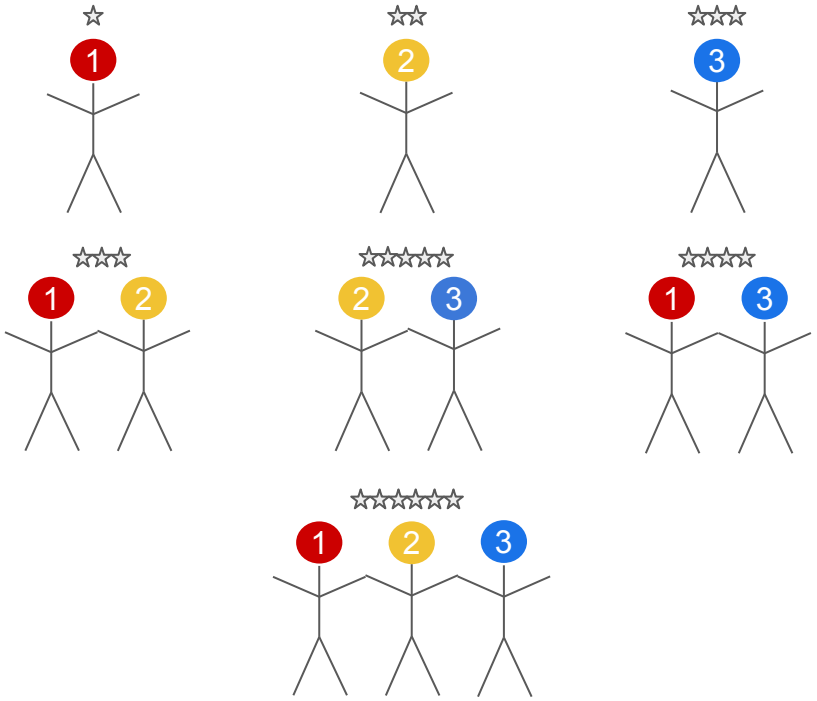
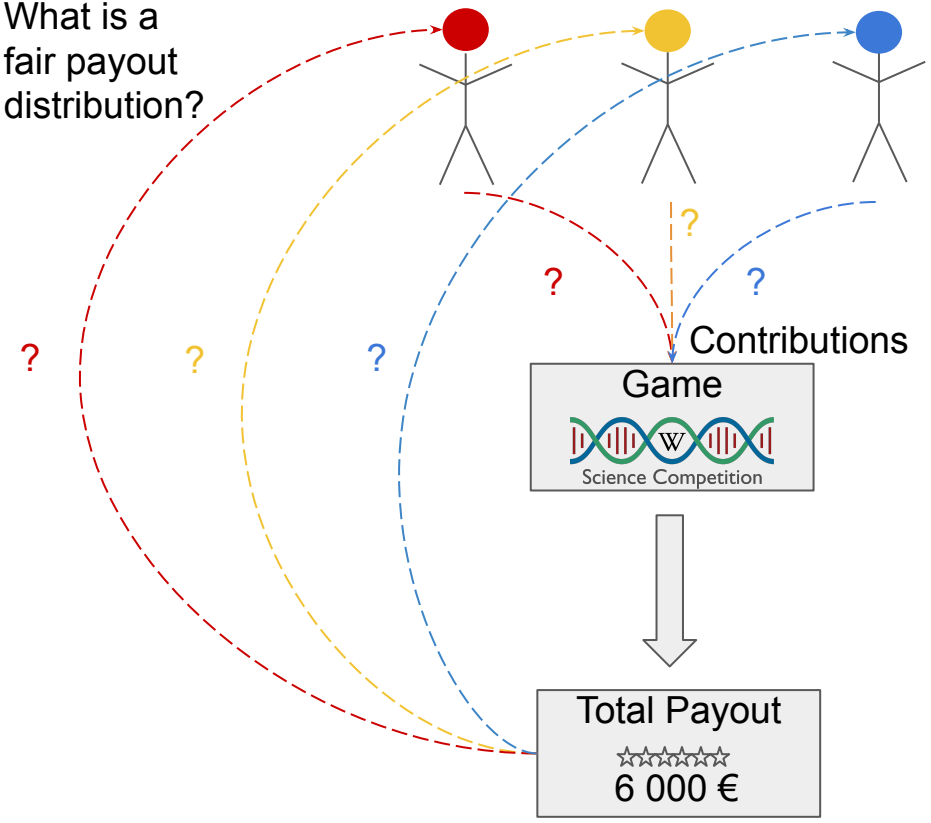
## Players do not interact
(payouts ☆ add up in each coalition)

