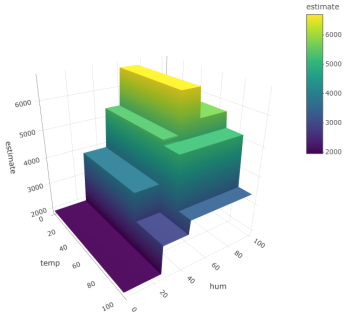
# Interpretable Machine Learning

# Interpretable Models 1
# Rule-based Models

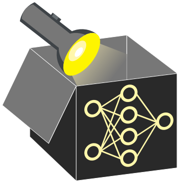



**Learning goals**

- Decision trees
- RuleFit
- Decision rules

# DECISION TREES

▸ "Breiman et al." 1984



**Idea**: Partition data into axis-aligned regions via greedy search for feature cut points (minimizing a split criterion), then predict a constant mean $c_m$ in each leaf region $\mathcal{R}_m$:

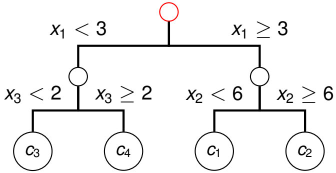
$$\hat{f}(x) = \sum_{m=1}^{M} c_m \mathbb{1}_{\{x \in \mathcal{R}_m\}}$$

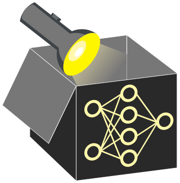
# DECISION TREES
▸ "Breiman et al." 1984

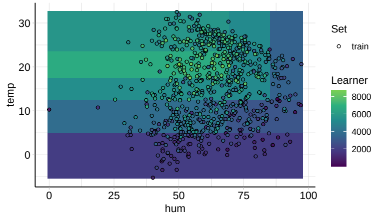
**Idea**: Partition data into axis-aligned regions via greedy search for feature cut points (minimizing a split criterion), then predict a constant mean $c_m$ in each leaf region $\mathcal{R}_m$:

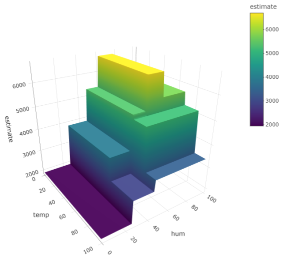
$$\hat{f}(x) = \sum_{m=1}^{M} c_m \mathbb{1}_{\{x \in \mathcal{R}_m\}}$$

- Applicable to regression and classification
- Models interactions and non-linear effects
- Handles mixed feat, spaces & missing values

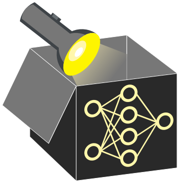


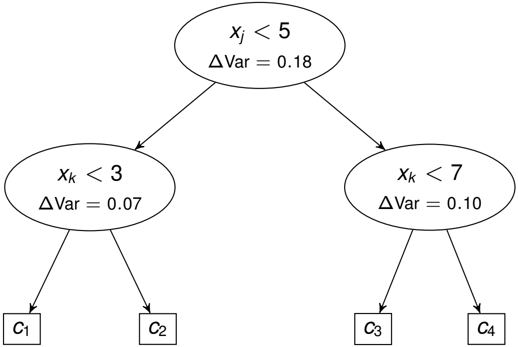


# INTERPRETATION OF TREE-BASED MODELS



- Interpretation via path of decision rules along tree branches
- **Feature importance** (quantifies how often and how usefully $x_j$ is used):
  - For each split on feature $x_j$, record the decrease in the split criterion
  - Aggregate this over the tree: sum or avg. over all splits involving $x_j$
  - Split criterion: variance (regression), Gini index / entropy (classif.)



