

## Source: Chapter 02 of DIP, 3E: Digital Image Fundamentals

- Elements of Visual Perception
  - Structure of Human Eye
  - Image Formation in the Eye
  - Brightness Adaptation and Discrimination
- Light and the Electromagnetic Spectrum

- Image Sensing and Acquisition
  - Image Acquisition Using a Single Sensor
  - Image Acquisition Using Sensor Strips
  - Image Acquisition Using Arrays
  - A Simple Image Formation Model

- Some Basic Relationships between Pixels
  - Neighbors of a Pixel
  - Adjacency, Connectivity, Regions, and Boundaries
  - Distance Measures

# Mathematical Tools used in DIP

Array versus Matrix Operations

Linear versus Non-Linear Operations

Arithmetic Operations

Set and Logical Operations

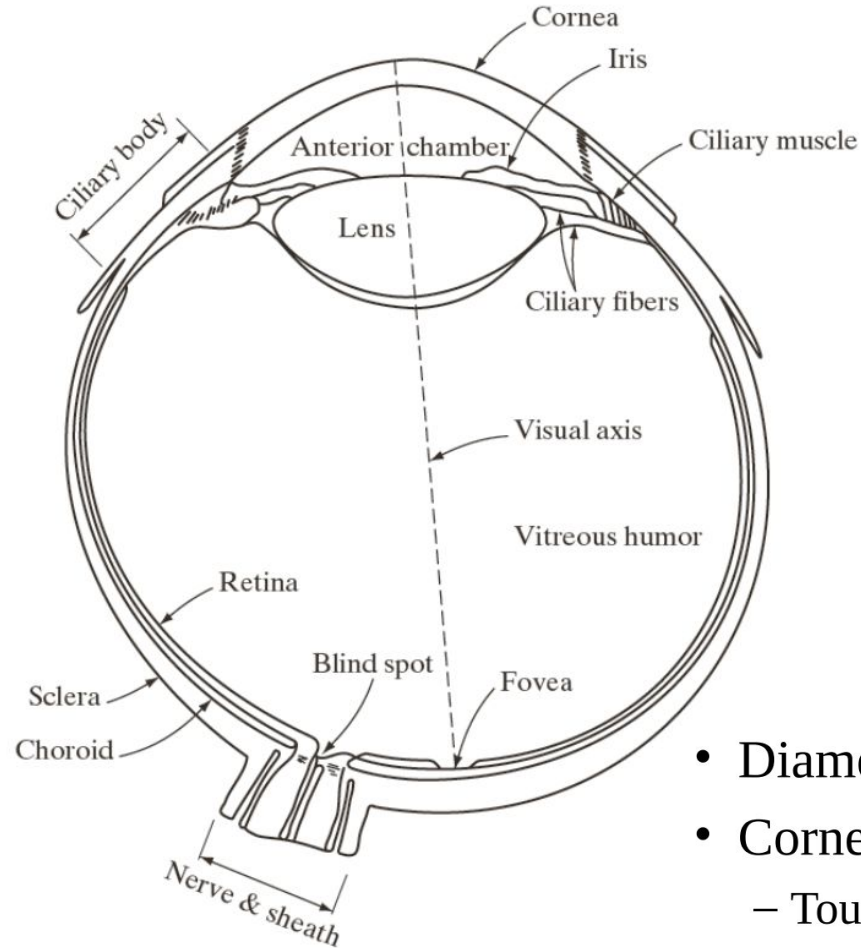
Vector and Matrix Operations

Image Transforms

Probabilistic Methods

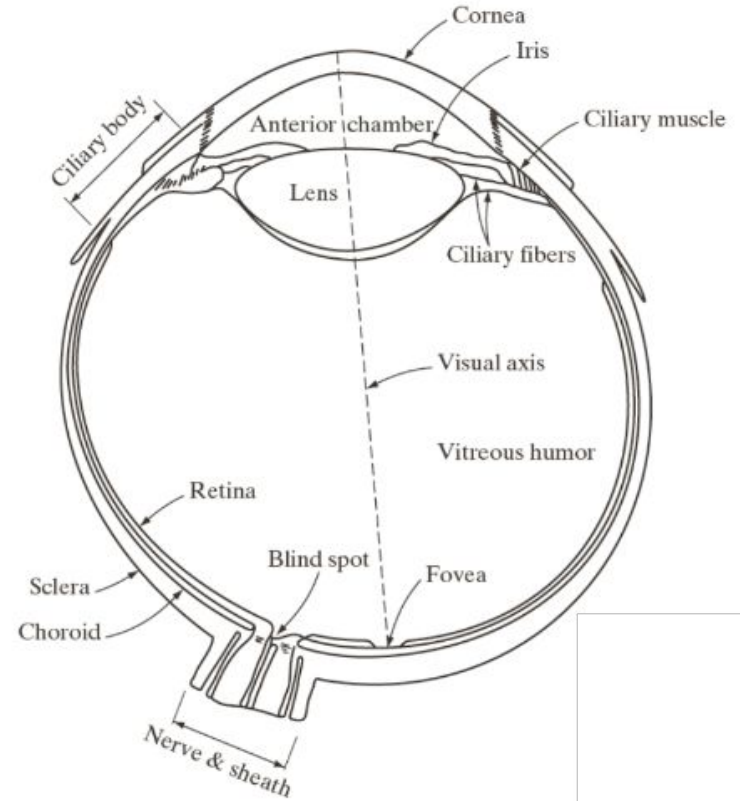
## FIGURE 2.1

Simplified diagram of a cross section of the human eye.



- Diameter of Eye: 20mm
- Cornea
  - Tough, Transparent Tissue

- Iris
  - Controls Aperture (Pupil)
  - Opening: 2mm to 8mm
- Lens
  - Concentric Layers of Fibroid Cells
  - 60% -70% water, 6% fat, protein
  - Absorbs 8% light
- Retina
  - Innermost membrane
  - Fovea
    - Circular Indentation of 1.5mm diameter
    - Modeled as 1.5mm X 1.5mm (CCD: 5mm X 5mm)
  - Cone & Rod Receptors
  - Blind Spot

