Lab 8

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Imports

In [1]:

```
import random
import shutil
import urllib.request
from functools import reduce, partial
from math import ceil
from pathlib import Path
from typing import List, Optional, Callable, Tuple, Dict

import matplotlib.pyplot as plt
import torch
from PIL import Image
from matplotlib_inline.backend_inline import set_matplotlib_formats
from torch import nn, Tensor
from torch.optim import Adam, Optimizer
from torch.utils.data import DataLoader, Dataset
from torchsummary import summary
from torchvision import transforms
from torchvision.models import vgg16

set_matplotlib_formats('png', 'pdf')
```

Exercise 1

NB: it is impractical to do this exercise on your laptop, even if you have a GPU. You are advised to work on [Google Colab (https://colab.research.google.com)](https://colab.research.google.com).

In this exercise we would like to build a classification system for 120 different breeds of dogs, based on commonplace pictures. The data is available on [Kaggle (https://www.kaggle.com/c/dog-breed-identification/data)](https://www.kaggle.com/c/dog-breed-identification/data) (permanent, requires an account) or [Dropbox (https://www.dropbox.com/s/l7b7l5fjwwj6ad2/dog-breed-identification.zip?dl=0)](https://www.dropbox.com/s/l7b7l5fjwwj6ad2/dog-breed-identification.zip?dl=0) (temporary, no login needed). Download it and unzip it, then put the contents in a folder named `.data/DOGBREED`. Otherwise, execute the code below.

In [2]:

```python
# If you have problems downloading from python, try from a browser.
url = 'https://www.dropbox.com/s/l7b7l5fjwwj6ad2/dog-breed-identification.zip?dl=1'
data_root = '.data'
dataset_dir = Path(data_root, 'DOGBREED')
file_name = 'dog-breed-identification.zip'
file_path = Path(data_root, file_name)

if not dataset_dir.exists():
    Path(data_root).mkdir(exist_ok=True)
    urllib.request.urlretrieve(url, file_path)
    shutil.unpack_archive(file_path, dataset_dir)
    file_path.unlink()
```

This dataset is composed of 10222 pictures in different resolutions and aspect rations. The smallest classes are Briad and Eskimo dog with only 66 images each, whereas the biggest class is the Scottish deerhound with 126 images.

Here are some sample images along with the relative label:

In [3]:

```python
with open(Path(dataset_dir, 'labels.csv')) as file:
    label_list = file.readlines()

label_list = [label_entry.strip().split(sep=',') for label_entry in label_list][1:]
print(label_list[:3])
```

```
[['000bec180eb18c7604dcecc8fe0dba07', 'boston_bull'], ['001513dfcb2ffa
fc82cccf4d8bbaba97', 'dingo'], ['001cdf01b096e06d78e9e5112d419397', 'p
ekinese']]
```

In [4]:

```
_, axs = plt.subplots(5, 5, figsize=(10, 10))
axs = axs.flatten()
for ax in axs:
    rand_file, rand_label = tuple(label_list[random.randint(0, len(label_list))])
    img = Image.open(Path(dataset_dir, 'train', rand_file + '.jpg'))
    ax.imshow(img)
    ax.set_title(rand_label)
    ax.axis('off')
plt.show()
```

This is a challenging problem since there are many classes and only few instances per class. Moreover, the images contain a lot of details that do not help to identify the dogs, but allow a network to easily memorize the data. We will first try to naively approach the problem and directly train a CNN on this dataset. After convincing ourselves that this is not going to work, we will use a pre-trained VGG16 (i.e., trained successfully on some other dataset) and fine-tune it to our data.

But first, we do a bit of data organizing to reserve some images for a validation subset. Afterwards, we will create a custom `Dataset` that conveniently loads images on the fly during our training procedure.

We will also convert our labels to integers and create a dict with the reverse mapping.

In [5]:

```
# Shuffle list of all keys and labels
random.shuffle(label_list)
val_size = len(label_list) // 5

# Split in training and validation sets
train_label_list = label_list[val_size:]
val_label_list = label_list[:val_size]

# Extract keys and labels
train_keys, train_labels = zip(*train_label_list)
val_keys, val_labels = zip(*val_label_list)

# Get all unique classes
unique_classes = set([label_entry[1] for label_entry in label_list])
print('Number of classes: {}'.format(len(unique_classes)))

# Create class mappings
num_to_class_mapping = {i: class_label for i, class_label in enumerate(unique_class
class_to_num_mapping = {class_label: i for i, class_label in enumerate(unique_class

# Convert string labels to integers
train_labels = [class_to_num_mapping[label] for label in train_labels]
val_labels = [class_to_num_mapping[label] for label in val_labels]
```

```
Number of classes: 120
```

In [6]:

```python
class DogBreedDataset(Dataset):
    def __init__(self,
                 keys: List[str],
                 labels: [int],
                 img_root: Optional[Path] = None,
                 transform: Optional[Callable] = None
                 ):
        """
        Initialize a dog breed dataset.

        :param keys: List of identifiers for the images.
        :param labels: List of labels for the identifiers.
        :param img_root: Path pointing to the image directory.
        :param transform: Transformation to apply on loaded image.
        """

        self.keys = keys
        self.labels = labels
        self.transform = transform

        if img_root is None:
            self.img_root = Path('.data', 'DOGBREED', 'train')
        else:
            self.img_root = img_root


    def __len__(self) -> int:
        return len(self.keys)

    @property
    def shape(self) -> Tuple:
        return self[0][0].shape

    def __getitem__(self, idx: int) -> Tuple[Tensor, Tensor]:
        key = self.keys[idx]
        img = (
            Image.open(Path(self.img_root, key + '.jpg'))
        )

        if self.transform is not None:
            img = self.transform(img)

        y = (
            torch.tensor(self.labels[idx], dtype=torch.long)
        )


        return img, y
```

