

Interpretable Machine Learning

Feature Importances 1

Leave One Covariate Out (LOCO)

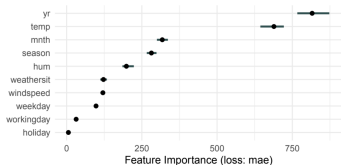
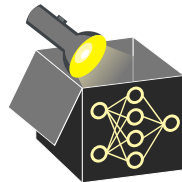


Figure: Bike Sharing Dataset

Learning goals

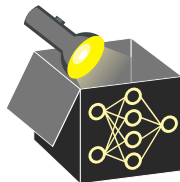
- Definition of LOCO
- Interpretation of LOCO

LOCO

► LEI_2018

► TIBSHIRANI_2018

LOCO idea: Remove the feature from data, refit model on reduced data, and measure the loss in performance compared to model fitted on complete data.

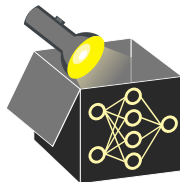


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- 1 Learn model on $\mathcal{D}_{\text{train}, -j}$ where feature x_j was removed, i.e.

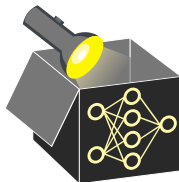
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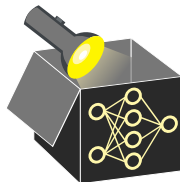
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$$\hat{f}_{-j} = \mathcal{I}(\mathcal{D}_{\text{train}, -j})$$
- 2 Compute the difference in local L_1 loss for each element in $\mathcal{D}_{\text{test}}$, i.e.
$$\Delta_j^{(i)} = \left| y^{(i)} - \hat{f}_{-j}(x_{-j}^{(i)}) \right| - \left| y^{(i)} - \hat{f}(x^{(i)}) \right| \text{ with } i \in \mathcal{D}_{\text{test}}$$



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- 3 Compute importance score by $\text{LOCO}_j = \text{med}(\Delta_j)$



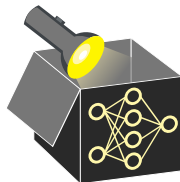
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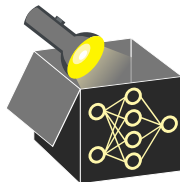
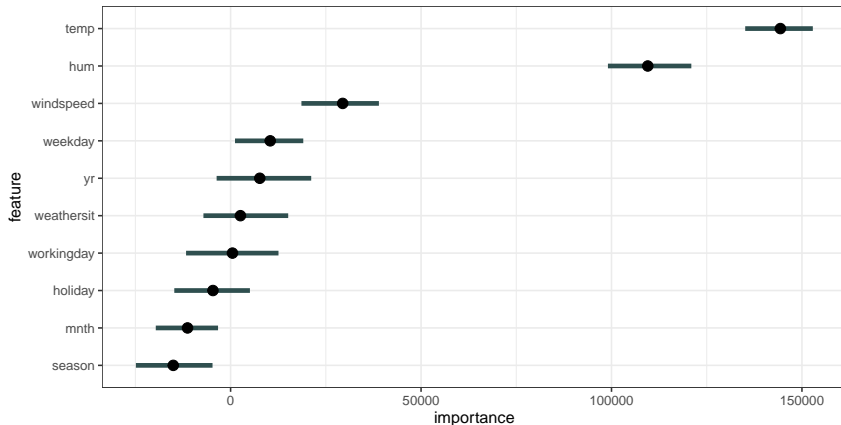
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The method can be generalized to other loss functions and aggregations. If we use mean instead of median we can rewrite LOCO as

$$\text{LOCO}_j = \mathcal{R}_{\text{emp}}(\hat{f}_{-j}) - \mathcal{R}_{\text{emp}}(\hat{f}).$$



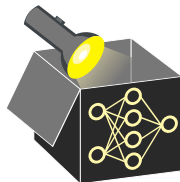
BIKE SHARING EXAMPLE



- Trained random forest (default hyperparams) on 70% of bike sharing data
- Performance measure: mean squared error (MSE)
- Computed LOCO on test set for all features, measuring increase in MSE
- temp was most important: removal increased MSE by approx. 140.000

INTERPRETATION OF LOCO

Interpretation: LOCO estimates the generalization error of the learner on a reduced dataset \mathcal{D}_{-j} .



