Deep Learning

Exercise Sheet 1

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1 In Class

1. **Classification and Regression**

   a) Give an example for a classification and a regression problem one could solve with supervised machine learning.

   b) Do you know a loss function that could be used for a classification problem? What kind of loss function could be used for a regression problem?

2. **Risk and Empirical Risk**

   Let $f : \mathcal{X} \to \mathcal{Y}$ be a model, $P_{x,y}$ the data underlying distribution, and $L(y, f(x))$ a loss function. Recall the definition of the risk $\mathcal{R}(f)$ and empirical risk $\mathcal{R}(f)_{\text{emp}}$, and show that

   $$\mathbb{E}[\mathcal{R}(f)_{\text{emp}}] = \mathcal{R}(f)$$

   Which property of the expectation is used? And what follows for $mathcalR(f)_{\text{emp}}$ from the central limit theorem?

3. **Linear Regression**

   a) Consider a linear regression model $y = X\theta$ with feature-data-matrix $X \in \mathbb{R}^{n \times (p+1)}$, parameters $\theta \in \mathbb{R}^{p+1}$, and label vector $y \in \mathbb{R}^n$. Derive the normal equations, that is, the parameters that minimize the mean squared error (MSE).

b) Show that minimizing the MSE and maximizing the log-likelihood (for linear regression with normal distributed noise) leads to the same parameters.

4. **Logistic Regression**

   Consider a logistic regression model

   $$\pi(x) = P(y = 1|x) = \frac{\exp(\theta^T x)}{1 + \exp(\theta^T x)}$$

   Derive an expression for the log-likelihood of θ given a data set $(x^{(1)}, y^{(1)}), \ldots, (x^{(n)}, y^{(n)})$.

5. **Underfitting and Overfitting**

   You are training three classifiers on a training set. The plots in Figure 1.1, show the evolution of their classification error on training and test set during training. Do the models underfit, overfit or model the data well?
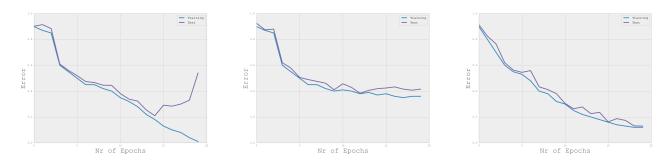

Figure 1.1: Training and test error during iterative optimization. Left: model 1, middle: model 2, right: model 3.

2 At Home

1. If you want to familiarise yourself with Python and its machine learning libraries, please go through the tutorial at `http://scikit-learn.org/stable/tutorial/basic/tutorial.html`.