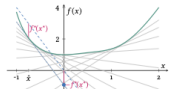
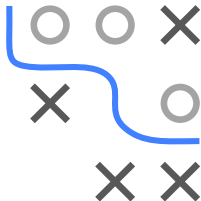


Other forms of duality



(a) The function f (solid) and the linear function of $x^* = -4$ (dashed) and its shifted tangent (dotted).



(b) The Fenchel conjugate f^* of f .

- Dual norms
- Conjugate functions
- Fenchel duality
- Examples in statistics

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CONSTRAINED MINIMIZATION AND DUAL NORMS

Consider the problem of norm minimization under linear constraints in its primal form:

$$\begin{array}{ll} \min_{\mathbf{x} \in \mathbb{R}^d} & \|\mathbf{x}\| \\ \text{s.t.} & \mathbf{G}\mathbf{x} = \mathbf{h}, \end{array}$$

where $\|\cdot\|$ is some norm function. For instance, if the norm is the $L1$ norm, this problem is the famous ► basis pursuit problem.

Question: Is there a more straightforward way to solve constrained optimization problems involving norms?



CONSTRAINED MINIMIZATION AND DUAL NORMS

/ 2

Here, the concept of the dual norm from functional analysis can be helpful.

Definition: Let $\|\mathbf{x}\|$ be the norm of \mathbf{x} . Then the dual norm $\|\mathbf{x}\|_*$ is defined as

$$\|\mathbf{x}\|_* = \max_{\|\mathbf{z}\| \leq 1} \mathbf{z}^T \mathbf{x}$$

Using this definition, one can show that if $\|\mathbf{x}\|$ is a norm and $\|\mathbf{x}\|_*$ is the dual norm of it, $\|\mathbf{z}^T \mathbf{x}\| \leq \|\mathbf{z}\| \|\mathbf{x}\|_*$ holds.

Examples: The dual norm of the L_p norm $\|\cdot\|_p$ is the L_q norm $\|\cdot\|_q$ where it holds that $1/p + 1/q = 1$.



CONSTRAINED PROBLEMS AND CONJUGATE FUNCTIONS