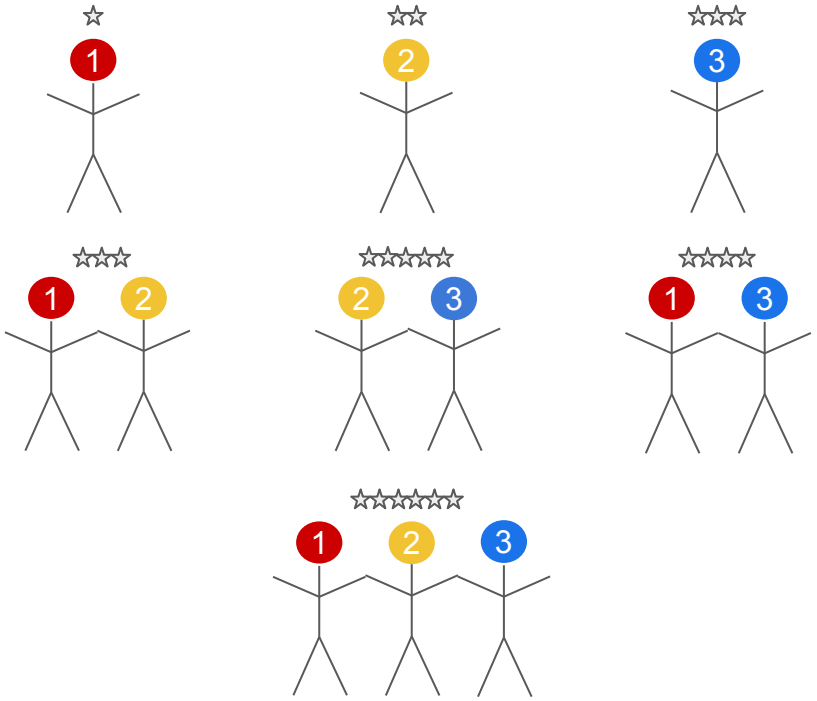



## Players do not interact

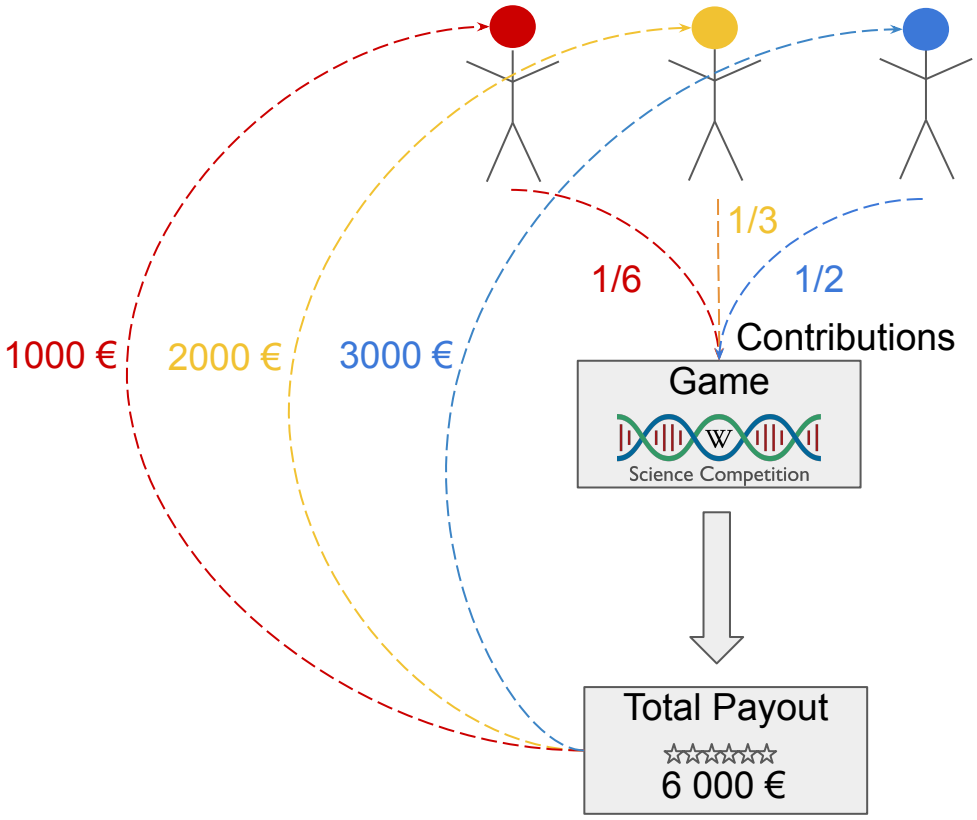
What is a fair payout distribution?



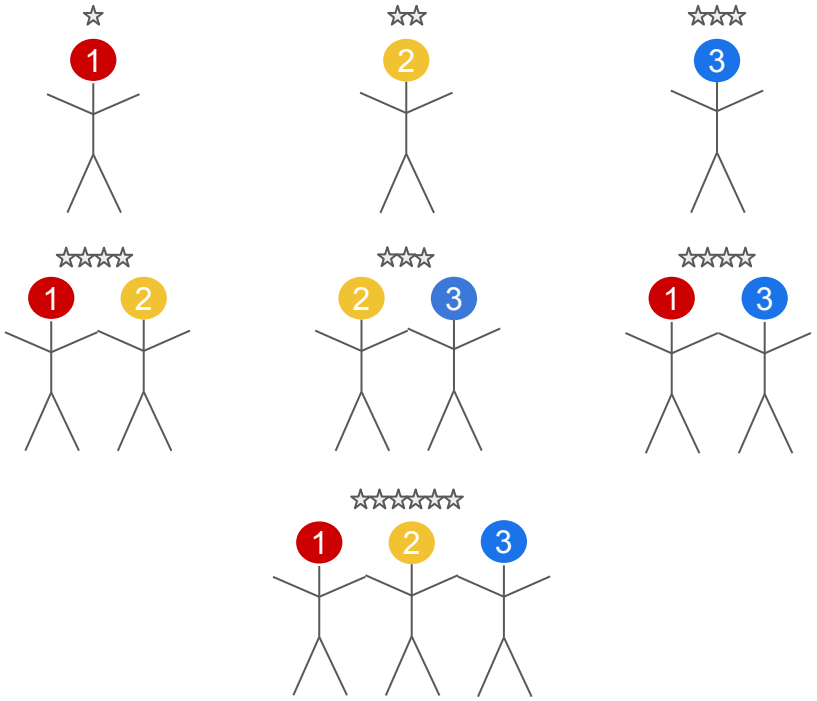
## Players do not interact
(payouts ☆ add up in each coalition)




## Players do not interact

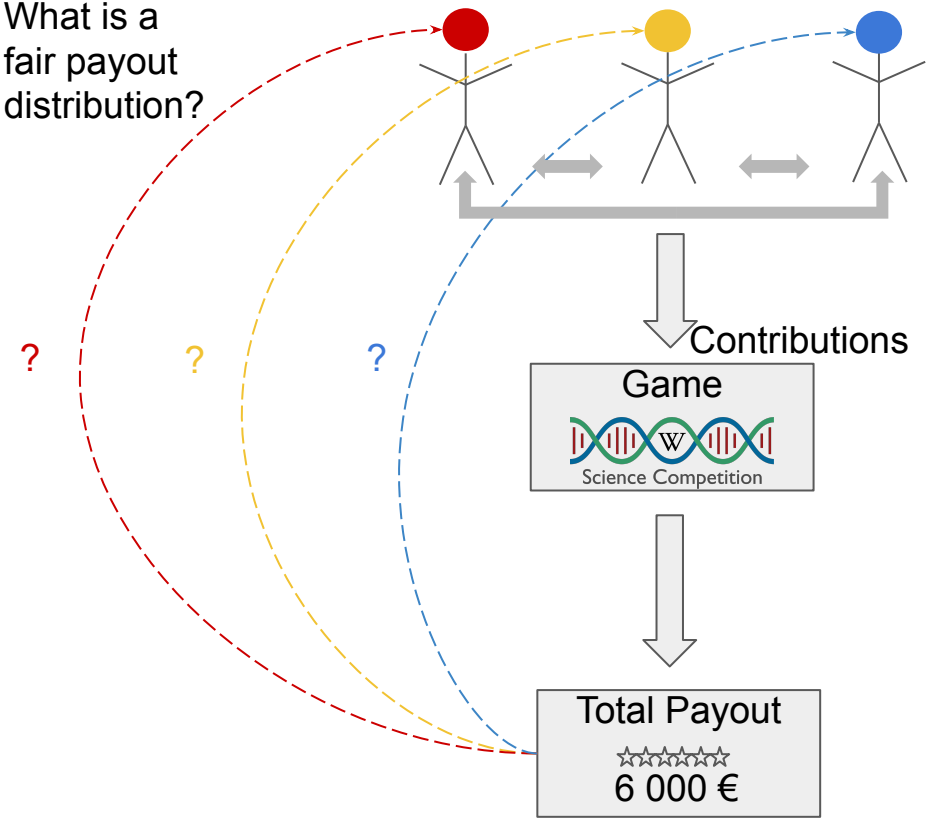


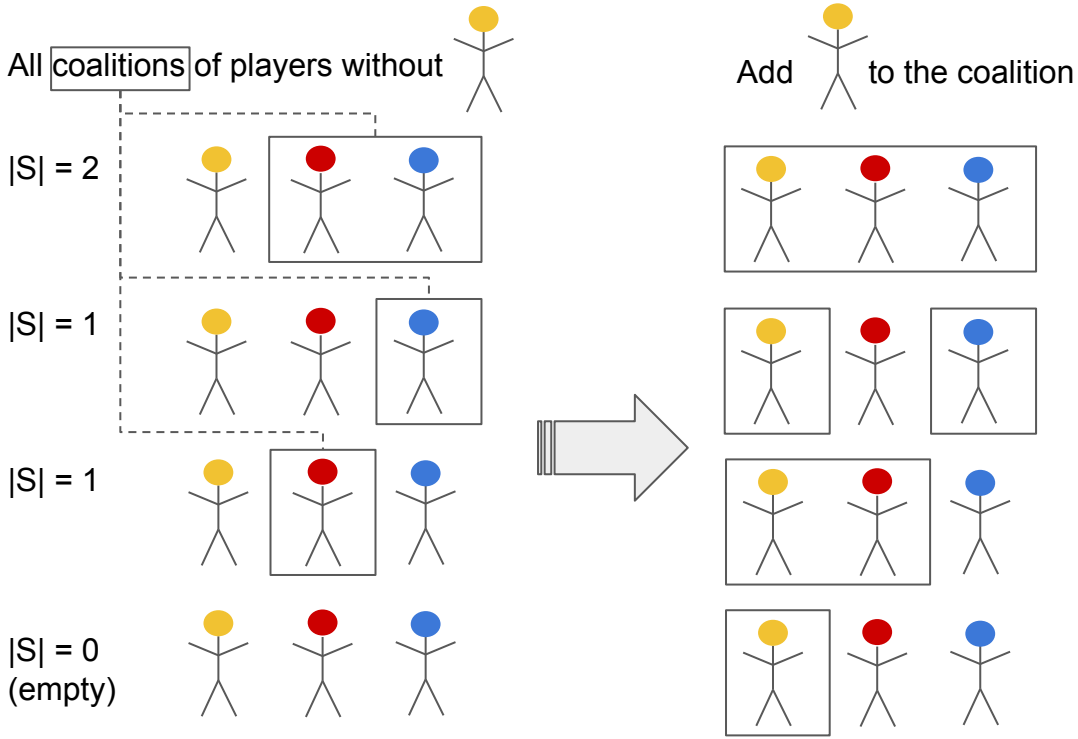
## Players interact
(payouts ☆ do not add up)




