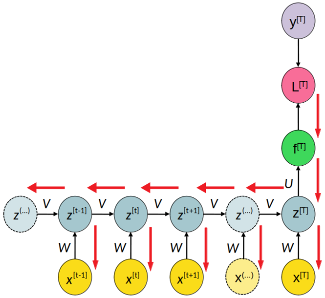
Deep Learning

Recurrent Neural Networks - Backpropogation



Learning goals

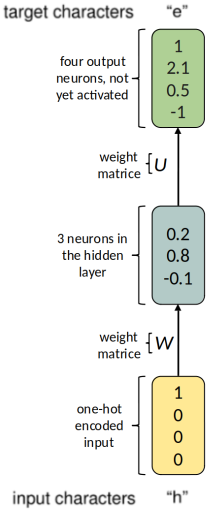
- How does Backpropagation work for RNNs?
- Exploding and Vanishing Gradients

SIMPLE EXAMPLE: CHARACTER LEVEL LANGUAGE MODEL

Task: Learn character probability distribution from input text

- Suppose we only had a vocabulary of four possible letters: “h”, “e”, “l” and “o”
- We want to train an RNN on the training sequence “hello”.
- This training sequence is in fact a source of 4 separate training examples:
    - The probability of “e” should be likely given the context of “h”
    - “l” should be likely in the context of “he”
    - “l” should also be likely given the context of “hel”
    - and “o” should be likely given the context of “hell”

SIMPLE EXAMPLE: CHARACTER LEVEL LANGUAGE MODEL



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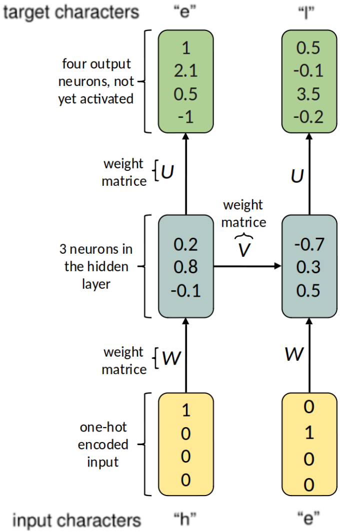

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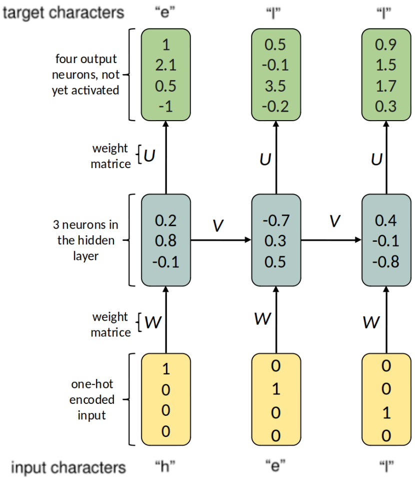
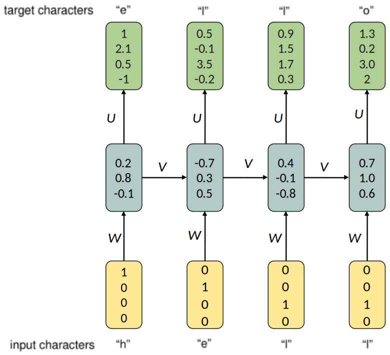


SIMPLE EXAMPLE: CHARACTER LEVEL LANGUAGE MODEL

The RNN has a 4-dimensional input and output. The exemplary hidden layer consists of 3 neurons. This diagram shows the activations in the forward pass when the RNN is fed the characters “hell” as input. The output contains confidences the RNN assigns for the next character.



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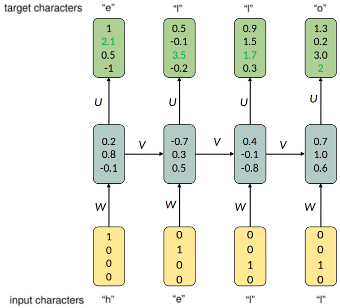
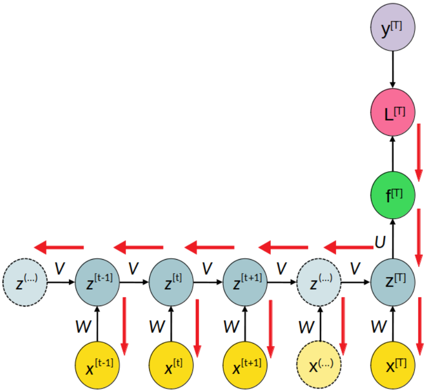


Our goal is to increase the confidence for the correct letters (green digits) and decrease the confidence of all others (we could also use a softmax activation to squash the digits to probabilities $\in [0, 1]$). How can we now train the network?

Backpropagation through time!

BACKPROPAGATION THROUGH TIME



- For training the RNN, we need to compute $\frac{dL}{du_{i,j}}$, $\frac{dL}{dv_{i,j}}$, and $\frac{dL}{dw_{i,j}}$.
- To do so, during backpropagation at time step t for an arbitrary RNN, we need to compute

$$\frac{dL}{d\mathbf{z}^{[1]}} = \frac{dL}{d\mathbf{z}^{[t]}} \frac{d\mathbf{z}^{[t]}}{d\mathbf{z}^{[t-1]}} \cdots \frac{d\mathbf{z}^{[2]}}{d\mathbf{z}^{[1]}}$$

