

Anisotropic diffusion for denoising and edge detection

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0.1 General principles

In real life objects have a small discrete number of semantic scales. If we take a look at a tree from a few feet, we'll see the trunk, some branches and the overall shape. At a finer resolution, we would be able to see the leaves, and at an even bigger resolution we would see the green veins on each leaf.

A picture too can have a range of different important scales. It can bear information at a low spatial frequency, while details will be held in the high frequencies of the picture.

When receiving a damaged picture, we might not be able to know what the most important "spectral scale" for the image is.

What we want to do then is the construct new images from the one we just received, by filtering its specter to only keep the component at a given scale. This way we will have a family of images generated from the first one, that each contain information on a given spatial scale. This family of functions is often generated by convoluting the image with a gaussian, or a LoG (Laplacian of Gaussian) kernel, the width of which is a function of a parameter ϵ . This notion is called a space-scale. It was made popular by Witkins at the beginning of the eighties, however the idea of a space-scale was already born in the works of japanese scientist Taizo Iijima, in 1959.

Convoluting with Gaussian kernels of increasing width is comparable to solving the diffusion problem, which general equation is:

$$I_t = \text{div}(c(x, y, t)\nabla I) \quad (1)$$

In isotropic situation $c(x, y, t)$ is only a function of time, and in the case of a time-constant environment c is a constant.

Here is the result of a simulation by M. Datar, that can be found in his paper on Anisotropic Diffusion (<http://www.cs.utah.edu/~manasi/coursework/cs7960/p2/project2.html>).

Gaussian convolution can generate a scale space that have some fine properties. However it also has some pretty obvious drawbacks, some of which are:

- Noise disappears but so do edges. The high peaks of the gradients tend to smoothen, as we can notice on the results of the diffusion process simulation from M. Datar.
- Edges are naturally shifted away from their true location when the

Figure 1: *The result of diffusion for a diffusion constant $c = 1$, at various times expressed in seconds*

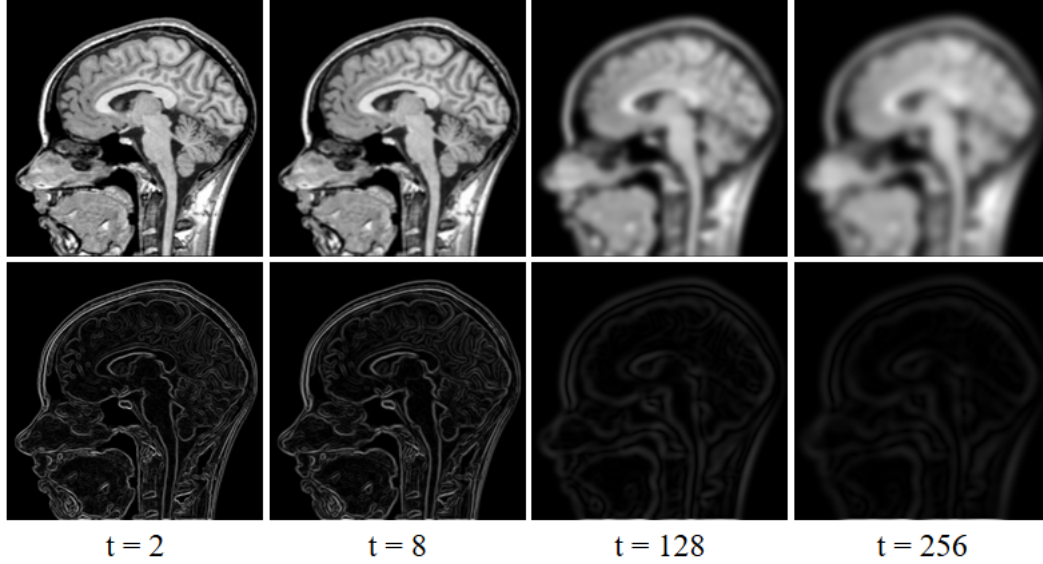


image is convoluted with an increasingly flat Gaussian kernel. That means that even if we recognize an edge in a coarse scale image, we won't be able to figure out the true position of that edge so easily. The only way to find this true location would be to track it through the space-scale, but it requires a lot of resources.

Note that the Lagrangian of Gaussian (LoG) is often used for the isotropic diffusion methods. It presents two advantages: it is linear, and it seems to be very close from the way mammals see.

