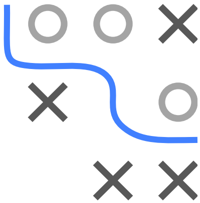
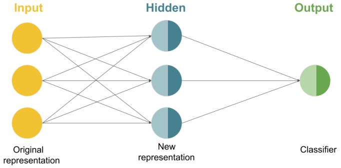
Introduction to Machine Learning



Neural Networks
Single hidden layer neural networks



Learning goals

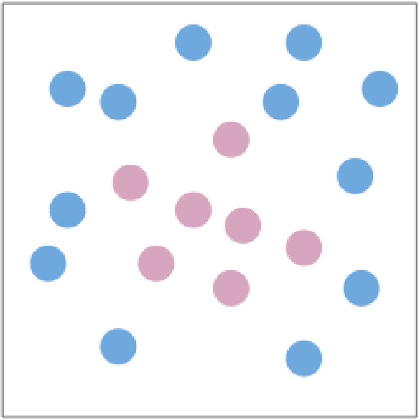
- Architecture of single hidden layer neural networks
- Representation learning/ understanding the advantage of hidden layers
- Typical (non-linear) activation functions

MOTIVATION

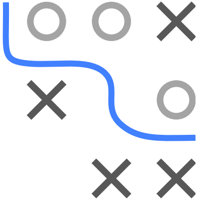
- The graphical representation of simple functions/models, like logistic regression, is useful because:
- Individual neurons can be used as building blocks of more complex functions.
- Networks of neurons can represent very complex hypothesis spaces.
- Most importantly, it allows us to define appropriate hypothesis spaces to learn functions that are common in practice in a data-efficient way (see Lin, Tegmark et al. 2016).

MOTIVATION

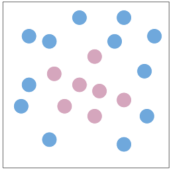
- Can a single neuron perform binary classification of these points?



MOTIVATION

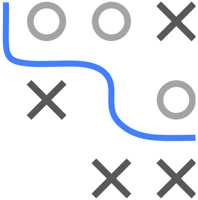


- As a single neuron is restricted to learning only linear decision boundaries, its performance on the following task is quite poor:

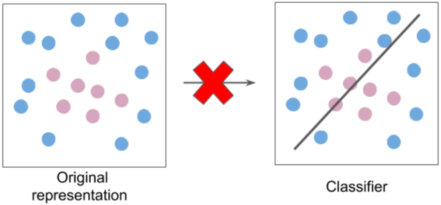


- However, the neuron can easily separate the classes if the original features are transformed (e.g., from Cartesian to polar coordinates):

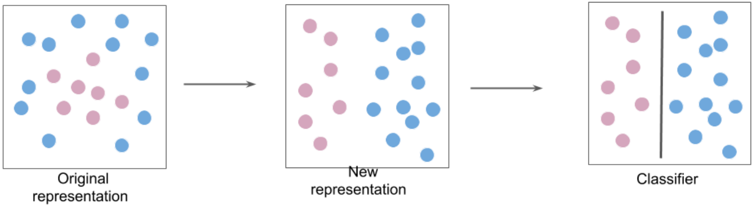
MOTIVATION



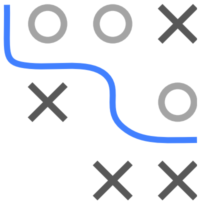
- Instead of classifying the data in the original representation,



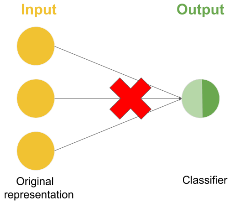
- we classify it in a new feature space.



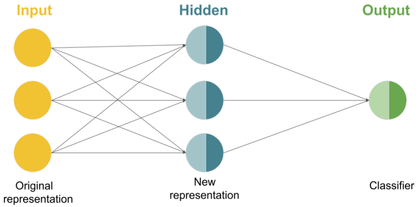
MOTIVATION



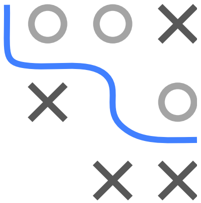
- Analogously, instead of a single neuron,



- we use more complex networks.



REPRESENTATION LEARNING



- It is *very* critical to feed a classifier the “right” features in order for it to perform well.
- Before deep learning took off, features for tasks like machine vision and speech recognition were “hand-designed” by domain experts. This step of the machine learning pipeline is called **feature engineering**.
- DL automates feature engineering. This is called **representation learning**.

