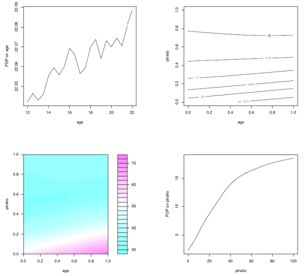
Interpretable Machine Learning

Functional Decompositions: Further Methods



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- Alternatives: Generalized fANOVA and ALE
- Advantages and relevance of functional decompositions

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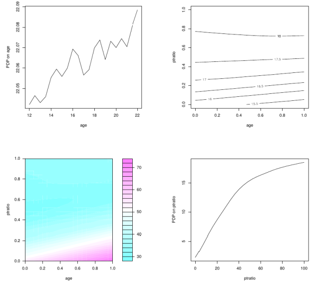

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$$\hat{f}(x_1, x_2, x_3) = -2x_1 - 2\sin(x_3) + |x_1|x_2 + 0.5x_2x_3 + 1.$$

$\rightsquigarrow$ Following two decompositions would both “make sense”:

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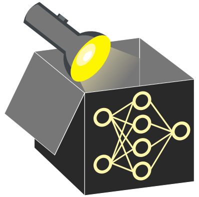

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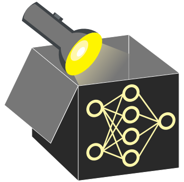

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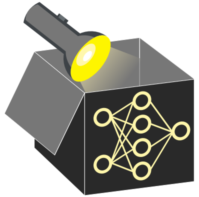

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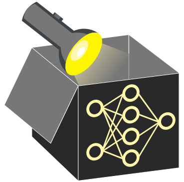

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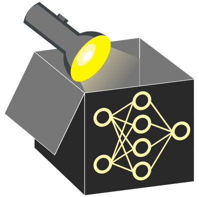

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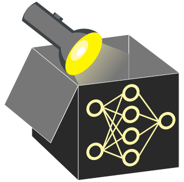

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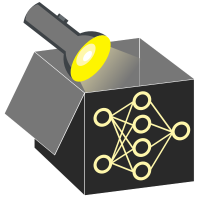

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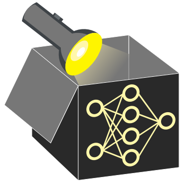

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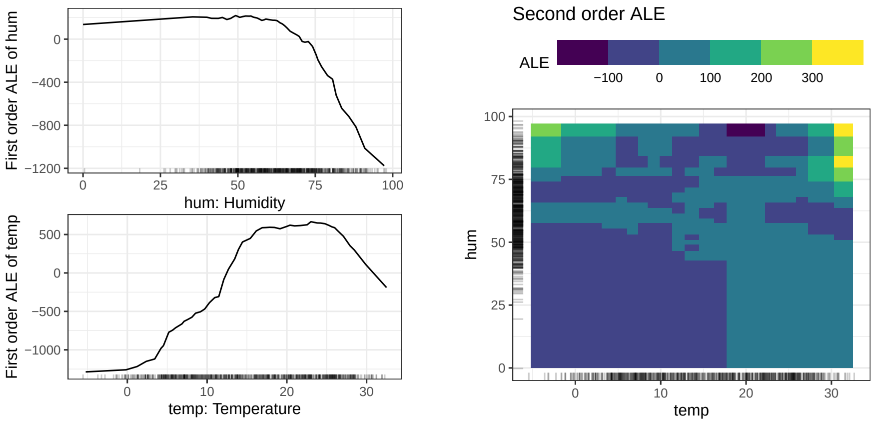

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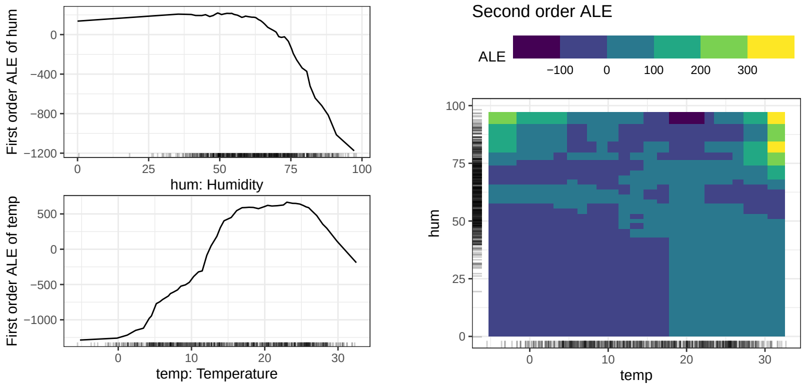

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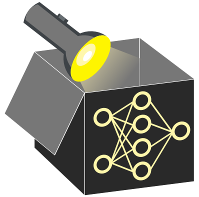

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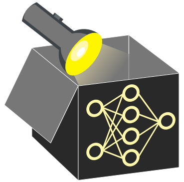

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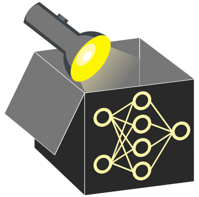

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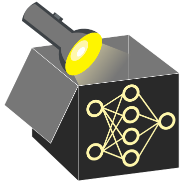

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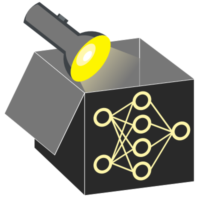

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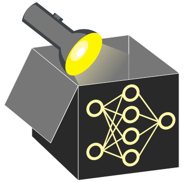

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