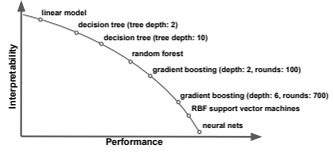
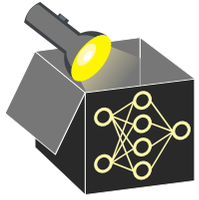
Interpretable Machine Learning

Introduction, Motivation, and History



Learning goals

- Why interpretability?
- Developments until now?
- Use cases for interpretability



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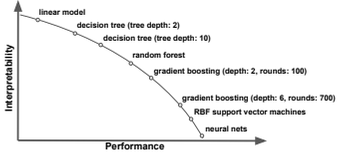

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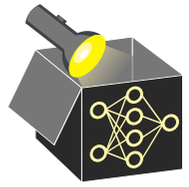
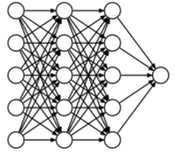
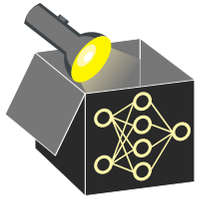


WHY INTERPRETABILITY?

- ML: huge potential to aid decision-making process due to its predictive performance
- ML models are black boxes, e.g., XGBoost, RBF SVM or DNNs
  $\rightsquigarrow$ too complex to be understood by humans
- Some applications are "learn to understand"





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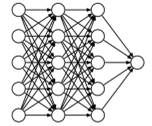

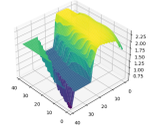

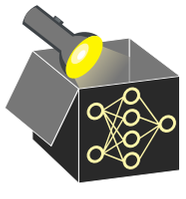

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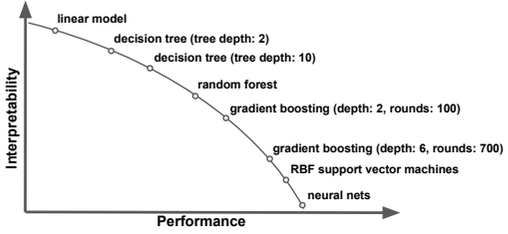
  1. hurts trust
  2. creates barriers

⇝ Many disciplines with required trust rely on traditional models, e.g., linear models, with less predictive performance



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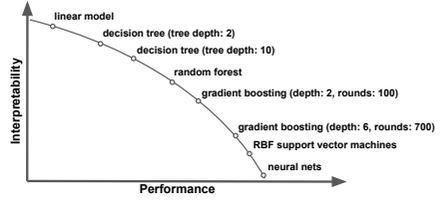
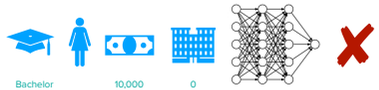
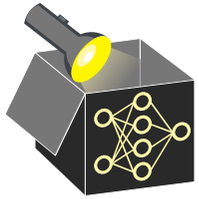


INTERPRETABILITY IN HIGH-STAKES DECISIONS

Examples of critical areas where decisions based on ML models can affect human life

- Credit scoring and insurance applications
  ▸ Society of Actuaries
  - Reasons for not granting a loan
  - Fraud detection in insurance claims





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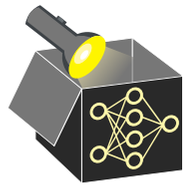

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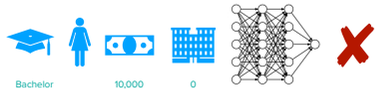

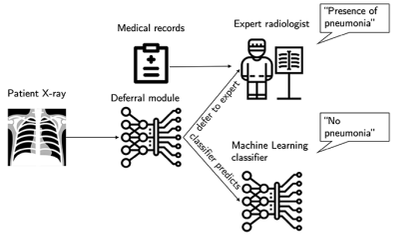

▸ Miliard (2020)

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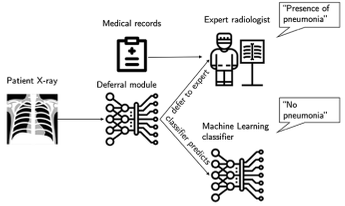

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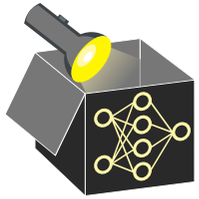
NEED FOR INTERPRETABILITY



Need for interpretability becoming increasingly important from a legal perspective

- General Data Protection Regulation (GDPR) requires for some applications that models have to be explainable ▸ Goodman & Flaxman (2017)
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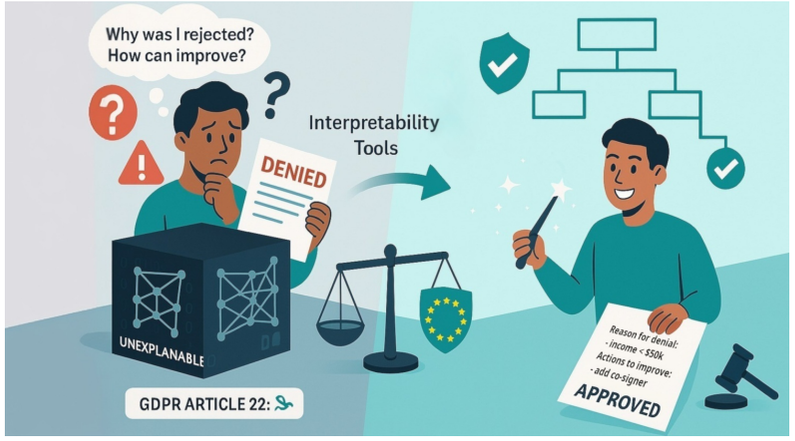

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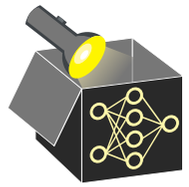

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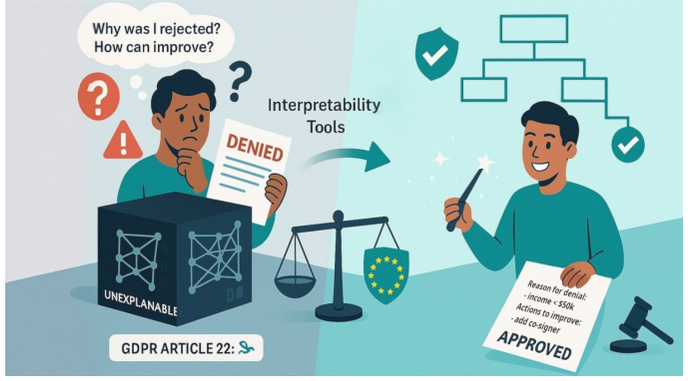
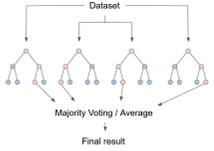


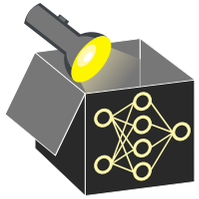
BRIEF HISTORY OF INTERPRETABILITY

- 18th and 19th century:
  Linear regression models (Gauss, Legendre, Quetelet)
- 1940s:
  Emergence of sensitivity analysis (SA)
- Middle of 20th century:
  Rule-based ML, incl. decision rules and decision trees
- 2001:
  Built-in feature importance measure of random forests
- >2010:
  Explainable AI (XAI) for deep learning
- >2015:
  IML as an independent field of research

▸ Carl Friedrich Gauss

▸ Wikipedia





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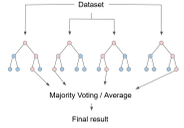
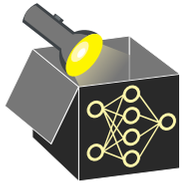
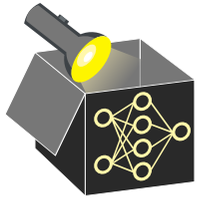
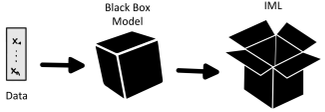




Interpretable Machine Learning



Interpretation Goals



Learning goals

Understand Interpretation Goals:

- Global insights (discovery)
- Improve model (debug and audit)
- Understand and control individual predictions
- Justification and fairness

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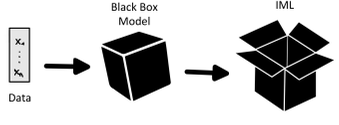
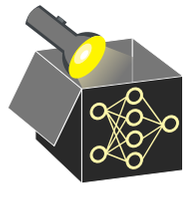
Interpretation Goals





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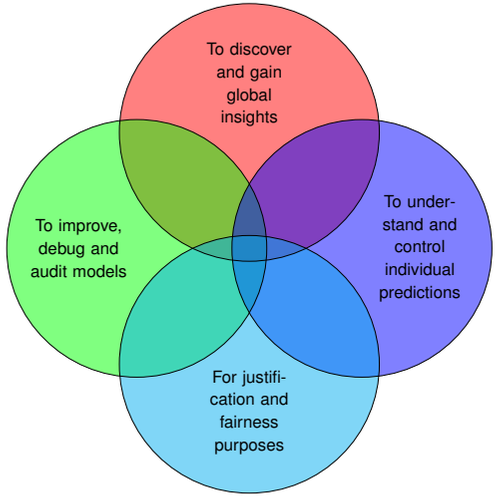

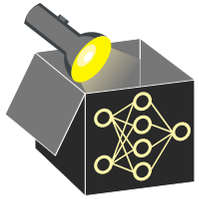

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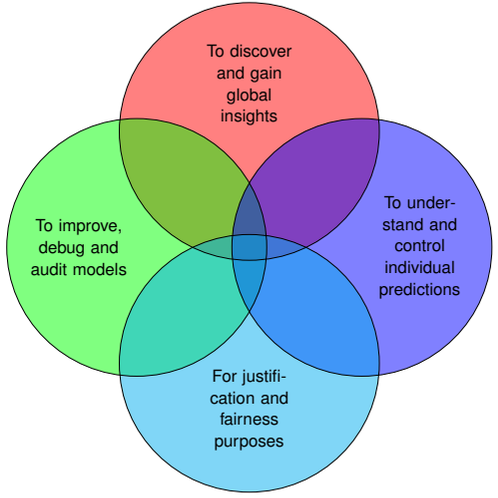

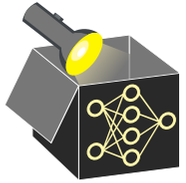

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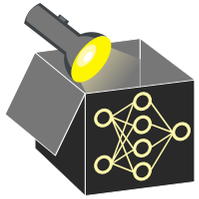
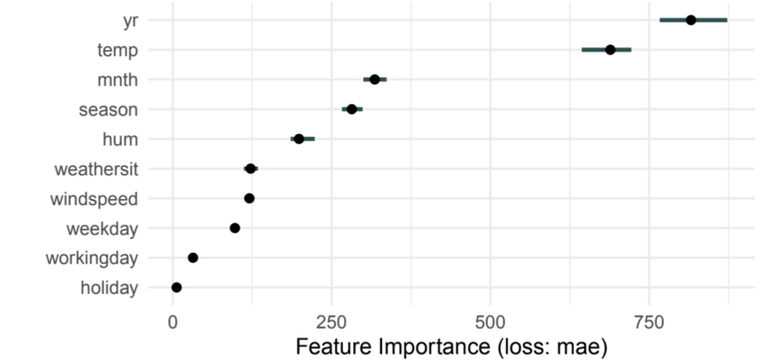
DISCOVER AND GAIN GLOBAL INSIGHTS



⇝ Gain insights about data, model, and underlying data-generating process

Example: Bike Sharing Dataset (predict number of bike rentals per day)
Exemplary question: Which feature influences model performance and how much?