## Players interact

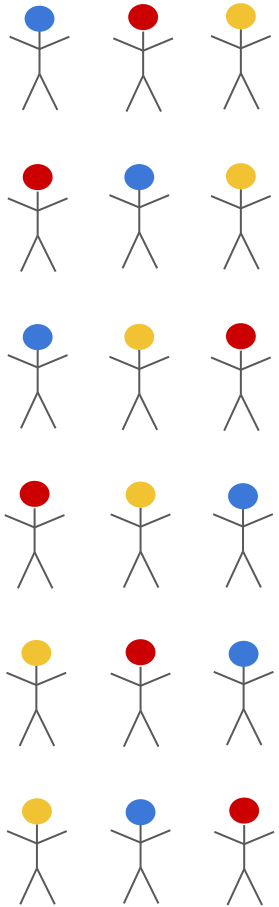
What is a fair payout distribution?






- Compute total payout of each coalition
- Compute difference in total payout for each coalition with and without player
- Sum up weighted differences in total payout

|P|! = 6 orders

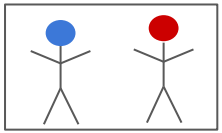


|P|! = 6 orders

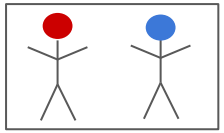
via orders

(players before “yellow” player)

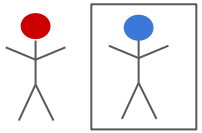
|S| = 2
weight = 1/6



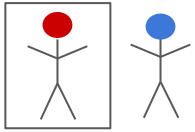
|S| = 2
weight = 1/6



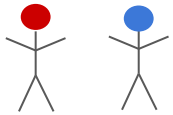
|S| = 1
weight = 1/6



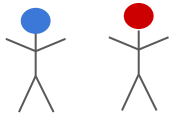
|S| = 1
weight = 1/6



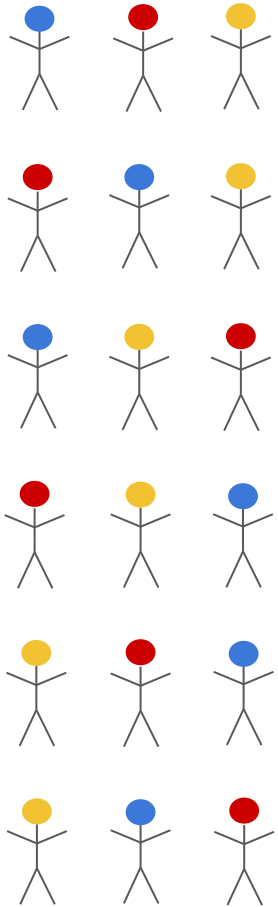
|S| = 0
weight = 1/6



|S| = 0
weight = 1/6



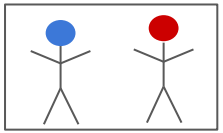
|P|! = 6 orders



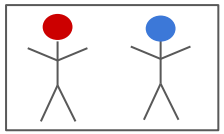
## via orders

(players before “yellow” player)

