Chapter 1 Differential Manifold

$$\mathbb{R}^m = \{x = (x^1, \cdots, x^m) \mid x^i \in \mathbb{R}, 1 \leqslant i \leqslant \infty\}.$$

$\forall x, y \in \mathbb{R}^m$, $\alpha \in \mathbb{R}$. 第一个结构

$$\begin{cases} (x+y)^i = x^i + y^i \\ (\alpha x)^i = \alpha x^i \end{cases}$$

i 第i个坐标

给定义：加法与数乘

得结构①：linear space.

$d(x,y) = \sqrt{\sum_{i=1}^{m} (x^i - y^i)^2}$ 第二个结构

It's easy to prove that

$$\begin{cases} (1) \quad d(x,y) \geqslant 0 \quad d(x,y) = 0 \Leftrightarrow x = y. \\ (2). \ d(x,y) = d(y,x) \\ (3) \quad d(x,z) \leqslant d(x,y) + d(y,z) \end{cases}$$

给定义：度量/距离

得结构②：matric space.

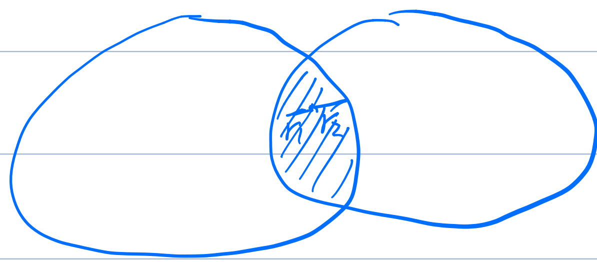
$\mathcal{T}_d = \{U \subset \mathbb{R}^m \mid \forall x \in U, \exists r>0, \text{ s.t. } B(x,r)\subset U\}$

注：每一点都是内点 $\Rightarrow$ 给出开集

It's easy to prove that

$$\begin{cases} (1)\ \ \phi, \mathbb{R}^m \in \mathcal{T}_d, \\ (2)\ \ U_1, U_2 \in \mathcal{T}_d \Rightarrow U_1 \cap U_2 \in \mathcal{T}_d. \\ (3)\ \ U_\alpha \in \mathcal{T}_d, \quad \alpha \in T \Rightarrow \bigcup_{\alpha\in T} U_\alpha \in \mathcal{T}_d. \end{cases}$$

第三个结构



$\cap$: $\forall x \in A_1 \cap A_2$, $\exists\, r_1, r_2 > 0$

s.t. $B(x, r_1) \subset A_1$ $\quad B(x, r_2) \subset A_2$

取 $r = \min(r_1, r_2) \Rightarrow B(x,r) \subset A_1 \cap A_2$

$\cup$: $\forall x \in A_1 \cup A_2$,

任取 $r = r_1$ 或 $r_2 \Rightarrow B(x,r) \subset A_1 \cup A_2$

注：有 matric 后，构造开球，诱导得自然拓扑 $\Rightarrow$ 得结构③：Topological space.

§1. The definition of differential Manifold.

Definition 1. A manifold M of dimension m is a 1° connected paracompact Hausdorff space for which 2° every point has a neighborhood U that is homeomorphic to an open subsets of $\mathbb{R}^m$

不连通: $X: U \cup V$ s.t. $U \cap V = \emptyset$

X能写成2个不相交的集合的并.

连通: 不 不连通

仿紧: 每个开覆盖有局部连续 ····

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mapping f同胚 $\boxed{X(\mathcal{T}_1) \xrightarrow{f} Y(\mathcal{T}_2)}$

若 f 满足 $\begin{cases} f: X \to Y & \text{continous} \\ f: 1 \leftrightarrow 1 & \text{有 inverse mapping} \\ f^{-1}: Y \to X & \text{continous} \end{cases}$

注：在同胚映射下，保持不变的性质 ⇒ 拓扑性质.
（即与形状无关的性质）

Such a homeomorphic

$$\varphi_U : U \to \varphi_U(U)$$

is called a (coordinate) chart

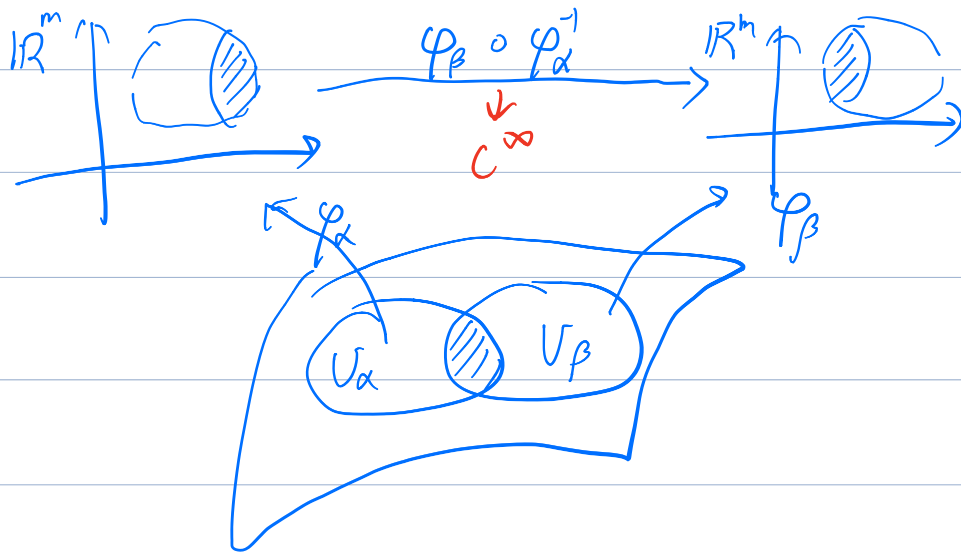
An atlas is a family $\{(U_\alpha, \varphi_\alpha) \mid \alpha \in I\}$ for which the U_α consistitute an open covering of M.

$$M \overset{\text{def}}{=} \bigcup_{\alpha \in I} U_\alpha$$ 单位分解

Definition 2. An atlas $\{(U_\alpha, \varphi_\alpha)\}$ on a manifold M is called differentiable if all chart transitions.

$$\varphi_\beta \circ \varphi_\alpha^{-1} : \varphi_\alpha(U_\alpha \cap U_\beta) \to \varphi_\beta(U_\alpha \cap U_\beta)$$

are differentiable of C^∞ (in case $U_\alpha \cap U_\beta \neq \emptyset$)



C^∞：各分量偏导都是 ∞阶可导