

$\forall \varepsilon > 0, \exists N, \text{ 当 } n \geq N \text{ 时}$

$$|f_n(x)| < \varepsilon \quad \forall x \in (0, \infty).$$

$\exists N > 0, \text{ s.t. } n > N \text{ 时}$

$$\forall x \in (0, \infty) \quad |f_n(x)| = \left| \frac{x}{1+x^n} \right| < \varepsilon$$

$$\frac{x}{1+x^n} \geq 0, \quad \frac{1+x^n - nx^n}{(1+x^n)^2} \geq 0 \Rightarrow$$

$$\Rightarrow x^n = \frac{1}{n-1}$$

$$x = \frac{1}{(n-1)^{\frac{1}{n}}}$$

$$\frac{x}{1+x^n} = \frac{x}{1+\frac{1}{n-1}} = \frac{x(n-1)}{n}$$

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$$\frac{(n-1)^{1-\frac{1}{n}}}{n} \geq \frac{\sqrt{2}}{2}$$

$$e^x \geq 1+x$$

$$\frac{(n-1)^{1-\frac{1}{n}}}{n} \geq \frac{\sqrt{2}}{2}$$

$$e^{\frac{1}{n} \ln(n-1)} \geq 1 + \frac{1}{n} \ln(n-1)$$

$$(n-1)^{\frac{1}{n}} \geq 1 + \frac{1}{n} \ln(n-1)$$

$$\geq 1 + \frac{1}{n} \ln(n-1)$$

$$= (1 - \frac{1}{n}) \cdot (n-1)^{-\frac{1}{n}} \cdot (n-1)^{1-\frac{1}{n}}$$

$\frac{1}{h^2}$