SINGLE HIDDEN LAYER NETWORKS

- **Single neurons** perform a 2-step computation:
  1. **Affine Transformation:** a weighted sum of inputs plus bias.
  2. **Activation:** a non-linear transformation on the weighted sum.

- **Single hidden layer networks** consist of two layers (without input layer):
  1. **Hidden Layer:** having a set of neurons.
  2. **Output Layer:** having one or more output neurons.

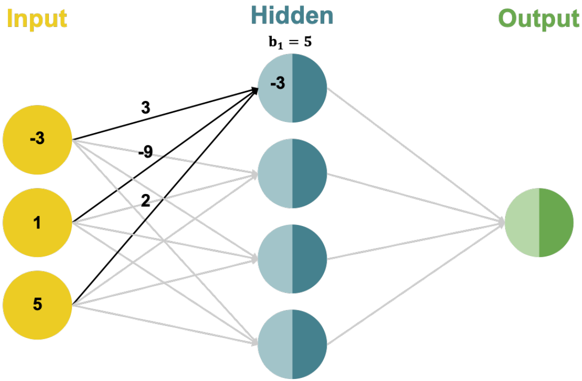
- Multiple inputs are simultaneously fed to the network.

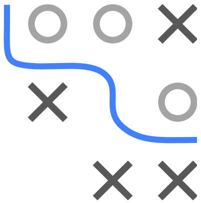
- Each neuron in the hidden layer performs a 2-step computation.

- The final output of the network is then calculated by another 2-step computation performed by the neuron in the output layer.

SINGLE HIDDEN LAYER NETWORKS: EXAMPLE

Each neuron in the hidden layer performs an **affine transformation** on the inputs:

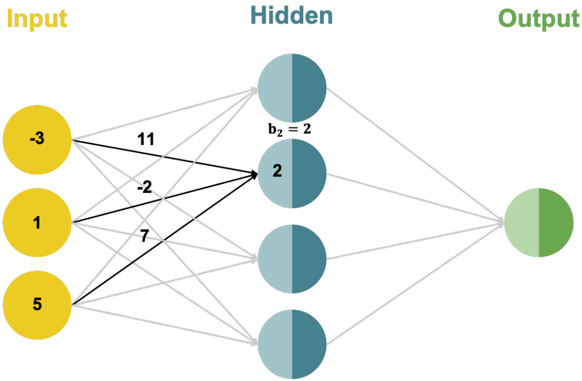


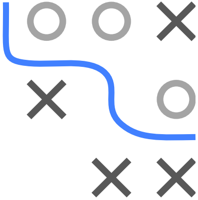


$$z_{1,\text{in}} = w_{11}x_1 + w_{21}x_2 + w_{31}x_3 + b_1$$
$$z_{1,\text{in}} = 3 * (-3) + (-9) * 1 + 2 * 5 + 5 = -3$$

SINGLE HIDDEN LAYER NETWORKS: EXAMPLE

Each neuron in the hidden layer performs an **affine transformation** on the inputs:

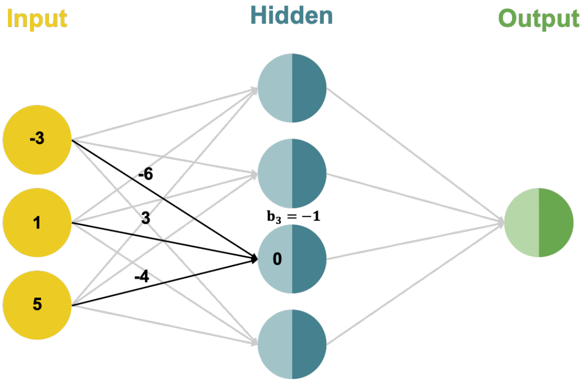


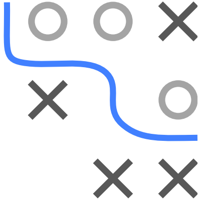


$$z_{2,\text{in}} = w_{12}x_1 + w_{22}x_2 + w_{32}x_3 + b_2$$
$$z_{2,\text{in}} = 11 * (-3) + (-2) * 1 + 7 * 5 + 2 = 2$$

SINGLE HIDDEN LAYER NETWORKS: EXAMPLE

Each neuron in the hidden layer performs an **affine transformation** on the inputs:

