COST CURVES

- Directly plot the misclassif costs / error (in terms of prior probs)
- Might be easier to interpret than ROC, especially in case of different misclassif costs or priors

Example:

- f_1 and f_2 with intersecting ROC curves
- f_2 dominates first, then f_1

BUT: Unclear for which thresholds, costs or class distribs f_2 better than f_1

ROC curves for f_1 and f_2

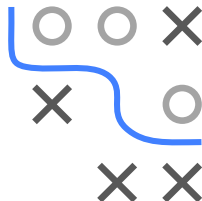Nathalie Japkowicz (2004): Evaluating Learning Algorithms : A Classification Perspective. (p. 125)

COST CURVES

Simplifying assumption: equal misclassif costs, i.e., $cost_{FN} = cost_{FP}$
$\Rightarrow$ Expected misclassif cost reduces to misclassif error rate
With law of total prob, we write error rate as function of π_+:

$$\rho_{MCE}(\pi_+) = (1 - \pi_+) \cdot \mathbb{P}(\hat{y} = 1 | y = 0) + \pi_+ \cdot \mathbb{P}(\hat{y} = 0 | y = 1)$$
$$= (1 - \pi_+) \cdot FPR + \pi_+ \cdot FNR$$
$$= (FNR - FPR) \cdot \pi_+ + FPR$$

Confusion matrix

	True class	
	$y = 1$	$y = 0$
Pred. $\hat{y} = 1$	TP	FP
class $\hat{y} = 0$	FN	TN

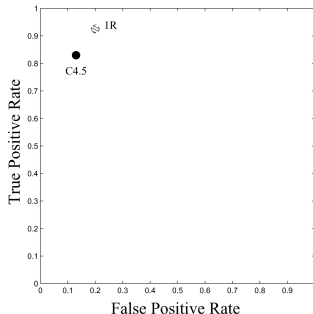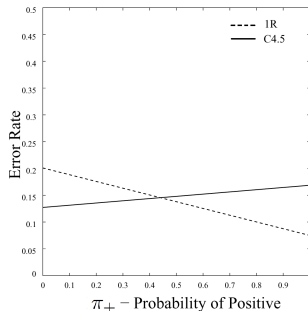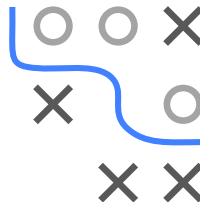Cost matrix

	True class	
	$y = 1$	$y = 0$
Pred. $\hat{y} = 1$	0	$cost_{FP}$
class $\hat{y} = 0$	$cost_{FN}$	0

COST CURVES

- Cost line of a classifier with slope $(FNR - FPR)$ and intercept FPR:

$$\rho_{MCE}(\pi_+) = (FNR - FPR) \cdot \pi_+ + FPR$$

- Cost curves are point–line duals of ROC curves, i.e., a single classifier is represented by a point in the ROC space and by a line in cost space

Chris Drummond and Robert C. Holte (2006): Cost curves: An improved method for visualizing classifier performance.
Machine Learning, 65, 95-130 (URL).

COST LINES

Cost line of a classifier with slope (*FNR* − *FPR*) and intercept *FPR*:
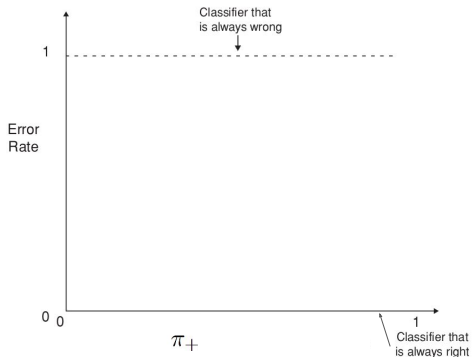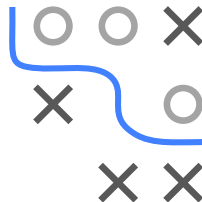
$$\rho_{MCE}(\pi_+) = (FNR - FPR) \cdot \pi_+ + FPR$$

- Hard classifiers are points (TPR, FPR) in ROC space
- The cost line of a classifier connects (π_+, ρ_{MCE})-points at $(0, FPR)$ and $(1, 1 - TPR)$
- Classifier 3 always dominates classifier 1
- Classifier 3 is better than classifier 2 when $\pi_+ < 0.7$

Cost lines plot different values of π_+ vs. $\rho_{MCE}(\pi_+)$
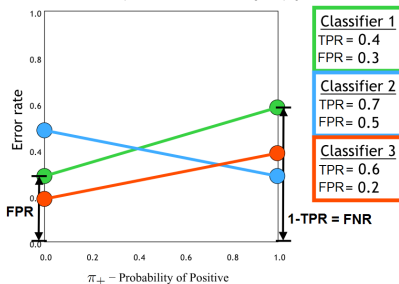
Classifier 1
TPR = 0.4
FPR = 0.3

