Interpretable Machine Learning

Leave One Covariate Out (LOCO)

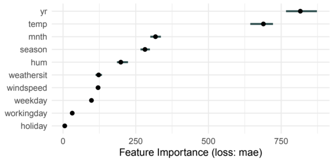

Figure: Bike Sharing Dataset

Learning goals

- Definition of LOCO
- Interpretation of LOCO

Interpretable Machine Learning

Feature Importances 1

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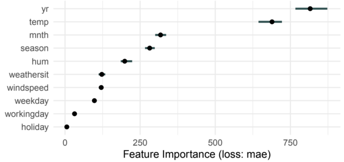

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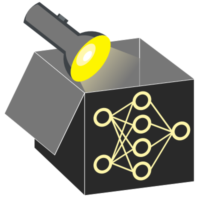

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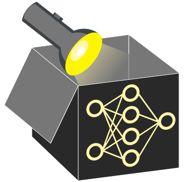

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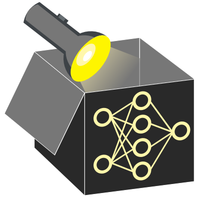

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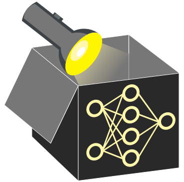

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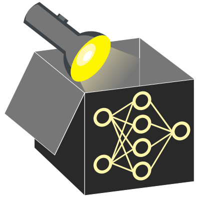

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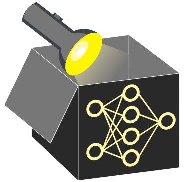

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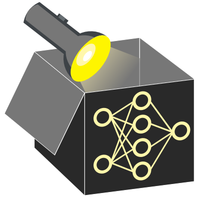

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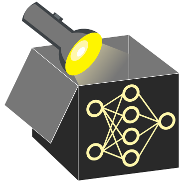

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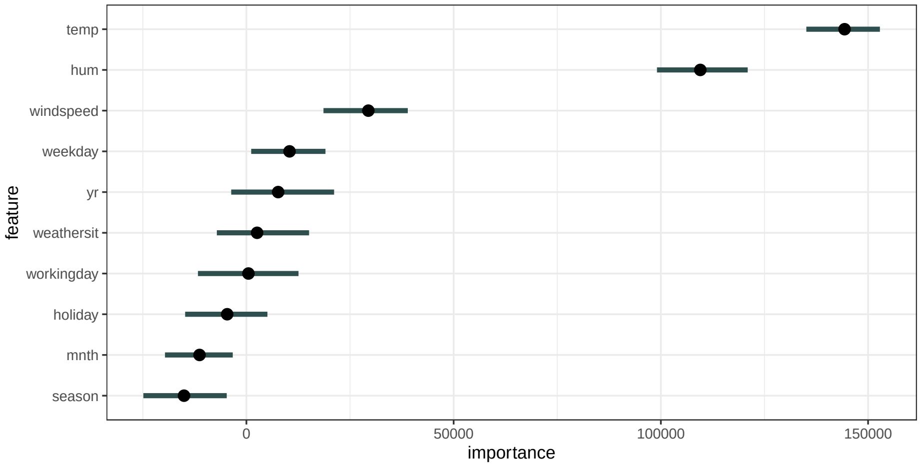

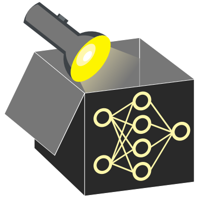

- Trained random forest (default hyperparameters) on 70% of bike sharing data
- Performance measure: mean squared error (MSE)
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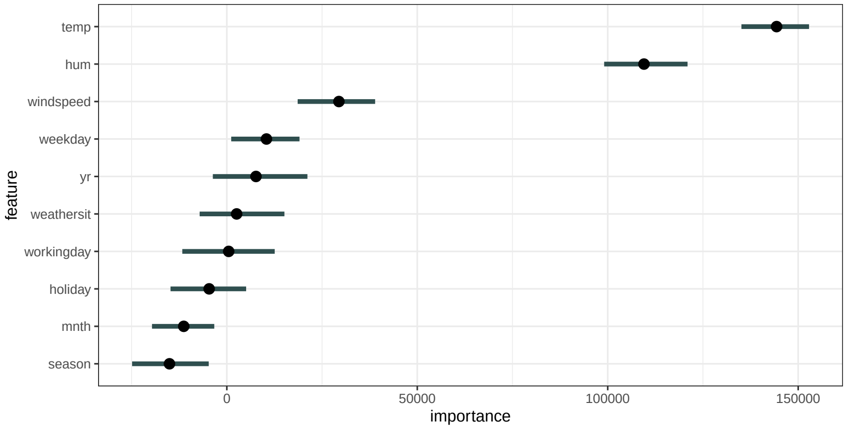