Data preparation

As this dataset is fairly small, we can generate more synthetic images by applying random transformations to the images we have. You might have noticed that our `DogBreedDataset` already takes `transform` as an argument, where we will pass some functions, that transform our images during training. Everytime a new batch of data is requested, the augmentations are randomly applied on-the-fly. This saves a lot of memory, at the price of larger computational resources needed.

This technique is called *data augmentation*, and it can greatly help in reducing overfitting on small datasets.

For this task we can use the `transforms` module of `torchvision` (check the docs [here (https://pytorch.org/vision/stable/transforms.html)](https://pytorch.org/vision/stable/transforms.html)). To chain multiple transformations, we use `Compose` .

We now want to randomly perform the following transformations to each image:

- Flip horizontally
- Rotation of at most 30 degrees
- Change brightness, contrast and saturation (`ColorJitter`)
- Change perspective (`RandomPerspective`)
- Resize to 224×224
- Convert from PIL image to a tensor with a range 0 to 1 (now 0 to 255).

We do not use random augmentation for the validation images except for centering and scaling. Why?

In [7]:

```
# Resizing to a square with varying aspect ratios per image can be a bit tricky.
# We define a small custom transformation, where we will pad the input to a square
# and then do the resizing.
# More info: https://discuss.pytorch.org/t/how-to-resize-and-pad-in-a-torchvision-t
class ResizeToSquare:
    def __init__(self, size: int):
        self.resize_transform = transforms.Resize(size)

    def __call__(self, img: Image) -> Image:
        img = self._do_square_padding(img)
        return self.resize_transform(img)

    @staticmethod
    def _do_square_padding(img: Image) -> Image:
        max_wh = max(img.size)
        p_left, p_top = [(max_wh - s) // 2 for s in img.size]
        p_right, p_bottom = [max_wh - (s+pad) for s, pad in zip(img.size, [p_left,
        padding = (p_left, p_top, p_right, p_bottom)
        return transforms.functional.pad(img, padding, 0, 'constant')

train_transforms = transforms.Compose([
    transforms.RandomHorizontalFlip(),
    transforms.RandomRotation(degrees=30),
    transforms.ColorJitter(
        brightness=0.5,
        saturation=0.5,
        contrast=0.5,
        hue=0.2
    ),
    transforms.RandomPerspective(),
    ResizeToSquare(size=224),
    transforms.ToTensor()
])

val_transforms = transforms.Compose([
    ResizeToSquare(size=224),
    transforms.ToTensor()
])
```

We can now finally create our `DogBreedDataset` objects:

In [8]:

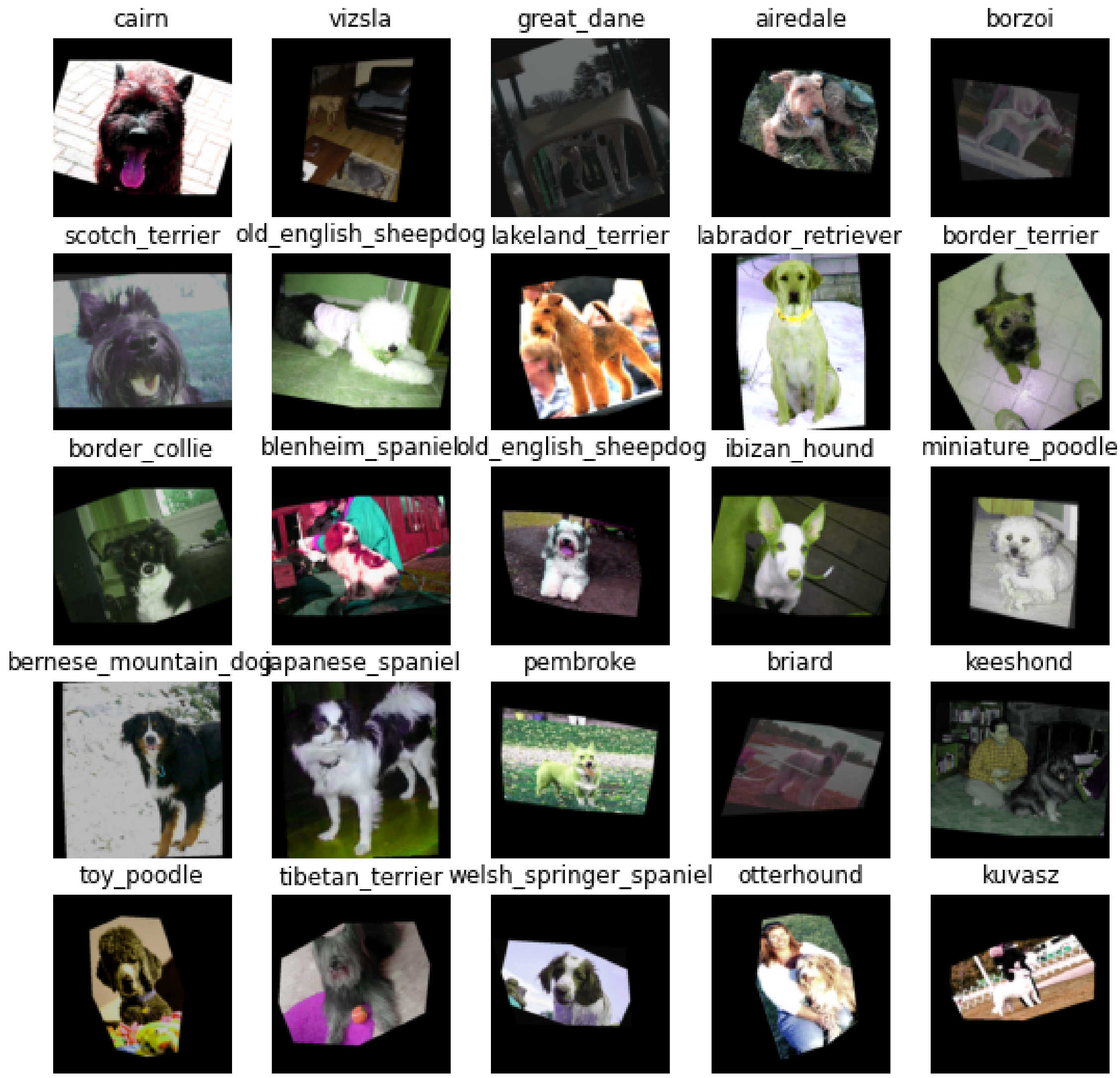
```
train_dataset = DogBreedDataset(
    keys=train_keys,
    labels=train_labels,
    transform=train_transforms
)

val_dataset = DogBreedDataset(
    keys=val_keys,
    labels=val_labels,
    transform=val_transforms
)
```

