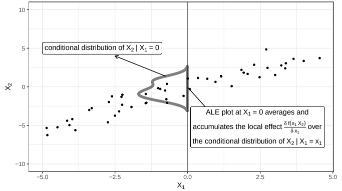
Interpretable Machine Learning

Accumulated Local Effect (ALE): Introduction



Learning goals

- PD plots and its extrapolation issue
- M plots and its omitted-variable bias
- Understand ALE plots

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Feature Effects

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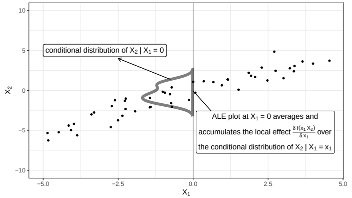

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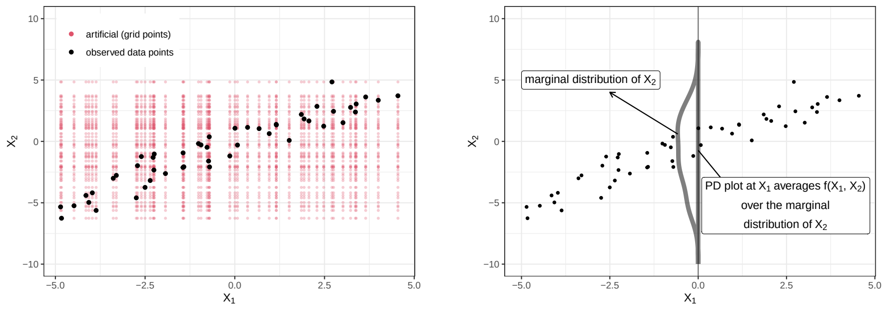
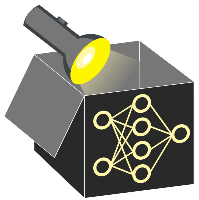
MOTIVATION - CORRELATED FEATURES





- PD plots **average over predictions** of artificial points that are out of distribution/ unlikely (red)
  $\Rightarrow$ Can lead to misleading / biased interpretations, especially if model also contains interactions
- Not wanted if interest is to interpret effects within data distribution

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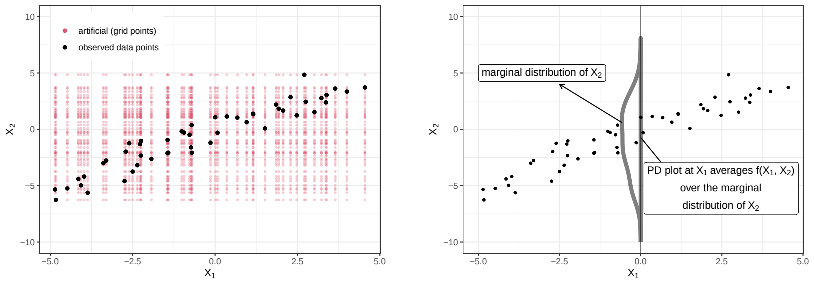

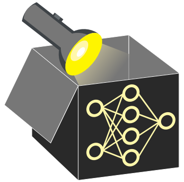

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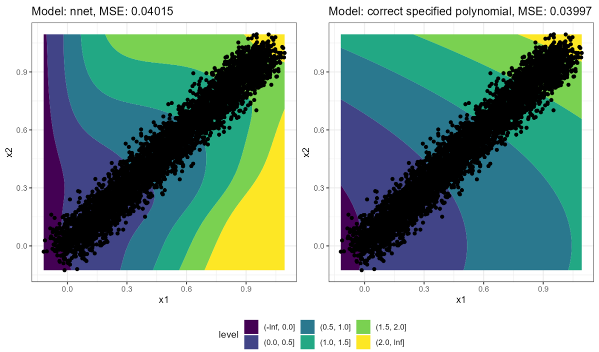
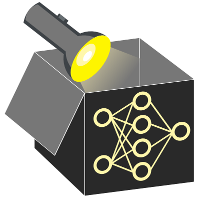
$$y = x_1 + x_2^2 + \epsilon, \text{ where } x_1 = x + \epsilon_1,\ x_2 = x + \epsilon_2 \text{ and } \epsilon_1, \epsilon_2 \sim N(0, 0.05).$$



- Test error (MSE) of NN is comparable to other models
- NN contains interactions (see complex pred. surface)



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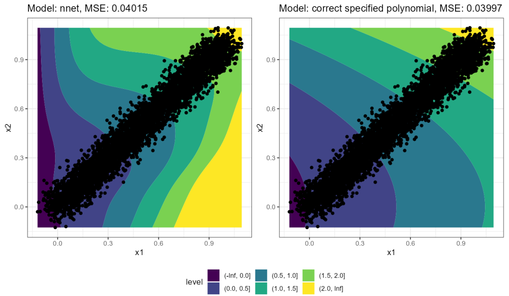

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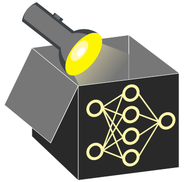