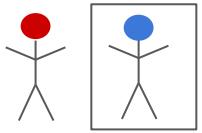
|S| = 2
weight = 1/6



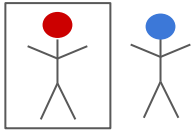
|S| = 2
weight = 1/6



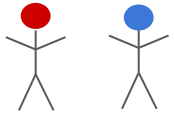
|S| = 1
weight = 1/6



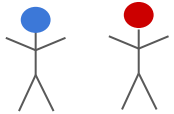
|S| = 1
weight = 1/6



|S| = 0
weight = 1/6

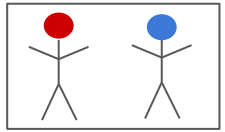


|S| = 0
weight = 1/6

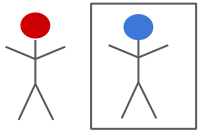


## via sets

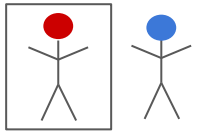
|S| = 2
weight = 2/6



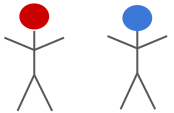
|S| = 1
weight = 1/6

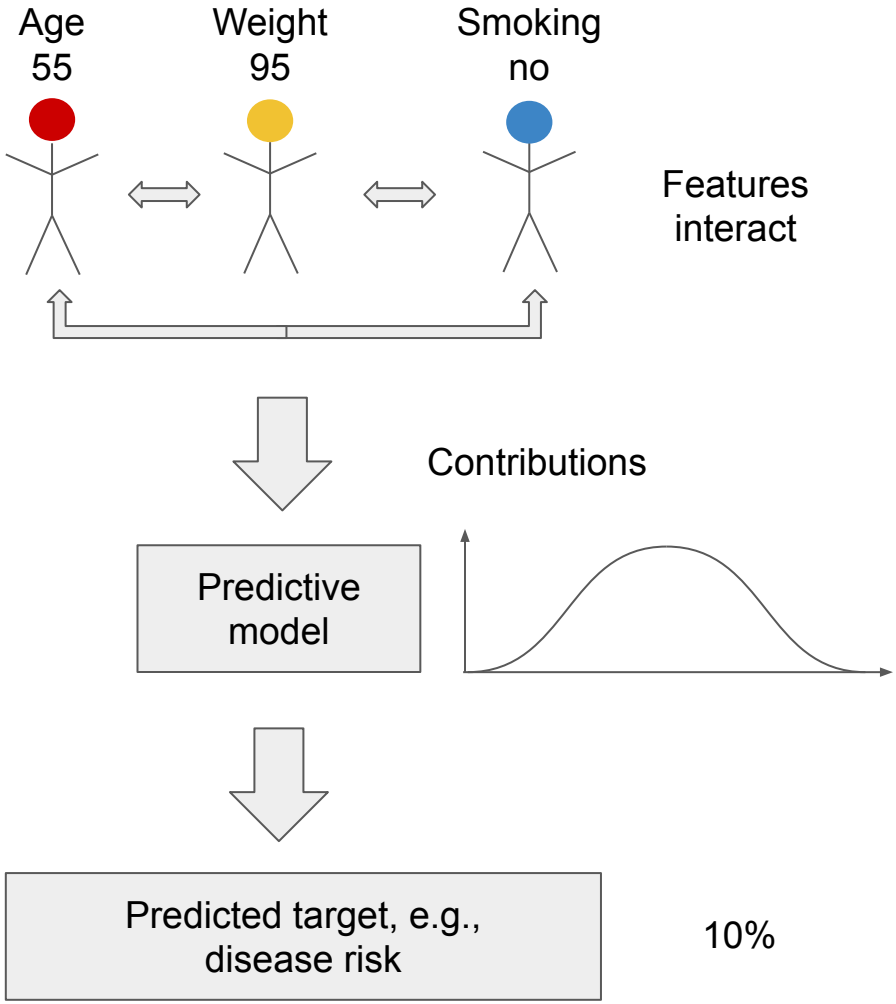


