Chepter 1. Integration.

$f: \mathbb{R}^1 \to \mathbb{R}^1 \quad [a,b] \subset \mathbb{R}^1.$

Riemman Integral . approximate by sums of dx-forms

$$\sum_{i=1}^{n} f(t_i)\, m(E_i)$$

where $E_1, E_2, \cdots, E_n$, disjoint intervals. whose union is $[a,b]$.

$m(E_i)$: the length of E_i

(因要求一致收敛，不满意)

(逐项积分一致收敛，逐项求导要求导函数一致收敛)

① 是否一致收敛

② 是否可积 $\Leftrightarrow$ 几乎处处连续

(可积 $\Rightarrow$ 有界，连续 $\Rightarrow$ 可积（闭区间连续函数一定一致连续，因此可积）

几乎处处连续：a.e. almost everywhere.

定义连续需先逐点定义 $\lim_{x\to x_0} f(x)=f(x_0)$

逐点：pointwise. $\Rightarrow$ 整个区间/整体

$\rightarrow$ 不连续集合 X, $\text{length}(X)=0$

$\mathbb{R}^n$ 上 length $\begin{cases} n=1 & 长 \\ n=2 & 面 \end{cases}$ $n=\cdots\cdots$ 测度

$\Rightarrow$ 大量函数在 Riemman Integration 下不可积

因此，将 Riemman Integration 推广到 Lebesgue Integration

特点：① 原有性质保留、② 原某些不可积也可积

③. 随意交换次序 . ④ ......

Lebesgue Integration

$E_i \rightarrow$ a larger class, so-called "measurable sets".

The class of functions under consideration

$\longrightarrow$ "measurable sets"

几乎处处连续函数 → 可测函数

$$\int_{[a,b]} f(x)\, dx$$

The crutial set-theorieties properties involved.

1° the union and intersation of any countable family of measurables sets are measurable → X 可数：$X \overset{1-1}{\longleftrightarrow} \mathbb{N}$

$A_1 \cup A_2 \cup \cdots \cup A_n \cup \cdots$ (只有可数才能这么写)

2° So is the complement of every measurable set

3° "Countable Additivity"

"length" → $m(E_1 \cup E_2 \cup \cdots \cup E_n \cup \cdots) = m(E_1) + m(E_2)$

"measure" ↗ $+ \cdots + m(E_n) + \cdots$

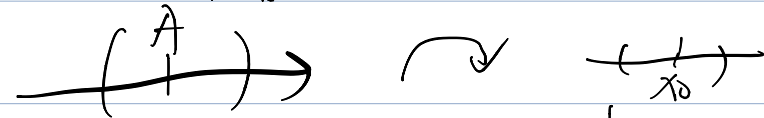
§1 The Concept of Measurability

Topological space, open set, continuous functions

Measurable space, measurable set, measurable functions.

数学分析的基础是极限

$\lim\limits_{x\to x_0} f(x)=A$ / $\lim\limits_{x\to x_0^+} f(x)=+\infty$ ......



$\to$ $\forall \varepsilon>0\ \exists \delta>0, 0<|x-x_0|<\delta$ 有 $|f(x)-A|<\varepsilon$,

A任一邻域,存在x_0的邻域, A邻域内任一点可对应 x_0 邻域内任一点(开映回为开)

$\mathbb{R}^n$上拓扑: $\{U\subset \mathbb{R}^n \mid U \text{ is any open set}\}$

$\Rightarrow$ 任意集合X的拓扑: $\{U\subset X \mid U \text{ is an open set}\}$

where U satisfies ......