Interpretable Machine Learning

Shapley Additive Global Importance (SAGE)

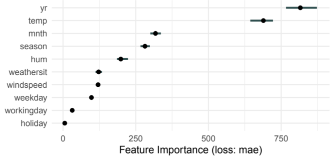

Figure: Bike Sharing Dataset

Learning goals

- How SAGE fairly distributes importance
- Definition of SAGE value function
- Difference SAGE value function and SAGE values
- Marginal and Conditional SAGE

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Feature Importances 1

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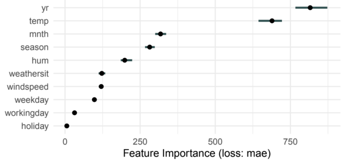

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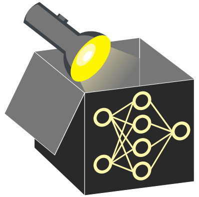
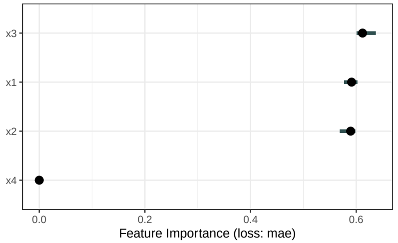
CHALLENGE: FAIR ATTRIBUTION OF IMPORTANCE



Recap:

- Data: $x_1, \ldots, x_4$ uniformly sampled from $[-1, 1]$
- DGP: $y := x_1 x_2 + x_3 + \epsilon_Y$ with $\epsilon_Y \sim N(0, 1)$
- Model: $\hat{f}(x) \approx x_1 x_2 + x_3$



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Reason: PFI assesses importance given that all remaining features are preserved. If we first permute x_1 and then x_2, permutation of x_2 would have no effect on the performance (and vice versa).

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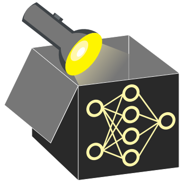
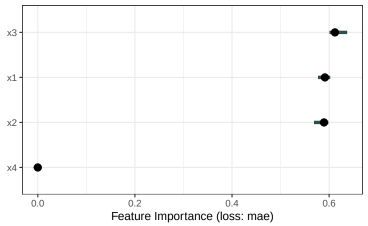
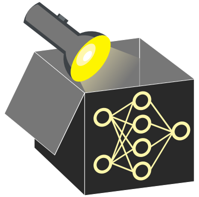




SAGE IDEA ▸ Covert et al. (2020)



SAGE: Use Shapley values to compute a fair attribution of importance (via model performance)

Idea:

- Feature importance attribution can be regarded as cooperative game
  $\rightsquigarrow$ features jointly contribute to achieve a certain model performance
- Players: features
- Payoff to be fairly distributed: model performance
- Surplus contribution of a feature depends on the coalition of features that are already accessible by the model

Note:

- Similar idea (called SFIMP) was proposed in ▸ Casalicchio et al. (2018)
- Definition based on model refits was proposed in context of feature selection in ▸ Cohen et al. (2007)

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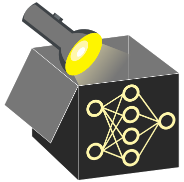

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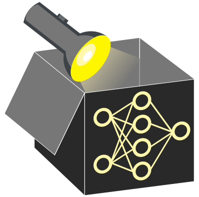

Removal Idea: To deprive information of the non-coalition features $-S$ from the model, marginalize the prediction function over the features $-S$ to be "dropped".

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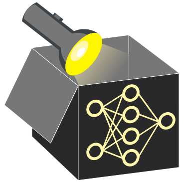

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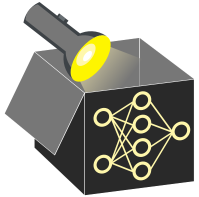

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$\leadsto$ Quantify the predictive power of a coalition S in terms of reduction in risk
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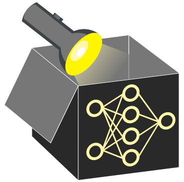

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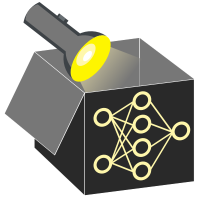

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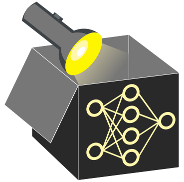

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SAGE - MARGINAL AND CONDITIONAL SAMPLING

When computing the marginalized prediction $\hat{f}_S(x_S)$, the “dropped” features can be sampled from

- the marginal distribution $\mathbb{P}(x_{-S}) \Rightarrow$ marginal SAGE
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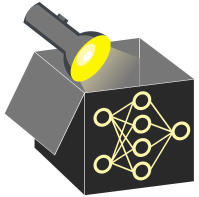

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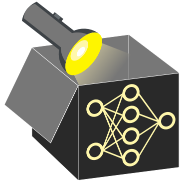

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Example:

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  $x_1 = \epsilon_1$
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  $x_3 = x_2 + \epsilon_3$ (all ϵ_j i.i.d.)
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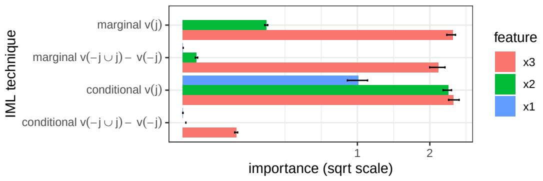

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- Conditional $v(j)$ are also nonzero for features that are not used by $\hat{f}$ (e.g., due to correlation)
- For conditional value function v, the difference $v(-j \cup j) - v(-j)$ quantifies the unique contribution of x_j over remaining features x_{-j}
  $\Rightarrow$ Since $y \perp\!\!\!\perp x_1, x_2 | x_3$, only $v(\{1,2,3\}) - v(\{1,2\})$ is nonzero (i.e., for feature $j = 3$)

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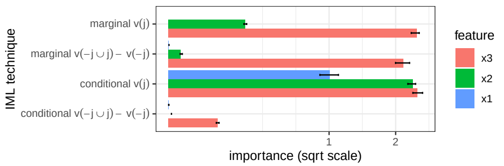

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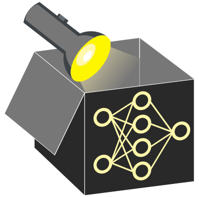

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- can be computed by averaging the contribution of x_j over all feature orderings
- for feature permutation τ, the contribution of j in the set S_j^τ is given as $v(S_j^\tau \cup \{j\}) - v(S_j^\tau)$
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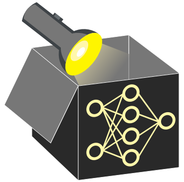

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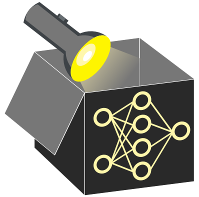

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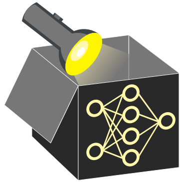

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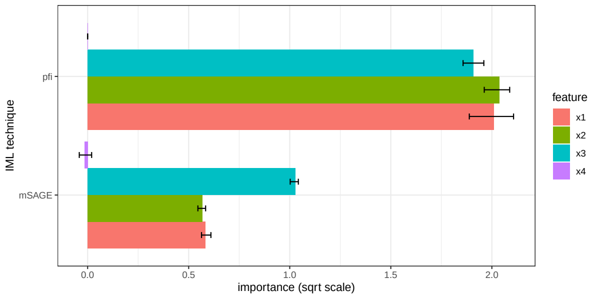

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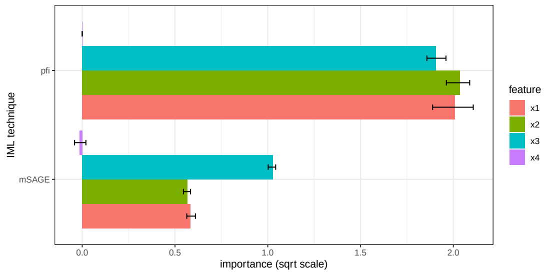

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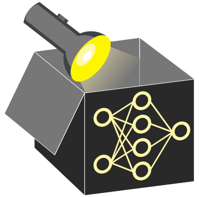

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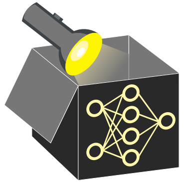

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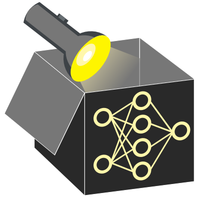

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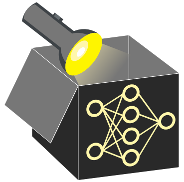
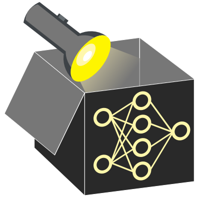


IMPLICATIONS MARGINAL SAGE VALUES



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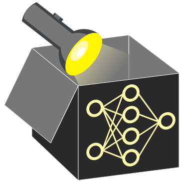

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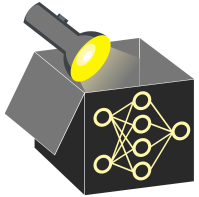

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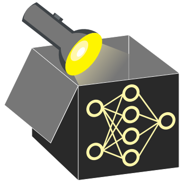

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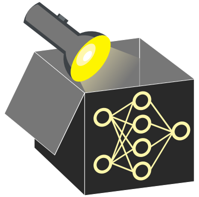
DEEP DIVE: SHAPLEY AXIOMS FOR SAGE

The Shapley axioms can be translated into properties of SAGE. The interpretation depends on whether conditional or marginal sampling is used.



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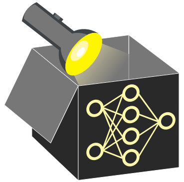

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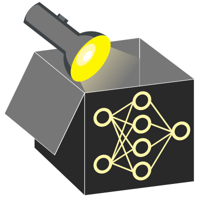

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