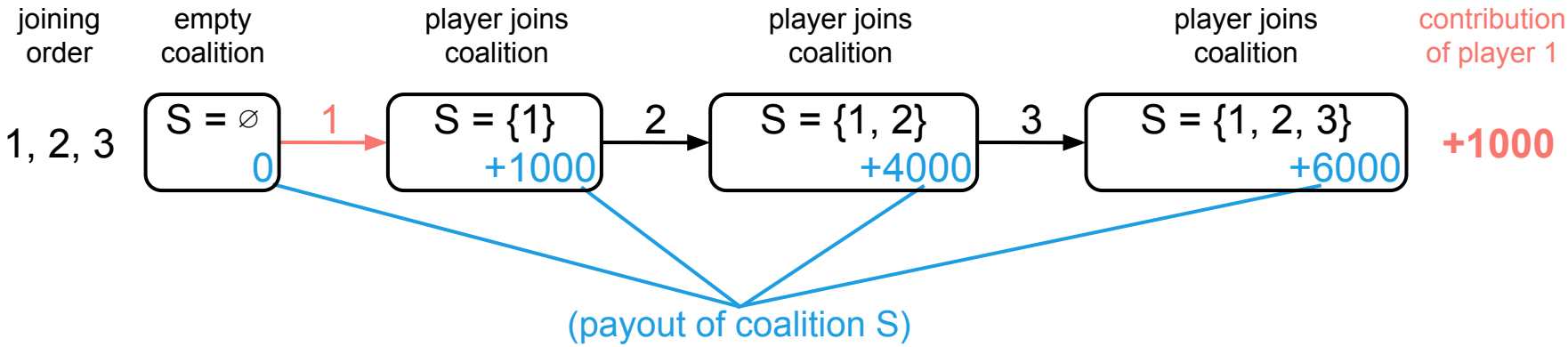
|S| = 1
weight = 1/6

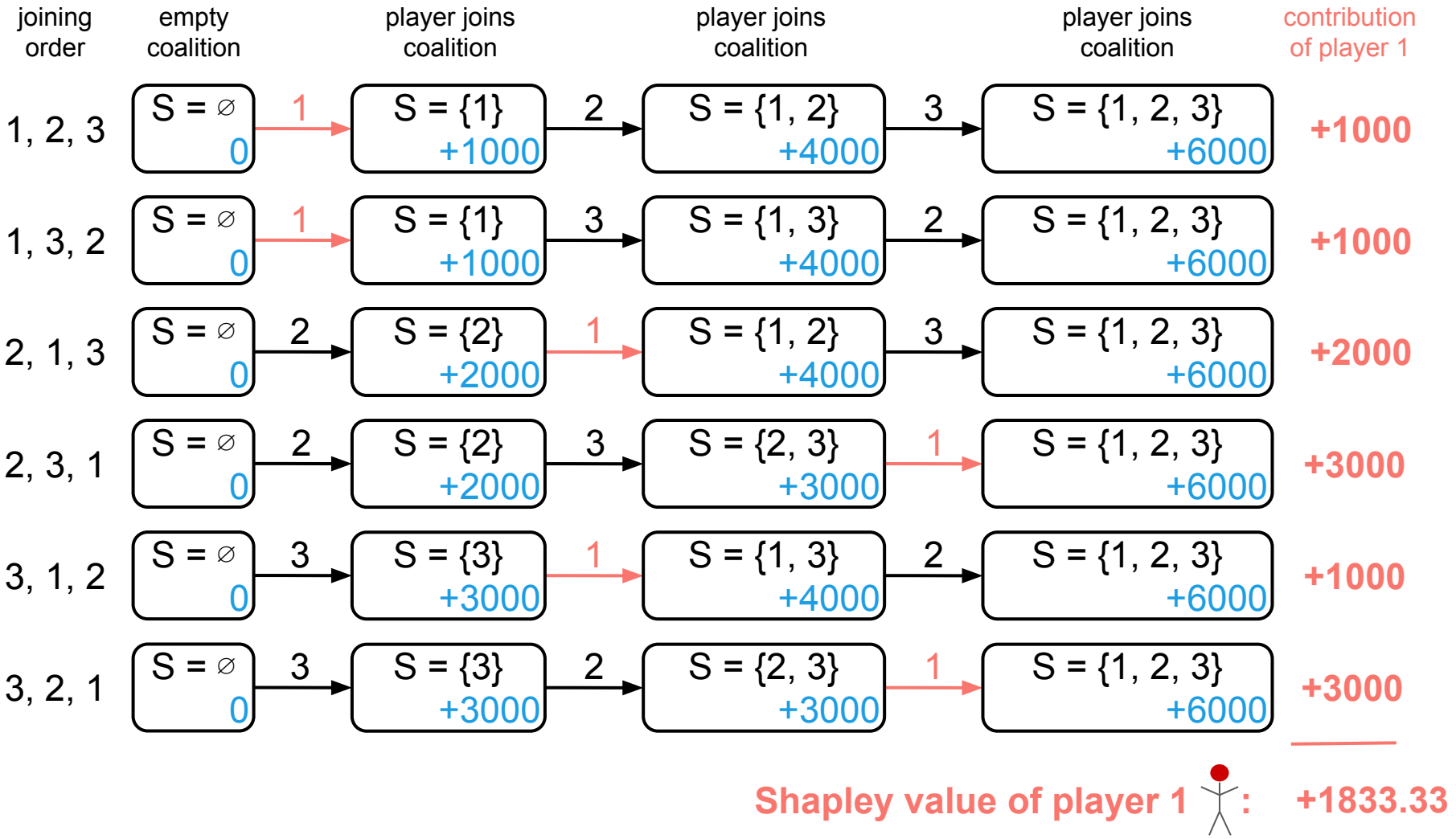


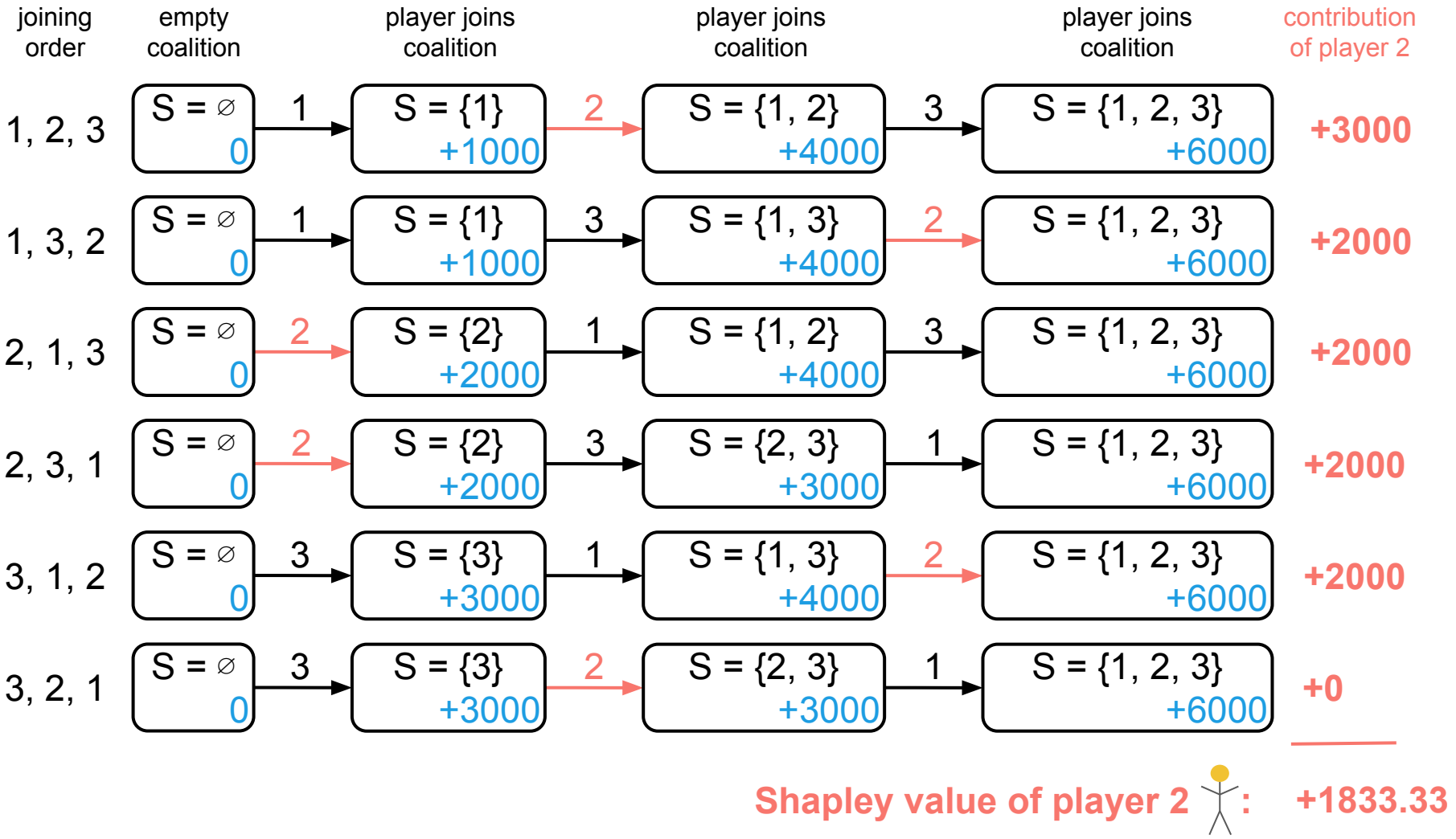
|S| = 0
weight = 2/6

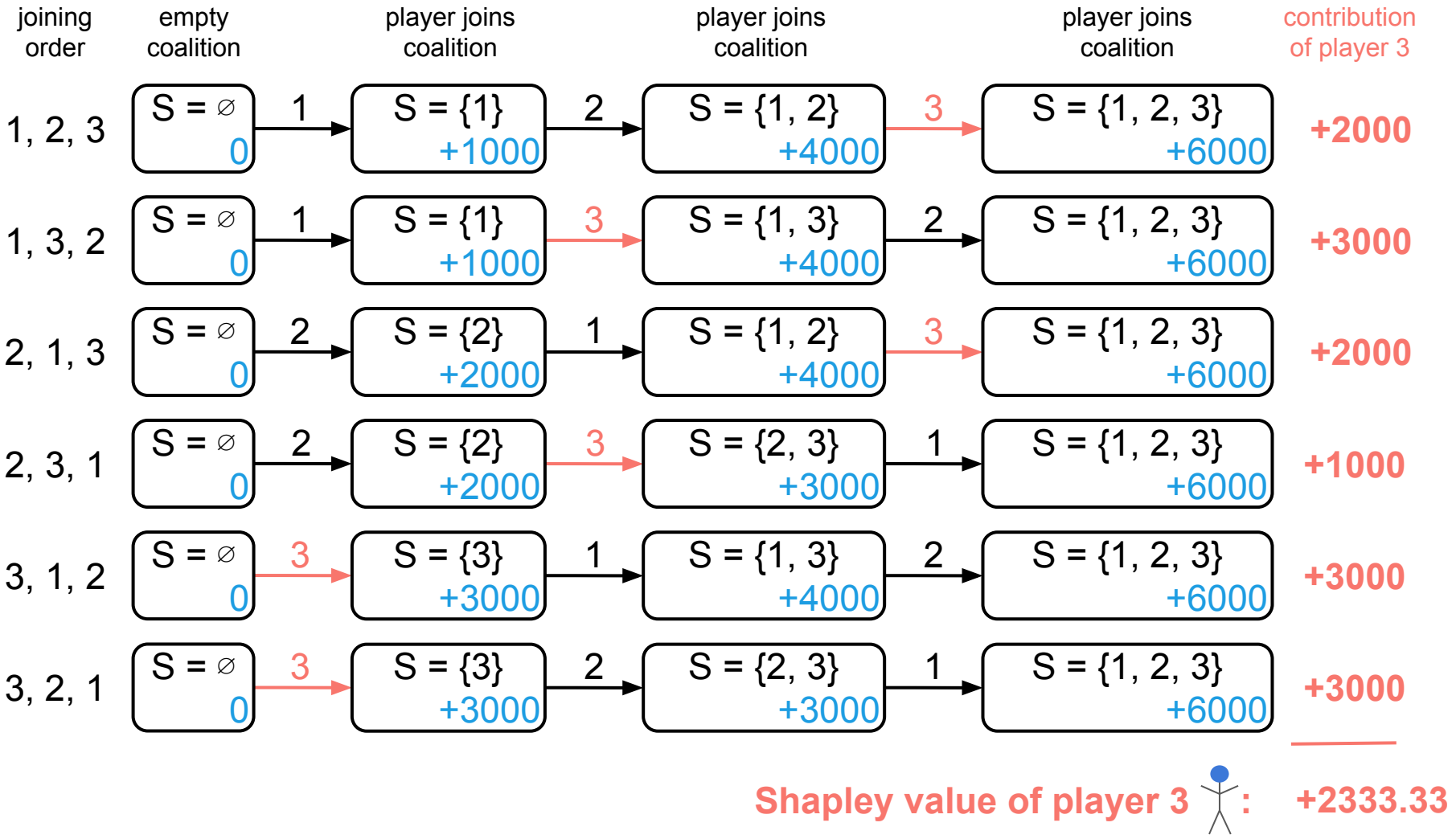



Age
55
Weight
95
Smoking
no
Features
interact
Contributions
Predictive
model
Predicted target, e.g.,
disease risk
10%

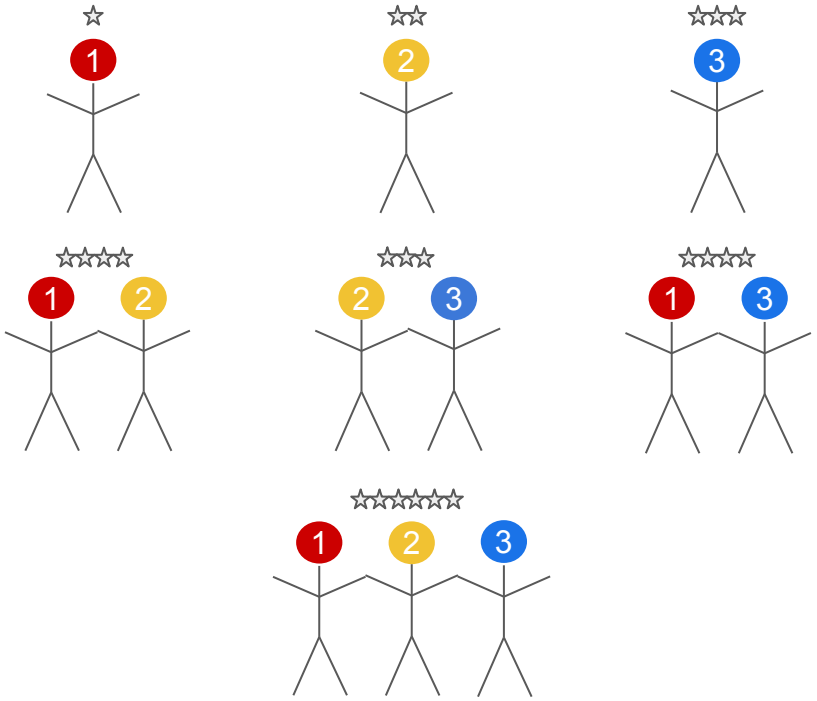

joining order
empty coalition
player joins coalition
player joins coalition
player joins coalition
contribution of player 1
1, 2, 3
S = ∅
0
1
S = {1}
+1000
2
S = {1, 2}
+4000
3
S = {1, 2, 3}
+6000
+1000
(payout of coalition S)


