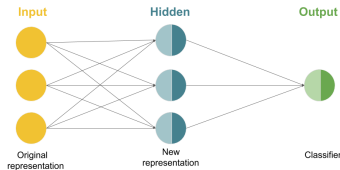


Introduction to Machine Learning

Neural Networks

Single hidden layer neural networks



Learning goals

- Architecture of single hidden layer neural networks
- Representation learning/ understanding the advantage of hidden layers
- Typical (non-linear) activation functions

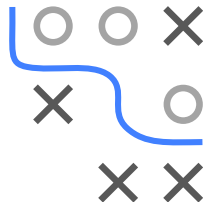
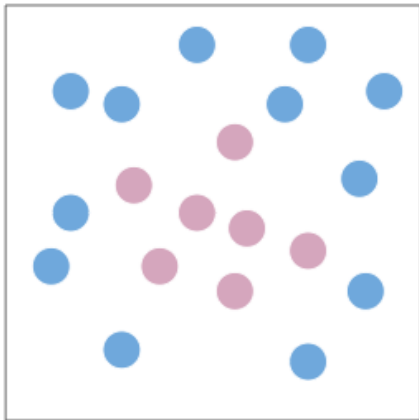
MOTIVATION

- The graphical way of representing simple functions/models, like logistic regression. Why is that useful?
- Because individual neurons can be used as building blocks of more complicated functions.
- Networks of neurons can represent extremely complex hypothesis spaces.
- Most importantly, it allows us to define the “right” kinds of hypothesis spaces to learn functions that are common in our universe in a data-efficient way (see Lin, Tegmark et al. 2016).



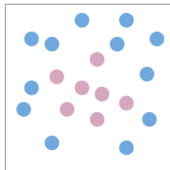
MOTIVATION

- Can a single neuron perform binary classification of these points?

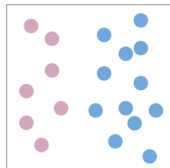


MOTIVATION

- As a single neuron is restricted to learning only linear decision boundaries, its performance on the following task is quite poor:

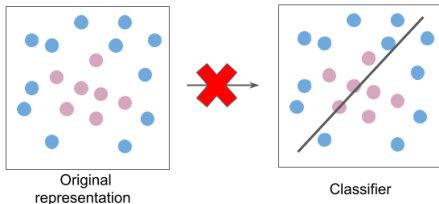


- However, the neuron can easily separate the classes if the original features are transformed (e.g., from Cartesian to polar coordinates):

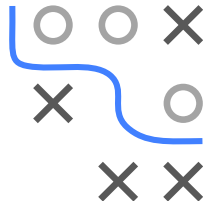


MOTIVATION

- Instead of classifying the data in the original representation,

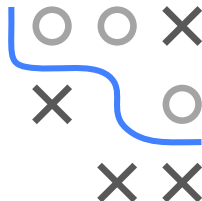


- we classify it in a new feature space.



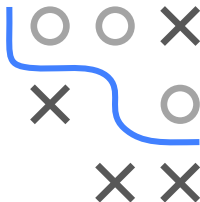
REPRESENTATION LEARNING

- It is *very* critical to feed a classifier the “right” features in order for it to perform well.
- Before deep learning took off, features for tasks like machine vision and speech recognition were “hand-designed” by domain experts. This step of the machine learning pipeline is called **feature engineering**.
- DL automates feature engineering. This is called **representation learning**.



