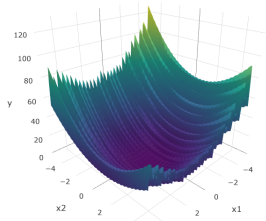
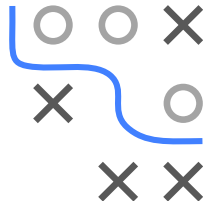


Optimization in Machine Learning

Multi-Start Optimization



Learning goals

- Multimodal functions
- Basins of Attractions
- Simple multi-start procedure

MOTIVATION

- So far: derivative-free methods for *unimodal* objective function (exception: simulated annealing)
- With multimodal objective functions, methods converge to **local minima**.
- Optimum found may differ for different starting values $\mathbf{x}^{[0]}$



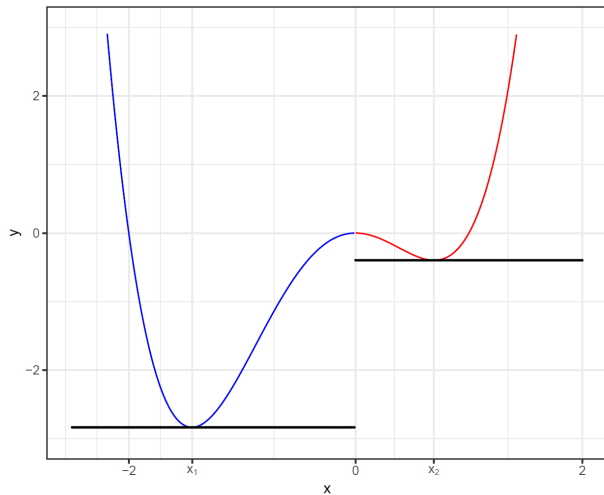
Attraction areas:

- Let f_1^*, \dots, f_k^* be local minimum values of f with $f_i^* \neq f_j^* \quad \forall i \neq j$.
- Notation: $A(\mathbf{x}^{[0]})$ denotes result of algorithm A started at $\mathbf{x}^{[0]}$
- Then: Set

$$\mathcal{A}(f_i^*, A) = \{\mathbf{x} : A(\mathbf{x}) = f_i^*\}$$

is called *attraction area/basin of attraction* of f_i^* for algorithm A

ATTRACTION AREAS

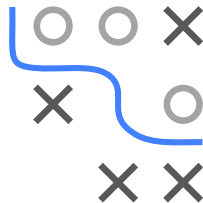


basin of attraction paths

— x_1

— x_2

● optimum found by Nelder-Mead

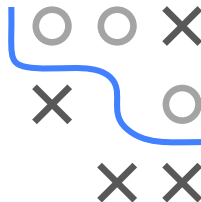
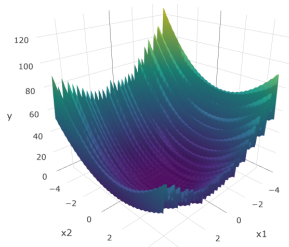


MULTI-STARTS

Levy function:

$$f(\mathbf{x}) = \sin^2(3\pi x_1) + (x_1 - 1)^2[1 + \sin^2(3\pi x_2)] + (x_2 - 1)^2[1 + \sin^2(2\pi x_2)]$$

- Global minimum: $f(\mathbf{x}^*) = 0$ at $\mathbf{x}^* = (1, 1)^\top$
- Optimize f by BFGS method with random starting point in $[-2, 2]^2$ and collect result
- Repeat 100 times



Distribution of results (y values):

##	Min.	1st Qu.	Median	Mean	3rd Qu.	Max.
##	0.0000	0.1099	0.5356	2.4351	1.9809	18.3663

MULTI-STARTS / 3

BFGS with Multistart gives us the true minimum of the Levy function:

```
iters = 20 # number of starts
xbest = c(runif(1, -2, 2), runif(1, -2, 2))

for (i in 1:iters) {
  x1 = runif(1, -2, 2)
  x2 = runif(1, -2, 2)
  res = optim(par = c(x1, x2), fn = f, method = "BFGS")
}

if (res$value < f(xbest)) {
  xbest = res$par
}

xbest
## [1] 1 1
```