Here are some examples of how the augmented images look:

In [9]:

```
_, axs = plt.subplots(5, 5, figsize=(10, 10))
axs = axs.flatten()
for ax in axs:
    img, label = train_dataset[random.randint(0, len(train_dataset))]
    ax.imshow(img.permute(1, 2, 0))
    ax.set_title(num_to_class_mapping[int(label)])
    ax.axis('off')
plt.show()
```

```
/home/tobias/dl-exercise/.venv/lib/python3.8/site-packages/torchvisio
n/transforms/functional.py:594: UserWarning: torch.lstsq is deprecated
in favor of torch.linalg.lstsq and will be removed in a future PyTorch
release.
torch.linalg.lstsq has reversed arguments and does not return the QR d
ecomposition in the returned tuple (although it returns other informat
ion about the problem).
To get the qr decomposition consider using torch.linalg.qr.
The returned solution in torch.lstsq stored the residuals of the solut
ion in the last m - n columns of the returned value whenever m > n. In
torch.linalg.lstsq, the residuals in the field 'residuals' of the retu
rned named tuple.
The unpacking of the solution, as in
X, _ = torch.lstsq(B, A).solution[:A.size(1)]
should be replaced with
X = torch.linalg.lstsq(A, B).solution (Triggered internally at  ../ate
n/src/ATen/LegacyTHFunctionsCPU.cpp:389.)
  res = torch.lstsq(b_matrix, a_matrix)[0]
```



Define a Network

After preparing the data we define a network architecture. There are a lot of possible architectures. A good start might be a slightly smaller version of the famous VGG16 architecture. It consists of 4 blocks of 2 convolutional layers followed by one max pooling step, then two fully connected layers of size 512 are used, for a total of around 5 million weight parameters.

Global average pooling is used instead of flattening to reduce the number of parameters of the network. It takes the average of every input channel, so that a tensor of shape 14x14x512 results in a vector of 512 elements, each of which is the average of the corresponding 14x14 slice.

Also let's define our device. The exercise will run without a GPU but it is probably not feasible as computation on CPU would be magnitudes slower.

In [10]:

```
device = torch.device('cuda' if torch.cuda.is_available() else 'cpu')
```

In [11]:

```
model = nn.Sequential(
   # Block 1
    nn.Conv2d(in_channels=3, out_channels=64, kernel_size=(3, 3), padding=1),
    nn.ReLU(),
    nn.Conv2d(in_channels=64, out_channels=64, kernel_size=(3, 3), padding=1),
    nn.ReLU(),
    nn.MaxPool2d(2),

    # Block 2
    nn.Conv2d(in_channels=64, out_channels=128, kernel_size=(3, 3), padding=1),
    nn.ReLU(),
    nn.Conv2d(in_channels=128, out_channels=128, kernel_size=(3, 3), padding=1),
    nn.ReLU(),
    nn.MaxPool2d(2),

    # Block 3
    nn.Conv2d(in_channels=128, out_channels=256, kernel_size=(3, 3), padding=1),
    nn.ReLU(),
    nn.Conv2d(in_channels=256, out_channels=256, kernel_size=(3, 3), padding=1),
    nn.ReLU(),
    nn.MaxPool2d(2),

    # Block 4
    nn.Conv2d(in_channels=256, out_channels=512, kernel_size=(3, 3), padding=1),
    nn.ReLU(),
    nn.Conv2d(in_channels=512, out_channels=512, kernel_size=(3, 3), padding=1),
    nn.ReLU(),
    nn.MaxPool2d(2),

    # Flatten by global averaging.
    nn.AdaptiveAvgPool2d((1, 1)),
    nn.Flatten(),
    nn.Linear(in_features=512, out_features=512),
    nn.ReLU(),
    nn.Linear(in_features=512, out_features=512),
    nn.ReLU(),

    # Output for 120 classes.
    nn.Linear(in_features=512, out_features=120),
).to(device)
```

