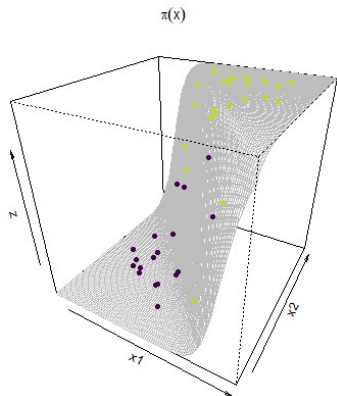
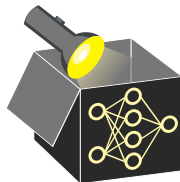


Interpretable Machine Learning

Generalized Linear Models (GLMs)



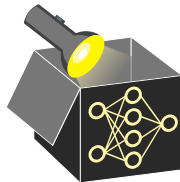
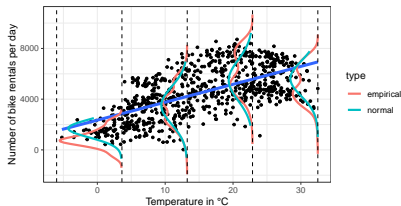
Learning goals

- Definition of GLMs
- Logistic regression as example
- Interpretation in logistic regression

Problem: Target variable given feat not always normally distributed

~> LM not suitable

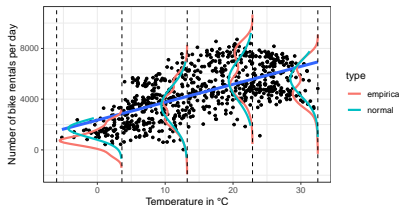
- Target is binary (e.g., disease classif.)
~> Bernoulli / Binomial distribution
- Target is count variable
(e.g., number of sold products)
~> Poisson distribution
- Time until an event occurs
(e.g., time until death)
~> Gamma distribution



Problem: Target variable given feat not always normally distributed

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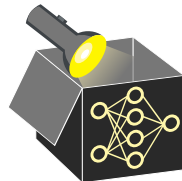
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Solution: GLMs - extend LMs by allowing other distrib.-s from exp. family

$$g(\mathbb{E}(y | \mathbf{x})) = \mathbf{x}^\top \boldsymbol{\theta} \Leftrightarrow \mathbb{E}(y | \mathbf{x}) = g^{-1}(\mathbf{x}^\top \boldsymbol{\theta})$$

- Link function g links linear predictor $\mathbf{x}^\top \boldsymbol{\theta}$ to expectation of distrib. of $y | \mathbf{x}$
↪ LM is special case: Gaussian distrib. for $y | \mathbf{x}$ with g as identity func.
- Link function g and distribution need to be specified
- High-order and interaction effects can be manually added as in LMs
- Note: Interpretation of weights depend on link function and distribution



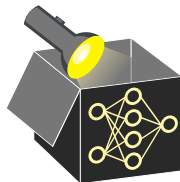
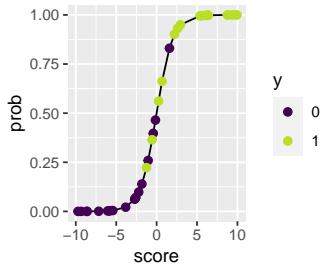
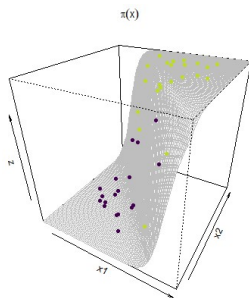
GLM - LOGISTIC REGRESSION

- Logistic regression $\hat{=}$ GLM with Bernoulli distribution and logit link function:

$$g(x) = \log\left(\frac{x}{1-x}\right) \Rightarrow g^{-1}(x) = \frac{1}{1 + \exp(-x)}$$

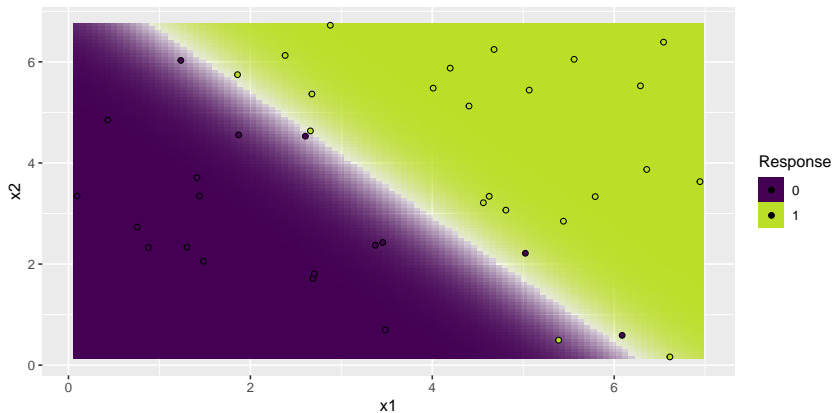
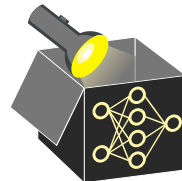
- Models probabilities for binary classification by