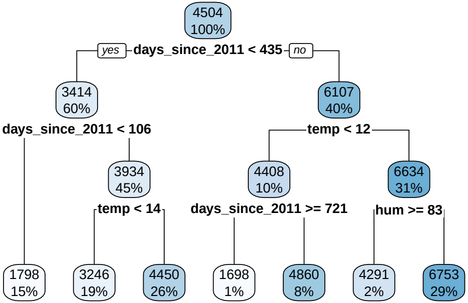

- Each $\Delta$Var is assigned to the splitting feature
- Feature importance = sum of all $\Delta$Var for that feat.:

  $x_j$**:** $0.18$
  $x_k$**:** $0.07 + 0.10 = 0.17$

# DECISION TREES - EXAMPLE

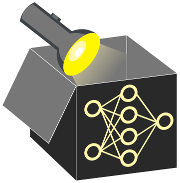
- Fit decision tree with tree depth of 3 on bike data
- E.g., mean prediction for the first 105 days since 2011 is 1798
  $\rightsquigarrow$ Applies to $\hat{=}15\%$ of the data (leftmost branch)
- `days_since_2011`: highest feat. importance (explains most of variance)

| Feature | Importance |
|---|---|
| days_since_2011 | 79.53 |
| temp | 17.55 |
| hum | 2.92 |



# UNBIASED RECURSIVE PARTITIONING

▸ "Hothorn" 2006 ▸ "Zeileis" 2008 ▸ "Strobl" 2007

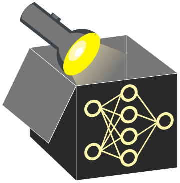


**Problems** with CART (Classification and Regression Trees):

1. Selection bias towards high-cardinal/continuous features
2. Splits on any improvement, regardless of significance
   $\rightsquigarrow$ prone to overfitting

# UNBIASED RECURSIVE PARTITIONING

▶ "Hothorn" 2006 ▶ "Zeileis" 2008 ▶ "Strobl" 2007



**Problems** with CART (Classification and Regression Trees):

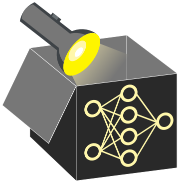
1. Selection bias towards high-cardinal/continuous features
2. Splits on any improvement, regardless of significance
   $\rightsquigarrow$ prone to overfitting

**Unbiased recursive partitioning** via conditional inference trees (`ctree`) or model-based recursive partitioning (`mob`):

1. Separate selection of **feature used for splitting** and **split point**
2. Hypothesis test as stopping criteria

# UNBIASED RECURSIVE PARTITIONING

▶ "Hothorn" 2006 ▶ "Zeileis" 2008 ▶ "Strobl" 2007



**Problems** with CART (Classification and Regression Trees):

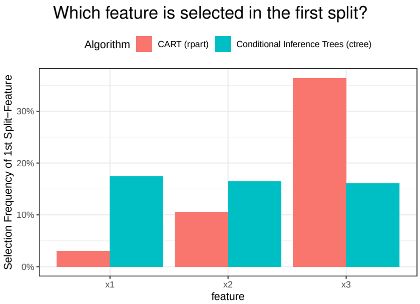
1. Selection bias towards high-cardinal/continuous features
2. Splits on any improvement, regardless of significance
   $\rightsquigarrow$ prone to overfitting

**Unbiased recursive partitioning** via conditional inference trees (`ctree`) or model-based recursive partitioning (`mob`):

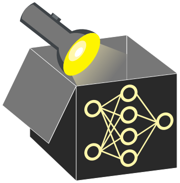
1. Separate selection of **feature used for splitting** and **split point**
2. Hypothesis test as stopping criteria

**Example (selection bias)**:
Simulate data ($n = 200$), $Y \sim N(0, 1)$ and 3 features of different cardinality indep. from $Y$ (repeat 500 times):

- $X_1 \sim Binom(n, \frac{1}{2})$
- $X_2 \sim M(n, (\frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}))$
- $X_3 \sim M(n, (\frac{1}{8}, \frac{1}{8}, \frac{1}{8}, \frac{1}{8}, \frac{1}{8}, \frac{1}{8}, \frac{1}{8}, \frac{1}{8}))$



# UNBIASED RECURSIVE PARTITIONING



Differences to CART:

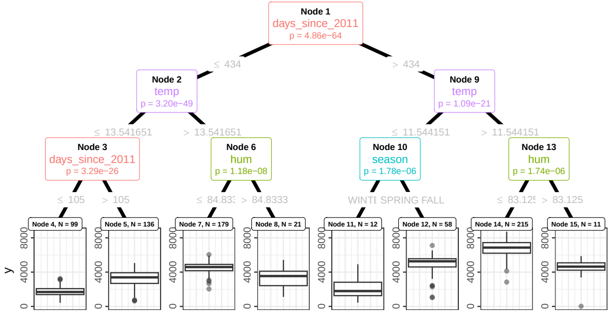
- Two-step approach (finds 1. most significant split feat., 2. best split point)
- Parametric model (e.g. LM instead of constant) can be fitted in leaf nodes
- Significance of split (p-value) given in each node
- `ctree` and `mob` differ in hypothesis test used for selecting the split feature (independence test vs. fluctuation test) and how to find the best split point