- Year (`yr`) and Temperature (temp) most important features
- Holiday (`holiday`) less important (Can we drop it?)

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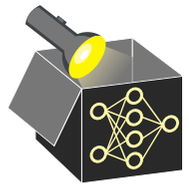

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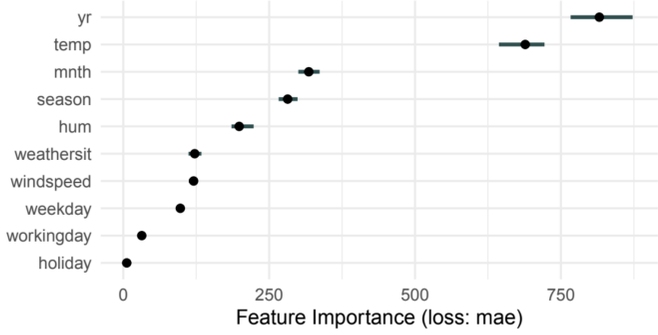

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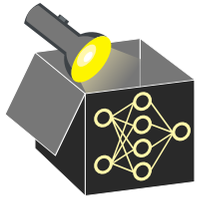

⇝ Insights help to identify flaws (in data or model), which can be corrected

Example: Neural Net Tank ▸ gwern.net

Cautionary tale (never actually happened):

- Train a neural network to detect tanks
- Good fit on training data
- Application outside training data: failure

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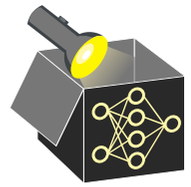

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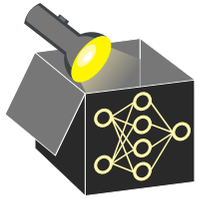

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- Reasons vary depending on input
  ⇝ NN based decision on irrelevant pixels

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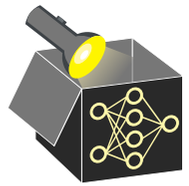

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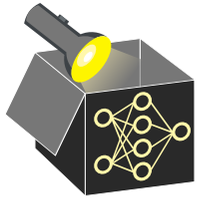
IMPROVE, DEBUG AND AUDIT MODELS



⇝ Insights help to identify flaws (in data or model), which can be corrected

Example: Neural Net Tank ▸ gwern.net

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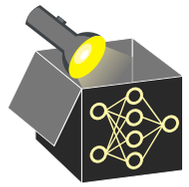

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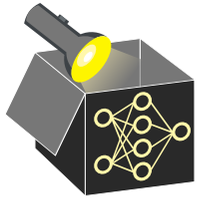

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Comment on tank example:

"We made exactly the same mistake in one of my projects on insect recognition. We photographed 54 classes of insects. Specimens had been collected, identified, and placed in vials. Vials were placed in boxes sorted by class. I hired student workers to photograph the specimens.

Naturally they ***did this one box at a time; hence, one class at a time.*** *Photos were taken in alcohol.* ***Bubbles would form in the alcohol. Different bubbles on different days.*** *The learned classifier was surprisingly good. But a* ***saliency map revealed that it was reading the bubble patterns*** *and ignoring the specimens.*

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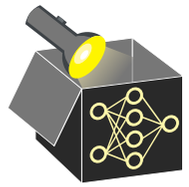

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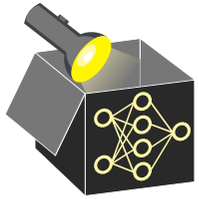

- Nearly all computer programs have bugs
  - ⇝ Minimizing such bugs extremely relevant
- Process with multiple steps to locate, understand and solve a problem
  - ⇝ Classical debugging
- **In ML** we have a program (learner) writing another program (model)
- How to debug or audit programs which contain ML models?
- Based on a single cross-val score?

⇝ Being able to interpret your model will always be helpful – if possible!

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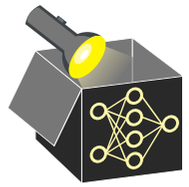

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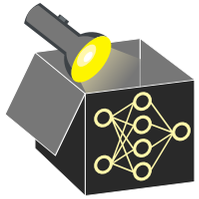
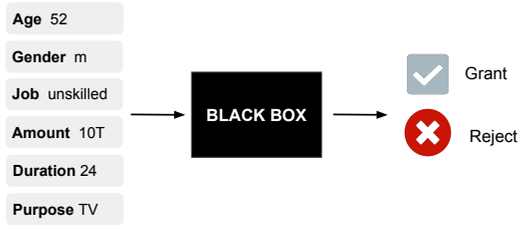
UNDERSTAND & CONTROL INDIVIDUAL PREDICTIONS



⇝ Explaining individual decisions can prevent unwanted actions based on the model

Example: Credit Risk Application
x: customer and credit information; *y*: grant or reject credit



Questions:

- Why was the credit rejected?
- Is it a fair decision?
- **How should x be changed so that the credit is accepted?**

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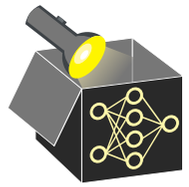

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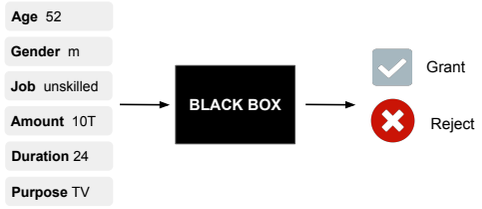

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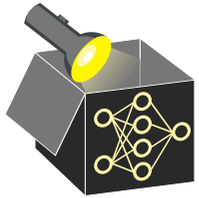

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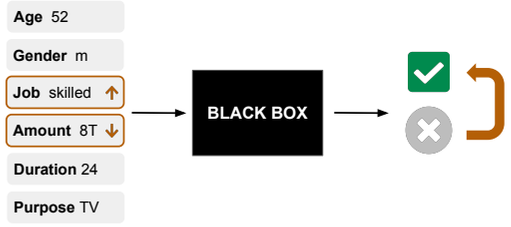

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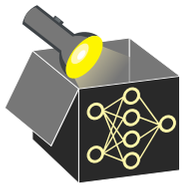

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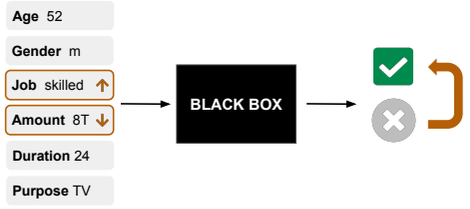

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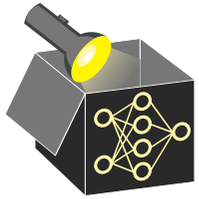
JUSTIFICATION AND FAIRNESS



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Example: COMPAS

- COMPAS: Correctional Offender Management Profiling for Alternative Sanctions
- Commercial tool used in courts to assess a defendant's risk of re-offending
- Predicts **recidivism risk**:
  - Likelihood of an individual with a past offense is arrested again
  - Features: race, gender, age, number of prior prison sentences, ...
  - Output: COMPAS score from 1 (low risk) to 10 (high risk) risk of recidivism
- Based on a questionnaire completed by the defendant

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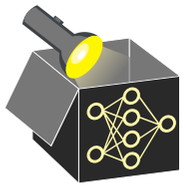

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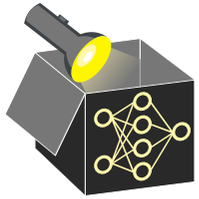
JUSTIFICATION AND FAIRNESS: COMPAS

Larson et al. 2016



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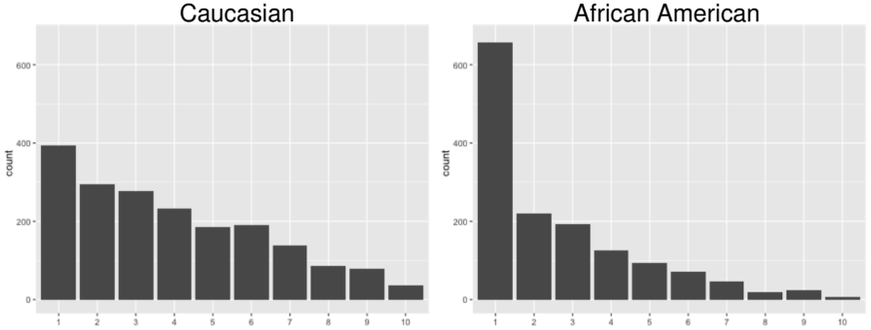
Descriptive data analysis of the target (COMPAS score) by a feature encoding race:



COMPAS score: 1 (low risk) to 10 (high risk)

⇝ Model skewed towards low risk for Caucasians
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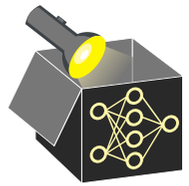
⇝ Use IML to investigate if and how much the model uses the defendant's race

JUSTIFICATION AND FAIRNESS: COMPAS

LARSON



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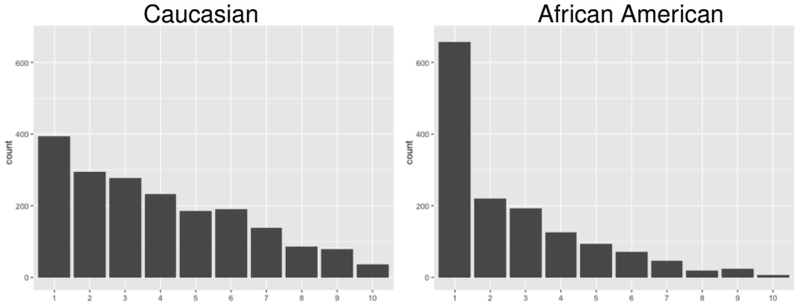

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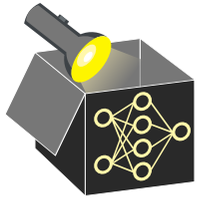
▸ Alvarez-Melis and Jaakkola 2018



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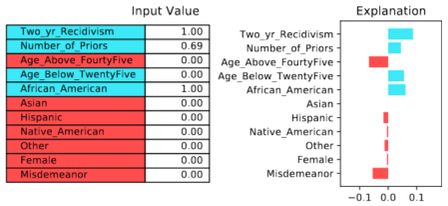
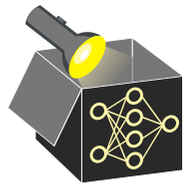


- Pick a defendant
- LIME quantifies a feature's impact on the defendant's COMPAS score
- `African_American` has a large positive weight on COMPAS score
- Occurs for many individuals, see ▸ XAI Stories ⇝ Suggests racial bias

JUSTIFICATION AND FAIRNESS: COMPAS

▸ ALVAREZ_MELIS_JAAKKOLA



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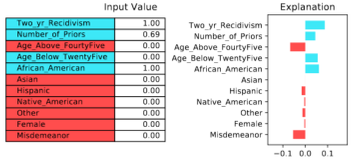

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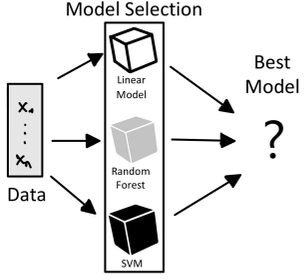
Interpretable Machine Learning

Dimensions of Interpretability



Learning goals

- Difference between intrinsic, model-specific, and model-agnostic interpretability
- Different types of explanations
- Local, global, and regional explanations
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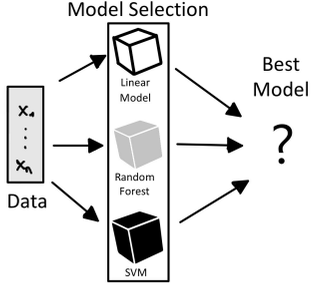

