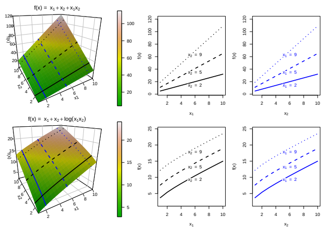
Interpretable Machine Learning

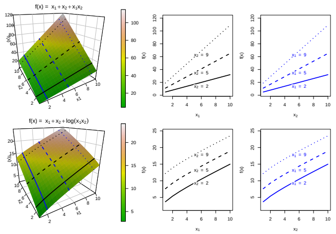
Feature Interactions



Learning goals

- Feature interactions
- Difference to feature dependencies

Interpretable Machine Learning
Feature Interactions



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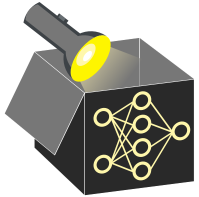
FEATURE INTERACTIONS



- Feature dependencies concern data distribution
- Feature interactions may occur in structure of **both** model or DGP (e.g., functional relationship between X and $\hat{f}(X)$ or X and $Y = f(X)$)
  $\leadsto$ Feature dependencies may lead to feature interactions in a model

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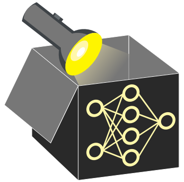

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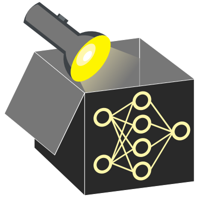

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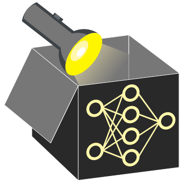

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- Interactions: A feature's effect on the prediction depends on other features
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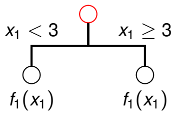
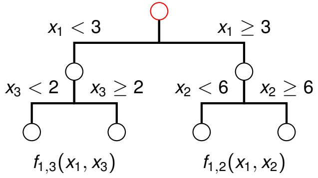


No interaction



Interactions: x_1 and x_3,
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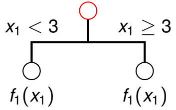

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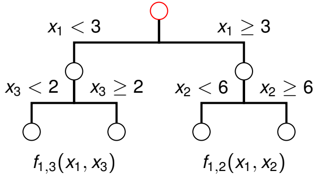

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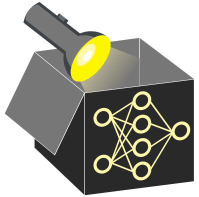
FEATURE INTERACTIONS ▸ Friedman and Popescu (2008)



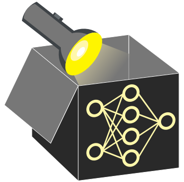
Definition: A function $f(\mathbf{x})$ contains an interaction between x_j and x_k if a difference in $f(\mathbf{x})$-values due to changes in x_j will also depend on x_k, i.e.:

$$\mathbb{E}\left[\frac{\partial^2 f(\mathbf{x})}{\partial x_j \partial x_k}\right]^2 > 0$$

$\Rightarrow$ If x_j and x_k do not interact, $f(\mathbf{x})$ is sum of 2 functions, each independent of x_j, x_k:

$$f(\mathbf{x}) = f_{-j}(x_1, \dots, x_{j-1}, x_{j+1}, \dots, x_p) + f_{-k}(x_1, \dots, x_{k-1}, x_{k+1}, \dots, x_p)$$

FEATURE INTERACTIONS ▸ FRIEDMAN_POPESCU



Definition: A function $f()$ contains an interaction between x_j and x_k if a difference in $f()$-values due to changes in x_j will also depend on x_k, i.e.:

$$\mathbb{E}\left[\frac{\partial^2 f()}{\partial x_j \partial x_k}\right]^2 > 0$$

$\Rightarrow$ If x_j and x_k don't interact, $f()$ is sum of 2 functions, each indep. of x_j, x_k:

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FEATURE INTERACTIONS

Example: $f(\mathbf{x}) = x_1 + x_2 + x_1 \cdot x_2$ (not separable)

$$\mathbb{E}\left[\frac{\partial^2(x_1+x_2+x_1 \cdot x_2)}{\partial x_1 \partial x_2}\right]^2 = \mathbb{E}\left[\frac{\partial(1+x_2)}{\partial x_2}\right]^2 = 1 > 0$$