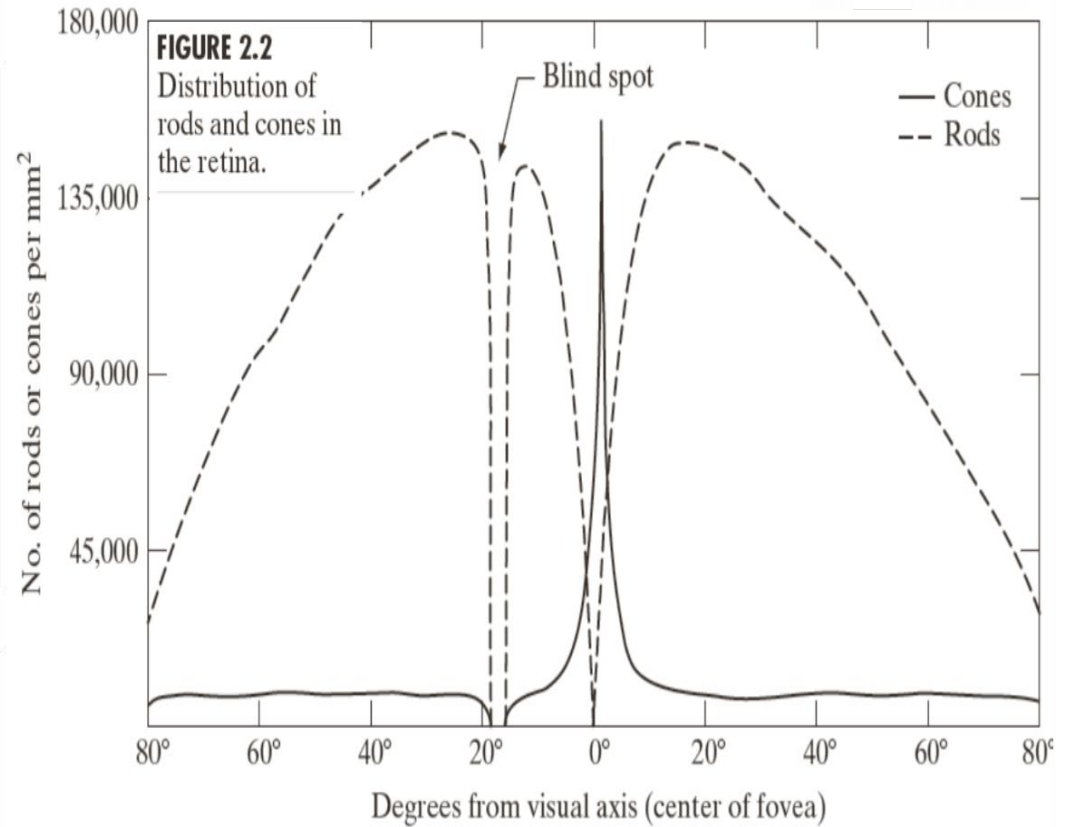


- Cone Receptors (Color)

- Connected to single nerve, localized around Fovea
  - 1,50,000 elements / mm<sup>2</sup>
- 6 to 7 million
- Photopic or Bright-light Vision

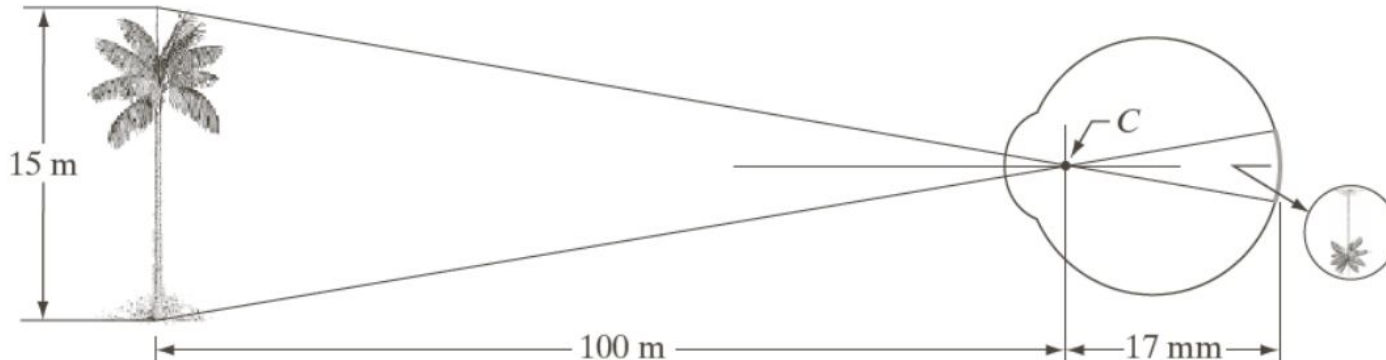
- Rod Receptors (Black-and-White)

- Connected to multiple nerves, distributed
- 75 to 150 million
- Scotopic or Dim-light Vision



# Image Formation in Camera & Eye

- What is the fundamental difference?
  - Camera
    - Fixed Focal Length of Lens
    - Variable Distance between Lens
  - Eye
    - Variable Focal Length of Lens (14mm to 17mm)
    - Fixed Distance between Lens and Film (17mm)