0.2 Anisotropic diffusion and the Perona & Malik study

In the general case the diffusion factor $c(x, y, t)$ is a function of space and time. Perona & Malik worked on a specific range of solutions, generated from two classes of diffusion factors:

$$c_{\kappa}(\|\nabla I\|) = \exp\left(-\frac{\|\nabla I\|}{\kappa}\right)^2 \quad (2)$$

and

$$c_{\kappa}(\|\nabla I\|) = \frac{1}{1 + \left(\frac{\|\nabla I\|}{\kappa}\right)^2} \quad (3)$$

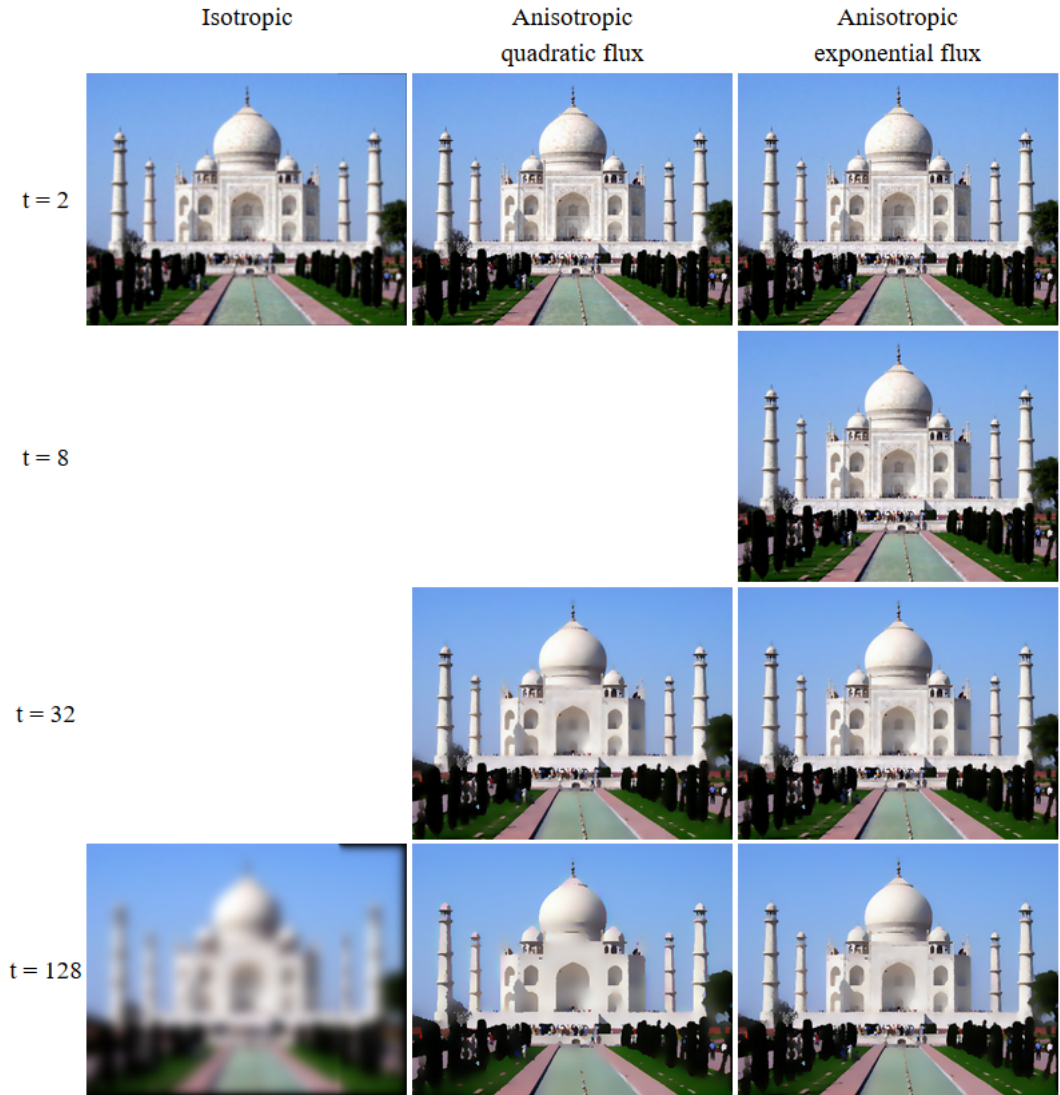
In both cases we can see that the spots with a high gradient will generate a low diffusion factor, which means that diffusion will not occur much around the edges. Those two diffusion functions are the result of compromises between blurring the spots with little intensity variations while preserving the edges.

Note that the κ parameter plays an important role: when high, only very important gradients will induce a low diffusion factor. That means that the asymptotic situation $\kappa \rightarrow 0$ is equivalent to isotropic diffusion. A low value for κ leads to very localised diffusion, where the intensity level is almost constant.

From this anisotropic diffusion we can generate anisotropic scale space that have good properties.

As can be seen, the low_resolution image that was generated through anisotropic diffusion make it to preserve the most important edges. The major gradients in the image are preserved, some are even enhanced.

Figure 2: *A picture that was diffused through a classic gaussian convolution kernel (on the left) and an anisotropic kernel (respectively corresponding to quadratic and exponential diffusion functions)*



0.3 Anisotropic filters for denoising and edge detection

Anisotropic diffusion is an effective process for denoising (through blurring) while preserving the edges (which cannot be done with a mere gaussian kernel convolution).