Classifier 2
TPR = 0.7
FPR = 0.5

Classifier 3
TPR = 0.6
FPR = 0.2

COST LINES - EXAMPLE

- Horizontal dashed line: worst classifier (100% error rate for all π_+)
  $\Rightarrow FNR = FPR = 1$
- x-axis: perfect classifier (0% error rate for all π_+) $\Rightarrow FNR = FPR = 0$
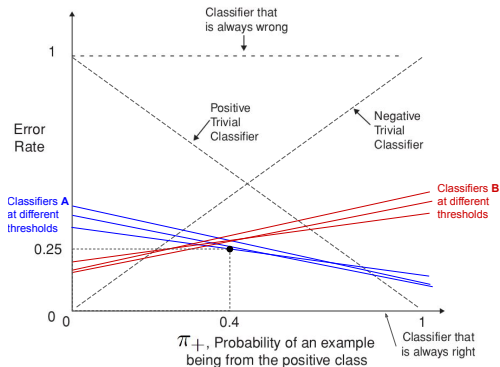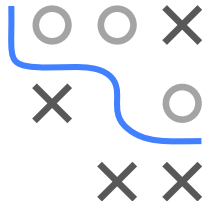

$$\rho_{MCE} = (FNR - FPR) \cdot \pi_+ + FPR$$

Confusion matrix

		True class	
		$y = 1$	$y = 0$
Pred.	$\hat{y} = 1$	TP	FP
class	$\hat{y} = 0$	FN	TN

Classifier that is always wrong

1

Error Rate

0 0

π_+

1
Classifier that is always right

COST LINES - EXAMPLE

- Horizontal dashed line: worst classifier (100% error rate for all π_+)
  $\Rightarrow FNR = FPR = 1$
- x-axis: perfect classifier (0% error rate for all π_+) $\Rightarrow FNR = FPR = 0$
- Dashed diagonal lines: trivial classifiers, i.e., ascending diagonal always predicts
  negative instances ($\Rightarrow FNR = 1$ and $FPR = 0$) and vice versa

$$\rho_{MCE} = (FNR - FPR) \cdot \pi_+ + FPR$$

Confusion matrix

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COST LINES - EXAMPLE

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- Descending/ascending bold lines: two families of classifiers A and B (represented by points in their respective ROC curves)
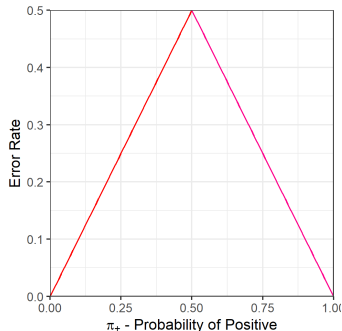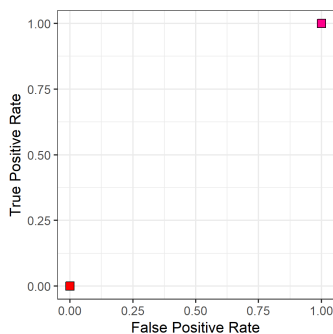

$$\rho_{MCE} = (FNR - FPR) \cdot \pi_+ + FPR$$

Confusion matrix

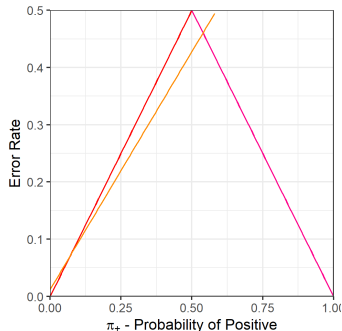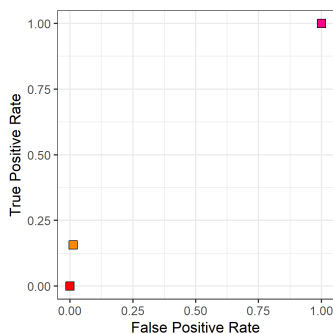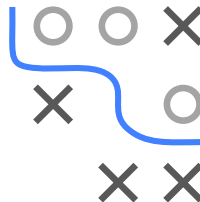|		True class	
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VISUALIZE COST CURVE - LOWER ENVELOPE

- Left: ROC = TPR & FPR of a classifier for different prob thresholds
- Right: Corresponding cost lines
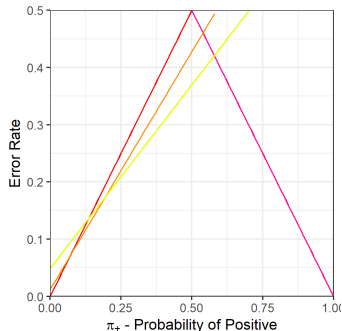- Duality: For every ROC point we can construct the CC line, and vice versa.
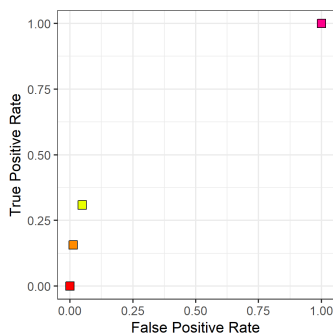
VISUALIZE COST CURVE - LOWER ENVELOPE

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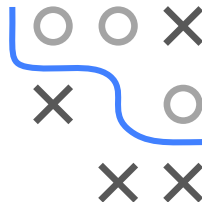- Duality: For every ROC point we can construct the CC line, and vice versa.

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- Duality: For every ROC point we can construct the CC line, and vice versa.

VISUALIZE COST CURVE - LOWER ENVELOPE

- Left: ROC = TPR & FPR of a classifier for different prob thresholds
- Right: Corresponding cost lines
- Duality: For every ROC point we can construct the CC line, and vice versa.
- **Cost curve** (right, black) is lower envelope of **cost lines**
  $\hat{=}$ pointwise minimum of error rate (as function of π_+)