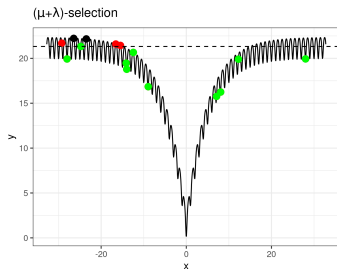
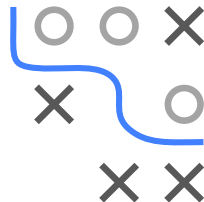


# Optimization in Machine Learning

## Evolutionary Algorithms ES / Numerical Encodings



### Learning goals

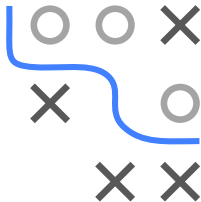
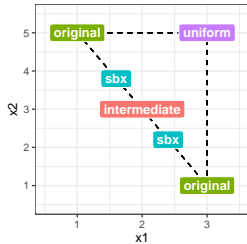
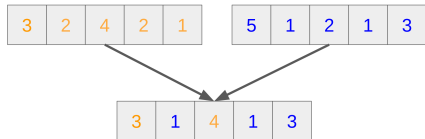
- Recombination
- Mutation
- A few simple examples

# RECOMBINATION FOR NUMERIC

Options for recombination of two individuals  $\mathbf{x}, \tilde{\mathbf{x}} \in \mathbb{R}^d$ :

- **Uniform crossover:** Choose gene  $j$  of parent 1 with probability  $p$  and of parent 2 with probability  $1 - p$
- **Intermediate recombination:** Offspring is created from mean of two parents:  $\frac{1}{2}(\mathbf{x} + \tilde{\mathbf{x}})$
- **Simulated Binary Crossover (SBX):** generate **two** offspring

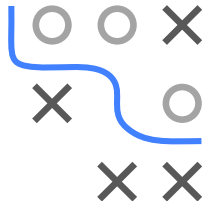
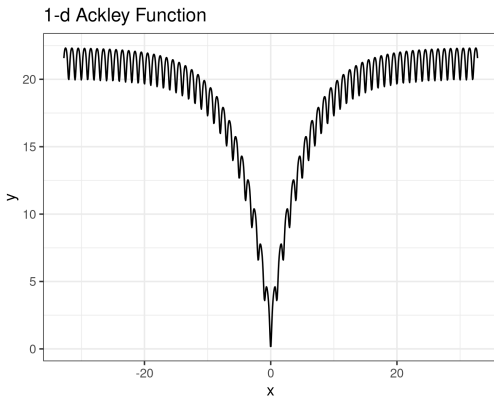
$$\bar{\mathbf{x}} \pm \frac{1}{2}\beta(\tilde{\mathbf{x}} - \mathbf{x}), \quad \bar{\mathbf{x}} = \frac{1}{2}(\mathbf{x} + \tilde{\mathbf{x}}), \quad \beta \in [0, 1] \text{ uniformly at random}$$





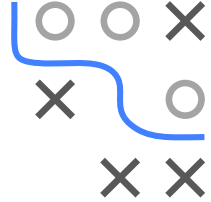
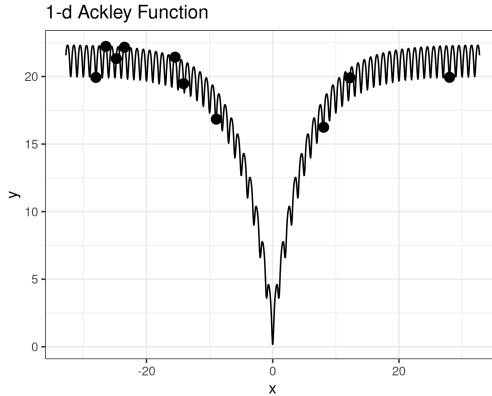
# EXAMPLE OF AN EVOLUTIONARY ALGORITHM

(Simple) EA on 1-dim Ackley function on  $[-30, 30]$ . Usually, for optimizing a function  $f : \mathbb{R}^d \rightarrow \mathbb{R}$ , individuals are encoded as real vectors  $\mathbf{x} \in \mathbb{R}^d$ .