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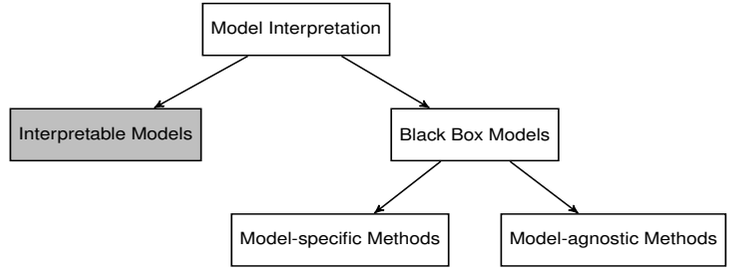
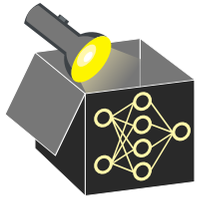
INTRINSIC, MODEL-SPECIFIC, MODEL-AGNOSTIC





Intrinsically Interpretable Models:

- Simple model structure (e.g., weighted sum or tree)
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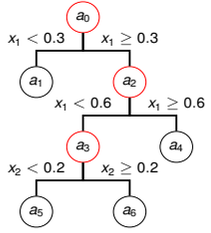

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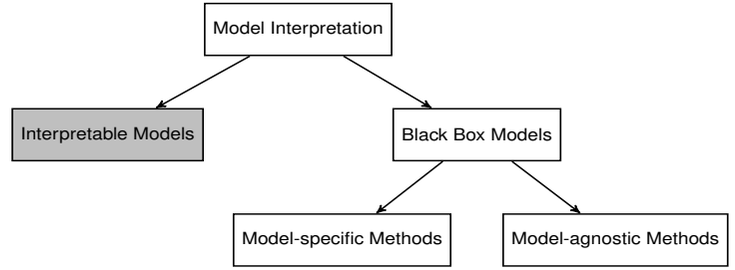

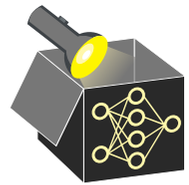

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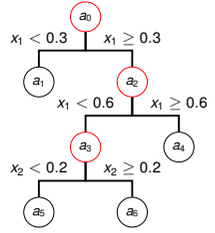

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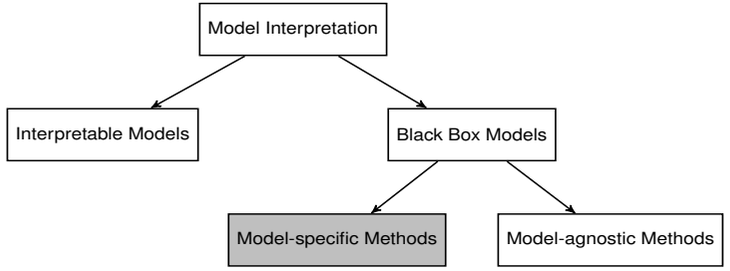
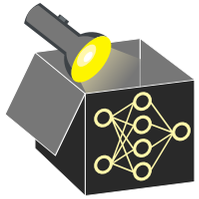
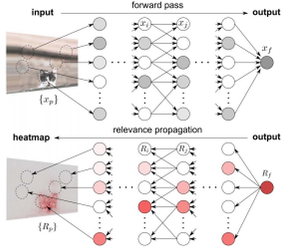




Model-specific Methods:

- Designed for specific model types (e.g., NNs)
- Examples: Gini importance of tree-based models, Layer-wise relevance propagation (LRP)
- Pro: Exploit model structure
- Con: Restricted to specific model class



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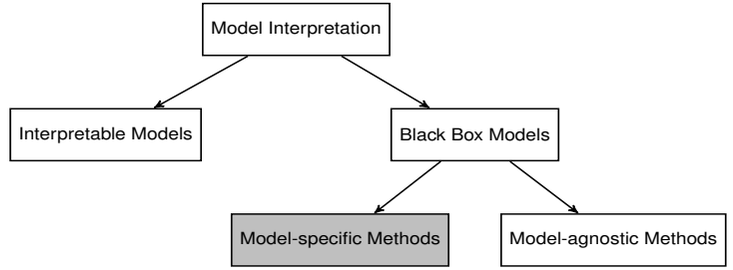
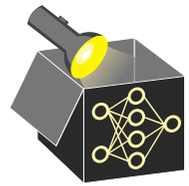
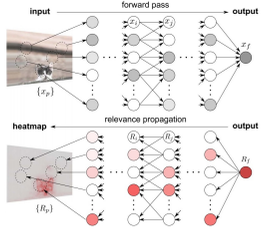




Model-specific Methods:

- Designed for specific model types (e.g., NNs)
- Examples:
  Gini importance of tree-based models,
  Layer-wise relevance propagation (LRP)
- Pro: Exploit model structure
- Con: Restricted to specific model class



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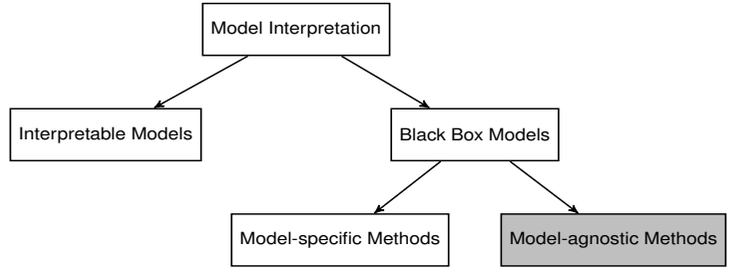

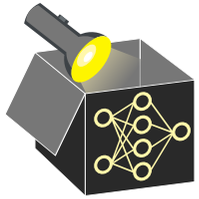

Model-agnostic Methods:

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  ⇝ Unknown which model is best / deployed
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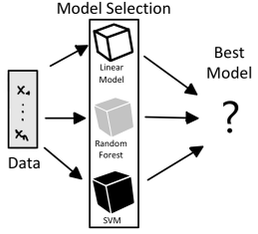

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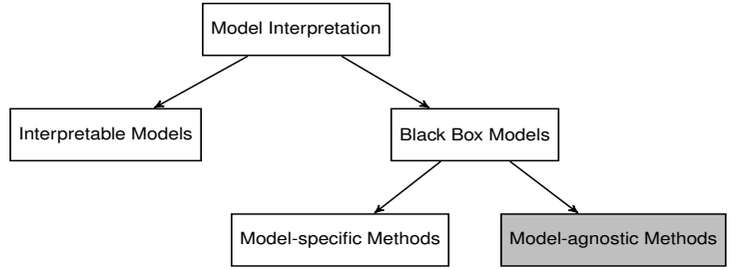

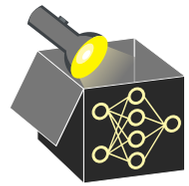

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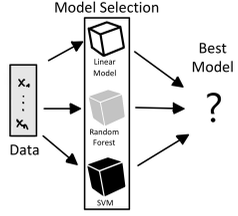
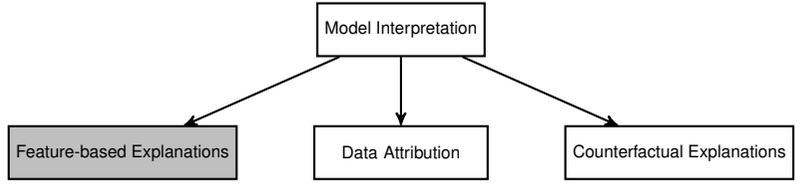
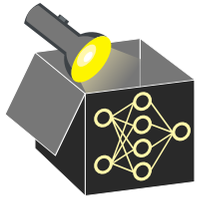


TYPES OF EXPLANATIONS





Feature-based Explanations:

- Analyze the role of individual features in model behavior.
- Types of feature-based explanations:
  - Feature Importance
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- Common principle: Vary or perturb feature values and observe changes in predictions, variance, or performance.

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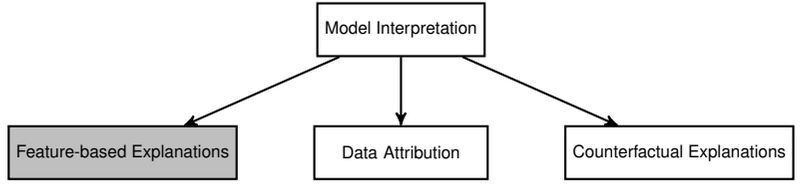

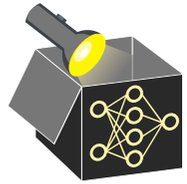

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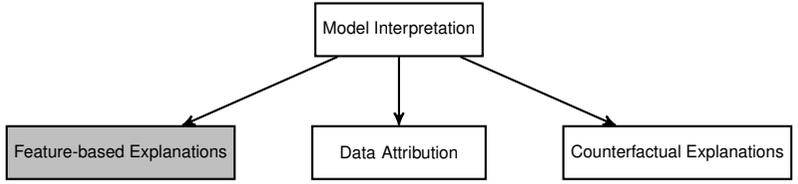

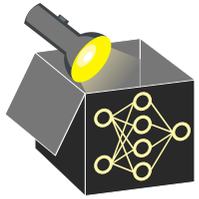

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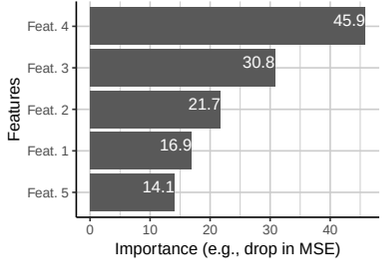

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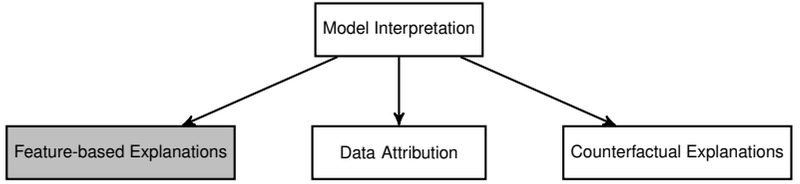

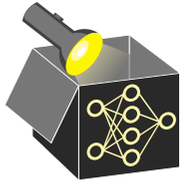

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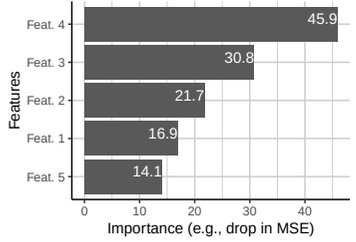

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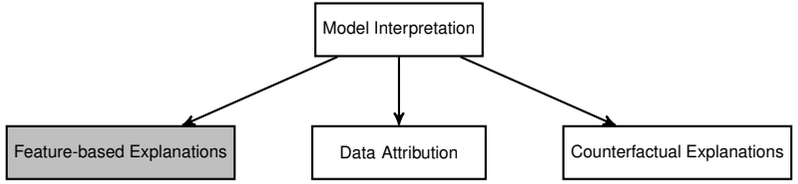

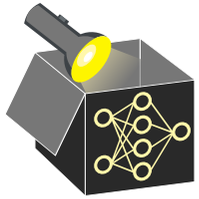

Feature Effects indicate changes (direction and magnitude) in model prediction due to changes in feature values.