LONG-TERM DEPENDENCIES

- Here, $\mathbf{z}^{[t]} = \sigma(\mathbf{V}^\top \mathbf{z}^{[t-1]} + \mathbf{W}^\top \mathbf{x}^{[t]} + \mathbf{b})$
- It follows that:

$$\frac{d\mathbf{z}^{[t]}}{d\mathbf{z}^{[t-1]}} = \text{diag}(\sigma'(\mathbf{V}^\top \mathbf{z}^{[t-1]} + \mathbf{W}^\top \mathbf{x}^{[t]} + \mathbf{b}))\mathbf{V}^\top = \mathbf{D}^{[t-1]}\mathbf{V}^\top$$

$$\frac{d\mathbf{z}^{[t-1]}}{d\mathbf{z}^{[t-2]}} = \text{diag}(\sigma'(\mathbf{V}^\top \mathbf{z}^{[t-2]} + \mathbf{W}^\top \mathbf{x}^{[t-1]} + \mathbf{b}))\mathbf{V}^\top = \mathbf{D}^{[t-2]}\mathbf{V}^\top$$

$$\vdots$$

$$\frac{d\mathbf{z}^{[2]}}{d\mathbf{z}^{[1]}} = \text{diag}(\sigma'(\mathbf{V}^\top \mathbf{z}^{[1]} + \mathbf{W}^\top \mathbf{x}^{[2]} + \mathbf{b}))\mathbf{V}^\top = \mathbf{D}^{[1]}\mathbf{V}^\top$$

$$\frac{dL}{d\mathbf{z}^{[1]}} = \frac{dL}{d\mathbf{z}^{[t]}} \frac{d\mathbf{z}^{[t]}}{d\mathbf{z}^{[t-1]}} \cdots \frac{d\mathbf{z}^{[2]}}{d\mathbf{z}^{[1]}} = \mathbf{D}^{[t-1]}\mathbf{D}^{[t-2]} \ldots \mathbf{D}^{[1]}(\mathbf{V}^\top)^{t-1}$$

LONG-TERM DEPENDENCIES

- In general, for an arbitrary time-step $i < t$ in the past, $\frac{d\mathbf{z}^{[t]}}{d\mathbf{z}^{[i]}}$ will contain the term $(\mathbf{V}^\top)^{t-i}$ (this follows from the chain rule).
- Based on the largest eigenvalue of $\mathbf{V}^\top$, the presence of the term $(\mathbf{V}^\top)^{t-i}$ can either result in vanishing or exploding gradients.
- This problem is quite severe for RNNs (as compared to feedforward networks) because the **same** matrix $\mathbf{V}^\top$ is multiplied several times. Click here
- As the gap between t and i increases, the instability worsens.
- It is thus quite challenging for RNNs to learn long-term dependencies. The gradients either **vanish** (most of the time) or **explode** (rarely, but with much damage to the optimization).
- That happens simply because we propagate errors over very many stages backwards.

LONG-TERM DEPENDENCIES

Many values > 1:
exploding gradient

Gradient clipping to
scale big gradient

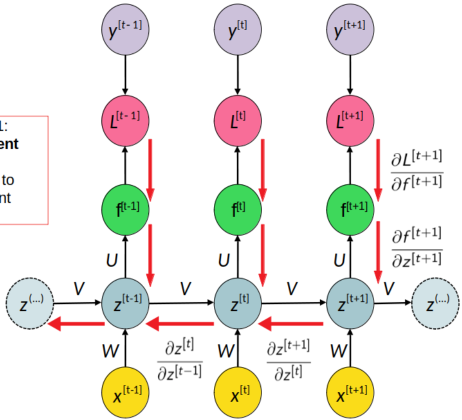


Figure: Exploding gradients

LONG-TERM DEPENDENCIES

- Recall, that we can counteract exploding gradients by implementing gradient clipping.
- To avoid exploding gradients, we simply clip the norm of the gradient at some threshold *h* (see chapter 4):

$$\text{if } ||\nabla W|| > \text{h} : \nabla W \leftarrow \frac{h}{||\nabla W||} \nabla W$$

LONG-TERM DEPENDENCIES

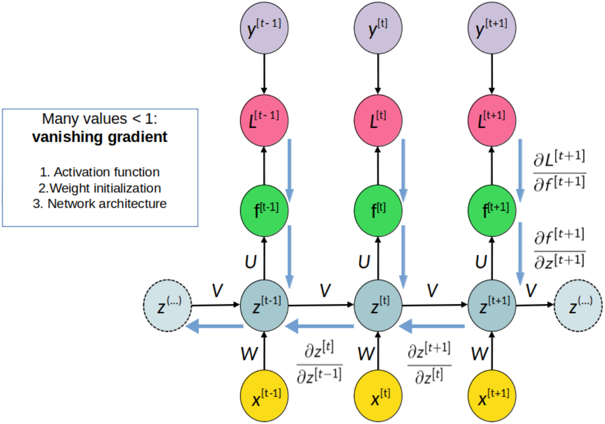


Figure: Vanishing gradients

LONG-TERM DEPENDENCIES

- Even for a stable RNN (gradients not exploding), there will be exponentially smaller weights for long-term interactions compared to short-term ones and a more sophisticated solution is needed for this vanishing gradient problem (discussed in the next chapters).
- The vanishing gradient problem heavily depends on the choice of the activation functions.
  - Sigmoid maps a real number into a “small” range (i.e. $[0, 1]$) and thus even huge changes in the input will only produce a small change in the output. Hence, the gradient will be small.
  - This becomes even worse when we stack multiple layers.
  - We can avoid this problem by using activation functions which do not “squash” the input.
  - The most popular choice is ReLU with gradients being either 0 or 1, i.e., they never saturate and thus don’t vanish.
  - The downside of this is that we can obtain a “dead” ReLU.

REFERENCES

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