In [12]:

```
summary(model, input_size=(3, 224, 224), device=str(device))
```

```
----------------------------------------------------------------
        Layer (type)               Output Shape         Param #
================================================================
            Conv2d-1         [-1, 64, 224, 224]           1,792
              ReLU-2         [-1, 64, 224, 224]               0
            Conv2d-3         [-1, 64, 224, 224]          36,928
              ReLU-4         [-1, 64, 224, 224]               0
         MaxPool2d-5         [-1, 64, 112, 112]               0
            Conv2d-6        [-1, 128, 112, 112]          73,856
              ReLU-7        [-1, 128, 112, 112]               0
            Conv2d-8        [-1, 128, 112, 112]         147,584
              ReLU-9        [-1, 128, 112, 112]               0
        MaxPool2d-10          [-1, 128, 56, 56]               0
           Conv2d-11          [-1, 256, 56, 56]         295,168
             ReLU-12          [-1, 256, 56, 56]               0
           Conv2d-13          [-1, 256, 56, 56]         590,080
             ReLU-14          [-1, 256, 56, 56]               0
        MaxPool2d-15          [-1, 256, 28, 28]               0
           Conv2d-16          [-1, 512, 28, 28]       1,180,160
             ReLU-17          [-1, 512, 28, 28]               0
           Conv2d-18          [-1, 512, 28, 28]       2,359,808
             ReLU-19          [-1, 512, 28, 28]               0
        MaxPool2d-20          [-1, 512, 14, 14]               0
AdaptiveAvgPool2d-21            [-1, 512, 1, 1]               0
          Flatten-22                  [-1, 512]               0
           Linear-23                  [-1, 512]         262,656
             ReLU-24                  [-1, 512]               0
           Linear-25                  [-1, 512]         262,656
             ReLU-26                  [-1, 512]               0
           Linear-27                  [-1, 120]          61,560
================================================================
Total params: 5,272,248
Trainable params: 5,272,248
Non-trainable params: 0
----------------------------------------------------------------
Input size (MB): 0.57
Forward/backward pass size (MB): 195.26
Params size (MB): 20.11
Estimated Total Size (MB): 215.94
----------------------------------------------------------------
```

Train the network

As usual, let's define an optimizer, loss function, dataloaders and training loop.

In [13]:

```
epochs = 10
batch_size = 32
num_workers = 16

loss = (
    nn.CrossEntropyLoss()
)

optimizer = (
    Adam(model.parameters())
)


train_loader = DataLoader(
    dataset=train_dataset,
    batch_size=batch_size,
    shuffle=True,
    num_workers=num_workers,
)

val_loader = DataLoader(
    dataset=val_dataset,
    batch_size=batch_size,
    num_workers=num_workers,
)

def train(
        model: nn.Module,
        loss: nn.Module,
        optimizer: Optimizer,
        train_loader: DataLoader,
        val_loader: DataLoader,
        epochs: int
) -> Dict:
    # Intermediate results during training will be saved here.
    # This allows plotting the training progress afterwards.
    metrics: Dict = {
        'train_loss': [],
        'train_acc': [],
        'val_loss': [],
        'val_acc': [],
    }

    num_train_batches = ceil(len(train_loader.dataset) / batch_size)
    num_val_batches = ceil(len(val_loader.dataset) / batch_size)

    for ep in range(1, epochs + 1):
        total_loss = 0
        num_correct = 0

        for batch_idx, (x, y) in enumerate(train_loader):
            x = x.to(device)
            y = y.to(device)

            y_hat = model(x)
            batch_loss = loss(y_hat, y)

            optimizer.zero_grad()
```

```
            batch_loss.backward()
            optimizer.step()

            if batch_idx % 10 == 0:
                print('TRAINING BATCH:\t({:5} / {:5})\tLOSS:\t{:.3f}'
                      .format(batch_idx, num_train_batches, float(batch_loss) / bat

            total_loss += float(batch_loss)
            num_correct += int(torch.sum(torch.argmax(y_hat, dim=1) == y))


        ep_train_loss = total_loss / len(train_loader.dataset)
        ep_train_acc = num_correct / len(train_loader.dataset)

        # Reset counters
        total_loss = 0
        num_correct = 0

        for batch_idx, (x, y) in enumerate(val_loader):
            x = x.to(device)
            y = y.to(device)

            with torch.no_grad():
                y_hat = model(x)
                batch_loss = loss(y_hat, y)

            if batch_idx % 10 == 0:
                print('VALIDATION BATCH:\t({:5} / {:5})\tLOSS:\t{:.3f}'
                      .format(batch_idx, num_val_batches, float(batch_loss) / batch

            total_loss += float(batch_loss)
            num_correct += int(torch.sum(torch.argmax(y_hat, dim=1) == y))

        ep_val_loss = total_loss / len(val_loader.dataset)
        ep_val_acc = num_correct / len(val_loader.dataset)

        metrics['train_loss'].append(ep_train_loss)
        metrics['train_acc'].append(ep_train_acc)
        metrics['val_loss'].append(ep_val_loss)
        metrics['val_acc'].append(ep_val_acc)

        print('EPOCH:\t{:5}\tTRAIN LOSS:\t{:.3f}\tTRAIN ACCURACY:\t{:.3f}\tVAL LOSS
              '{:.3f}\tVAL ACCURACY:\t{:.3f}'
              .format(ep, ep_train_loss, ep_train_acc,ep_val_loss, ep_val_acc,
                      end='\r'))
    return metrics

metrics = train(model, loss, optimizer, train_loader, val_loader, epochs)
```