- Model-agnostic methods: ICE curves, PD plots . . .
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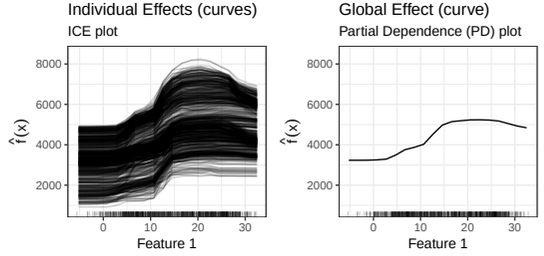
- Further examples: ALE, SHAP, and LIME



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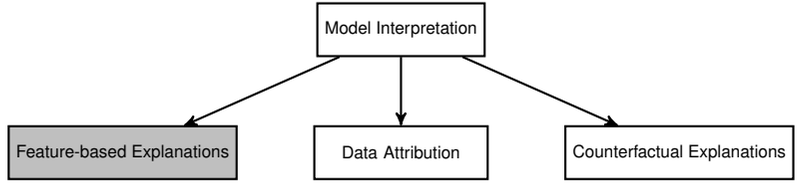

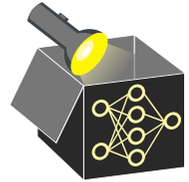

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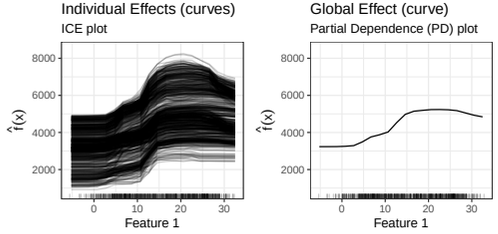

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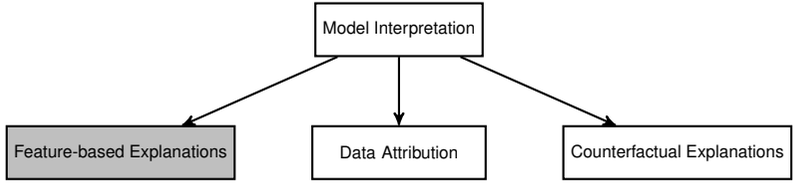
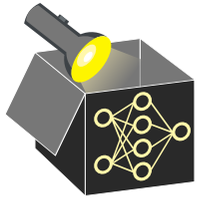
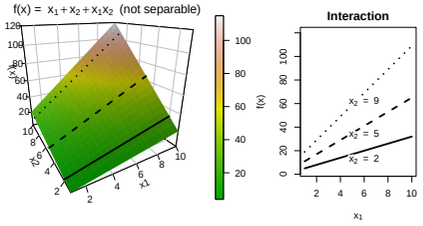




Feature Interaction: How combinations of features jointly affect predictions.

- Model-agnostic methods: Friedman's H-statistic
- Pendant in linear models: Coefficients of interaction terms θ_{jk}



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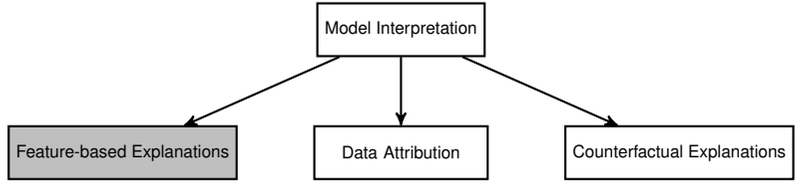

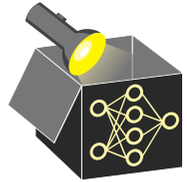

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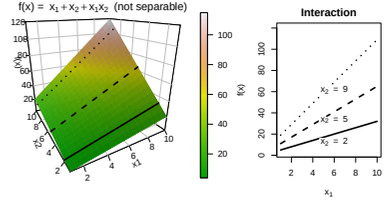

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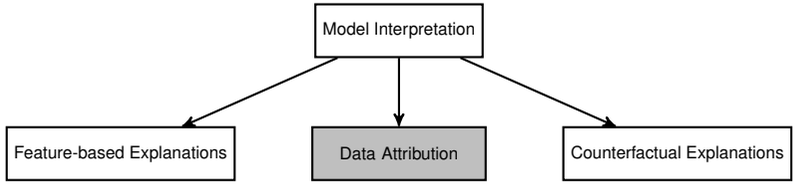

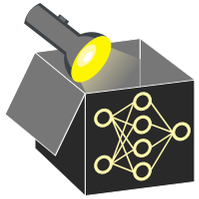

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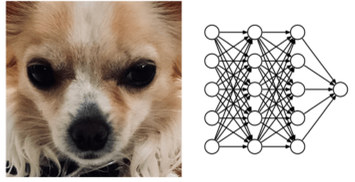
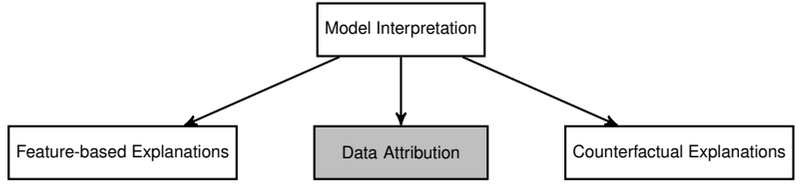
Example: A model should distinguish muffins and dogs.

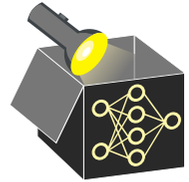
Question: Why does it misclassify this dog image (test point) as a muffin?



Muffin

TYPES OF EXPLANATIONS





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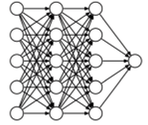

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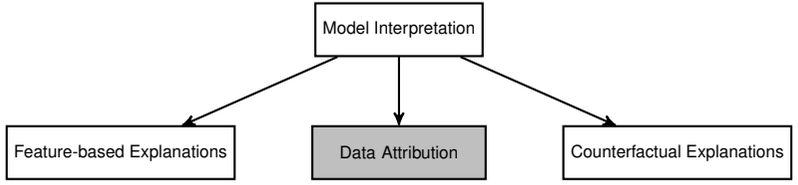

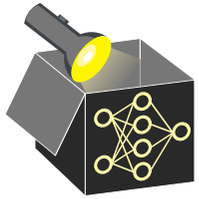

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Approach: Measure how perturbations to training instances affect prediction or loss.

- ⇝ Influential training instances drive prediction of test points.
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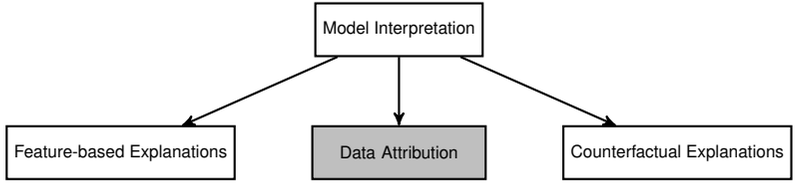

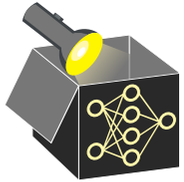

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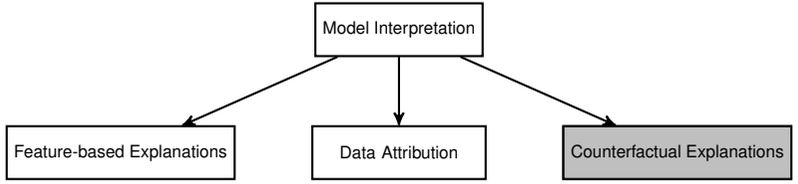
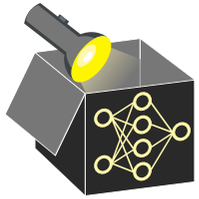
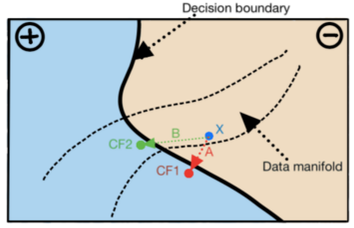




Counterfactual Explanations:

- Identify smallest necessary change in feature values so that a desired outcome is predicted
- Contrastive explanations
- Diverse counterfactuals
- Feasible & actionable explanations



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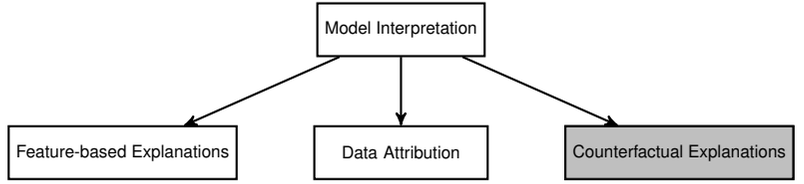

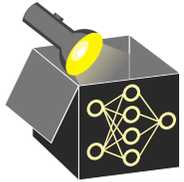

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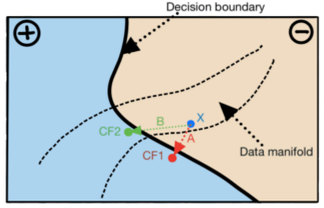
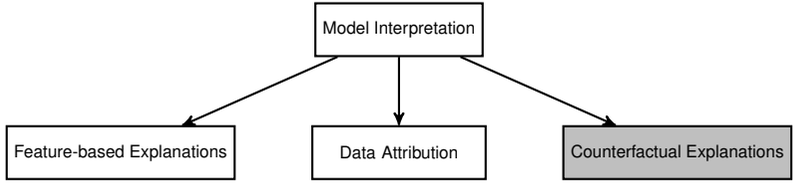
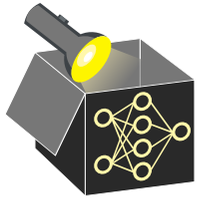
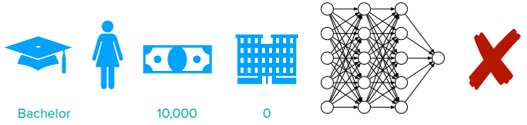


TYPES OF EXPLANATIONS

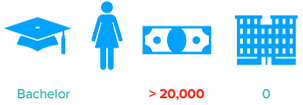




Example (loan application):



What can a person do to obtain a favorable prediction from a given model ?



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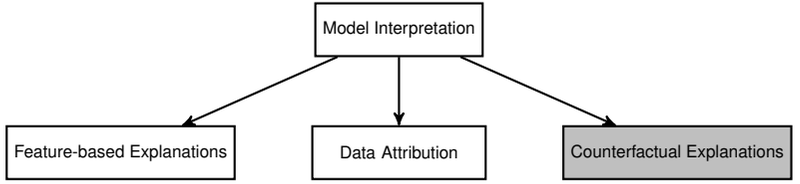

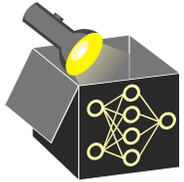

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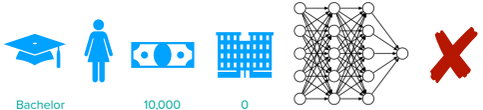

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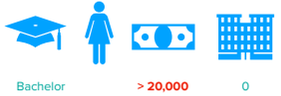

LOCAL, GLOBAL, AND REGIONAL EXPLANATIONS

Local – Explain model behavior for **single instances**:

- Provide nuanced instance-specific insights
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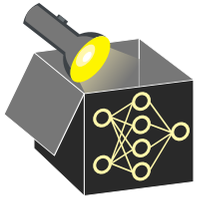
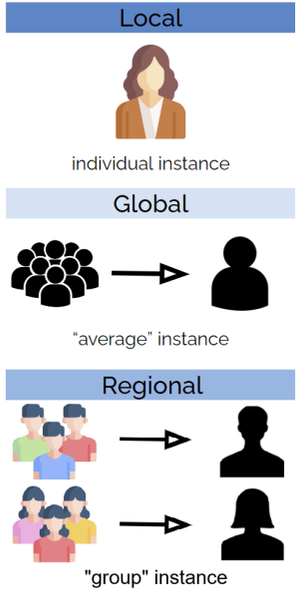
- Examples: Counterfactuals, LIME, SHAP, ICE





