

# CCS WITH TRUE COSTS

Assume unequal misclassification costs, i.e.,  $cost_{FN} \neq cost_{FP}$  and generalize error rate to **expected costs** (as function of  $\pi_+$ ):

$$Costs(\pi_+) = (1 - \pi_+) \cdot FPR \cdot cost_{FP} + \pi_+ \cdot FNR \cdot cost_{FN}$$

Maximum of expected costs happens when

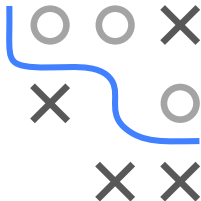
$$FPR = FNR = 1 \Rightarrow Costs_{max} = (1 - \pi_+) \cdot cost_{FP} + \pi_+ \cdot cost_{FN}$$

Consider **normalized costs** (as function of  $\pi_+$ ):

$$\begin{aligned} Costs_{norm}(\pi_+) &= \frac{(1 - \pi_+) \cdot FPR \cdot cost_{FP} + \pi_+ \cdot FNR \cdot cost_{FN}}{(1 - \pi_+) \cdot cost_{FP} + \pi_+ \cdot cost_{FN}} \\ &= \frac{(1 - \pi_+) \cdot cost_{FP} \cdot FPR}{(1 - \pi_+) \cdot cost_{FP} + \pi_+ \cdot cost_{FN}} + \frac{\pi_+ \cdot cost_{FN} \cdot FNR}{(1 - \pi_+) \cdot cost_{FP} + \pi_+ \cdot cost_{FN}} \end{aligned}$$

Let "probability times cost"  $PC(+)$  be normalized version of  $\pi_+ \cdot cost_{FN}$ :

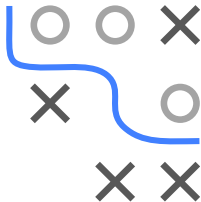
$$PC(+) = \frac{\pi_+ \cdot cost_{FN}}{(1 - \pi_+) \cdot cost_{FP} + \pi_+ \cdot cost_{FN}} \text{ and } 1 - PC(+) = \frac{(1 - \pi_+) \cdot cost_{FP}}{(1 - \pi_+) \cdot cost_{FP} + \pi_+ \cdot cost_{FN}}$$



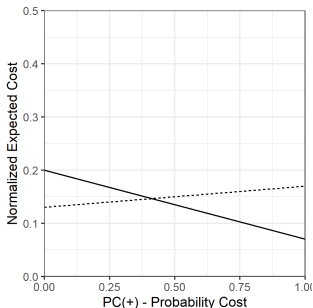
# CCS WITH TRUE COSTS / 2

To obtain cost lines, we need a function with slope  $(FNR - FPR)$  and intercept  $FPR \Rightarrow$  Rewrite  $Costs_{norm}(\pi_+)$  as function of  $PC(+)$ :

$$\begin{aligned} Costs_{norm}(PC(+)) &= (1 - PC(+)) \cdot FPR + PC(+)) \cdot FNR \\ &= (FNR - FPR) \cdot PC(+) + FPR \\ &= \begin{cases} FPR, & \text{if } PC(+) = 0 \\ FNR, & \text{if } PC(+) = 1 \end{cases} \end{aligned}$$



- Plot is similar to simplified case with  $cost_{FN} = cost_{FP}$
- Axes' labels and their interpretation have changed
- Normalized cost vs. "probability times cost"



## COMPARE WITH TRIVIAL CLASSIFIERS

- Operating range of a classifier is a set of  $PC(+)$  values (operating points) where classifier performs better than both trivial classifiers
- Intersection of cost curves and trivial classifiers' diagonals determine operating range
- At any  $PC(+)$  value, the vertical distance of trivial diagonal to a classifier's cost curve within operating range shows advantage in performance (normalized costs) of classifier

**Example:** Dotted lines are operating range of a classifier (here:  $[0.14, 0.85]$ )