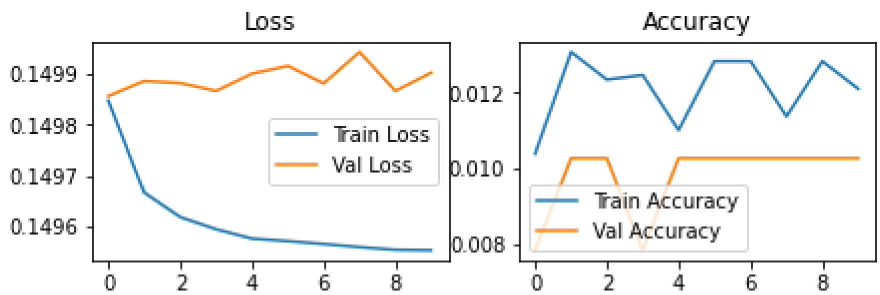
```
EPOCH:      1   TRAIN LOSS:     0.150   TRAIN ACCURACY: 0.010   VAL
LOSS:   0.150   VAL ACCURACY:   0.008
EPOCH:      2   TRAIN LOSS:     0.150   TRAIN ACCURACY: 0.013   VAL
LOSS:   0.150   VAL ACCURACY:   0.010
EPOCH:      3   TRAIN LOSS:     0.150   TRAIN ACCURACY: 0.012   VAL
LOSS:   0.150   VAL ACCURACY:   0.010
EPOCH:      4   TRAIN LOSS:     0.150   TRAIN ACCURACY: 0.012   VAL
LOSS:   0.150   VAL ACCURACY:   0.008
EPOCH:      5   TRAIN LOSS:     0.150   TRAIN ACCURACY: 0.011   VAL
LOSS:   0.150   VAL ACCURACY:   0.010
EPOCH:      6   TRAIN LOSS:     0.150   TRAIN ACCURACY: 0.013   VAL
```

```
LOSS:    0.150   VAL ACCURACY:   0.010
EPOCH:       7   TRAIN LOSS:     0.150   TRAIN ACCURACY: 0.013   VAL
LOSS:    0.150   VAL ACCURACY:   0.010
EPOCH:       8   TRAIN LOSS:     0.150   TRAIN ACCURACY: 0.011   VAL
LOSS:    0.150   VAL ACCURACY:   0.010
EPOCH:       9   TRAIN LOSS:     0.150   TRAIN ACCURACY: 0.013   VAL
LOSS:    0.150   VAL ACCURACY:   0.010
EPOCH:      10   TRAIN LOSS:     0.150   TRAIN ACCURACY: 0.012   VAL
LOSS:    0.150   VAL ACCURACY:   0.010
```

We recycle the plotting function of lab 6 to plot the training progress:

In [14]:

```python
def get_training_progress_plot(
        train_losses: List[float],
        train_accs: List[float],
        val_losses: List[float],
        val_accs: List[float],
) -> None:
    fig, (ax1, ax2) = plt.subplots(1, 2, figsize=(7, 2))

    ax1.set_title('Loss')
    ax1.plot(train_losses, label='Train Loss')
    ax1.plot(val_losses, label='Val Loss')
    ax1.legend()

    ax2.set_title('Accuracy')
    ax2.plot(train_accs, label='Train Accuracy')
    ax2.plot(val_accs, label='Val Accuracy')
    ax2.legend()

get_training_progress_plot(
    metrics['train_loss'],
    metrics['train_acc'],
    metrics['val_loss'],
    metrics['val_acc'],
)
```



Using a pretrained network

Even with the aid of data augmentation, the network does not perform good. This can be explained by the fact that the images are quite diverse in relation to the size of the training set. Data augmentation can only bring you so far, and even with the aid of regularization the task would be difficult.

One popular trick to overcome this difficulty, known as *pre-training*, is to use another CNN that has been trained on a different, larger dataset, for a related task. Most of the weights of this network are then frozen (i.e., will not be updated), and the last few layers (the "head") are replaced with new, freshly re-initialized ones and learned from scratch. What is the rationale behind freezing and unfreezing the weights?

PyTorch offers a variety of [pretrained models to download (https://pytorch.org/vision/stable/models.html)](https://pytorch.org/vision/stable/models.html).

After obtaining VGG16, our plan is to:

```
1. Get the body of the downloaded net (accessible over `features` attribut
e)
2. Create an extra head.
3. Train only the weights of the head.
```

In [15]:

```
vgg_body = vgg16(pretrained=True, progress=False).features.to(device)
vgg_head = nn.Sequential(
    nn.AdaptiveAvgPool2d((1, 1)),
    nn.Flatten(),
    nn.Linear(in_features=512, out_features=512),
    nn.ReLU(),
    nn.Linear(in_features=512, out_features=512),
    nn.ReLU(),
    nn.Linear(in_features=512, out_features=120),
).to(device)

head_optimizer = (
    Adam(vgg_head.parameters())
)
```

The documentation of VGG16 reveals that input images should be normalized with a given range. Thus, we need to adjust the transforms of our datasets:

In [16]:

```
normalize_transform = transforms.Normalize(
    mean=[0.485, 0.456, 0.406],
    std=[0.229, 0.224, 0.225]
)

# Note: The clean way would be creating new datasets instead of modifiying internal
train_dataset.transform.transforms.append(normalize_transform)
val_dataset.transform.transforms.append(normalize_transform)
```

In the next step, we modify our train function to implement the desired behaviour. We train only the head for 10 epochs and then allow optimization of everything.