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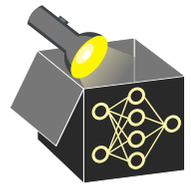

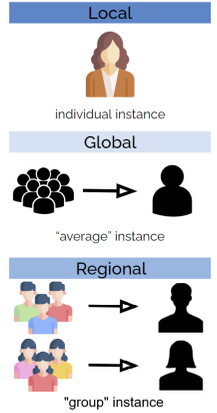

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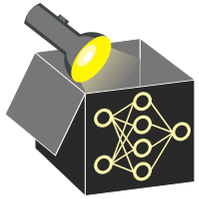

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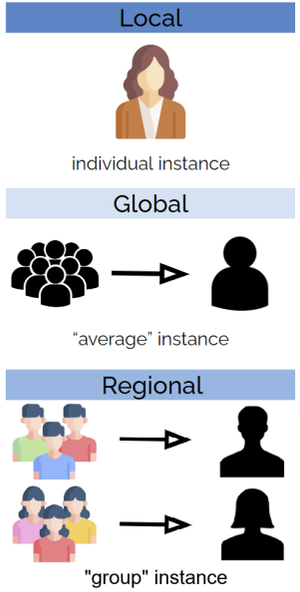

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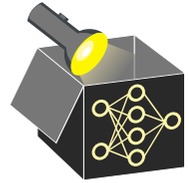

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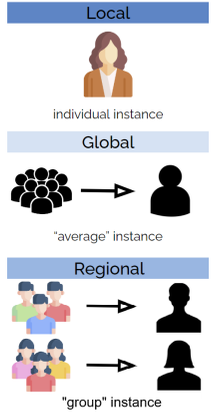

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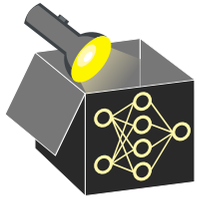

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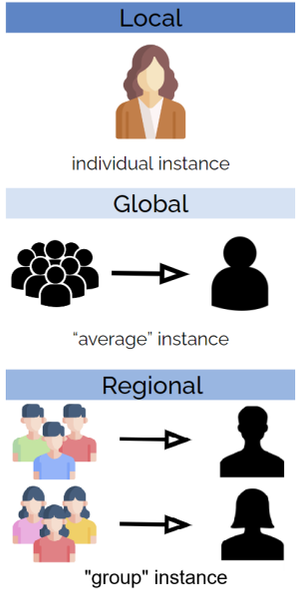

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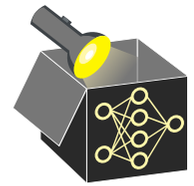

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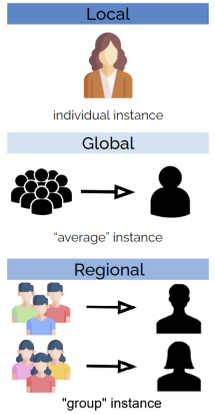

LOCAL, GLOBAL, AND REGIONAL EXPLANATIONS

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  ⇝ Detailed but cluttered/obscure pattern
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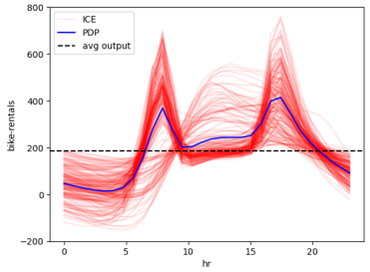

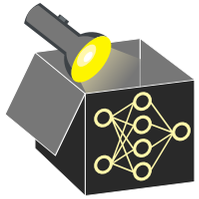

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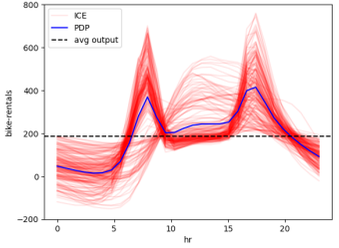

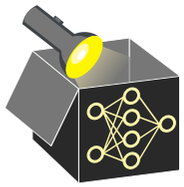

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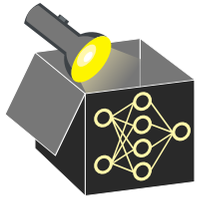

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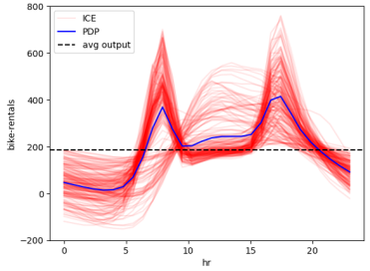

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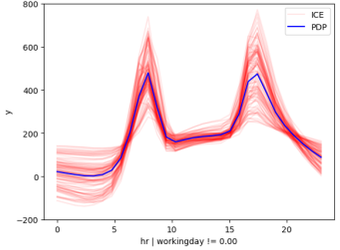
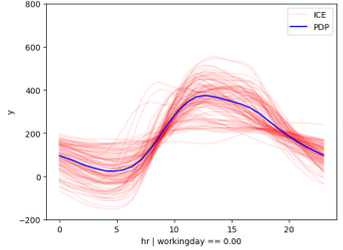
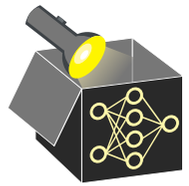




Region 2 (non-working day)

LOCAL, GLOBAL, REGIONAL EXPLANATIONS



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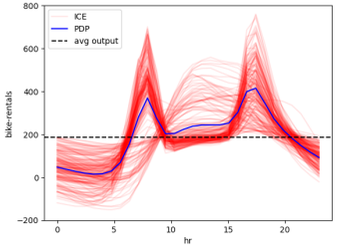

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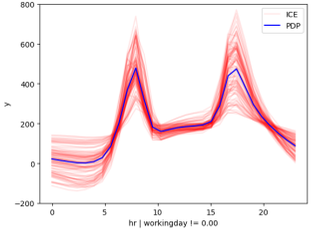

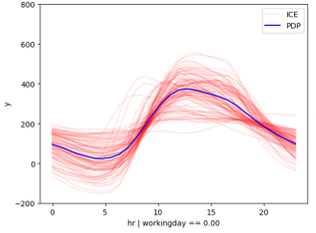

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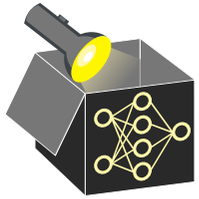
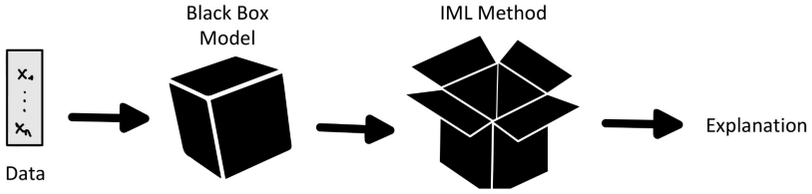
FIXED MODEL VS. REFITS



- Input of global interpretation methods: model + data, output: explanations
  ⇝ Explanations can be viewed as statistical estimators



- Situation in ML: Deployed model is trained on all available data
  ⇝ No unseen test data left to, e.g., reliably estimate performance
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  ⇝ But: Some IML methods rely on measuring loss requiring unseen test data
- Alternative: Explain the inducer that created the model (instead of a fixed model)
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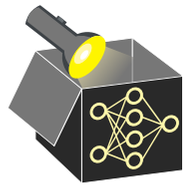

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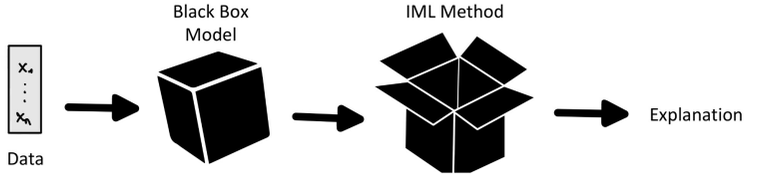

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	Research Question	Objects of analysis
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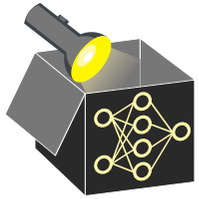

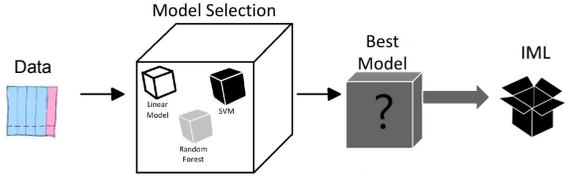

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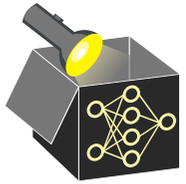

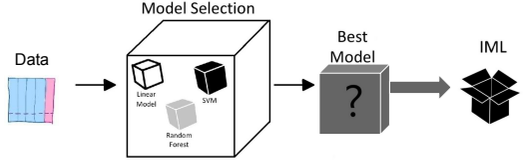

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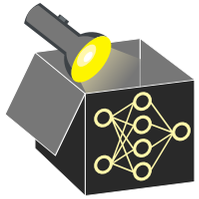

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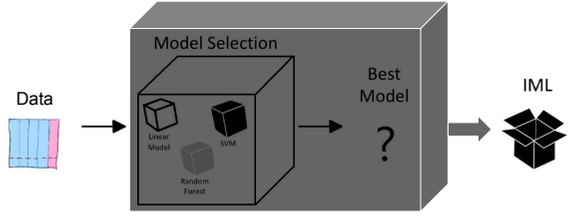
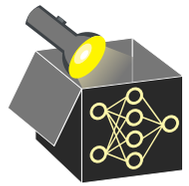


LEVELS OF INTERPRETABILITY



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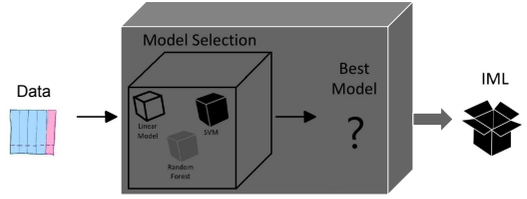

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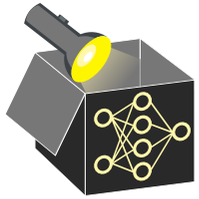

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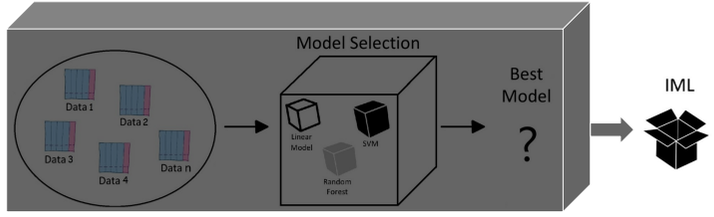

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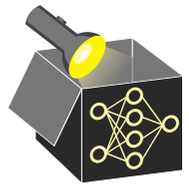

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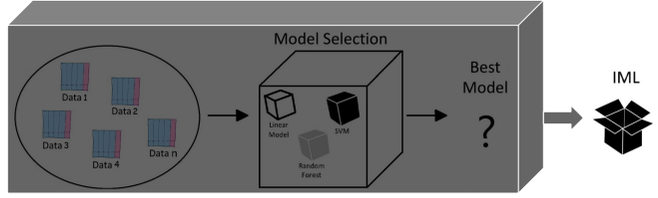
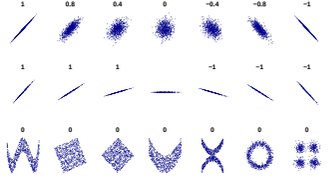


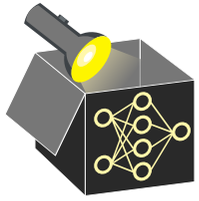
Interpretable Machine Learning

Correlation and Dependencies



Learning goals

- Pearson correlation
- Coefficient of determination R^2
- Mutual information
- Correlation vs. dependence