joining order
empty coalition
player joins coalition
player joins coalition
player joins coalition
contribution of player 1
1, 2, 3
S = ∅
0
1
S = {1}
+1000
2
S = {1, 2}
+4000
3
S = {1, 2, 3}
+6000
+1000
1, 3, 2
S = ∅
0
1
S = {1}
+1000
3
S = {1, 3}
+4000
2
S = {1, 2, 3}
+6000
+1000
2, 1, 3
S = ∅
0
2
S = {2}
+2000
1
S = {1, 2}
+4000
3
S = {1, 2, 3}
+6000
+2000
2, 3, 1
S = ∅
0
2
S = {2}
+2000
3
S = {2, 3}
+3000
1
S = {1, 2, 3}
+6000
+3000
3, 1, 2
S = ∅
0
3
S = {3}
+3000
1
S = {1, 3}
+4000
2
S = {1, 2, 3}
+6000
+1000
3, 2, 1
S = ∅
0
3
S = {3}
+3000
2
S = {2, 3}
+3000
1
S = {1, 2, 3}
+6000
+3000
Shapley value of player 1 :
+1833.33

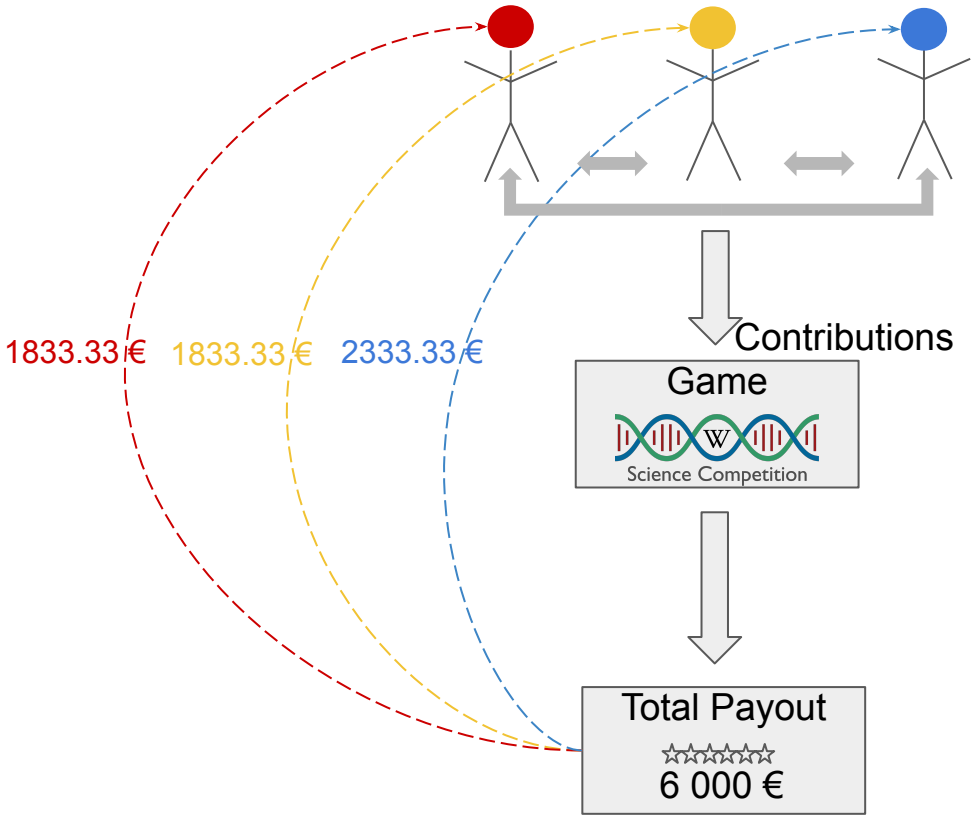

joining order
empty coalition
player joins coalition
player joins coalition
player joins coalition
contribution of player 2
1, 2, 3
S = ∅ 0
1
S = {1} +1000
2
S = {1, 2} +4000
3
S = {1, 2, 3} +6000
+3000
1, 3, 2
S = ∅ 0
1
S = {1} +1000
3
S = {1, 3} +4000
2
S = {1, 2, 3} +6000
+2000
2, 1, 3
S = ∅ 0
2
S = {2} +2000
1
S = {1, 2} +4000
3
S = {1, 2, 3} +6000
+2000
2, 3, 1
S = ∅ 0
2
S = {2} +2000
3
S = {2, 3} +3000
1
S = {1, 2, 3} +6000
+2000
3, 1, 2
S = ∅ 0
3
S = {3} +3000
1
S = {1, 3} +4000
2
S = {1, 2, 3} +6000
+2000
3, 2, 1
S = ∅ 0
3
S = {3} +3000
2
S = {2, 3} +3000
1
S = {1, 2, 3} +6000
+0
Shapley value of player 2 :
+1833.33


joining order
empty coalition
player joins coalition
player joins coalition
player joins coalition
contribution of player 3
1, 2, 3
S = ∅ 0
1
S = {1} +1000
2
S = {1, 2} +4000
3
S = {1, 2, 3} +6000
+2000
1, 3, 2
S = ∅ 0
1
S = {1} +1000
3
S = {1, 3} +4000
2
S = {1, 2, 3} +6000
+3000
2, 1, 3
S = ∅ 0
2
S = {2} +2000
1
S = {1, 2} +4000
3
S = {1, 2, 3} +6000
+2000
2, 3, 1
S = ∅ 0
2
S = {2} +2000
3
S = {2, 3} +3000
1
S = {1, 2, 3} +6000
+1000
3, 1, 2
S = ∅ 0
3
S = {3} +3000
1
S = {1, 3} +4000
2
S = {1, 2, 3} +6000
+3000
3, 2, 1
S = ∅ 0
3
S = {3} +3000
2
S = {2, 3} +3000
1
S = {1, 2, 3} +6000
+3000
Shapley value of player 3 :
+2333.33