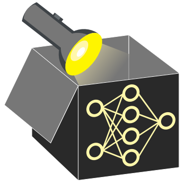

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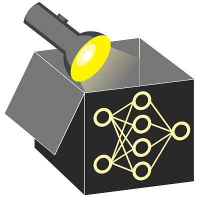

Interpretation: LOCO estimates the generalization error of the learner on a reduced dataset $\mathcal{D}_{-j}$.

Can we get insight into whether the ...

1. feature x_j is causal for the prediction $\hat{y}$?
   - In general, no also because we refit the model (counterexample next slide)
2. feature x_j contains prediction-relevant information?
   - In general, no (counterexample on the next slide)
3. model requires access to x_j to achieve its prediction performance?
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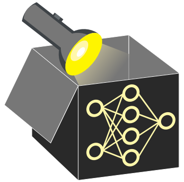

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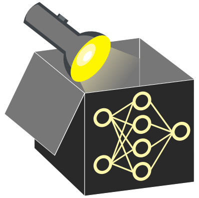

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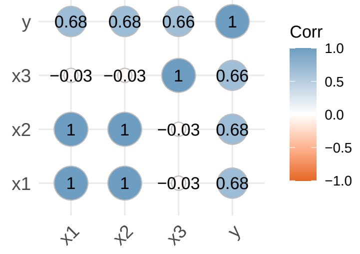

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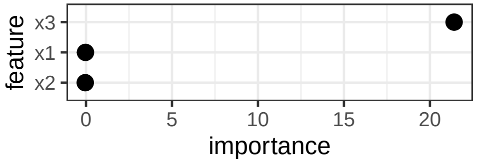

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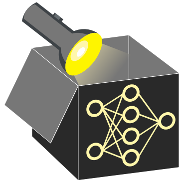

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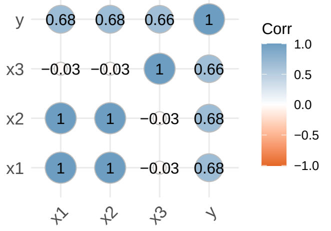

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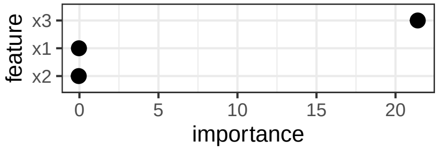

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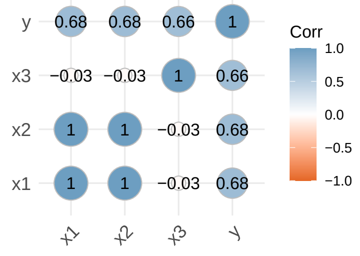

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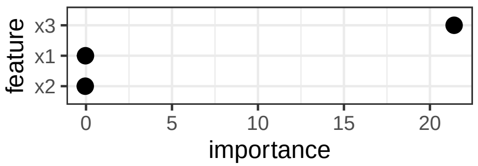

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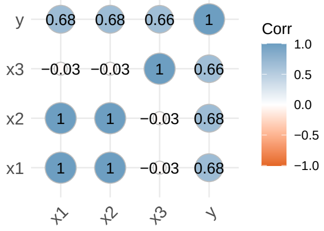

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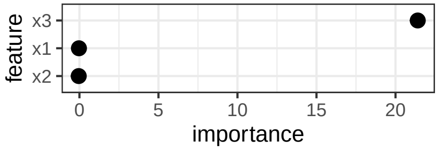

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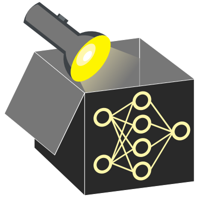

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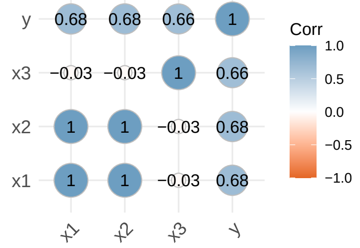