# UNBIASED RECURSIVE PARTITIONING

Differences to CART:

- Two-step approach (finds 1. most significant split feat., 2. best split point)
- Parametric model (e.g. LM instead of constant) can be fitted in leaf nodes
- Significance of split (p-value) given in each node
- `ctree` and `mob` differ in hypothesis test used for selecting the split feature (independence test vs. fluctuation test) and how to find the best split point

**Example** (`ctree`): Bike data (constant model in final nodes)



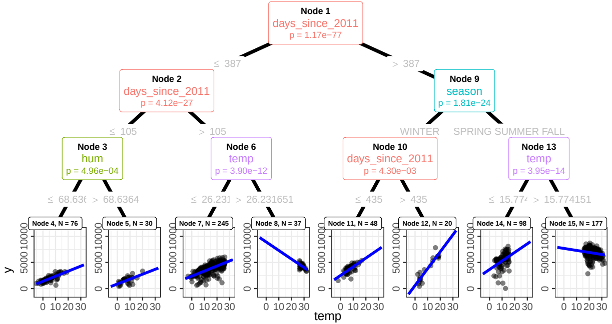

Train MSE:
758,844 (`ctree`)
742,244 (`mob`)

# UNBIASED RECURSIVE PARTITIONING

Differences to CART:

- Two-step approach (finds 1. most significant split feat., 2. best split point)
- Parametric model (e.g. LM instead of constant) can be fitted in leaf nodes
- Significance of split (p-value) given in each node
- `ctree` and `mob` differ in hypothesis test used for selecting the split feature (independence test vs. fluctuation test) and how to find the best split point

**Example** (`mob`): Bike data (linear model with `temp` in final nodes)



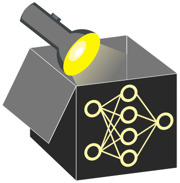

Train MSE:
758,844 (`ctree`)
742,244 (`mob`)

# OTHER RULE-BASED MODELS

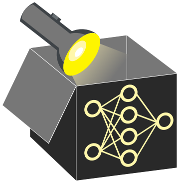
## Decision Rules ▸ "Holte" 1993

- Flat list of simple "if – then" statements
  $\rightsquigarrow$ very intuitive and easy-to-interpret
- Mainly devised for classification (support for regression is limited)
- Numeric features are typically discretised

| | | |
|---|---|---|
| IF $x_1 \leq 2.3$ AND $x_4 =$ "A" | THEN | y = 1 |
| ELSE IF $x_2 > 5.0$ | THEN | y = 2 |
| ELSE | | y = 3 |



# OTHER RULE-BASED MODELS



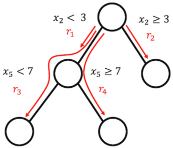
## Decision Rules ▸ "Holte" 1993

- Flat list of simple "if – then" statements
  $\rightsquigarrow$ very intuitive and easy-to-interpret
- Mainly devised for classification
  (support for regression is limited)
- Numeric features are typically discretised

| | | |
|---|---|---|
| IF $x_1 \leq 2.3$ AND $x_4 =$ "A" | THEN | y = 1 |
| ELSE IF $x_2 > 5.0$ | THEN | y = 2 |
| ELSE | | y = 3 |

## RuleFit ▸ "Friedman and Popescu" 2008

- Extract binary rules $r_m(\mathbf{x}) \in \{0, 1\}$ from many shallow trees (one per root-to-leaf path)
- Fit an $L_1$-regularized LM
  $\hat{f}(\mathbf{x}) = \beta_0 + \sum_m \beta_m r_m(\mathbf{x}) + \sum_j \gamma_j x_j$
- Regularization retains only a few rules
  $\Rightarrow$ sparse, non-linear, interaction-aware
- Coefficients relate to rule/feature importance




▸ "Molnar" 2022