## Players interact
(payouts ☆ do not add up)

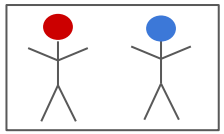



## Players interact

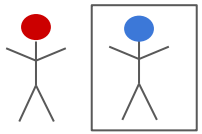


## via sets

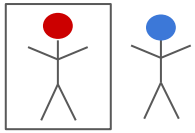
|S| = 2
weight = 2/6



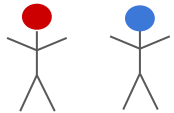
|S| = 1
weight = 1/6



|S| = 1
weight = 1/6

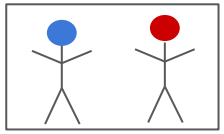


|S| = 0
weight = 2/6

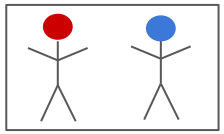


## via orders

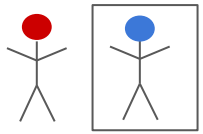
|S| = 2
weight = 1/6



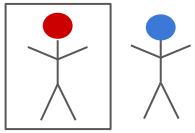
|S| = 2
weight = 1/6



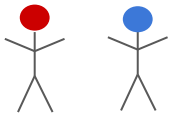
|S| = 1
weight = 1/6



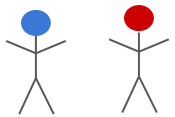
|S| = 1
weight = 1/6



|S| = 0
weight = 1/6

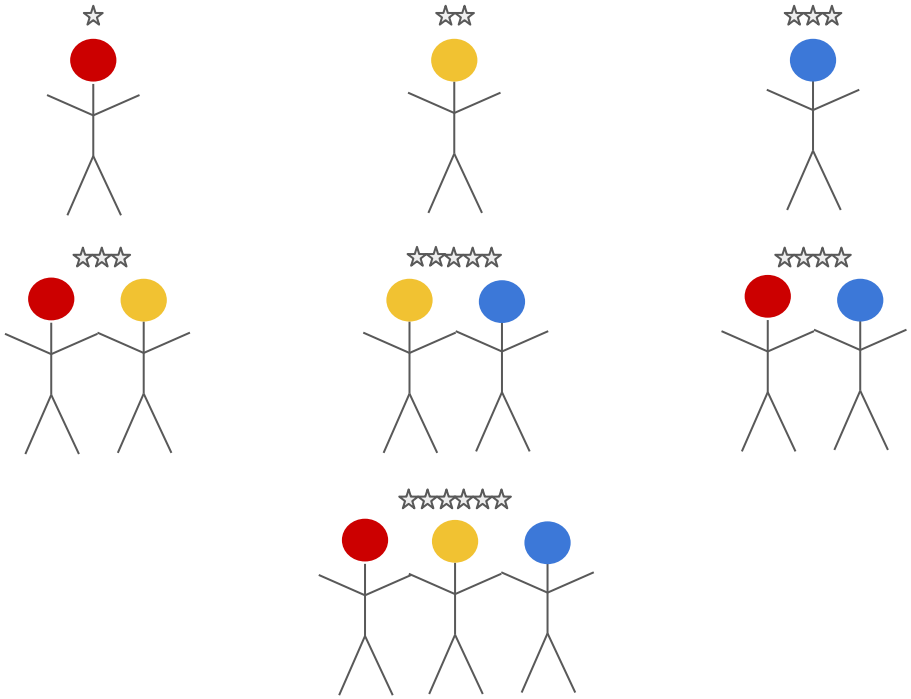


|S| = 0
weight = 1/6

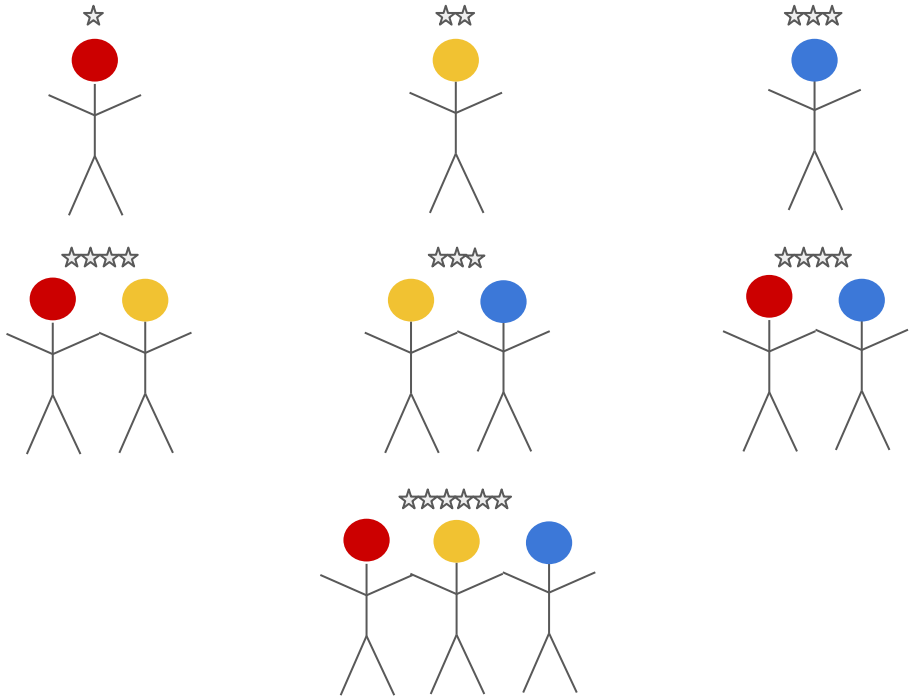


## Players do not interact

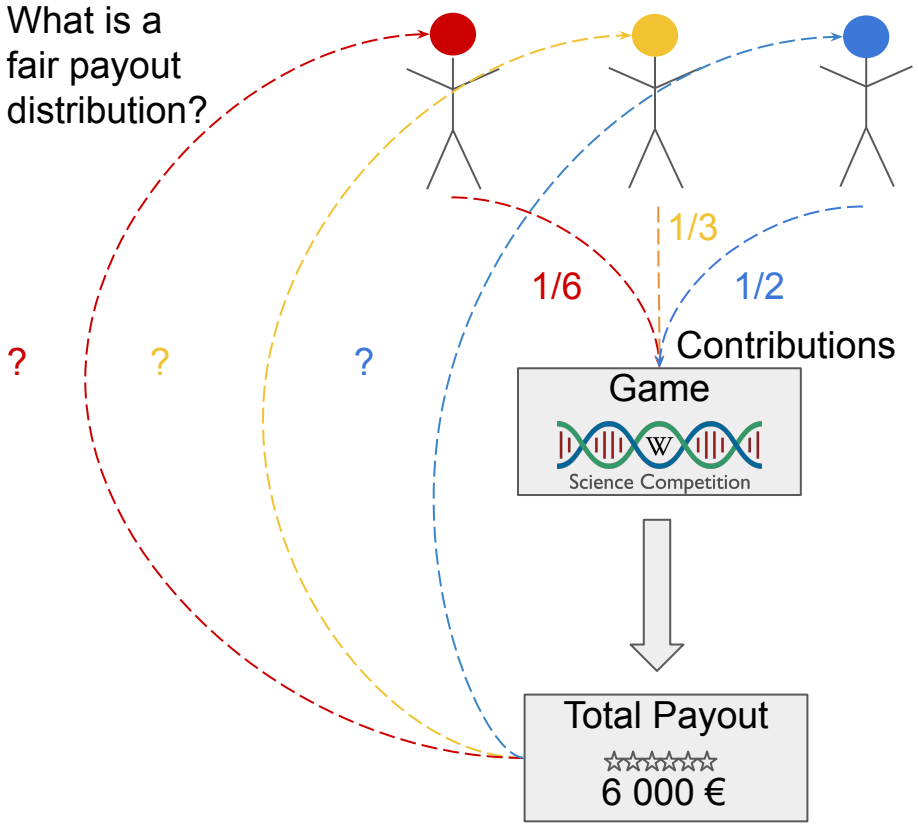
(payouts ☆ add up in each coalition)