$$\pi(\mathbf{x}) = \mathbb{E}(y \mid \mathbf{x}) = P(y = 1) = g^{-1}(\mathbf{x}^\top \boldsymbol{\theta}) = \frac{1}{1 + \exp(-\mathbf{x}^\top \boldsymbol{\theta})}$$



GLM - LOGISTIC REGRESSION

- Typically, we set the threshold to 0.5 to predict classes, e.g.,
 - Class 1 if $\pi(\mathbf{x}) > 0.5$
 - Class 0 if $\pi(\mathbf{x}) \leq 0.5$

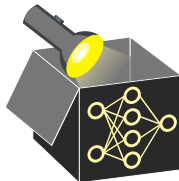


GLM - LOG. REGRESSION - INTERPRETATION

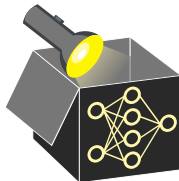
- **Recall:** Odds is ratio of two probabilities, odds ratio is ratio of two odds
- Weights θ_j are interpreted linear as in LM (but w.r.t. log-odds)
 \rightsquigarrow difficult to comprehend

$$\text{log-odds} = \log \left(\frac{\pi(\mathbf{x})}{1 - \pi(\mathbf{x})} \right) = \log \left(\frac{P(y = 1)}{P(y = 0)} \right) = \theta_0 + \theta_1 x_1 + \dots + \theta_p x_p$$

Interpretation: Changing x_j by one unit, changes log-odds of class 1 compared to class 0 by θ_j



GLM - LOG. REGRESSION - INTERPRETATION



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- Odds for cls 1 vs. cls 0: $\text{odds} = \frac{\pi(\mathbf{x})}{1 - \pi(\mathbf{x})} = \exp(\theta_0 + \theta_1 x_1 + \dots + \theta_p x_p)$
- Instead of interpreting changes w.r.t. log-odds, it is more common to use *odds ratio*

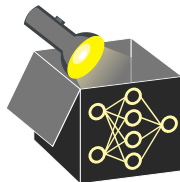
$$= \frac{\text{odds}_{x_j+1}}{\text{odds}} = \frac{\exp(\theta_0 + \theta_1 x_1 + \dots + \theta_j(x_j + 1) + \dots + \theta_p x_p)}{\exp(\theta_0 + \theta_1 x_1 + \dots + \theta_j x_j + \dots + \theta_p x_p)} = \exp(\theta_j)$$

Interpretation: Changing x_j by one unit, changes the **odds ratio** for class 1 (compared to class 0) by the **factor** $\exp(\theta_j)$

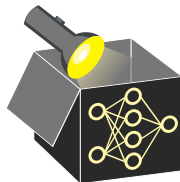
GLM - LOGISTIC REGRESSION - EXAMPLE

- Create a binary target variable for bike rental data:
 - Class 1: “high number of rentals” $> 70\%$ quantile (i.e., $\text{cnt} > 5531$)
 - Class 0: “low to medium number of rentals” (i.e., $\text{cnt} \leq 5531$)
- Fit a logistic regression model (GLM with Bernoulli distri. and logit link)

	Weights	SE	p-value
(Intercept)	-8.52	1.21	0.00
seasonSPRING	1.74	0.60	0.00
seasonSUMMER	-0.86	0.77	0.26
seasonFALL	-0.64	0.55	0.25
temp	0.29	0.04	0.00
hum	-0.06	0.01	0.00
windspeed	-0.09	0.03	0.00
days_since_2011	0.02	0.00	0.00

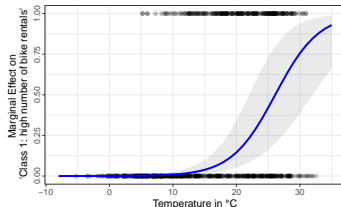


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Interpretation

- If temp increases by 1°C , odds ratio for class 1 increases by factor $\exp(0.29) = 1.34$ compared to class 0, c.p. ($\hat{=}$ “high number of bike rentals” now 1.34 times more likely)