

# Exercise Sheet: Boosting with Decision Stumps

## Exercise: Two Iterations of Componentwise Boosting

You are given a boosting model using decision stumps (depth-1 trees) as base learners. The goal is to approximate a target function  $y$  over a single feature  $x_1 \in [0, 2]$ . The boosting algorithm proceeds as follows with step size  $\nu = 1$ .

### Iteration 1

- Base learner: decision stump splitting at  $x_1 > 0.3$
- Predictions:  $f_1(x_1) = 1 \cdot \mathbb{I}(x_1 \leq 0.3) + 2 \cdot \mathbb{I}(x_1 > 0.3)$
- Initial model:  $\hat{f}^{[1]}(x_1) = f_1(x_1)$

### Iteration 2

- Residuals:  $r^{[1]} = y - \hat{f}^{[1]}$
- Fit a new stump to residuals: split at  $x_1 > 1.2$
- Predictions:  $f_2(x_1) = 0 \cdot \mathbb{I}(x_1 \leq 1.2) + (-1) \cdot \mathbb{I}(x_1 > 1.2)$
- Updated model:  $\hat{f}^{[2]}(x_1) = \hat{f}^{[1]}(x_1) + f_2(x_1)$

### Tasks

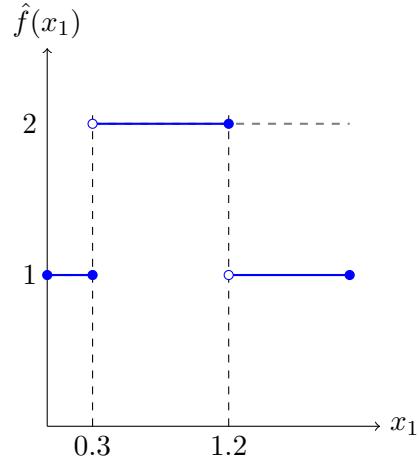
1. Derive the predictor in the second iteration  $\hat{f}^{[2]}(x_1)$  and sketch  $\hat{f}^{[2]}(x_1)$ .
2. Compute  $\hat{f}^{[2]}(x_1)$  for  $x_1 = 0.2, 0.8, 1.5$ .

## Solution

### Final Predictor

$$\hat{f}^{[2]}(x_1) = 1 \cdot \mathbb{I}(x_1 \leq 0.3) + 2 \cdot \mathbb{I}(0.3 < x_1 \leq 1.2) + 1 \cdot \mathbb{I}(x_1 > 1.2)$$

TikZ sketch:



Step-by-step evaluations:

- $x_1 = 0.2: \hat{f}^{[1]} = 1, f_2 = 0, \hat{f}^{[2]} = 1$
- $x_1 = 0.8: \hat{f}^{[1]} = 2, f_2 = 0, \hat{f}^{[2]} = 2$
- $x_1 = 1.5: \hat{f}^{[1]} = 2, f_2 = -1, \hat{f}^{[2]} = 1$