In [17]:

```
epochs = 25

def train(
        head: nn.Module,
        body: nn.Module,
        loss: nn.Module,
        head_optimizer: Optimizer,
        train_loader: DataLoader,
        val_loader: DataLoader,
        epochs: int
) -> Dict:
    # Intermediate results during training will be saved here.
    # This allows plotting the training progress afterwards.
    metrics: Dict = {
        'train_loss': [],
        'train_acc': [],
        'val_loss': [],
        'val_acc': [],
    }

    num_train_batches = ceil(len(train_loader.dataset) / batch_size)
    num_val_batches = ceil(len(val_loader.dataset) / batch_size)

    for ep in range(1, epochs + 1):
        total_loss = 0
        num_correct = 0

        for batch_idx, (x, y) in enumerate(train_loader):
            x = x.to(device)
            y = y.to(device)

            with torch.no_grad():
                x_features = body(x)
            y_hat = head(x_features)
            batch_loss = loss(y_hat, y)

            head_optimizer.zero_grad()
            batch_loss.backward()
            head_optimizer.step()

            if batch_idx % 10 == 0:
                print('TRAINING BATCH:\t({:5} / {:5})\tLOSS:\t{:.3f}'
                      .format(batch_idx, num_train_batches, float(batch_loss) / bat

            total_loss += float(batch_loss)
            num_correct += int(torch.sum(torch.argmax(y_hat, dim=1) == y))


        ep_train_loss = total_loss / len(train_loader.dataset)
        ep_train_acc = num_correct / len(train_loader.dataset)

        # Reset counters
        total_loss = 0
        num_correct = 0

        for batch_idx, (x, y) in enumerate(val_loader):
            x = x.to(device)
            y = y.to(device)
```