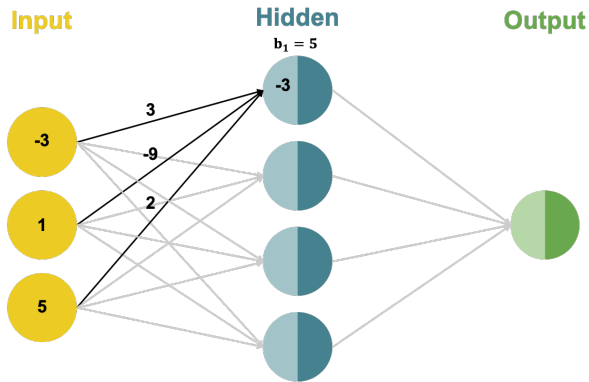
SINGLE HIDDEN LAYER NETWORKS

- **Single neurons** perform a 2-step computation:
 - ❶ **Affine Transformation:** a weighted sum of inputs plus bias.
 - ❷ **Activation:** a non-linear transformation on the weighted sum.
- **Single hidden layer networks** consist of two layers (without input layer):
 - ❶ **Hidden Layer:** having a set of neurons.
 - ❷ **Output Layer:** having one or more output neurons.
- Multiple inputs are simultaneously fed to the network.
- Each neuron in the hidden layer performs a 2-step computation.
- The final output of the network is then calculated by another 2-step computation performed by the neuron in the output layer.



SINGLE HIDDEN LAYER NETWORKS: EXAMPLE

Each neuron in the hidden layer performs an **affine transformation** on the inputs:



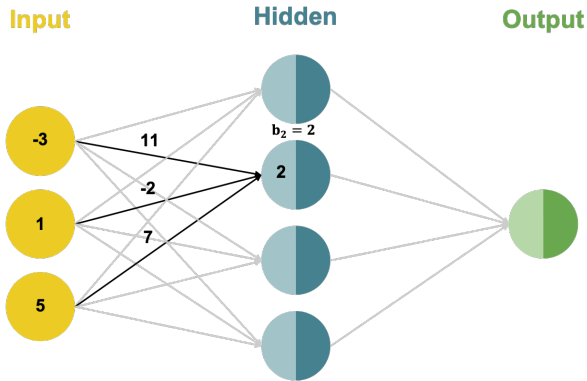
$$Z_{1,in} = w_{11}x_1 + w_{21}x_2 + w_{31}x_3 + b_1$$

$$Z_{1,in} = 3 * (-3) + (-9) * 1 + 2 * 5 + 5 = -3$$

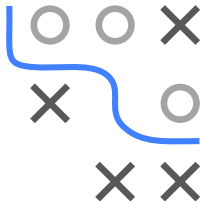


SINGLE HIDDEN LAYER NETWORKS: EXAMPLE

Each neuron in the hidden layer performs an **affine transformation** on the inputs:

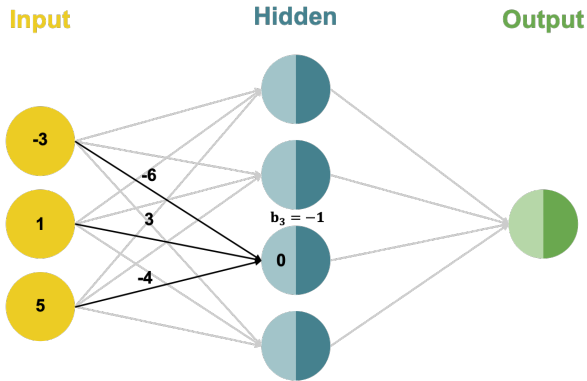


$$Z_{2,in} = w_{12}x_1 + w_{22}x_2 + w_{32}x_3 + b_2$$
$$Z_{2,in} = 11 * (-3) + (-2) * 1 + 7 * 5 + 2 = 2$$



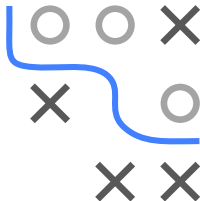
SINGLE HIDDEN LAYER NETWORKS: EXAMPLE

Each neuron in the hidden layer performs an **affine transformation** on the inputs:



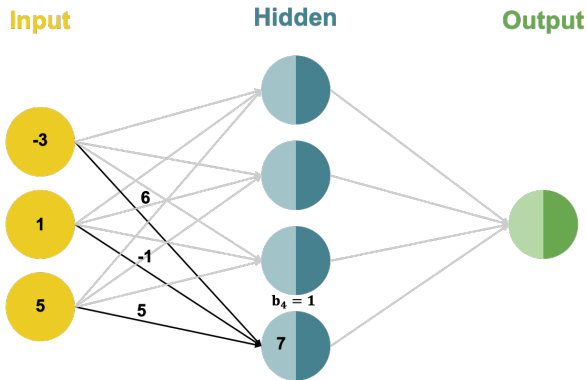
$$Z_{3,\text{in}} = w_{13}x_1 + w_{23}x_2 + w_{33}x_3 + b_3$$

$$Z_{3,\text{in}} = (-6) * (-3) + 3 * 1 + (-4) * 5 - 1 = 0$$



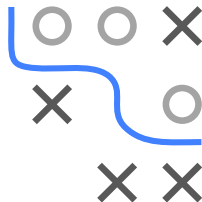
SINGLE HIDDEN LAYER NETWORKS: EXAMPLE

Each neuron in the hidden layer performs an **affine transformation** on the inputs:



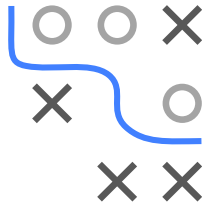
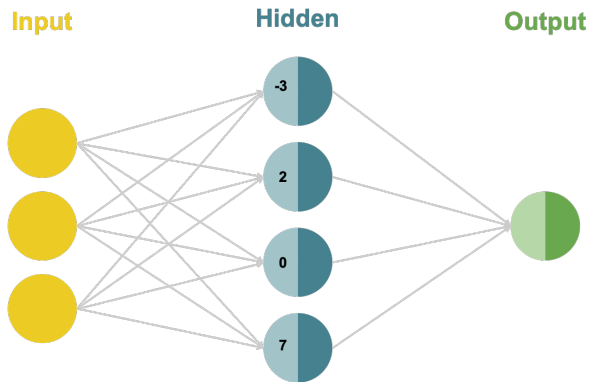
$$Z_{4,\text{in}} = w_{14}x_1 + w_{24}x_2 + w_{34}x_3 + b_4$$

$$z_{4,\text{in}} = 6 * (-3) + (-1) * 1 + 5 * 5 + 1 = 7$$



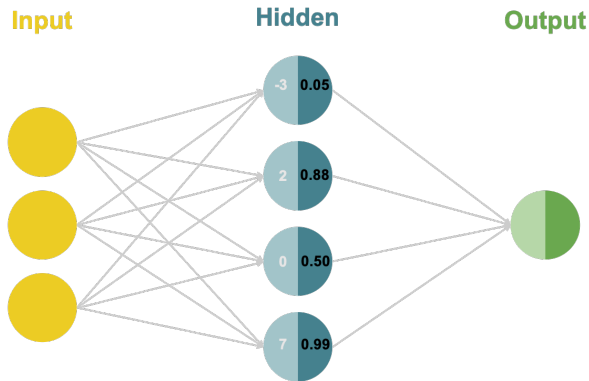
SINGLE HIDDEN LAYER NETWORKS: EXAMPLE

Each neuron in the hidden layer performs an **affine transformation** on the inputs:

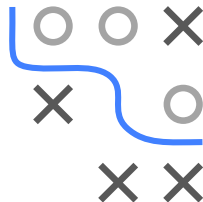


SINGLE HIDDEN LAYER NETWORKS: EXAMPLE

Each hidden neuron performs a non-linear **activation** transformation on the weight sum:

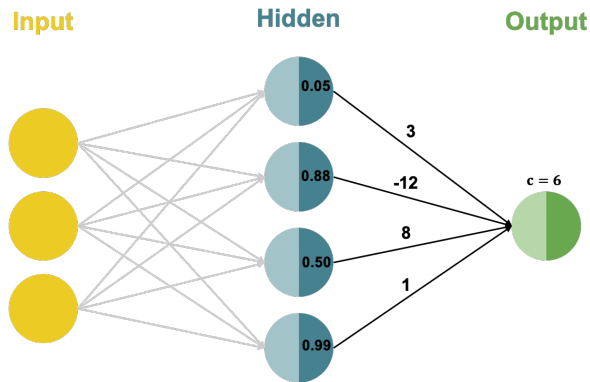


$$z_{i,out} = \sigma(z_{i,in}) = \frac{1}{1 + e^{-z_{i,in}}}$$

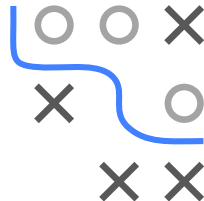


SINGLE HIDDEN LAYER NETWORKS: EXAMPLE

The output neuron performs an **affine transformation** on its inputs:

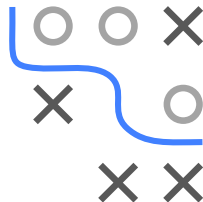
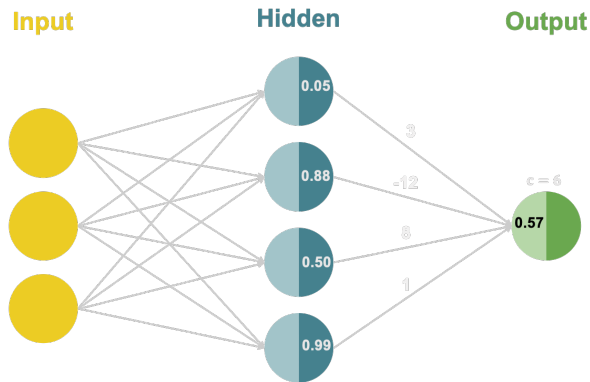


$$\hat{f}_{in} = u_1 z_{1,out} + u_2 z_{2,out} + u_3 z_{3,out} + u_4 z_{4,out} + c$$



SINGLE HIDDEN LAYER NETWORKS: EXAMPLE

The output neuron performs an **affine transformation** on its inputs:

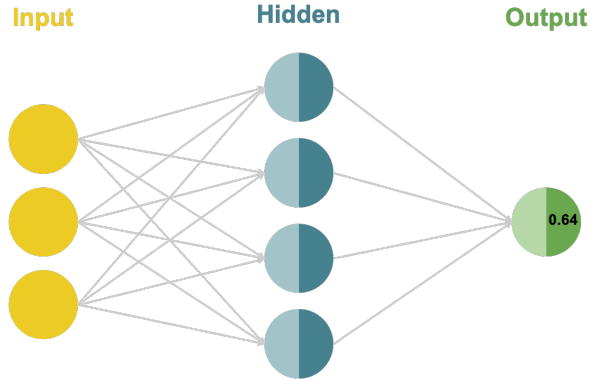


$$f_{in} = u_1 z_{1,out} + u_2 z_{2,out} + u_3 z_{3,out} + u_4 z_{4,out} + c$$

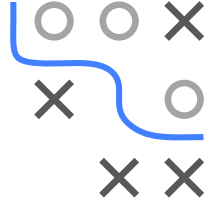
$$f_{in} = 3 * 0.05 + (-12) * 0.88 + 8 * 0.50 + 1 * 0.99 + 6 = 0.57$$

SINGLE HIDDEN LAYER NETWORKS: EXAMPLE

The output neuron performs a non-linear **activation** transformation on the weight sum:



$$f_{\text{out}} = \sigma(f_{\text{in}}) = \frac{1}{1 + e^{-f_{\text{in}}}}$$
$$f_{\text{out}} = \frac{1}{1 + e^{-0.57}} = 0.64$$



HIDDEN LAYER: ACTIVATION FUNCTION

- If the hidden layer does not have a non-linear activation, the network can only learn linear decision boundaries.
- A lot of different activation functions exist.



HIDDEN LAYER: ACTIVATION FUNCTION II

- **Sigmoid Activation Function:** The sigmoid function can be used even in the hidden layer.

$$\sigma(v) = \frac{1}{1 + \exp(-v)}$$