Interpretable Machine Learning – 2 / 6

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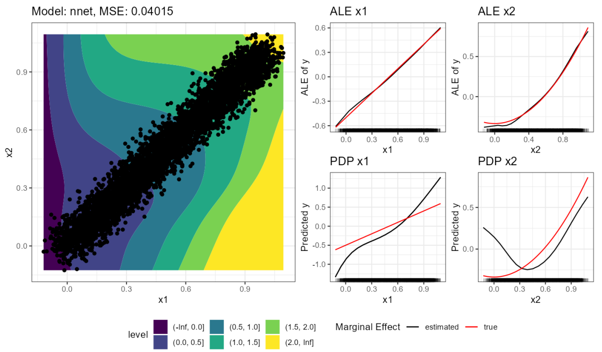

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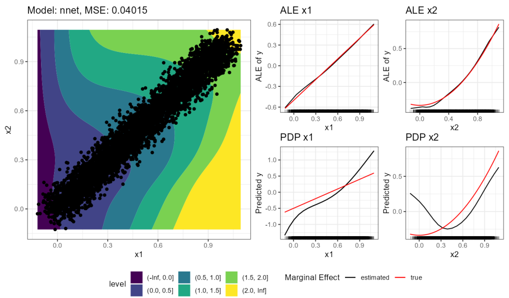

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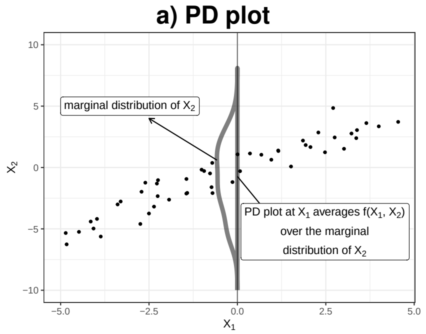

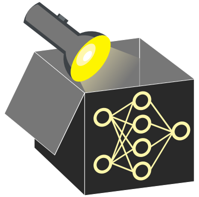

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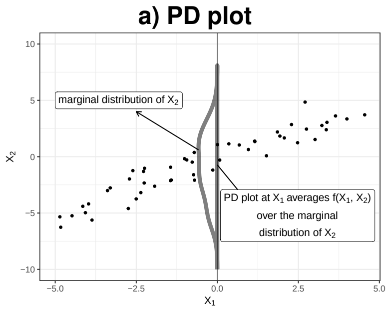

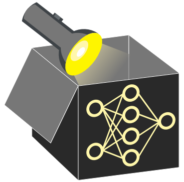

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Interpretable Machine Learning – 3 / 6

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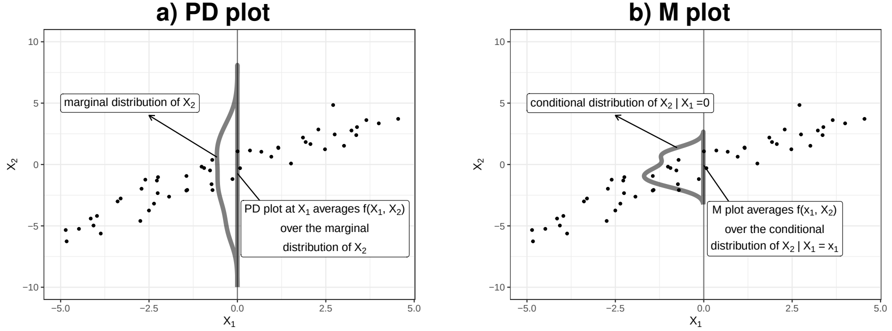

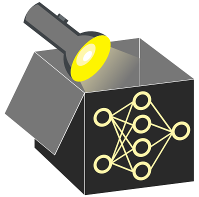

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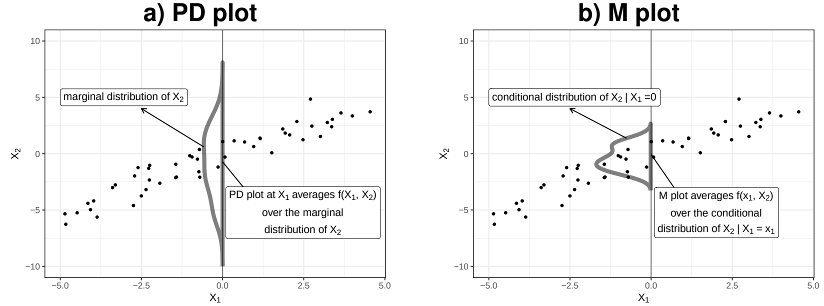

