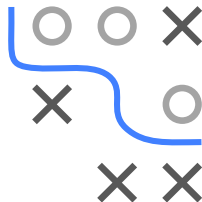


Introduction to Machine Learning

Evaluation

Simple Measures for Classification



Learning goals

- Know the definitions of misclassification error rate (MCE) and accuracy (ACC)
- Understand the entries of a confusion matrix
- Understand the idea of costs
- Know definitions of Brier score and log loss

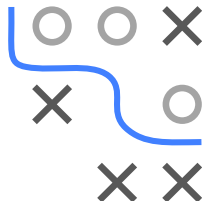
		True Class y	
		+	-
Pred.	+	True Positive (TP)	False Positive (FP)
\hat{y}	-	False Negative (FN)	True Negative (TN)

LABELS: MCE & ACC

The **misclassification error rate (MCE)** counts the number of incorrect predictions and presents them as a rate:

$$\rho_{MCE} = \frac{1}{m} \sum_{i=1}^m [y^{(i)} \neq \hat{y}^{(i)}] \in [0, 1].$$

MCE		
y		\hat{y}
	?	
	≠	
	?	
	≠	
	?	
	≠	



Accuracy (ACC) is defined in a similar fashion for correct classifications:

$$\rho_{ACC} = \frac{1}{m} \sum_{i=1}^m [y^{(i)} = \hat{y}^{(i)}] \in [0, 1].$$

ACC		
y		\hat{y}
	?	
	=	
	?	
	=	
	?	
	=	

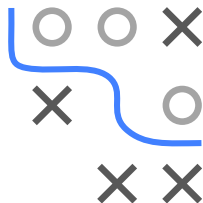
- If the data set is small this can be brittle.
- MCE says nothing about how good/skewed predicted probabilities are.
- Errors on all classes are weighted equally, which is often inappropriate.

LABELS: CONFUSION MATRIX

Much better than reducing prediction errors to a simple number is tabulating them in a **confusion matrix**:

- true classes in columns,
- predicted classes in rows.

We can nicely see class sizes (predicted/true) and where errors occur.



		True classes				
		setosa	versicolor	virginica	error	<i>n</i>
Predicted classes	setosa	50	0	0	0	50
	versicolor	0	46	4	4	50
	virginica	0	4	46	4	50
	error	0	4	4	8	-
<i>n</i>		50	50	50	-	150

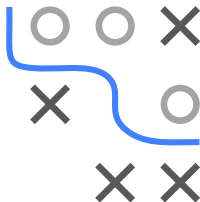
LABELS: CONFUSION MATRIX

- In binary classification, we typically call one class "positive" and the other "negative".
- The positive class is the more important, often smaller one.

		True Class y	
		+	-
Pred.	+	True Positive (TP)	False Positive (FP)
\hat{y}	-	False Negative (FN)	True Negative (TN)

e.g.,

- **True Positive** (TP) means that an instance is classified as positive that is really positive (correct prediction).
- **False Negative** (FN) means that an instance is classified as negative that is actually positive (incorrect prediction).



LABELS: COSTS

We can also assign different costs to different errors via a **cost matrix**.

$$Costs = \frac{1}{n} \sum_{i=1}^n C[y^{(i)}, \hat{y}^{(i)}]$$

Example: Depending on certain features (age, income, profession, ...) a bank wants to decide whether to grant a 10,000 EUR loan.

Predict if a person is solvent (yes / no).

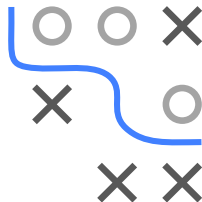
Should the bank lend them the money?

Exemplary costs:

Loss in event of default: 10,000 EUR

Income through interest paid: 100 EUR

		True classes	
		solvent	not solvent
Predicted classes	solvent	0	10,000
	not solvent	100	0

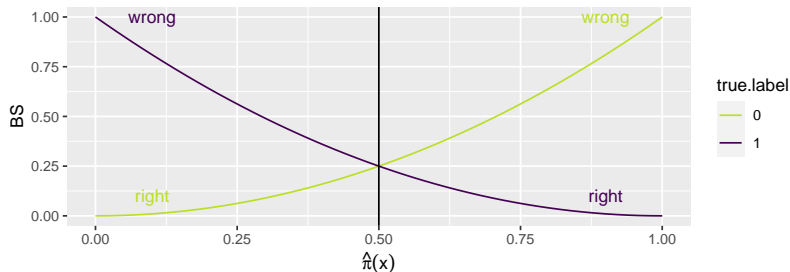


PROBABILITIES: BRIER SCORE

Measures squared distances of probabilities from the true class labels:

$$\rho_{BS} = \frac{1}{m} \sum_{i=1}^m \left(\hat{\pi}^{(i)} - y^{(i)} \right)^2$$

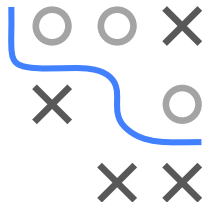
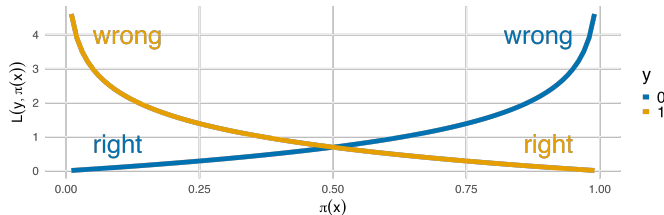
- Fancy name for MSE on probabilities.
- Usual definition for binary case; $y^{(i)}$ must be encoded as 0 and 1.



PROBABILITIES: LOG-LOSS

Logistic regression loss function, a.k.a. Bernoulli or binomial loss, $y^{(i)}$ encoded as 0 and 1.

$$\rho_{LL} = \frac{1}{m} \sum_{i=1}^m \left(-y^{(i)} \log \left(\hat{\pi}^{(i)} \right) - \left(1 - y^{(i)} \right) \log \left(1 - \hat{\pi}^{(i)} \right) \right).$$



- Optimal value is 0, “confidently wrong” is penalized heavily.
- Multi-class version: $\rho_{LL,MC} = -\frac{1}{m} \sum_{i=1}^m \sum_{k=1}^g o_k^{(i)} \log \left(\hat{\pi}_k^{(i)} \right).$