

$\forall \varepsilon > 0, \exists N, \text{ s.t. } n \geq N \text{ 时}$

$$|f_n(x)| < \varepsilon \quad \forall x \in (0, \infty).$$

$\exists N > 0, \text{ s.t. } n > N \text{ 时}$

$$\text{设 } |f_n(x)| = \left| \frac{x}{1+x^n} \right| < \varepsilon \quad \forall x \in (0, \infty)$$

$$\frac{x}{1+x^n} \geq 0, \quad \frac{1+x^n - nx^n}{(1+x^n)^2} = 0$$

$$\Rightarrow x^n = \frac{1}{n}$$

$$\text{设 } x =$$

$$\frac{x}{1+x^n} \cdot \frac{(n-1)^{\frac{1}{n}}}{n}$$

$$\frac{x}{1+x^n}$$

$$\frac{(n-1)^{\frac{1}{n}}}{n} = \frac{x}{1 + \frac{1}{n-1}} = \frac{x(n-1)}{n}$$

$$\frac{(n-1)^{1-\frac{1}{n}}}{n} = \frac{\sqrt{2}}{2}$$

$$e^x \geq 1+x$$

$$\frac{(n-1)^{1-\frac{1}{n}}}{n}$$

$$e^{\frac{1}{n} \ln(n-1)}$$

$$\geq 1 + \frac{1}{\sqrt{n}} \ln(n-1)$$

$$(n-1)^{1-\frac{1}{n}}$$

$$\rightarrow (1 - \frac{1}{n}) \cdot (n-1)^{-\frac{1}{n}} \cdot \underline{(n-1)^{1-\frac{1}{n}}}$$