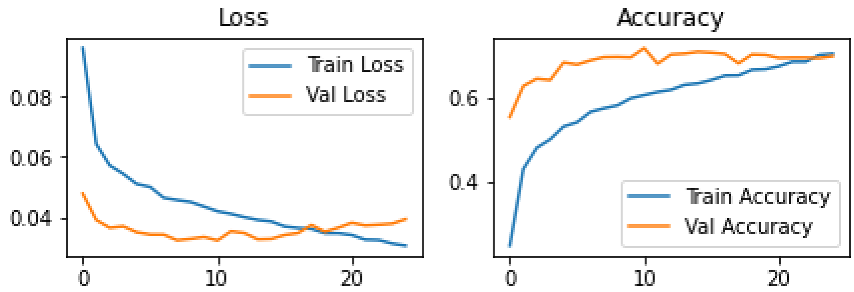
```python
            with torch.no_grad():
                y_hat = head(body(x))
                batch_loss = loss(y_hat, y)

            if batch_idx % 10 == 0:
                print('VALIDATION BATCH:\t({:5} / {:5})\tLOSS:\t{:.3f}'
                      .format(batch_idx, num_val_batches, float(batch_loss) / batch

            total_loss += float(batch_loss)
            num_correct += int(torch.sum(torch.argmax(y_hat, dim=1) == y))

        ep_val_loss = total_loss / len(val_loader.dataset)
        ep_val_acc = num_correct / len(val_loader.dataset)

        metrics['train_loss'].append(ep_train_loss)
        metrics['train_acc'].append(ep_train_acc)
        metrics['val_loss'].append(ep_val_loss)
        metrics['val_acc'].append(ep_val_acc)

        print('EPOCH:\t{:5}\tTRAIN LOSS:\t{:.3f}\tTRAIN ACCURACY:\t{:.3f}\tVAL LOSS
              '{:.3f}\tVAL ACCURACY:\t{:.3f}'
              .format(ep, ep_train_loss, ep_train_acc,ep_val_loss, ep_val_acc,
                      end='\r'))
    return metrics

metrics = train(
    head=vgg_head,
    body=vgg_body,
    head_optimizer=head_optimizer,
    loss=loss,
    train_loader=train_loader,
    val_loader=val_loader,
    epochs=epochs
)
```

```
EPOCH:       1   TRAIN LOSS:     0.096   TRAIN ACCURACY: 0.247   VAL
LOSS:   0.048   VAL ACCURACY:   0.553
EPOCH:       2   TRAIN LOSS:     0.064   TRAIN ACCURACY: 0.429   VAL
LOSS:   0.039   VAL ACCURACY:   0.627
EPOCH:       3   TRAIN LOSS:     0.057   TRAIN ACCURACY: 0.480   VAL
LOSS:   0.036   VAL ACCURACY:   0.645
EPOCH:       4   TRAIN LOSS:     0.054   TRAIN ACCURACY: 0.500   VAL
LOSS:   0.037   VAL ACCURACY:   0.641
EPOCH:       5   TRAIN LOSS:     0.051   TRAIN ACCURACY: 0.531   VAL
LOSS:   0.035   VAL ACCURACY:   0.683
EPOCH:       6   TRAIN LOSS:     0.050   TRAIN ACCURACY: 0.541   VAL
LOSS:   0.034   VAL ACCURACY:   0.679
EPOCH:       7   TRAIN LOSS:     0.046   TRAIN ACCURACY: 0.566   VAL
LOSS:   0.034   VAL ACCURACY:   0.688
EPOCH:       8   TRAIN LOSS:     0.046   TRAIN ACCURACY: 0.574   VAL
LOSS:   0.032   VAL ACCURACY:   0.696
EPOCH:       9   TRAIN LOSS:     0.045   TRAIN ACCURACY: 0.581   VAL
LOSS:   0.033   VAL ACCURACY:   0.697
EPOCH:      10   TRAIN LOSS:     0.043   TRAIN ACCURACY: 0.598   VAL
LOSS:   0.033   VAL ACCURACY:   0.696
EPOCH:      11   TRAIN LOSS:     0.042   TRAIN ACCURACY: 0.606   VAL
LOSS:   0.032   VAL ACCURACY:   0.718
EPOCH:      12   TRAIN LOSS:     0.041   TRAIN ACCURACY: 0.613   VAL
LOSS:   0.035   VAL ACCURACY:   0.681
EPOCH:      13   TRAIN LOSS:     0.040   TRAIN ACCURACY: 0.618   VAL
```

```
LOSS:    0.035   VAL ACCURACY:    0.703
EPOCH:      14   TRAIN LOSS:      0.039   TRAIN ACCURACY: 0.630   VAL
LOSS:    0.033   VAL ACCURACY:    0.705
EPOCH:      15   TRAIN LOSS:      0.039   TRAIN ACCURACY: 0.634   VAL
LOSS:    0.033   VAL ACCURACY:    0.709
EPOCH:      16   TRAIN LOSS:      0.037   TRAIN ACCURACY: 0.642   VAL
LOSS:    0.034   VAL ACCURACY:    0.707
EPOCH:      17   TRAIN LOSS:      0.036   TRAIN ACCURACY: 0.652   VAL
LOSS:    0.035   VAL ACCURACY:    0.704
EPOCH:      18   TRAIN LOSS:      0.036   TRAIN ACCURACY: 0.653   VAL
LOSS:    0.037   VAL ACCURACY:    0.682
EPOCH:      19   TRAIN LOSS:      0.035   TRAIN ACCURACY: 0.666   VAL
LOSS:    0.035   VAL ACCURACY:    0.703
EPOCH:      20   TRAIN LOSS:      0.035   TRAIN ACCURACY: 0.667   VAL
LOSS:    0.036   VAL ACCURACY:    0.702
EPOCH:      21   TRAIN LOSS:      0.034   TRAIN ACCURACY: 0.674   VAL
LOSS:    0.038   VAL ACCURACY:    0.694
EPOCH:      22   TRAIN LOSS:      0.032   TRAIN ACCURACY: 0.685   VAL
LOSS:    0.037   VAL ACCURACY:    0.695
EPOCH:      23   TRAIN LOSS:      0.032   TRAIN ACCURACY: 0.685   VAL
LOSS:    0.037   VAL ACCURACY:    0.695
EPOCH:      24   TRAIN LOSS:      0.031   TRAIN ACCURACY: 0.701   VAL
LOSS:    0.038   VAL ACCURACY:    0.694
EPOCH:      25   TRAIN LOSS:      0.030   TRAIN ACCURACY: 0.704   VAL
LOSS:    0.039   VAL ACCURACY:    0.699
```

In [18]:

```
get_training_progress_plot(
    metrics['train_loss'],
    metrics['train_acc'],
    metrics['val_loss'],
    metrics['val_acc'],
)
```



As you can see, the results are much better now, and would keep improving if we had trained for longer.

Exercise 2

This exercise is about the receptive field of convolutional neural networks. For our purposes, the receptive field of a neuron in layer L contains the features in a preceding layer ℓ that affect the output of said neuron, with $\ell = 0$ being the input to the network. In other words, changing any value in a neuron's receptive field will change the output of that neuron. By going backwards from layer L, convolutions and pooling operations enlarge the receptive field of neurons at layer L, so that the deeper the network, the larger the receptive field of neurons at the end ofthe network.

Let $\mathbf{z}_\ell \in \mathbb{R}^{n_\ell}$ be the output of layer ℓ (and $\mathbf{z}_0$ the input), that is obtained with a one-dimensional convolution or pooling operation from $\mathbf{z}_{\ell-1}$ with a kernel of size k_ℓ and stride s_ℓ. Define r_ℓ to be the size of the receptive field in the ℓ-th layer of a neuron in layer L, i.e. the minimum width of the largest region that contains the elements in $\mathbf{z}_\ell$ that affect a generic element in $\mathbf{z}_L$. Note that this region can contain gaps, i.e. neurons that do not affect the output of the neuron in layer L, if they are in between neurons that do affect it.

Show that $r_{\ell-1}$ can be computed from r_ℓ as follows:

$$r_{\ell-1} = s_\ell \cdot r_\ell + k_\ell - s_\ell$$

You can consider padding to be infinite, or, equivalently, focus on the neurons in the middle of the layer, without analyzing what happens near the borders. Hint: consider the case $k_\ell = 1$ first.

