ECON 608 Spring 2023

Problem Set 2

The due date for this assignment is Tuesday, 02/14

Reading: Heaton and Lucas (1995), Cao (2017), Azinovic et al. (2022). Lecture slides on GDSGE toolbox and Cao et al. (2020)

1. **Deep Equilibrium Nets**: Use the machine learning algorithm in Azinovic et al. (2022) (discussed in Lecture 2) to solve for the Krusell and Smith (1998) model with a continuum of agents and both idiosyncratic and aggregate shocks. Compare the solution the the GDSGE solution.

2. **Active versus Passive Investors**: Consider an alternative version of the asset pricing model with two representative agents in Heaton and Lucas (1995). Assume that one agent - Agent 1 - is a passive investors who consumes labor endowment and a fraction $(1-\beta)$ of financial income every period:

$$\begin{aligned} c_t^1 &= Y_t^1 + (1-\beta)\left(\left(q_t^s + d_t\right)s_t^1 + b_t^1\right) \\ q_t^s s_{t+1}^1 &= 0.5\beta\left(\left(q_t^s + d_t\right)s_t^1 + b_t^1\right) \\ p_t^s b_{t+1}^1 &= 0.5\beta\left(\left(q_t^s + d_t\right)s_t^1 + b_t^1\right) \end{aligned}$$

   The other agent -Agent 2 - makes optimal consumption-investment decisions. Define sequential competitive equilibrium and recursive equilibrium with wealth shares as endogenous state variable in this economy and use GDSE (or machine learning) to solve for a recursive equilibrium using the parameters in the GDSGE Heaton and Lucas (1995) example (*Hint:* start with $\beta = 0.8$ and use the converged solution for a lower value of β as the initial guess for higher β). Simulate the model and plot the long run distribution of equity price. How is it different from the equity price distribution in the original Heaton and Lucas (1995).

3. **Option and Spot Market**s: Consider Example 4 in Cao (2017) and its GDSGE implementation in the website. Now assume that the agents can trade call options and stocks. At time t, agent $i \in \{O, P\}$ can either buy call options at strike price p_t^s or sell covered call option at the same strike. The budget constraint of the agents become:

$$c^i + p_t^s s_{t+1}^i + p_t^\phi \phi_t^i \leq (p_t^s + d_t)s_t^i + \max\{p_t^s + d_t - p_{t-1}^s, 0\}\phi_t^i + Y_t^i$$

   and $s_t^i \geq 0$ and

$$\phi_t^i + s_t^i \geq 0.$$

   Define sequential competitive equilibrium and recursive equilibrium with wealth shares as endogenous state variable in this economy and use GDSE (or machine learning) to solve for a recursive equilibrium using the parameters in the GDSGE Cao (2017) example. (*Hint:* in the consistency equation $\omega_{t+1}^i = \frac{(p_{t+1}^s + d_{t+1})s_{t+1}^i + \max\{p_{t+1}^s + d_{t+1} - p_t^s, 0\}\phi_{t+1}^i}{p_{t+1}^s + d_{t+1}}$, p_t^s enters as an unknown). Simulate the model and plot the long run distribution of equity price. How is it different from the equity price distribution when there is no option? For additional (theoretical and empirical) studies of the relation between option and spot market, read Garleanu et al. (2009).

References

Azinovic, M., L. Gaegauf, and S. Scheidegger (2022). Deep equilibrium nets. *International Economic Review 63*(4), 1471–1525.

Cao, D. (2017). Speculation and financial wealth distribution under belief heterogeneity. *The Economic Journal*.

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Garleanu, N., L. H. Pedersen, and A. M. Poteshman (2009, 02). Demand-Based Option Pricing. *The Review of Financial Studies 22*(10), 4259–4299.

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