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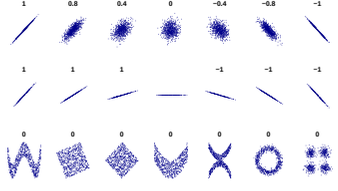

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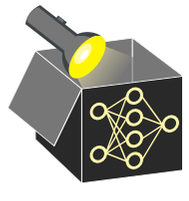
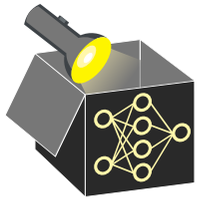
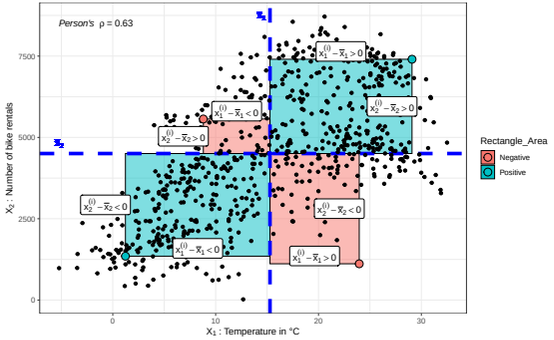


PEARSON'S CORRELATION COEFFICIENT ρ



Correlation often refers to Pearson's correlation (measures only **linear relationship**)

$$\rho(X_1, X_2) = \frac{\sum_{i=1}^{n} (x_1^{(i)} - \bar{x}_1) \cdot (x_2^{(i)} - \bar{x}_2)}{\sqrt{\sum_{i=1}^{n} (x_1^{(i)} - \bar{x}_1)^2 \sum_{i=1}^{n} (x_2^{(i)} - \bar{x}_2)^2}} \in [-1, 1]$$



Geometric interpretation of ρ:

- Numerator is sum of rectangle's area with width $x_1^{(i)} - \bar{x}_1$ and height $x_2^{(i)} - \bar{x}_2$
- Areas enter numerator with positive (**+**) or negative (**-**) sign, depending on position
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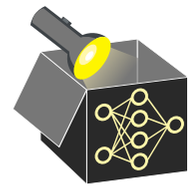

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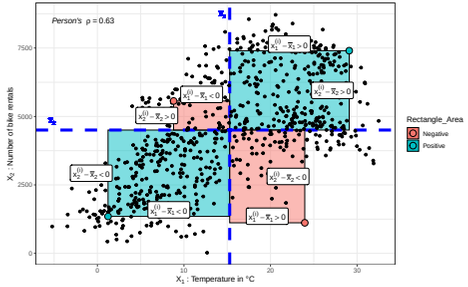

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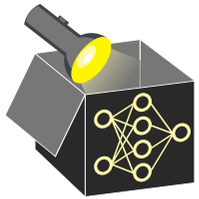

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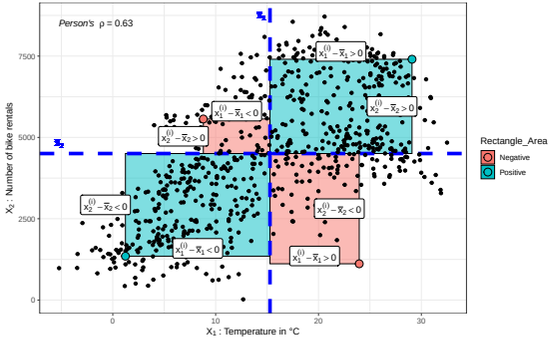

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- $\rho > 0$ if positive areas dominate negative areas $\rightsquigarrow X_1, X_2$ positive correlated
- $\rho < 0$ if negative areas dominate positive areas $\rightsquigarrow X_1, X_2$ negative correlated
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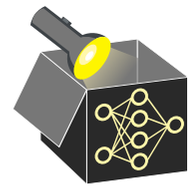

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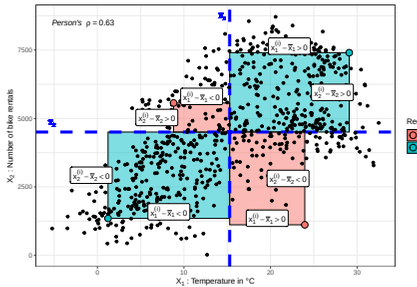

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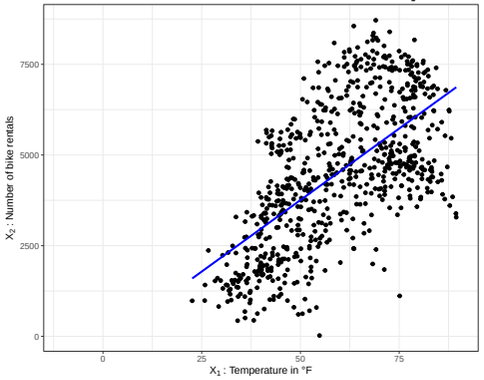
COEFFICIENT OF DETERMINATION R^2

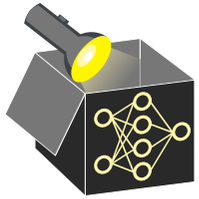
Another method to evaluate **linear dependency** between features is R^2



- Fit a linear model:
  $\hat{x}_2 = \hat{f}_{LM}(x_1) = \theta_0 + \theta_1 x_1$

$\rightsquigarrow$ Slope $\theta_1 = 0 \Rightarrow$ no dependence

$\rightsquigarrow$ Large slope $\Rightarrow$ strong dependence



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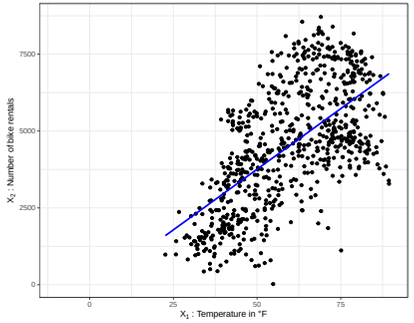

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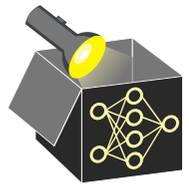

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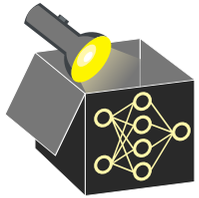

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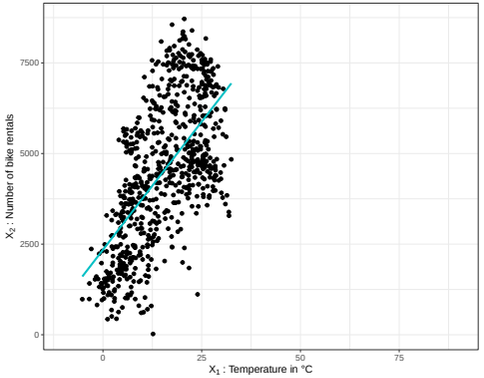

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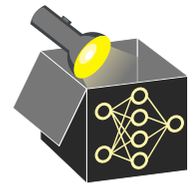

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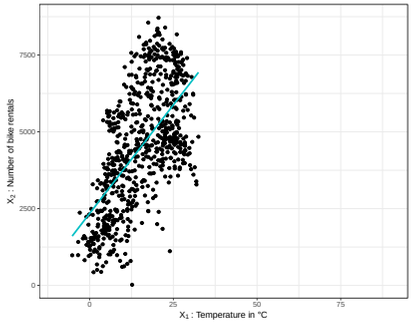

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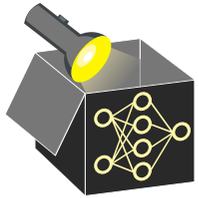

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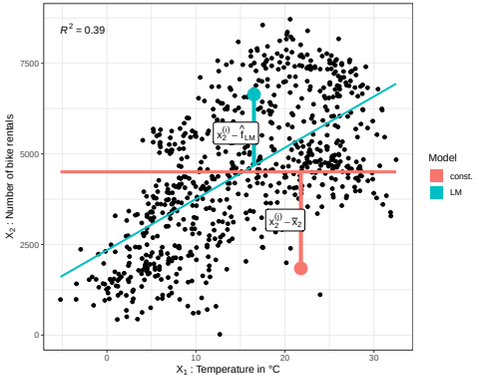

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$\Rightarrow$ Measure of fitting quality of LM: $R^2 = 1 - \frac{SSE_{LM}}{SSE_c} \in [0, 1]$
$\Rightarrow \rho(X_1, X_2) = R$

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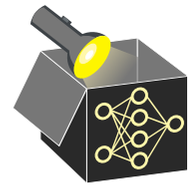

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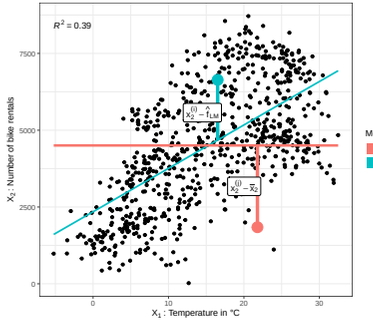

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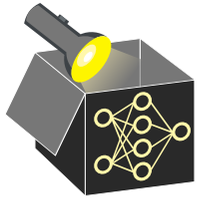
JOINT, MARGINAL AND CONDITIONAL DISTRIBUTION

For two discrete random variables X_1, X_2:

Joint distribution

$$p_{X_1,X_2}(x_1, x_2) = \mathbb{P}(X_1 = x_1, X_2 = x_2)$$

p_{X_1,X_2}	$\mathbb{P}(X_2 = 0)$	$\mathbb{P}(X_2 = 1)$	p_{X_1}
$\mathbb{P}(X_1 = 0)$	0.2	0.3	0.5
$\mathbb{P}(X_1 = 1)$	0.1	0.4	0.5
p_{X_2}	0.3	0.7	1



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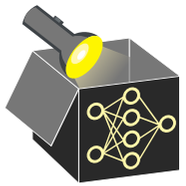
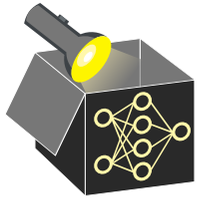
p_{X_2}	0.3	0.7	1



JOINT, MARGINAL AND CONDITIONAL DISTRIBUTION



For two discrete random variables X_1, X_2:

Joint distribution

$$p_{X_1,X_2}(x_1, x_2) = \mathbb{P}(X_1 = x_1, X_2 = x_2)$$

p_{X_1,X_2}	$\mathbb{P}(X_2 = 0)$	$\mathbb{P}(X_2 = 1)$	p_{X_1}
$\mathbb{P}(X_1 = 0)$	0.2	0.3	0.5
$\mathbb{P}(X_1 = 1)$	0.1	0.4	0.5
p_{X_2}	0.3	0.7	1

Marginal distribution

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$\rightsquigarrow$ In continuous case with integrals

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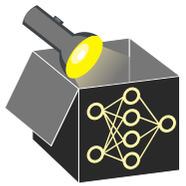

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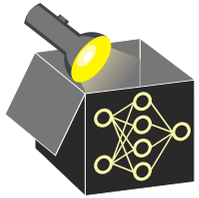

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$$p_{X_1|X_2}(x_1|x_2) = \mathbb{P}(X_1 = x_1|X_2 = x_2) = \frac{p_{X_1,X_2}(x_1, x_2)}{p_{X_2}(x_2)}$$

	$x_2 = 0$	$x_2 = 1$
$\mathbb{P}(X_1 = 0\|X_2 = x_2)$	0.67	0.43
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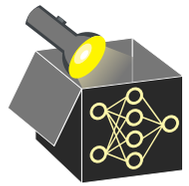
$\sum$	1	1

