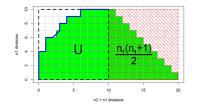
# **Introduction to Machine Learning**

# **Evaluation AUC & Mann-Whitney-U Test**



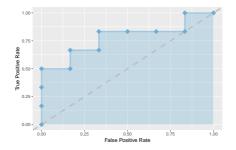


#### Learning goals

- Understand the rank-based nature of AUC
- See the connection between AUC and Mann-Whitney-U statistic

#### **AUC AS A RANK-BASED METRIC**

- The AUC metric is intimately related to the Mann-Whitney-U test, also known as Wilcoxon rank-sum test.
- This connection is best understood viewing the AUC from a slightly different angle: it is, in effect, a rank-based metric.
- Recall that, constructing the ROC curve, we count TP and FP.

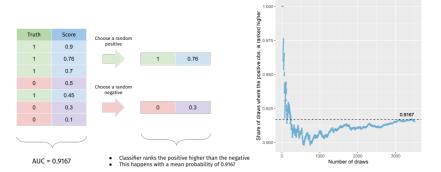


• The AUC abstracts from the actual classification scores and considers only their rank.



### **AUC AS A RANK-BASED METRIC /2**

- We can interpret the AUC as the probability of our classifier ranking a random positive observation higher than a random negative one.
- A perfect classifier will rank all positive above all negative observations, achieving AUC = 1.





#### **MANN-WHITNEY-U TEST**

- The Mann-Whitney-U test is a **non-parametric hypothesis test** on the difference in location between two samples  $X_1$ ,  $X_2$  of sizes  $n_1$  and  $n_2$ , respectively.
- Under the null,  $X_1$  and  $X_2$  follow the same (unknown) distribution  $\mathbb{P}$ , and for any pair of observations  $x_{1,1} \in X_1$ ,  $x_{2,1} \in X_2$  drawn at random from  $\mathbb{P}$ , the following statement holds:  $\mathbb{P}(x_{1,1} \in X_1) > \mathbb{P}(x_{2,1} \in X_2) = \mathbb{P}(x_{1,1} \in X_1) < \mathbb{P}(x_{2,1} \in X_2) = 0.5$ .
- The test statistic estimates the probability of a random sample from  $X_1$  ranking higher than one from  $X_2$  ( $R_1$  denoting the sum of ranks of the  $x_{1,i}$ ):

$$U = \frac{1}{n_1 n_2} \sum_{i=1}^{n_1} \sum_{i=1}^{n_2} \mathbb{I}[x_{1,i} > x_{2,j}] = R_1 - \frac{n_1(n_1 + 1)}{2}$$

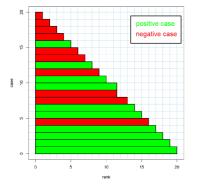
• For large samples, *U* is approximately normally distributed.

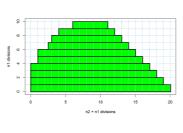


#### **AUC & MANN-WHITNEY-U TEST**

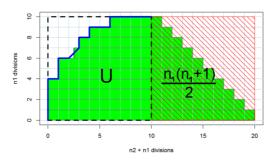
- We can directly interpret the AUC in the light of the U statistic.
- In order to see this, plot the ranks of all the scores as a stack of horizontal bars, and color them by label.
- Next, keep only the green ones, and slide them horizontally to get a nice even stairstep on the right edge:







# **AUC & MANN-WHITNEY-U TEST / 2**



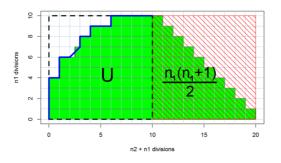


• Denoting the respective numbers of cases as  $n_+$  and  $n_-$ , we have:

$$U=R_{+}-rac{n_{+}(n_{+}+1)}{2}.$$

• The area of the green bars on the right is equal to  $\frac{n_+(n_++1)}{2}$ .

## **AUC & MANN-WHITNEY-U TEST / 3**





- *U*: area of the green bars on the left.
- $n_+ \cdot n_-$ : area of the dashed rectangle.
- $\Rightarrow$  AUC is *U* normalized to the unit square:

$$\mathsf{AUC} = \frac{\mathit{U}}{\mathit{n}_+ \cdot \mathit{n}_-}$$