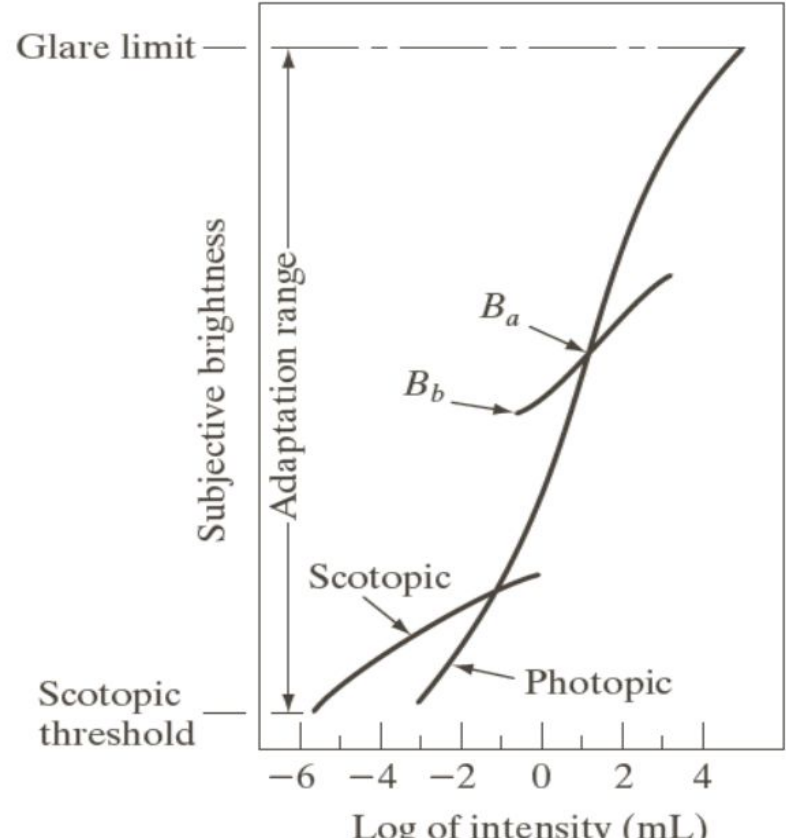
**FIGURE 2.3**

Graphical representation of the eye looking at a palm tree. Point C is the optical center of the lens.



- Brightness Adaptation & Discrimination
  - How do we perceive the intensity in an image?
- Range of Light Intensity Levels
  - $10^{10}$ : Scotopic Threshold to Glare Limit
  - $10^6$ : Photopic Vision
- Subjective Brightness
  - Brightness perceived by Vision
  - Log function of Light Intensity

**FIGURE 2.4**  
Range of  
subjective  
brightness  
sensations  
showing a  
particular



# What is Brightness Adaptation?

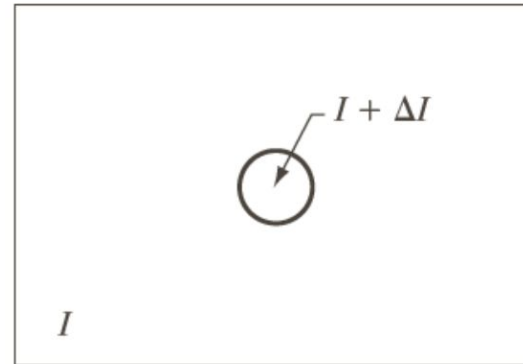
- Wide Range is supported
- Support is not simultaneous
- Changes overall sensitivity

## Brightness Adaptation Level

- Sensitivity at a given condition ( $B_a$ )
- Range gets restricted to a low end ( $B_b$ )
- High nd changes the level itself

