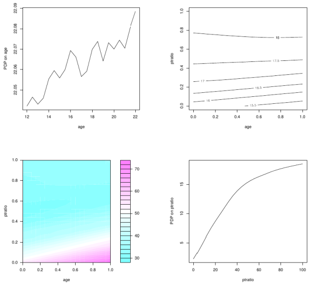
Interpretable Machine Learning

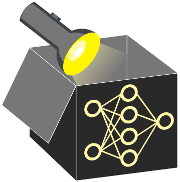
Functional Decompositions
Further Methods



Learning goals

- Limitations of classical fANOVA
- Alternatives: Generalized fANOVA and ALE
- Advantages and relevance of functional decompositions

LIMITATIONS OF CLASSICAL FANOVA



- Standard fANOVA builds on PD-functions
- *Remember:* Problems of PDPs for **correlated** / **dependent features**
- Here: Dependent features $\Longrightarrow$ Standard fANOVA does NOT fulfill vanishing conditions

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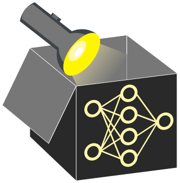

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Example

Assume dependency $2x_1^2 = x_2$ and

$$\hat{f}(x_1, x_2, x_3) = -2x_1 - 2\sin(x_3) + |x_1|x_2 + 0.5x_2x_3 + 1.$$

$\rightsquigarrow$ Following two decompositions would both "make sense":

$$\hat{f}(x_1, x_2, x_3) = \underbrace{1}_{g_\emptyset} + \underbrace{(-2x_1)}_{g_1(x_1)} + \underbrace{(-2\sin(x_3))}_{g_3(x_3)} + \underbrace{|x_1|x_2}_{g_{1,2}(x_1,x_2)} + \underbrace{0.5x_2x_3}_{g_{2,3}(x_2,x_3)}$$

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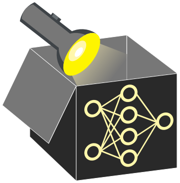

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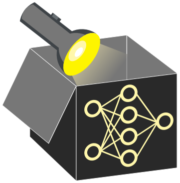
$\rightarrow$ Extreme example, but again: Problem of definition

ALTERNATIVE: GENERALIZED FUNCTIONAL ANOVA



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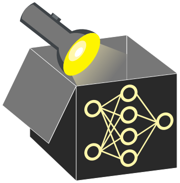

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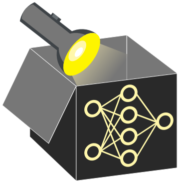

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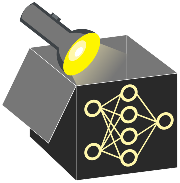

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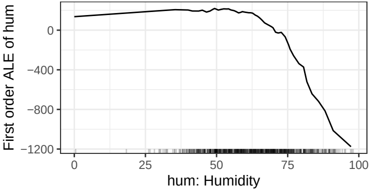
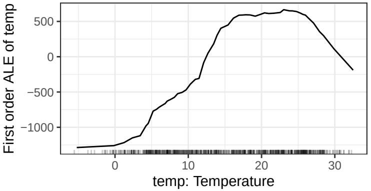
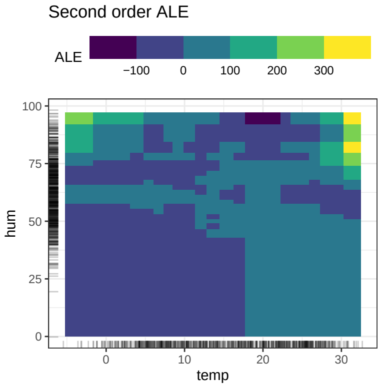
- **Problems:**
- Difficult to estimate, involves manual choice of a "weight function"
- Computationally very costly

REVISITING ALE PLOTS

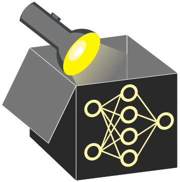
$$\hat{\tilde{f}}_{S,ALE}(x) = \sum_{k=1}^{k_S(x)} \frac{1}{n_S(k)} \sum_{i:\ x_S^{(i)} \in\ ]z_{k-1,S}, z_{k,S}]} \left[\hat{f}(z_{k,S}, \mathbf{x}_{-S}^{(i)}) - \hat{f}(z_{k-1,S}, \mathbf{x}_{-S}^{(i)}) \right]$$







ALE DECOMPOSITION



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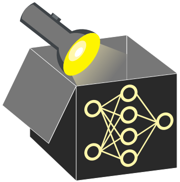

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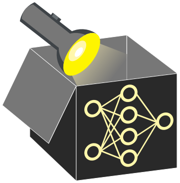
- **Advantages:** Handle dependencies well + computationally fast
- Constraints / orthogonality properties more complicated

$\Longrightarrow$ ALE decomp. theoretically more involved, but good alternative in practice

CONCLUSION: HOW USEFUL ARE FUNCTIONAL DECOMPOSITIONS?



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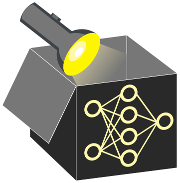

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  - Theoretical framework for general definition of interact.-s (H-statistic)
  - Theoretical background for many IML methods: GAMs and EBMs, ICE, PDPs and PD-functions, ALE plots, Shapley values, Feature importance methods (see later)

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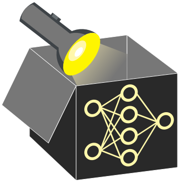

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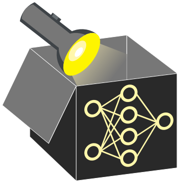

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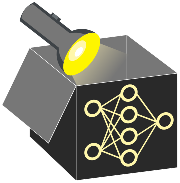

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Overall: Very important concept and theoretical background, explains idea behind many other methods