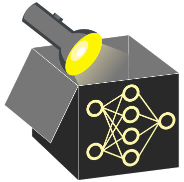

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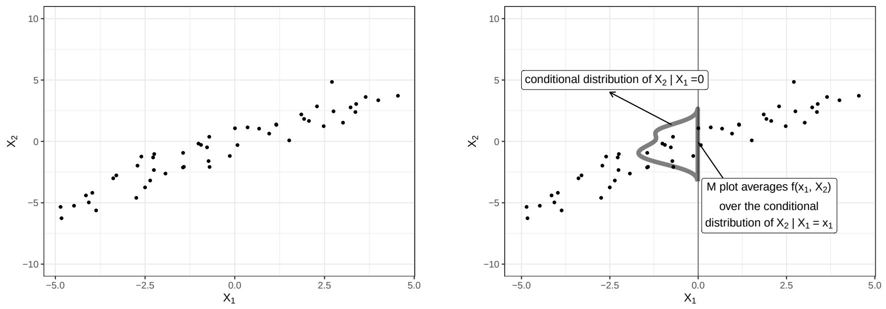

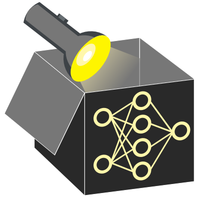

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  $\Rightarrow$ Averaging predictions close to data distribution avoid extrapolation issues
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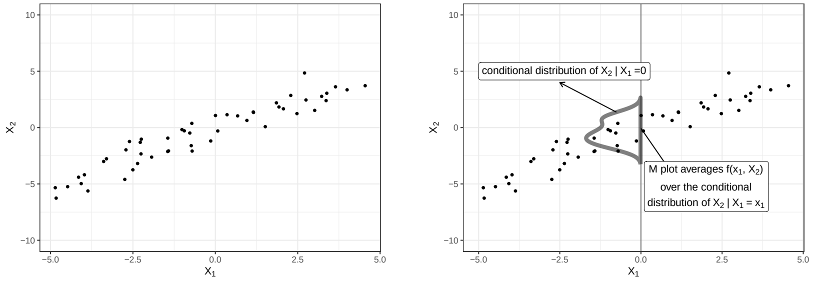

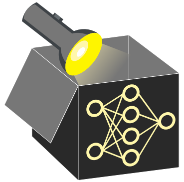

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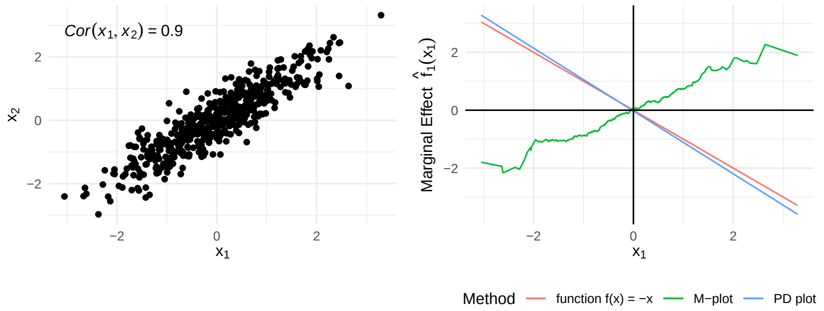

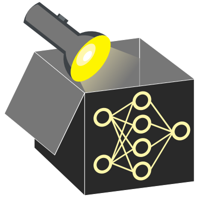

Illustration: Fit LM on 500 i.i.d. observations with features $x_1, x_2 \sim N(0, 1)$, $Cor(x_1, x_2) = 0.9$ and

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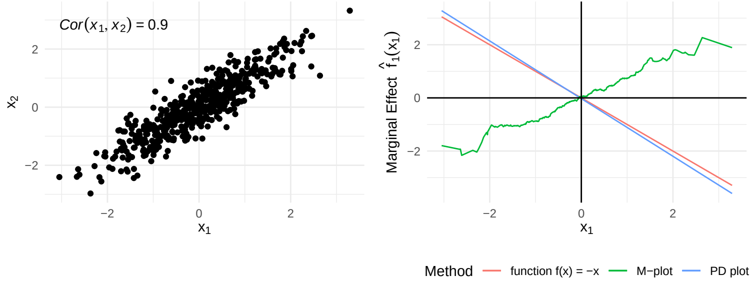

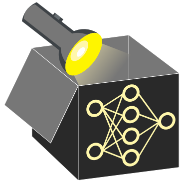

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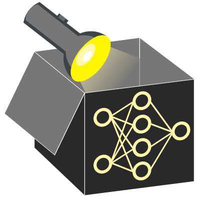

Idea: To remove unwanted effects of other features, take partial derivatives (local effects) of prediction function w.r.t. feature of interest and integrate (accumulate) them w.r.t. the same feature

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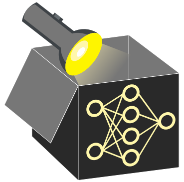

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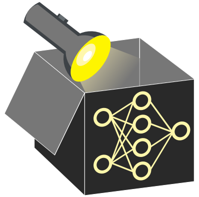

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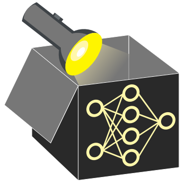

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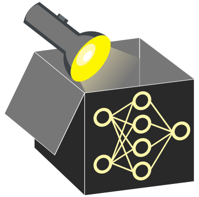

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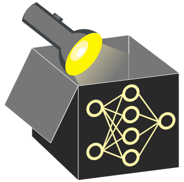

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