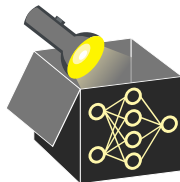
Can we get insight into whether the ...

- ❶ feature x_j is causal for the prediction \hat{y} ?
 - In general, no, also because we refit the model (counterexample on the next slide)
- ❷ feature x_j contains prediction-relevant information?
 - In general, no (counterexample on the next slide)
- ❸ model requires access to x_j to achieve its prediction performance?
 - Approximately, it provides insight into whether the *learner* requires access to x_j

INTERPRETATION OF LOCO

Example: Sample 1000 observations with

- $x_1, x_3 \sim N(0, 5)$, $x_2 = x_1 + \epsilon_2$ with $\epsilon_2 \sim N(0, 0.1)$
- $y = x_2 + x_3 + \epsilon$ with $\epsilon \sim N(0, 2)$
- Trained LM: $\hat{f}(x) = -0.02 - 1.02x_1 + 2.05x_2 + 0.98x_3$



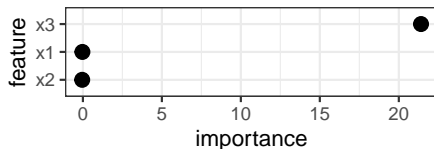
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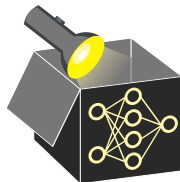
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Correlation matrix



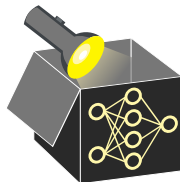
LOCO importance from LM trained on 70% of data, evaluated on remaining 30%



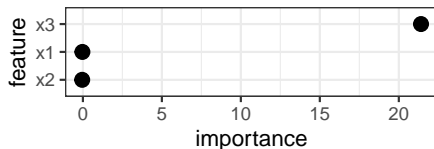
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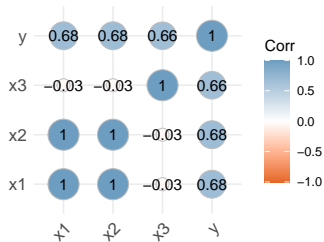
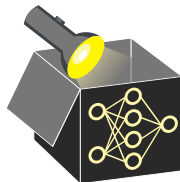
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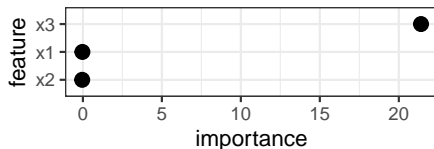
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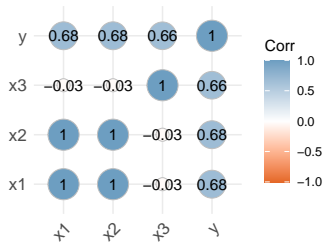
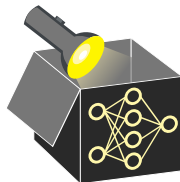
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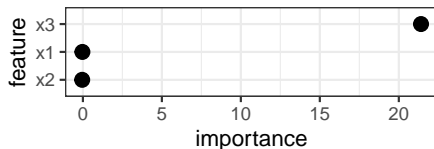
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- ⇒ We cannot infer (1) from LOCO (e.g. $\text{LOCO}_2 \approx 0$ but coef. of x_2 is 2.05)
- ⇒ We also can't infer (2), e.g., $\text{Cor}(x_2, y) = 0.68$ but $\text{LOCO}_2 \approx 0$
- ⇒ We can get insight into (3): x_2, x_1 highly corr. with $\text{LOCO}_1 = \text{LOCO}_2 \approx 0$
 - ↪ x_2 and x_1 take each others place if one of them is left out (unlike x_3)

PROS AND CONS

Pros:

- Requires (only?) one refitting step per feature for evaluation
- Easy to implement
- Testing framework available in [Lei 2018](#)

Cons:

- Provides insight into a learner on specific data, not a specific model
 - + for algorithm-level insight
 - for model-specific insights
- Model training is a random process and LOCO estimates can be noisy
 - ~> Limits inference on model and data, or multiple refittings necessary?
- Requires re-fitting the learner for each feature
 - ~> Computationally intensive compared to PFI

