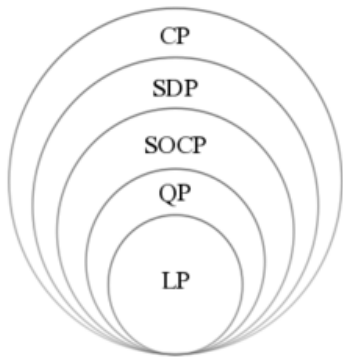


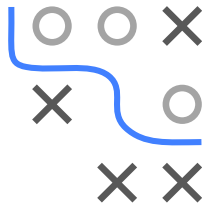
Optimization in Machine Learning

Algorithms for linear programs



Learning goals

- Definition
- Max. Likelihood
- Normal regression
- Risk Minimization



SIMPLEX ALGORITHM

The Simplex algorithm is the most important method for solving Linear programming. It was published in 1947 by Georg Dantzig.

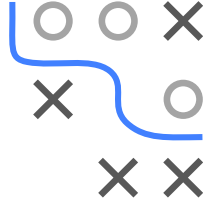
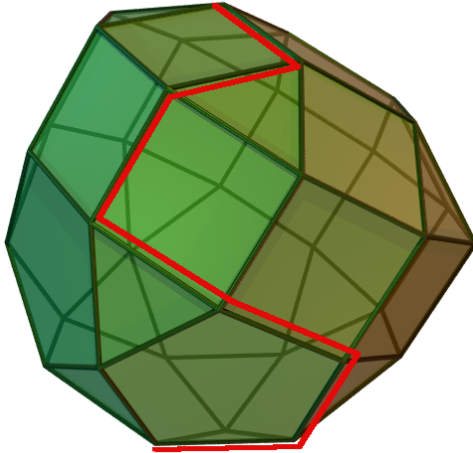
Basic idea: start from an arbitrary corner of the polytope. Run along this edge as long as the solution improves. Find a new edge, ...

Output: a path along the corners of the polytope that ends at the optimal point of the polytope.

Since linear programming is a **convex** optimization problem, the optimal corner found in this way is also a global optimum.



SIMPLEX ALGORITHM / 2



SIMPLEX ALGORITHM / 3

The simplex algorithm can be divided into two steps:

- **Phase I:** determination of a **starting point**
- **Phase II:** determination of the **optimal solution**

To be able to start, a starting point must first be found in **Phase I**, i.e. a feasible corner \mathbf{x}_0 .

In **phase II** this solution is iteratively improved by searching for an edge that improves the solution and running along it to the next corner.



SIMPLEX ALGORITHM / 4

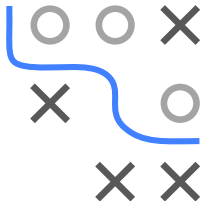
Phase I:

One way to find a starting point \mathbf{x}_0 is to solve a auxiliary linear problem with artificial variables ϵ :

$$\begin{array}{ll}\min_{\epsilon_1, \dots, \epsilon_m} & \sum_{i=1}^m \epsilon_i \\ \text{s.t.} & \mathbf{Ax} + \boldsymbol{\epsilon} \geq \mathbf{b} \\ & \epsilon_1, \dots, \epsilon_m \geq 0 \\ & \mathbf{x} \geq 0\end{array}$$

- A feasible starting point for the auxiliary problem is $\mathbf{x} = \mathbf{0}$ and

$$\epsilon_i = \begin{cases} 0 & \text{if } b_i < 0 \\ b_i & \text{if } b_i \geq 0 \end{cases}$$

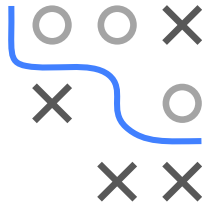
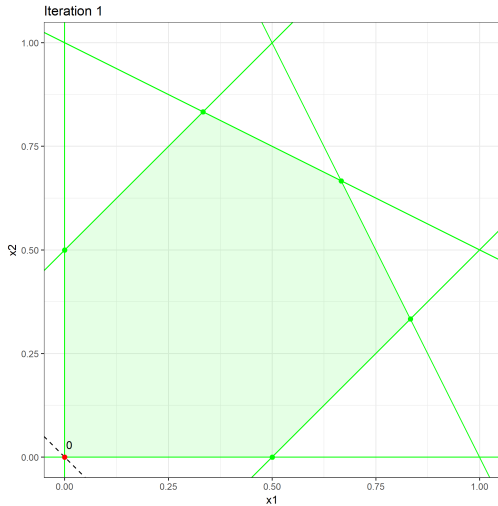


SIMPLEX ALGORITHM / 5

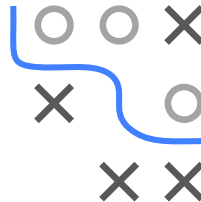
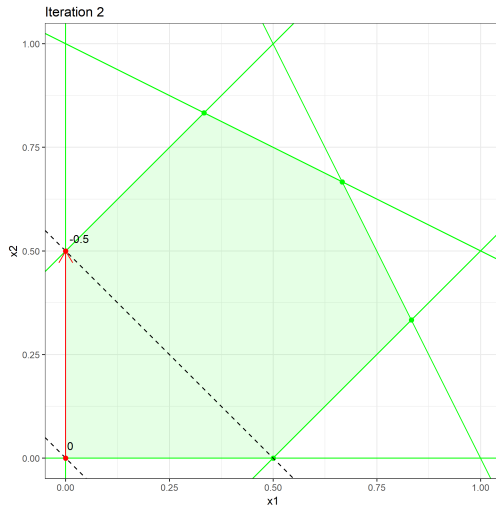
- We then apply phase II of the simplex algorithm to the auxiliary problem.
- If the original problem has a feasible solution, then the optimal solution of the auxiliary problem **must** be $\epsilon = (0, \dots, 0)$ (all artificial variables disappear) and the objective function is 0.
- If we find a solution with $\epsilon = \mathbf{0}$, then we have found a valid starting point.
- If we do not find a solution with $\epsilon = \mathbf{0}$, the problem can not be solved.



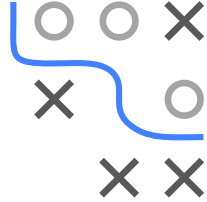
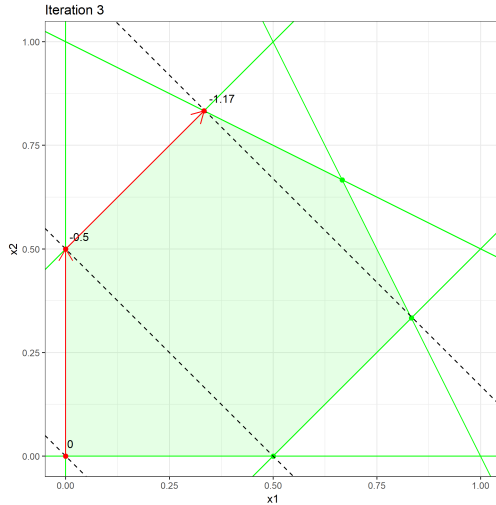
SIMPLEX ALGORITHM / 7



SIMPLEX ALGORITHM / 8



SIMPLEX ALGORITHM / 9



SIMPLEX ALGORITHM / 10