Then solve the recurrence to show that:

$$r_0 = \sum_{\ell=1}^{L} \left((k_\ell - 1) \prod_{i=1}^{\ell-1} s_i \right) + 1$$

with the base case being $r_L = 1$.

Compute the receptive field size of the pre-trained VGG16 architecture we used above, right before the global average pooling layer.

Now suppose to have a dilation of $d_\ell \geq 1$ at every layer. What is the new formula for r_0?

What is the most effective way to increase the size of the receptive field of a neural network?

Solution

Start with $k_\ell = 1$. Every neuron in r_ℓ, has a receptive field of 1, and we need to add $s_\ell - 1$ for the gap left between adjacent neurons, resulting in $r_{\ell-1} = r_\ell + (r_\ell - 1)(s_\ell - 1) = r_\ell s_\ell - s_\ell + 1$. If $k_\ell > 1$ and odd, we need to add $(k_\ell - 1)/2$ at both sides of the receptive field, resulting in

$$r_{\ell-1} = r_\ell s_\ell - s_\ell + 1 + 2 \cdot \frac{k_\ell - 1}{2} = r_\ell s_\ell + k_\ell - s_\ell$$

When k_ℓ is even, we need to add $k_\ell/2$ on one side, and $k_\ell/2 - 1$ on the other side; as this adds up to $k_\ell - 1$ again, the result is the same.

To solve the recurrence, we can unroll a few steps and try to spot a pattern. For ease of notation, we use $r, r', r'', r''', \ldots$ instead of $r_\ell, r_{\ell-1}, r_{\ell-2}, r_{\ell-3}, \ldots$, similarly for k_ℓ and s_ℓ.

$$\begin{aligned}
r' &= sr + k - s \\
r'' &= s'r' + k' - s' \\
&= s'sr + s'k - s's + k' - s' \\
r''' &= s''r'' + k'' - s'' \\
&= s''s'sr + s''s'k - s''s's + s''k' - s''s' + k'' - s'' \\
&= (r-1)s''s's + (k-1)s''s' + (k'-1)s'' + k'' \\
r'''' &= s'''r''' + k''' - s''' \\
&= (r-1)s'''s''s's + (k-1)s'''s''s' + (k'-1)s'''s'' + (k''-1)s''' + k'''
\end{aligned}$$

The next element would be:

$$
\begin{aligned}
r_{\ell-5} = (r_\ell - 1)s_{\ell-4}s_{\ell-3}s_{\ell-2}s_{\ell-1}s_\ell \\
&+ (k_\ell - 1)s_{\ell-4}s_{\ell-3}s_{\ell-2}s_{\ell-1} \\
&+ (k_{\ell-1} - 1)s_{\ell-4}s_{\ell-3}s_{\ell-2} \\
&+ (k_{\ell-2} - 1)s_{\ell-4}s_{\ell-3} \\
&+ (k_{\ell-3} - 1)s_{\ell-4} \\
&+ (k_{\ell-4} - 1) \\
&+ 1
\end{aligned}
$$

The pattern should be clear:

$$
r_{\ell-i} = (r_\ell - 1)\prod_{j=1}^{i} s_{\ell-i+j} + \sum_{j=1}^{i}\left((k_{\ell-i+j} - 1)\prod_{k=1}^{j-1} s_{\ell-i+k}\right) + 1
$$

With the convention that $\prod_{i=a}^{b} x_i = 1$ when $a > b$. Now using $i = \ell = L$, and remembering that $r_L = 1$, we find that:

$$
r_0 = \sum_{j=1}^{L}\left((k_j - 1)\prod_{k=1}^{j-1} s_k\right) + 1
$$

The pre-trained VGG16 has five blocks composed by two (first two blocks) or three (last three blocks) convolutions with kernel size 3 and stride 1 followed by a max pooling of kernel size 2 with stride 2. We can easily apply the recursive formula:

In [19]:

```
strides = [1,1,2, 1,1,2, 1,1,1,2, 1,1,1,2, 1,1,1,2]
kernels = [3,3,2, 3,3,2, 3,3,3,2, 3,3,3,2, 3,3,3,2]

def calc_receptive_field(r: int, l: int, kernels: List[int], strides: List[int]) ->
    return strides[l] * r + kernels[l] - strides[l]

func = partial(calc_receptive_field, kernels=kernels, strides=strides)
print(reduce(func, range(len(strides))))
```

212

Dilation can be seen as further gaps between the elements of every filter, so that a dilation of $d_j \geq 1$ leaves $d_j - 1$ gaps between elements of the filter. For example, a dilation of 2 means that a filter of size 3 will cover 5 elements in total, a dilation of 3 results in 7 elements, and so on. With this insight, we can replace k_j with $k_j d_j - d_j + 1$ in the formula above:

$$
r_0 = \sum_{j=1}^{L}\left((k_j d_j - d_j)\prod_{k=1}^{j} s_k\right) + 1
$$

According to the formula, the best way to increase the size of the receptive field of a network is to increase striding, because strides of successive layers get multiplied together. Note, however, that high strides means that, in practice, there will be large gaps in the receptive field, with many neurons not actually contributing to the neurons in the following layers. For this reason, neural networks since VGG16 have also become much deeper. Moreover, the receptive field is not the only parameter that affects performance, and having a large receptive field seems to be a necessary, but not sufficient condition for well-performing networks.