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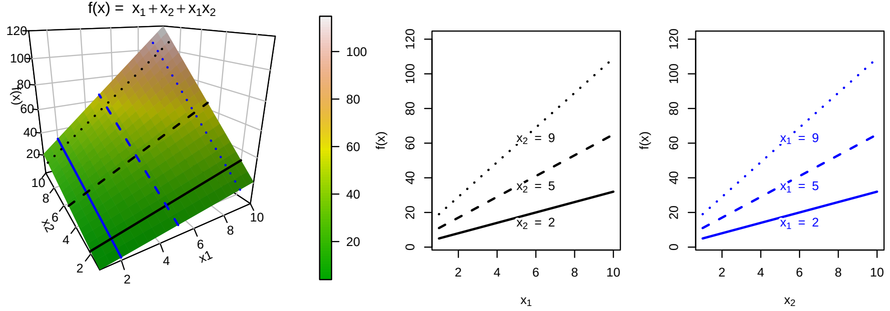

- Effect of x_1 on $f(\mathbf{x})$ varies with x_2 (and vice versa)

$\Rightarrow$ Different slopes

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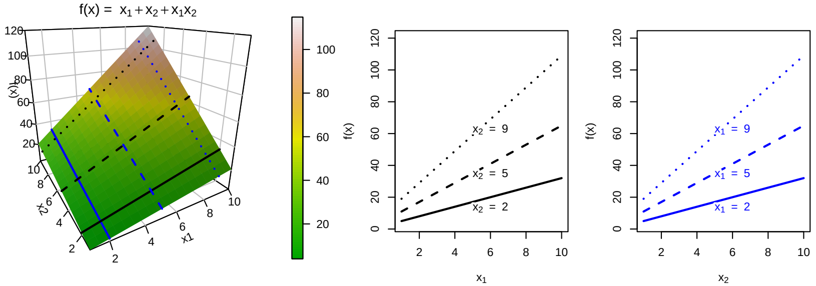

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Example of separable function:

$$f(\mathbf{x}) = x_1 + x_2 + \log(x_1 \cdot x_2) = x_1 + x_2 + \log(x_1) + \log(x_2)$$

$\Rightarrow f(\mathbf{x}) = f_1(x_1) + f_2(x_2)$ with $f_1(x_1) = x_1 + \log(x_1)$ and $f_2(x_2) = x_2 + \log(x_2)$

$\Rightarrow$ no interactions due to separability, also $\mathbb{E}\left[\frac{\partial^2 f(\mathbf{x})}{\partial x_1 \partial x_2}\right]^2 = 0$

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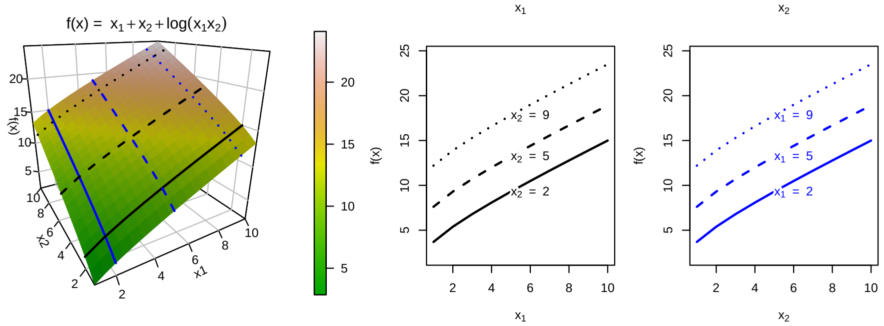

- Effect of x_1 on $f(\mathbf{x})$ stays the same for different x_2 values (and vice versa)

$\Rightarrow$ Parallel lines at different horizontal (blue) or vertical (black) slices

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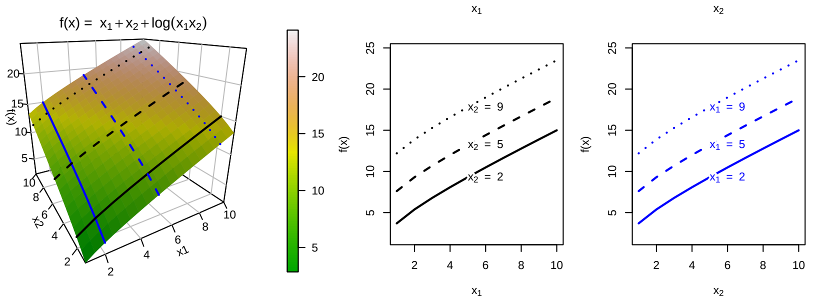

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