS

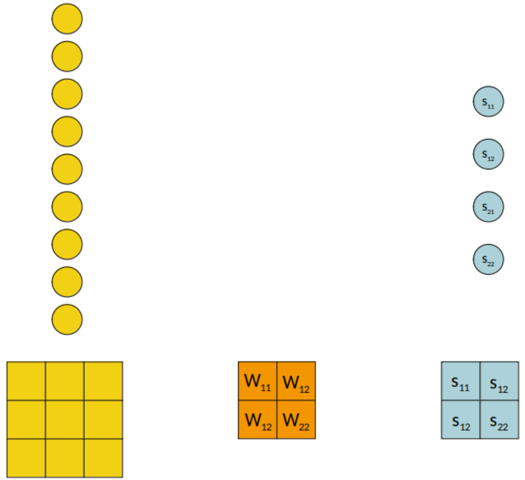


- In total, we obtain 36 connections!

SPARSE INTERACTIONS

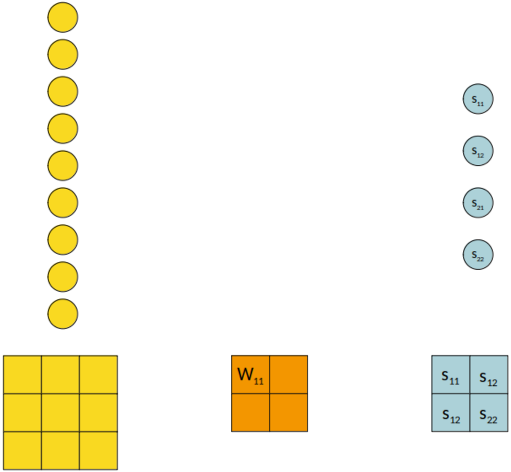
- What does that mean?
  - Our CNN has a **receptive field** of 4 neurons.
  - That means, we apply a “local search” for features.
  - A dense net on the other hand conducts a “global search”.
  - The receptive field of the dense net are 9 neurons.
- When processing images, it is more likely that features occur at specific locations in the input space.
- For example, it is more likely to find the eyes of a human in a certain area, like the face.
  - A CNN only incorporates the surrounding area of the filter into its feature extraction process.
  - The dense architecture on the other hand assumes that every single pixel entry has an influence on the eye, even pixels far away or in the background.

PARAMETER SHARING



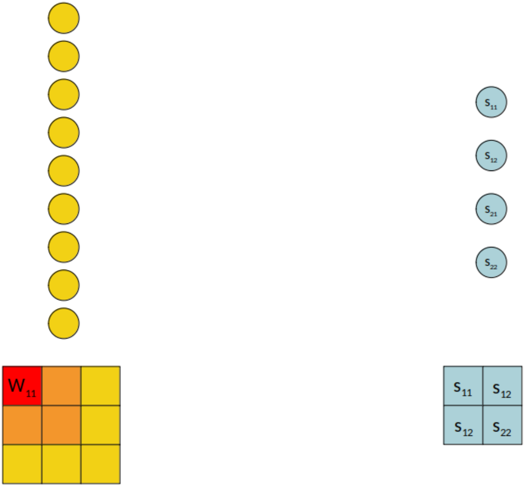
- For the next property we focus on the filter entries.

PARAMETER SHARING



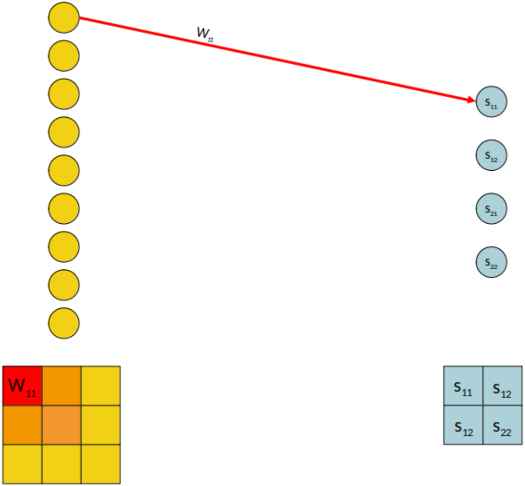
- In particular, we consider weight w_{11}

PARAMETER SHARING



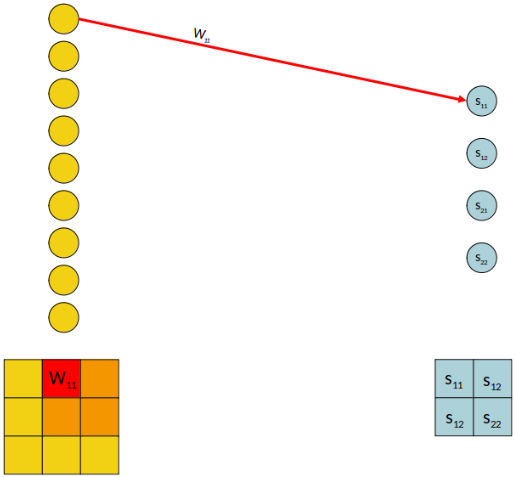
- As we move the filter to the first spatial location..