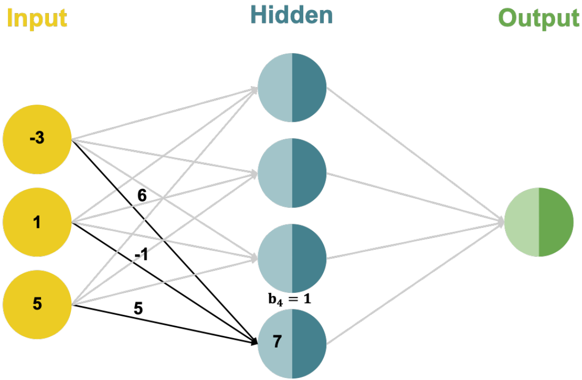


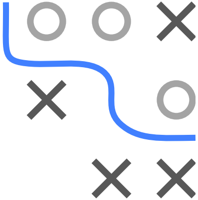


$$z_{3,\text{in}} = w_{13}x_1 + w_{23}x_2 + w_{33}x_3 + b_3$$
$$z_{3,\text{in}} = (-6) * (-3) + 3 * 1 + (-4) * 5 - 1 = 0$$

SINGLE HIDDEN LAYER NETWORKS: EXAMPLE

Each neuron in the hidden layer performs an **affine transformation** on the inputs:

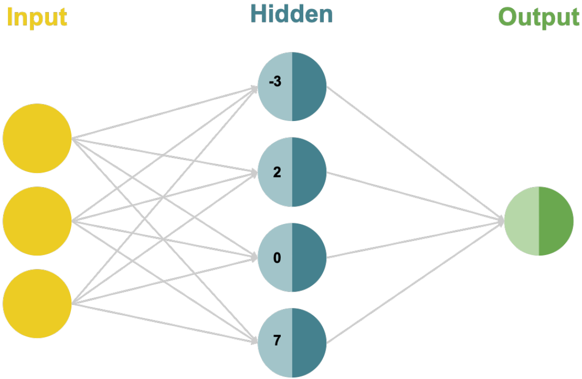


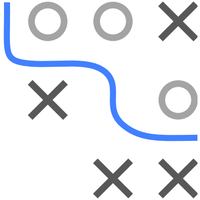


$$z_{4,\text{in}} = w_{14}x_1 + w_{24}x_2 + w_{34}x_3 + b_4$$
$$z_{4,\text{in}} = 6 * (-3) + (-1) * 1 + 5 * 5 + 1 = 7$$

SINGLE HIDDEN LAYER NETWORKS: EXAMPLE

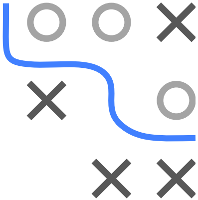
Each neuron in the hidden layer performs an **affine transformation** on the inputs:

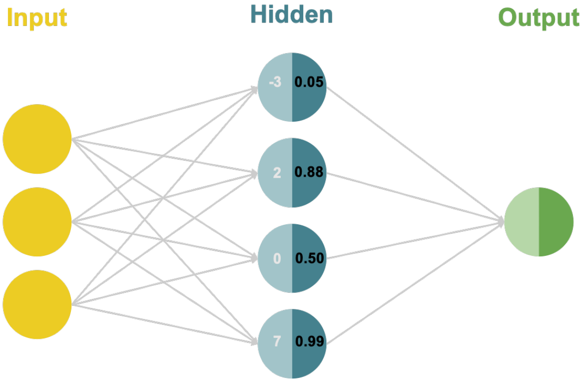




SINGLE HIDDEN LAYER NETWORKS: EXAMPLE

Each hidden neuron performs a non-linear **activation** transformation on the weight sum:

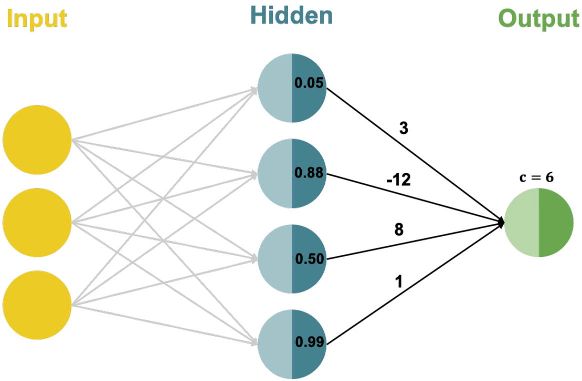


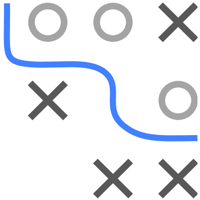


$$z_{\text{i,out}} = \sigma\left(z_{\text{i,in}}\right) = \frac{1}{1+e^{-z_{\text{in}}^{(i)}}}$$

SINGLE HIDDEN LAYER NETWORKS: EXAMPLE

The output neuron performs an **affine transformation** on its inputs:

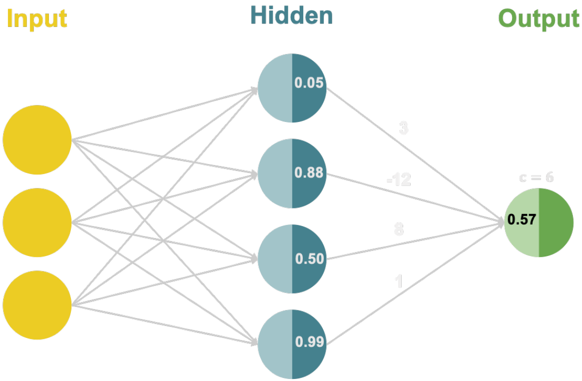


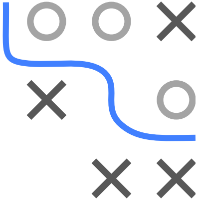


$$f_{\text{in}} = u_1 z_{1,\text{out}} + u_2 z_{2,\text{out}} + u_3 z_{3,\text{out}} + u_4 z_{4,\text{out}} + c$$

SINGLE HIDDEN LAYER NETWORKS: EXAMPLE

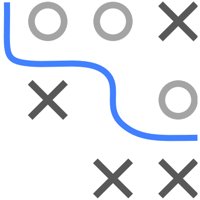
The output neuron performs an **affine transformation** on its inputs:



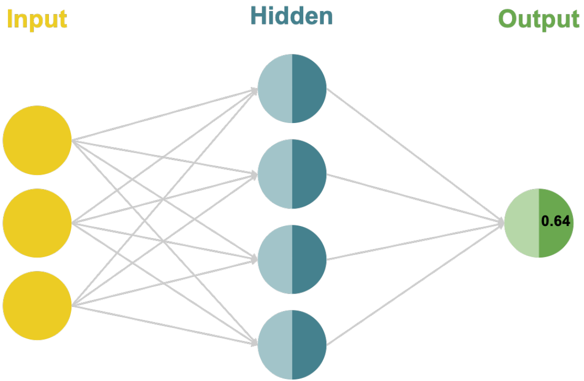


$$f_{\text{in}} = u_1 z_{1,\text{out}} + u_2 z_{2,\text{out}} + u_3 z_{3,\text{out}} + u_4 z_{4,\text{out}} + c$$
$$f_{\text{in}} = 3 * 0.05 + (-12) * 0.88 + 8 * 0.50 + 1 * 0.99 + 6 = 0.57$$

SINGLE HIDDEN LAYER NETWORKS: EXAMPLE

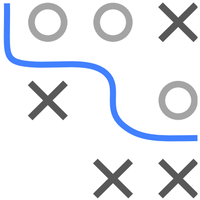


The output neuron performs a non-linear **activation** transformation on the weight sum:



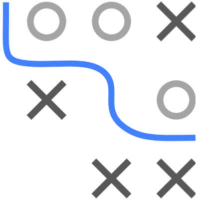
$$f_{\text{out}} = \sigma(f_{\text{in}}) = \frac{1}{1+e^{-f_{\text{in}}}}$$
$$f_{\text{out}} = \frac{1}{1+e^{-0.57}} = 0.64$$

HIDDEN LAYER: ACTIVATION FUNCTION



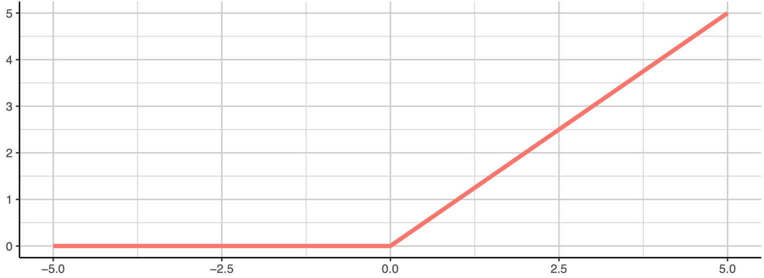
- If the hidden layer does not have a non-linear activation, the network can only learn linear decision boundaries.
- A lot of different activation functions exist.

HIDDEN LAYER: ACTIVATION FUNCTION I

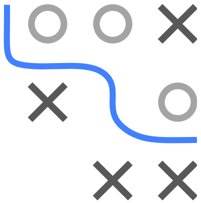


- **ReLU Activation:** Currently the most popular choice is the ReLU (rectified linear unit).

$$\sigma(v) = \max(0, v)$$



HIDDEN LAYER: ACTIVATION FUNCTION II



- **Sigmoid Activation Function:** The sigmoid function can be used even in the hidden layer.

$$\sigma(v) = \frac{1}{1 + \exp(-v)}$$