JOINT, MARGINAL AND CONDITIONAL DISTRIBUTION



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Dependence: Describes general dependence structure (e.g., non-lin. relationships)

- Definition: X_j, X_k independent $\Leftrightarrow$ joint distribution is product of marginals:

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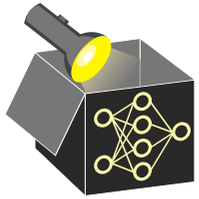

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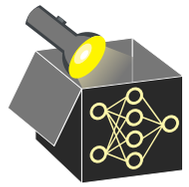

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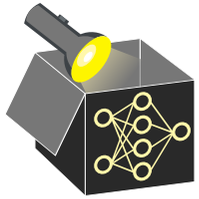

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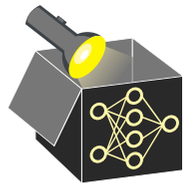

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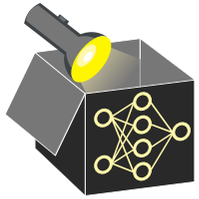

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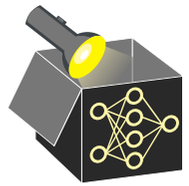

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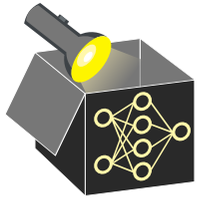

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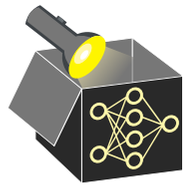

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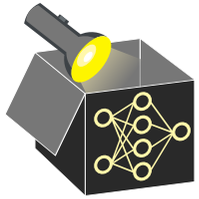

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$$MI(X_1, X_2) = \mathbb{E}_{p(x_1,x_2)}\left[log\left(\frac{p(x_1,x_2)}{p(x_1)p(x_2)}\right)\right]$$

- MI measures amount of "dependence" between features by looking how different the joint distribution is from pure independence $p(x_1, x_2) = p(x_1)p(x_2)$
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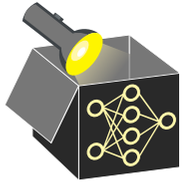
MUTUAL INFORMATION



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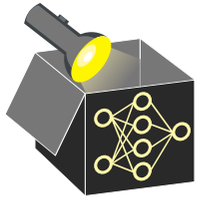
MUTUAL INFORMATION: EXAMPLE

For two discrete RV X_1 and Y:

$$MI(X_1; Y) = \mathbb{E}_{p(x_1,y)}\left[log\left(\frac{p(x_1,y)}{p(x_1)p(y)}\right)\right] = \sum_{x_1 \in \mathcal{X}_1}\sum_{y \in \mathcal{Y}} p(x_1,y) log\left(\frac{p(x_1,y)}{p(x_1)p(y)}\right)$$



$\mathbf{X}_1$	...	**Y**
yes	...	yes
yes	...	no
no	...	yes
no	...	no

	$\mathbb{P}(X_1 = \text{yes})$	$\mathbb{P}(X_1 = \text{no})$	p_Y
$\mathbb{P}(Y = \text{yes})$	0.25	0.25	0.5
$\mathbb{P}(Y = \text{no})$	0.25	0.25	0.5
p_{X_1}	0.5	0.5	1

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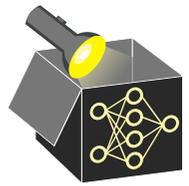

1	...	
yes	...	yes
yes	...	no
no	...	yes
no	...	no

	$\mathbb{P}(X_1 = \text{yes})$	$\mathbb{P}(X_1 = \text{no})$	p_Y
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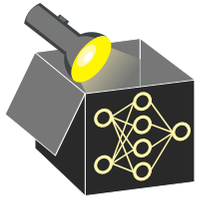

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X_1	...	Y
yes	...	yes
yes	...	no
no	...	yes
no	...	no

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p_{X_1}	0.5	0.5	1

$$\begin{aligned} MI(X_1; Y) &= 0.25\, log\left(\frac{0.25}{0.5 \cdot 0.5}\right) + 0.25\, log\left(\frac{0.25}{0.5 \cdot 0.5}\right) \\ &\quad + 0.25\, log\left(\frac{0.25}{0.5 \cdot 0.5}\right) + 0.25\, log\left(\frac{0.25}{0.5 \cdot 0.5}\right) \\ &= 0.25\, log\left(\frac{0.25}{0.25}\right) \cdot 4 \\ &= 0.25\, log\,(1) \cdot 4 = 0 \end{aligned}$$

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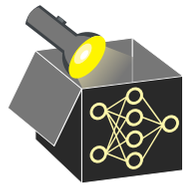

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1	...	
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no	...	yes
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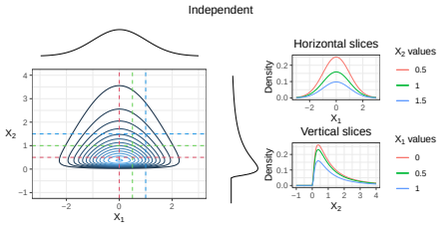
p_{X_1}	0.5	0.5	1

$$\begin{aligned} MI(X_1; Y) &= 0.25\, log\left(\frac{0.25}{0.5 \cdot 0.5}\right) + 0.25\, log\left(\frac{0.25}{0.5 \cdot 0.5}\right) \\ &\quad + 0.25\, log\left(\frac{0.25}{0.5 \cdot 0.5}\right) + 0.25\, log\left(\frac{0.25}{0.5 \cdot 0.5}\right) \\ &= 0.25\, log\left(\frac{0.25}{0.25}\right) \cdot 4 \\ &= 0.25\, log\,(1) \cdot 4 = 0 \end{aligned}$$

DEPENDENCE AND INDEPENDENCE

Example:



Conditional distributions at different vertical and horizontal slices (after normalizing area to 1) match their marginal distributions

$$\Rightarrow \mathbb{P}(X_1|X_2) = \mathbb{P}(X_1)$$
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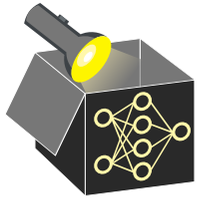

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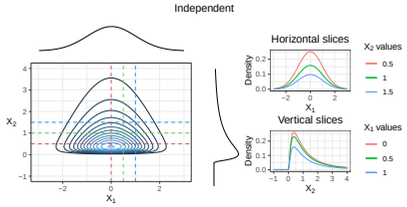

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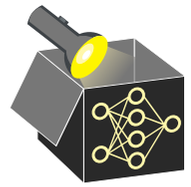

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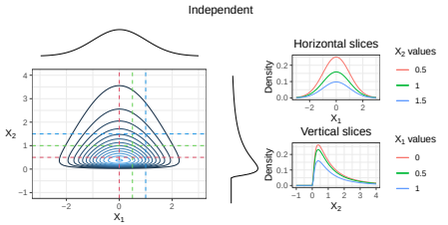

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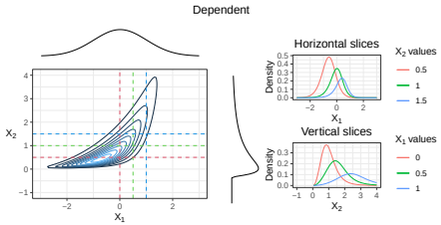

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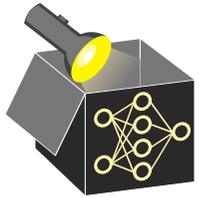

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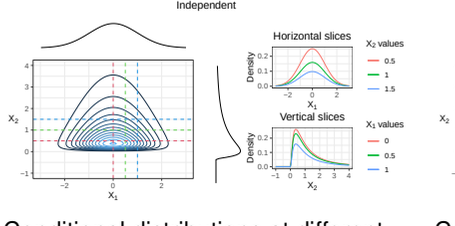

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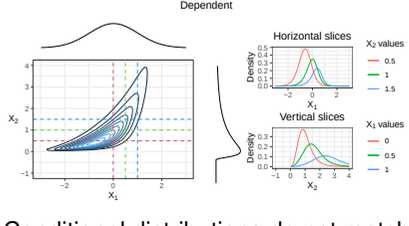

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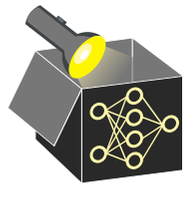
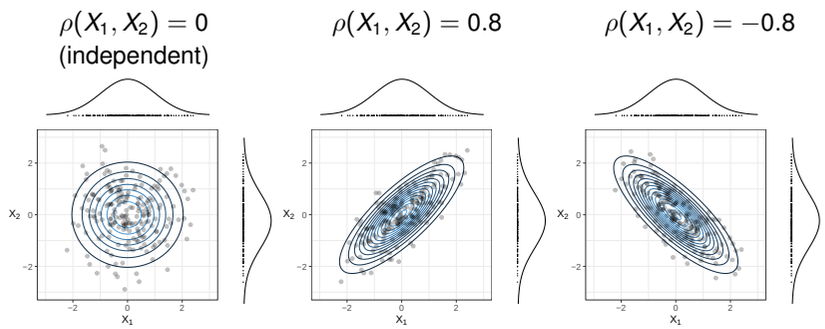
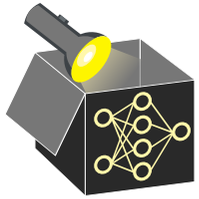


CORRELATION VS. DEPENDENCE

Illustration of bivariate normal distribution with different correlations $X_1, X_2 \sim N(0, 1)$





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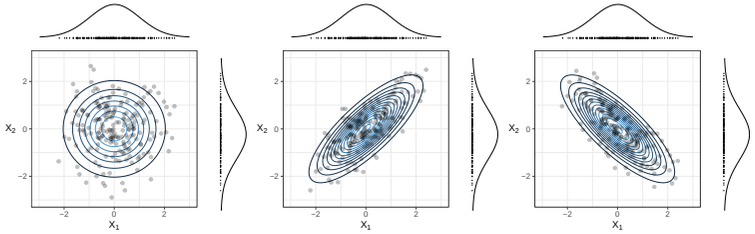

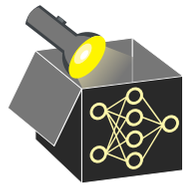

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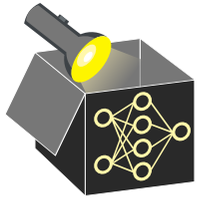

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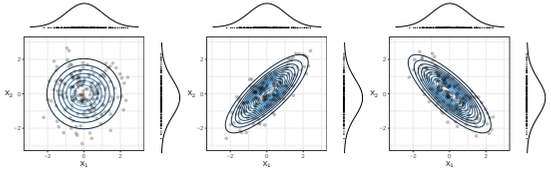
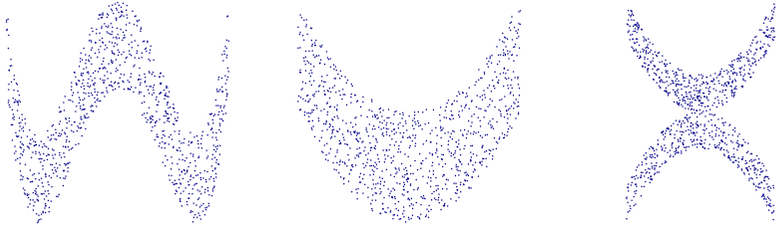


Examples with Pearson's correlation $\rho \approx 0$ but non-linear dependencies (MI $\neq 0$):

$\rho(X_1, X_2) = 0$, $MI(X_1, X_2) = 0.52$ $\quad \rho(X_1, X_2) = 0.01$, $MI(X_1, X_2) = 0.37$ $\quad \rho(X_1, X_2) = -0.06$, $MI(X_1, X_2) = 0.61$