PARAMETER SHARING



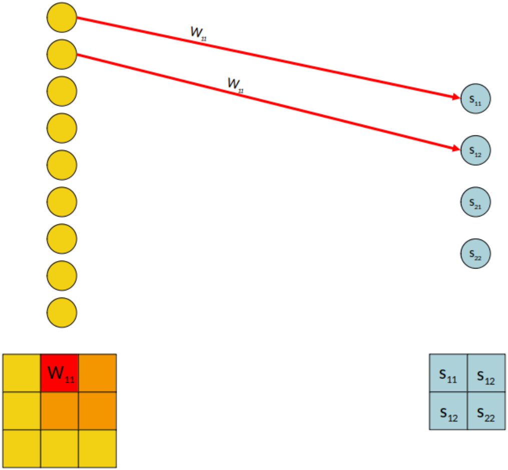
- ...we observe the following connection for weight w_{11}

PARAMETER SHARING



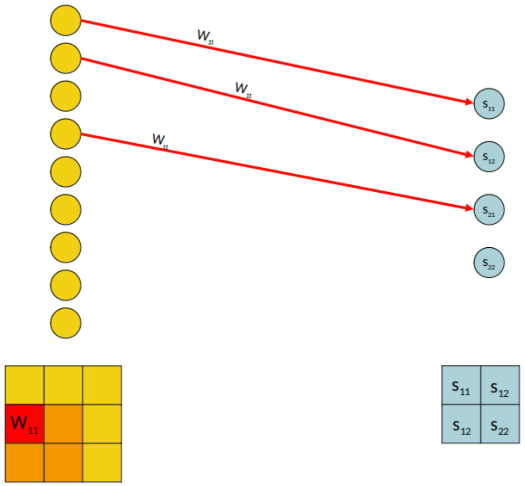
- Moving to the next location...

PARAMETER SHARING



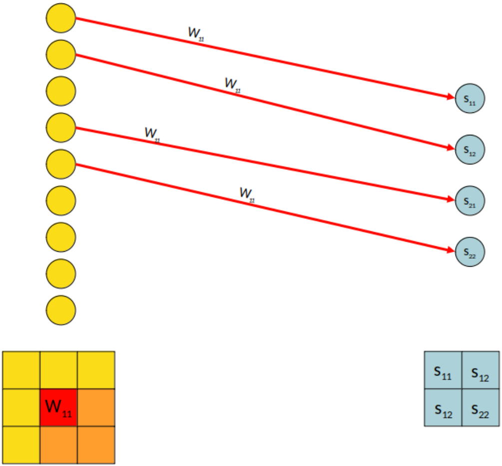
- ...highlights that we use the same weight more than once!

PARAMETER SHARING



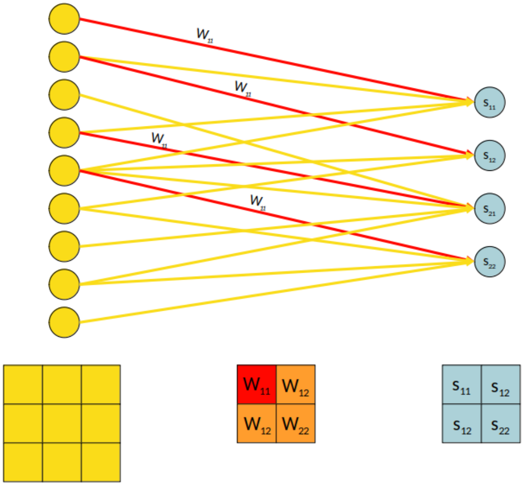
- Even three...

PARAMETER SHARING



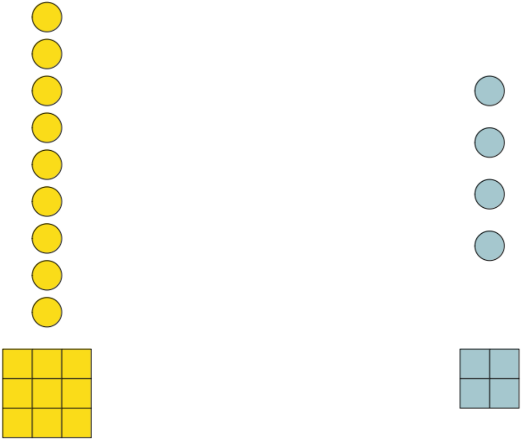
- And in total four times.

PARAMETER SHARING



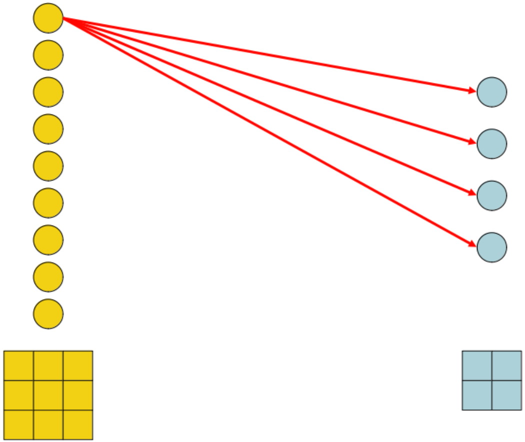
- All together, we have just used four weights.