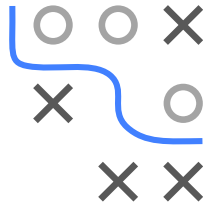
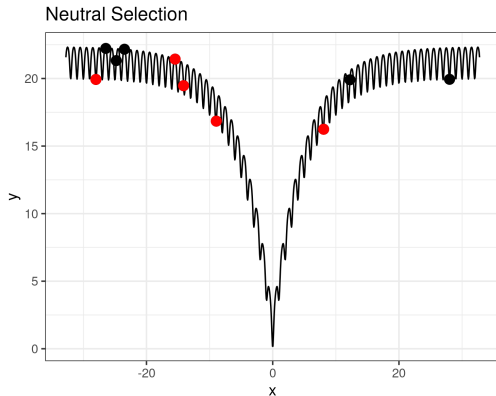
# EXAMPLE OF AN EVOLUTIONARY ALGORITHM

Random initial population with size  $\mu = 10$



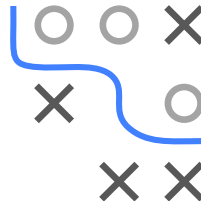
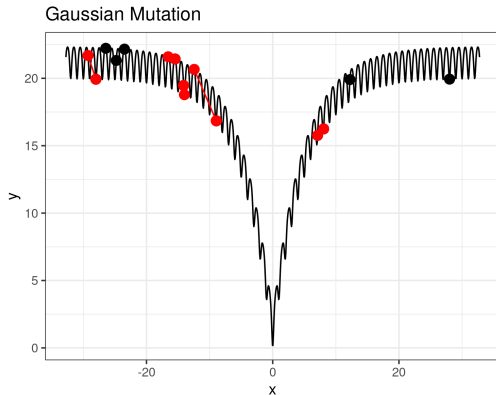
# EXAMPLE 1: ACKLEY FUNCTION

We choose  $\lambda = 5$  offsprings by neutral selection (red individuals).



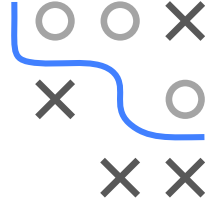
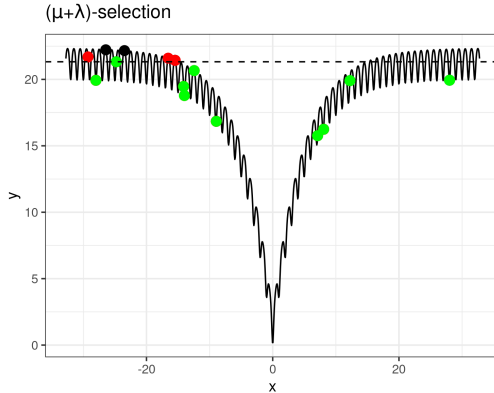
# EXAMPLE 1: ACKLEY FUNCTION

Use Gaussian mutation with  $\sigma = 2$ , but without recombination.



# EXAMPLE 1: ACKLEY FUNCTION

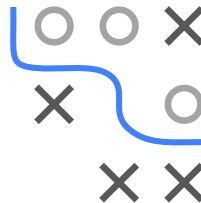
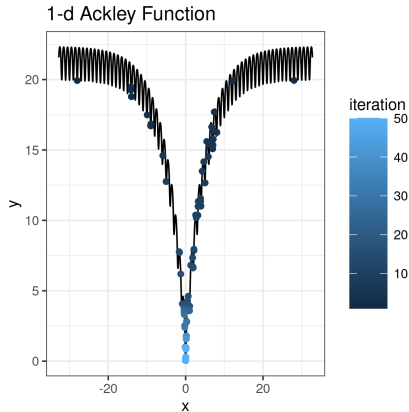
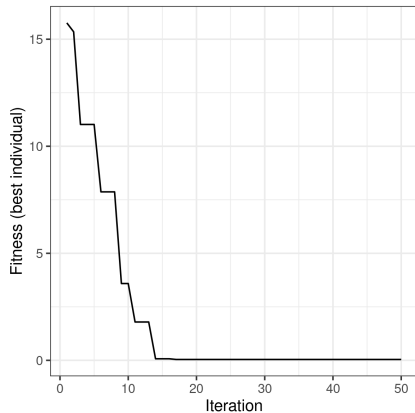
Use  $(\mu + \lambda)$  selection. Selected individuals are marked in green.





# EXAMPLE 1: ACKLEY FUNCTION

After 50 iterations:



## EXAMPLE 2: GRID OF BALLS

Consider a grid in which  $n$  balls with random radius are placed.



**Aim:** Find the circle with the largest possible radius in the grid that does **not** intersect with the other existing circles.

- What is the fitness function?
- How is the population defined?

Implementation: <https://juliabr.shinyapps.io/balls/>

## EXAMPLE 2: GRID OF BALLS / 2

In our example, the chromosome of an individual is the center of a circle, so the chromosomes are encoded as 2-dimensional real vectors  $\mathbf{x} = (x_1, x_2) \in \mathbb{R}^2$ .

The population  $P \subset \mathbb{R}^2$  is given as a set of circle centers.

The fitness function evaluates an individual  $\mathbf{x} \in P$  based on the distance to the nearest neighboring gray circle  $k$ .

$$f(\mathbf{x}) = \min_{k \in \text{Grid}} \text{distance}(k, \mathbf{x}),$$

where the distance is defined as 0 if a circle center is within the radius of a circle of the grid.

This function is to be maximized: we are looking for the largest circle that does not touch any of the gray circles.