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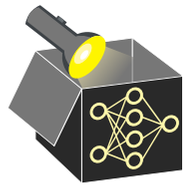

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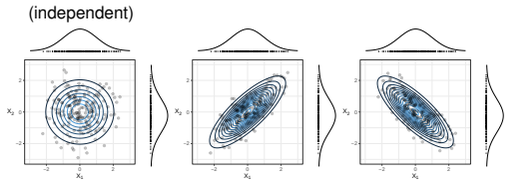

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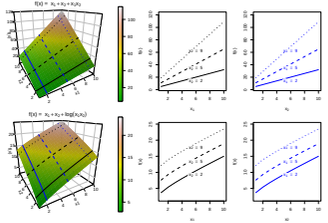
Interpretable Machine Learning

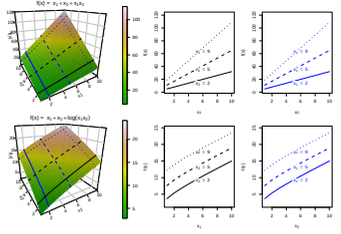
Feature Interactions



Learning goals

- Feature interactions
- Difference to feature dependencies

Interpretable Machine Learning
Feature Interactions



Learning goals

- Feature interactions
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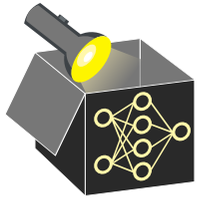
FEATURE INTERACTIONS

- Feature dependencies concern data distribution
- Feature interactions may occur in structure of **both** model or DGP (e.g., functional relationship between X and $\hat{f}(X)$ or X and $Y = f(X)$)
  $\rightsquigarrow$ Feature dependencies may lead to feature interactions in a model



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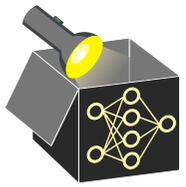
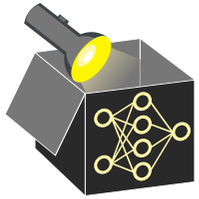


FEATURE INTERACTIONS



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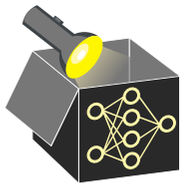

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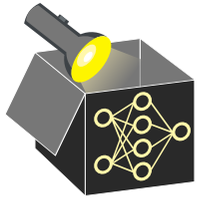

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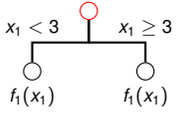
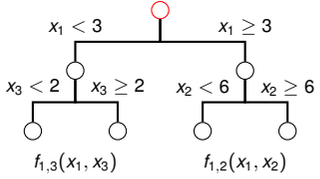
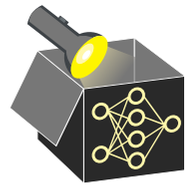


No interaction



Interactions: x_1 and x_3,
x_1 and x_2
No interactions: x_2 and x_3

FEATURE INTERACTIONS



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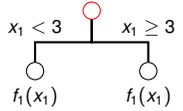
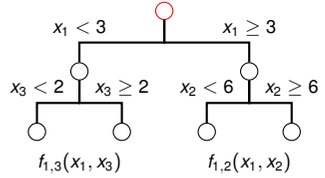


No interaction



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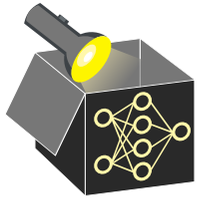
FEATURE INTERACTIONS

Friedman and Popescu (2008)



Definition: A function $f(\mathbf{x})$ contains an interaction between x_j and x_k if a difference in $f(\mathbf{x})$-values due to changes in x_j will also depend on x_k, i.e.:

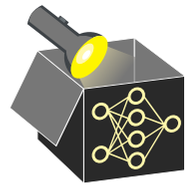
$$\mathbb{E}\left[\frac{\partial^2 f(\mathbf{x})}{\partial x_j \partial x_k}\right]^2 > 0$$

$\Rightarrow$ If x_j and x_k do not interact, $f(\mathbf{x})$ is sum of 2 functions, each independent of x_j, x_k:

$$f(\mathbf{x}) = f_{-j}(x_1, \ldots, x_{j-1}, x_{j+1}, \ldots, x_p) + f_{-k}(x_1, \ldots, x_{k-1}, x_{k+1}, \ldots, x_p)$$

FEATURE INTERACTIONS

FRIEDMAN_POPESCU



Definition: A function $f()$ contains an interaction between x_j and x_k if a difference in $f()$-values due to changes in x_j will also depend on x_k, i.e.:

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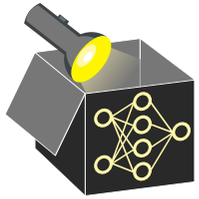
FEATURE INTERACTIONS

Example: $f(\mathbf{x}) = x_1 + x_2 + x_1 \cdot x_2$ (not separable)

$$\mathbb{E}\left[\frac{\partial^2(x_1+x_2+x_1 \cdot x_2)}{\partial x_1 \partial x_2}\right]^2 = \mathbb{E}\left[\frac{\partial(1+x_2)}{\partial x_2}\right]^2 = 1 > 0$$

$\Rightarrow$ interaction between x_1 and x_2



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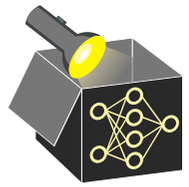
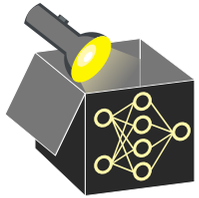


FEATURE INTERACTIONS



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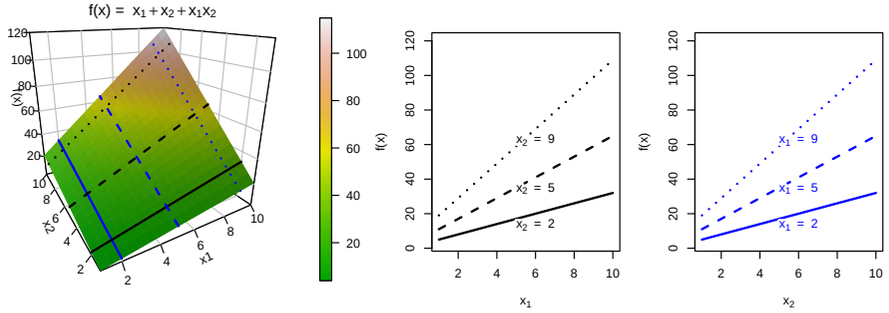
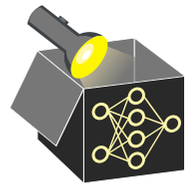


- Effect of x_1 on $f(\mathbf{x})$ varies with x_2 (and vice versa)

$\Rightarrow$ Different slopes

FEATURE INTERACTIONS



Example: $f() = x_1 + x_2 + x_1 \cdot x_2$ (not separable)

$$\mathbb{E}\left[\frac{\partial^2(x_1+x_2+x_1 \cdot x_2)}{\partial x_1 \partial x_2}\right]^2 = \mathbb{E}\left[\frac{\partial(1+x_2)}{\partial x_2}\right]^2 = 1 > 0$$

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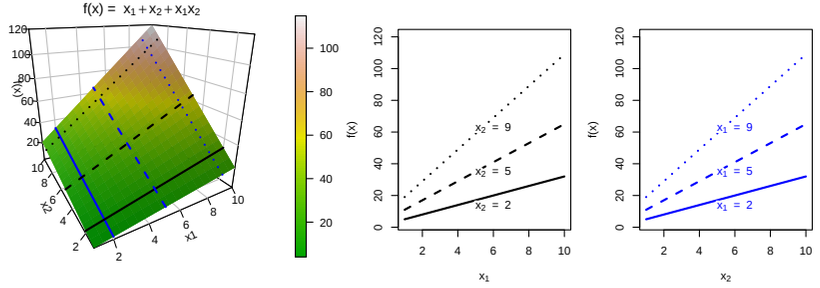


- Effect of x_1 on $f()$ varies with x_2 (and vice versa)

$\Rightarrow$ Different slopes

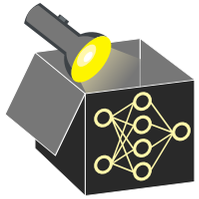
FEATURE INTERACTIONS

Example of separable function:

$$f(\mathbf{x}) = x_1 + x_2 + \log(x_1 \cdot x_2) = x_1 + x_2 + \log(x_1) + \log(x_2)$$

$\Rightarrow f(\mathbf{x}) = f_1(x_1) + f_2(x_2)$ with $f_1(x_1) = x_1 + \log(x_1)$ and $f_2(x_2) = x_2 + \log(x_2)$

$\Rightarrow$ no interactions due to separability, also $\mathbb{E}\left[\frac{\partial^2 f(\mathbf{x})}{\partial x_1 \partial x_2}\right]^2 = 0$



FEATURE INTERACTIONS

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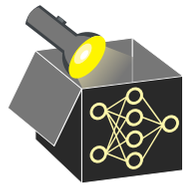
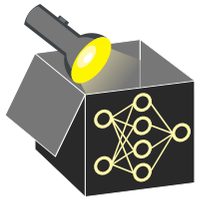


FEATURE INTERACTIONS



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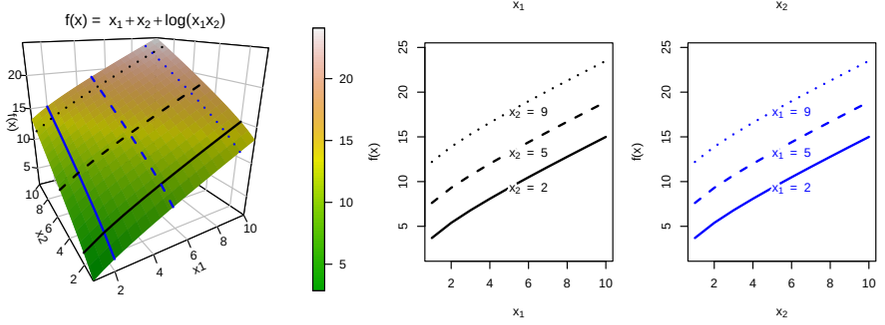
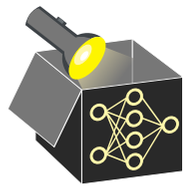


- Effect of x_1 on $f(\mathbf{x})$ stays the same for different x_2 values (and vice versa)

$\Rightarrow$ Parallel lines at different horizontal (blue) or vertical (black) slices

FEATURE INTERACTIONS



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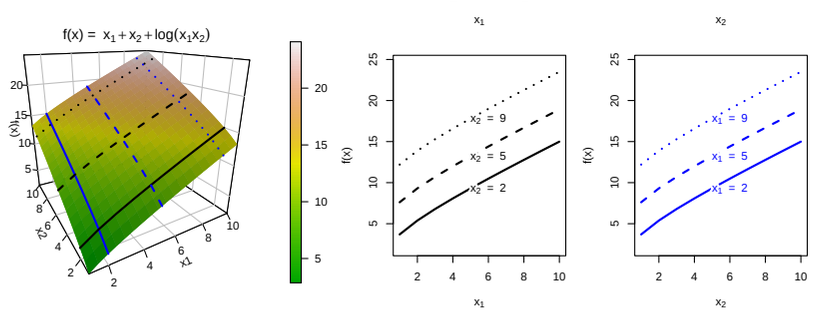

- Effect of x_1 on $f()$ stays the same for different x_2 values (and vice versa)

$\Rightarrow$ Parallel lines at different horizontal (blue) or vertical (black) slices