PARAMETER SHARING



- How many weights does a corresponding dense net use?

PARAMETER SHARING



- $9 \cdot 4 = 36$! That is 9 times more weights!

SPARSE CONNECTIONS AND PARAMETER SHARING

- Why is that good?
- Less parameters drastically reduce memory requirements.
- Faster runtime:
  - For m inputs and n outputs, a fully connected layer requires $m \times n$ parameters and has $\mathcal{O}(m \times n)$ runtime.
  - A convolutional layer has limited connections $k << m$, thus only $k \times n$ parameters and $\mathcal{O}(k \times n)$ runtime.
- But it gets even better:
  - Less parameters mean less overfitting and better generalization!

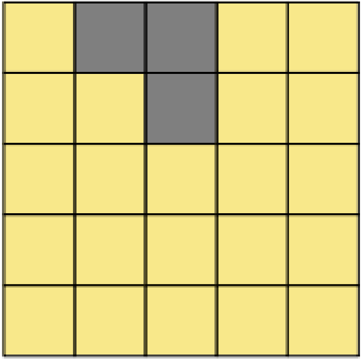
SPARSE CONNECTIONS AND PARAMETER SHARING

- Example: consider a color image with size 100×100.
- Suppose we would like to create one single feature map with a "same padding" (i.e. the hidden layer is of the same size).
  - Choosing a filter with size 5 means that we have a total of $5 \cdot 5 \cdot 3 = 75$ parameters (bias unconsidered).
  - A dense net with the same amount of "neurons" in the hidden layer results in

$$\underbrace{(100^2 \cdot 3)}_{\text{input}} \cdot \underbrace{(100^2)}_{\text{hidden layer}} = 300.000.000$$

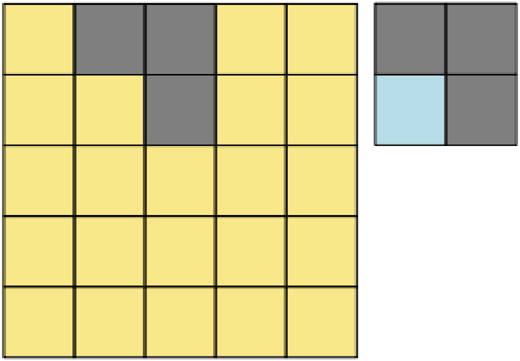
    parameters.
- Note that this was just a fictitious example. In practice we normally do not try to replicate CNN architectures with dense networks.

EQUIVARIANCE TO TRANSLATION



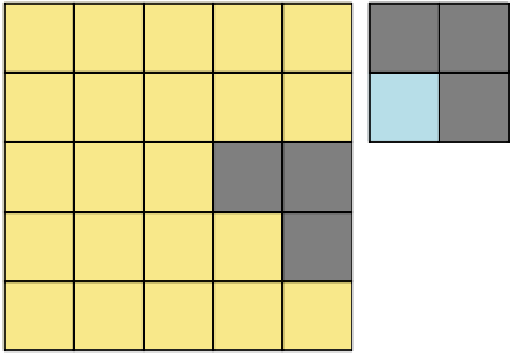
- Think of a specific feature of interest, here highlighted in grey.

EQUIVARIANCE TO TRANSLATION



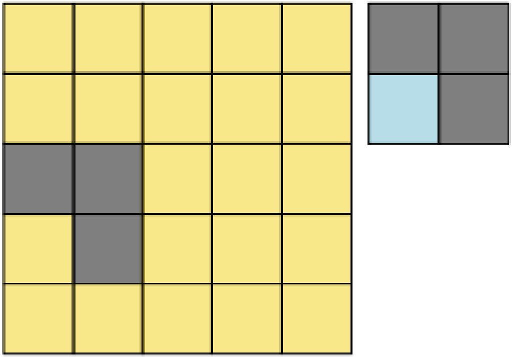
- Furthermore, assume we had a tuned filter looking for exactly that feature.

EQUIVARIANCE TO TRANSLATION



- The filter does not care at what location the feature of interest is located at.

EQUIVARIANCE TO TRANSLATION



- It is literally able to find it anywhere! That property is called **equivariance to translation.**

  Note: A function $f(x)$ is equivariant to a function g if $f(g(x)) = g(f(x))$.

NONLINEARITY IN FEATURE MAPS

- As in dense nets, we use activation functions on all feature map entries to introduce nonlinearity in the net.
- Typically rectified linear units (ReLU) are used in CNNs:
  - They reduce the danger of saturating gradients compared to sigmoid activations.
  - They can lead to *sparse activations*, as neurons ≤ 0 are squashed to 0 which increases computational speed.
- As seen in the last chapter, many variants of ReLU (Leaky ReLU, ELU, PReLU, etc.) exist.