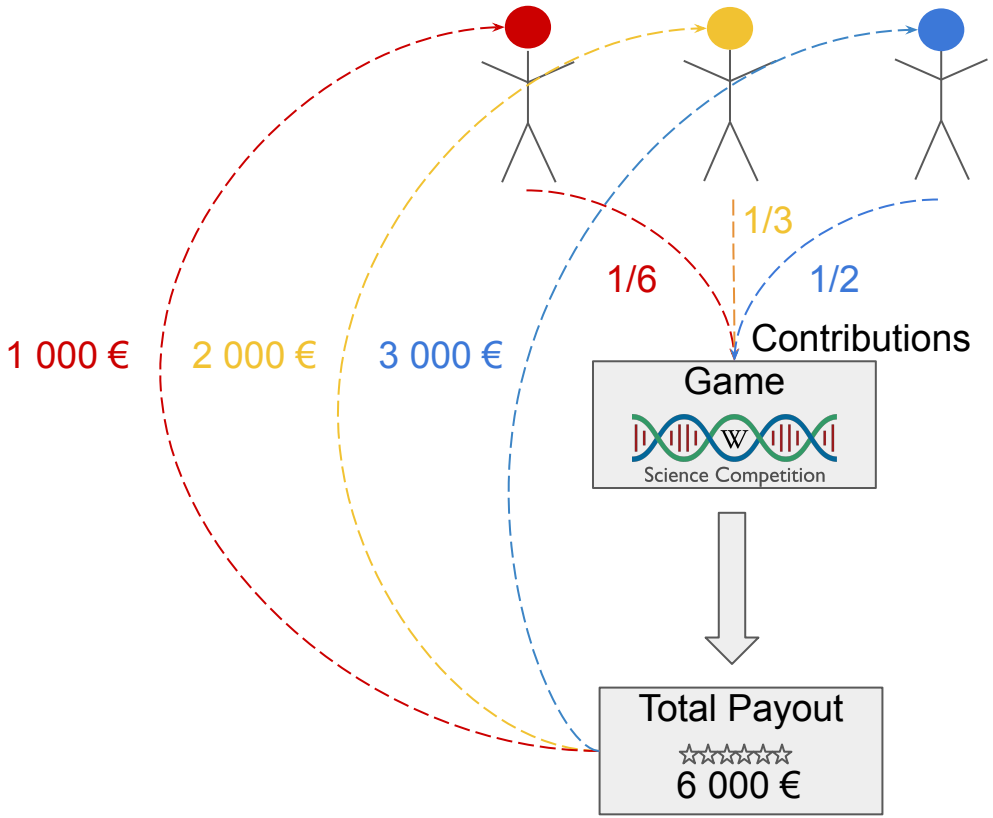
## Players interact



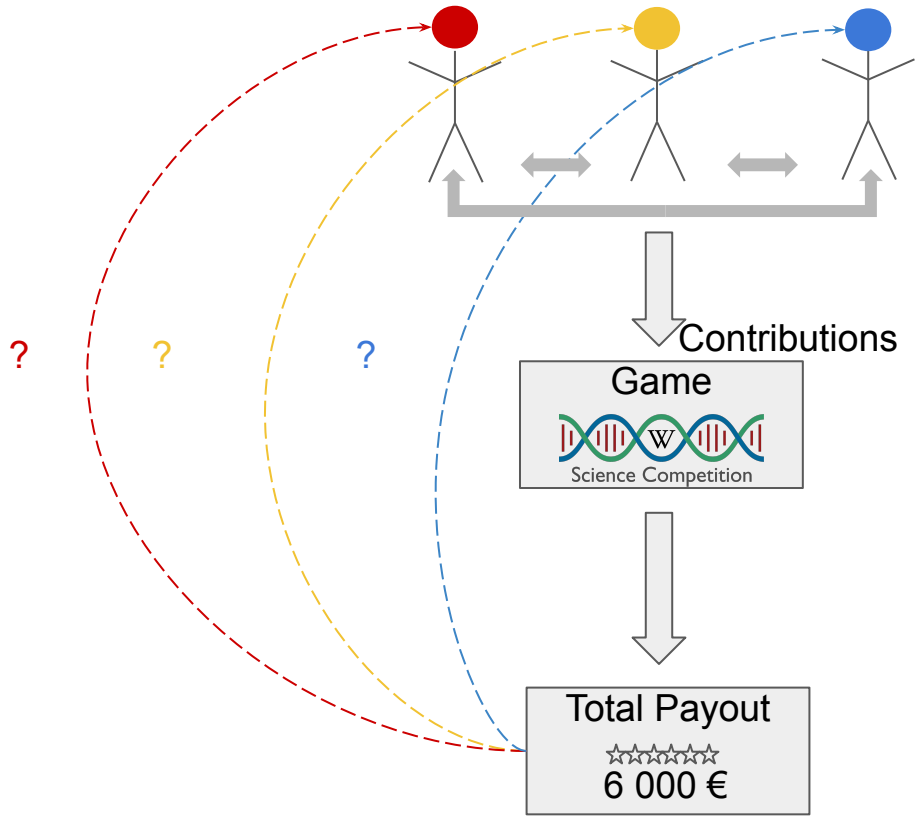
Players do not interact



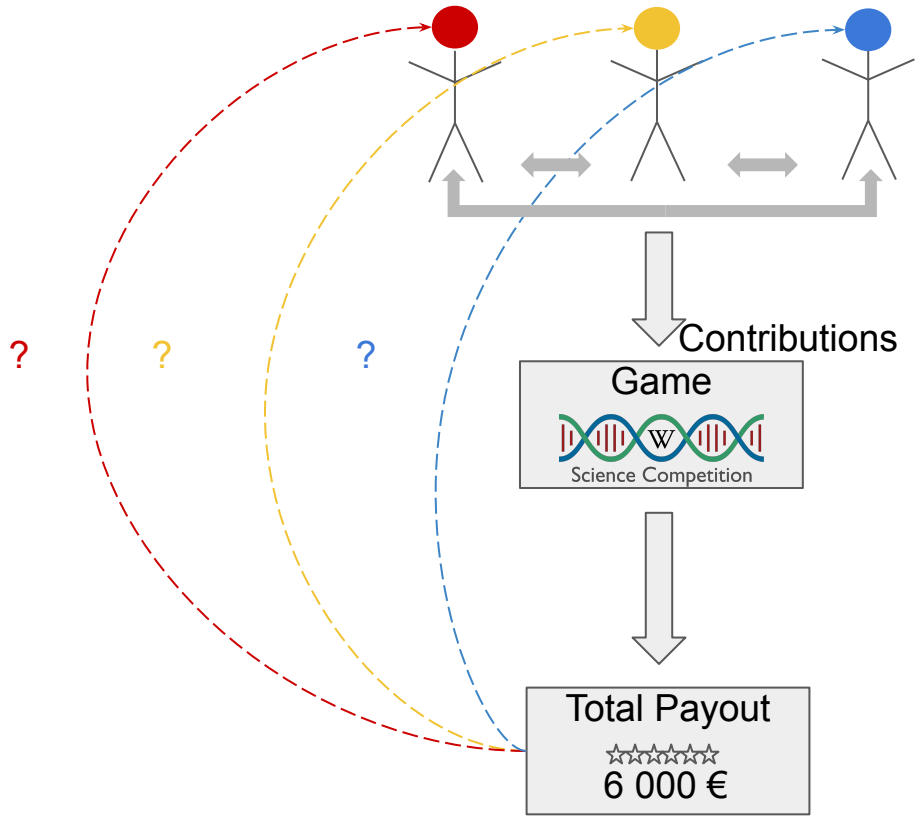
Players do not interact

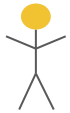


Players interact



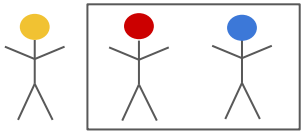
Players interact



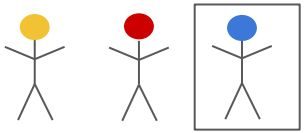
All coalitions of players without 

Add to the coalition

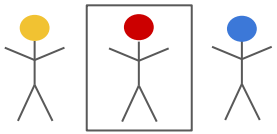
|S| = 2
weight = 2/6



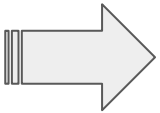
|S| = 1
weight = 1/6

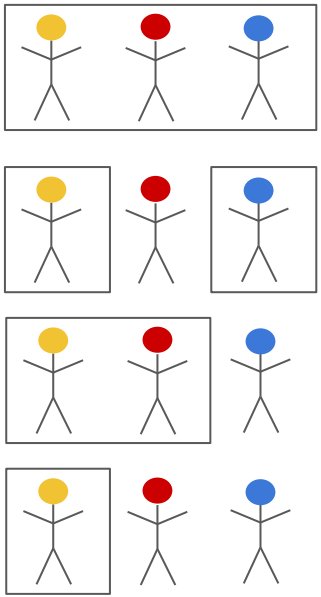


|S| = 1
weight = 1/6



|S| = 0
weight = 2/6
(empty coalition)





- Compute total payout of each coalition
- Compute difference in total payout for each coalition with and without player
- Sum up weighted differences in total payout