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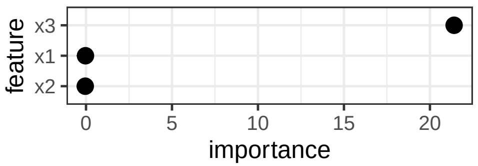

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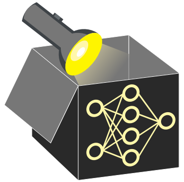

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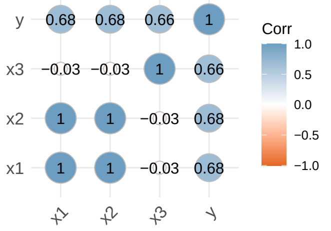

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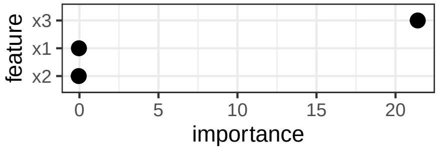

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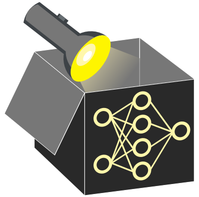

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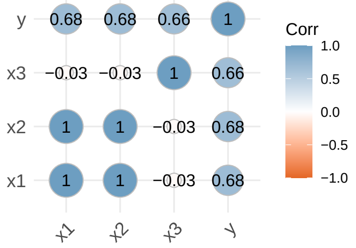
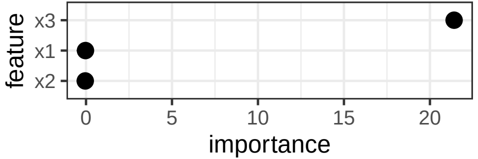


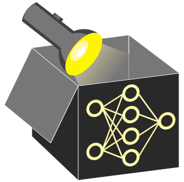
Correlation matrix



LOCO importance from LM trained on 70% of data, evaluated on remaining 30%

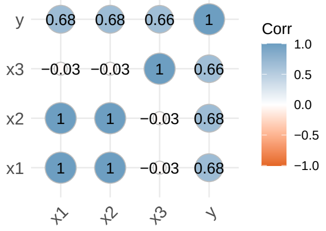
$\Rightarrow$ We cannot infer (1) from LOCO (e.g. $\text{LOCO}_2 \approx 0$ but coefficient of x_2 is 2.05)
$\Rightarrow$ We also can't infer (2), e.g., $Cor(x_2, y) = 0.68$ but $\text{LOCO}_2 \approx 0$
$\Rightarrow$ We can get insight into (3): x_2 and x_1 highly correlated with $\text{LOCO}_1 = \text{LOCO}_2 \approx 0$
$\rightsquigarrow$ x_2 and x_1 take each others place if one of them is left out (not the case for x_3)

INTERPRETATION OF LOCO

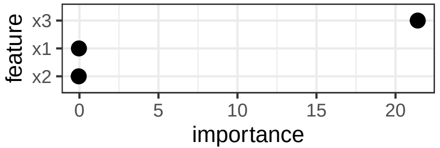


Example: Sample 1000 observations with

- $x_1, x_3 \sim N(0, 5)$, $x_2 = x_1 + \epsilon_2$ with $\epsilon_2 \sim N(0, 0.1)$
- $y = x_2 + x_3 + \epsilon$ with $\epsilon \sim N(0, 2)$
- Trained LM: $\hat{f}(x) = -0.02 - 1.02x_1 + 2.05x_2 + 0.98x_3$



Correlation matrix



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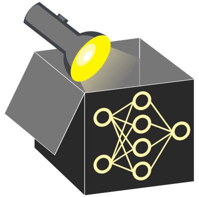
$\rightsquigarrow$ x_2 and x_1 take each others place if one of them is left out (unlike x_3)

PROS AND CONS



Pros:

- Requires (only?) one refitting step per feature for evaluation
- Easy to implement
- Testing framework available in ▸ Lei et al. (2018)

Cons:

- Provides insight into a learner on specific data, not a specific model
  $+$ for algorithm-level insight
  $-$ for model-specific insights
- Model training is a random process and LOCO estimates can be noisy
  $\rightsquigarrow$ Limits inference about on model and data, or multiple refittings necessary?
- Requires re-fitting the learner for each feature
  $\rightsquigarrow$ Computationally intensive compared to PFI

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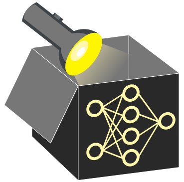

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