# What is Brightness Discrimination?

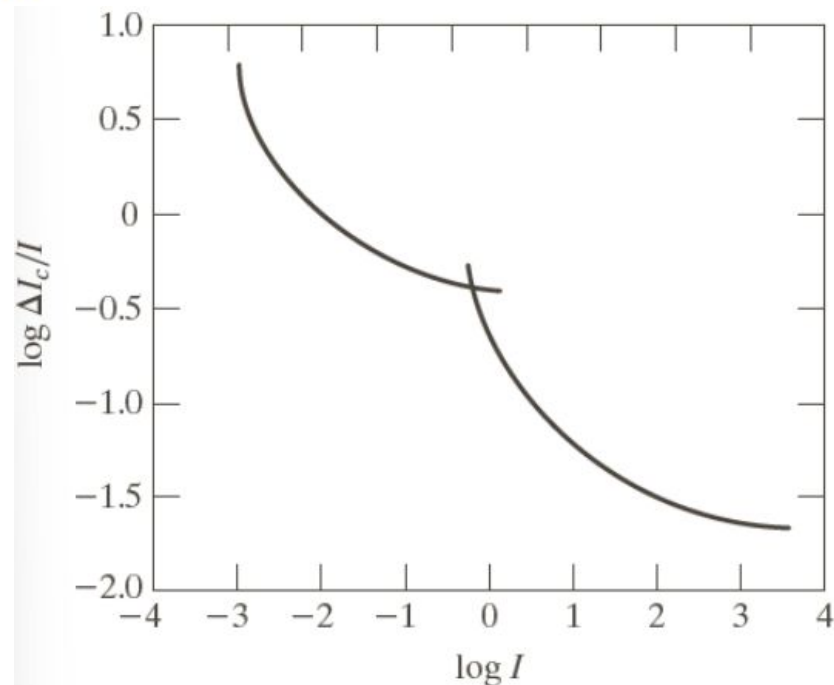
- Ability to discriminate between changes in light a specific adaptation level



**FIGURE 2.5** Basic experimental setup used to characterize brightness discrimination.

- Experiment for Weber Ratio
  - Uniform Back Illumination  $I$
  - Incremental Flash Illumination:  $\Delta I$
  - Subject Response: No / Yes
  - $\Delta I_c$  discriminable in 50% cases
  - Weber Ratio =  $\Delta I_c / I$

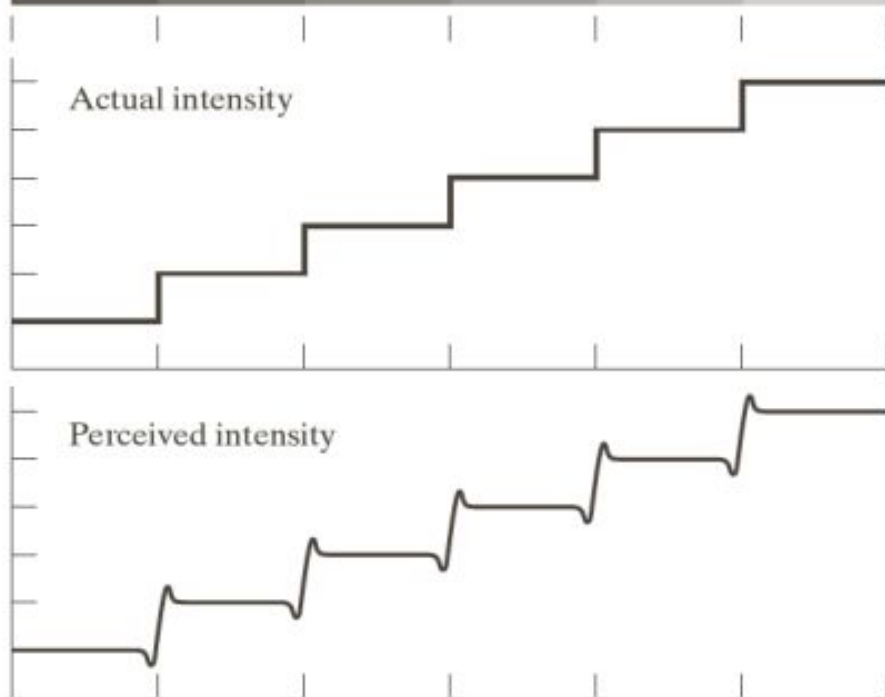
**FIGURE 2.6**  
Typical Weber  
ratio as a function  
of intensity.



- Weber Ratio
  - Small → Good Brightness Discrimination
  - Large → Poor Brightness Discrimination
  - Discontinuous functions for rod and cone behavior
- Continuity Behavior
  - Constant I
  - Uniformly varying  $\Delta I$
  - One / two dozens of intensities perceived at a time
- Perceived Brightness vs Intensity Phenomena
- Mach Bands (after Ernst Mach)
  - Undershoot / Overshoot around boundary of different intensity regions



a  
b  
c

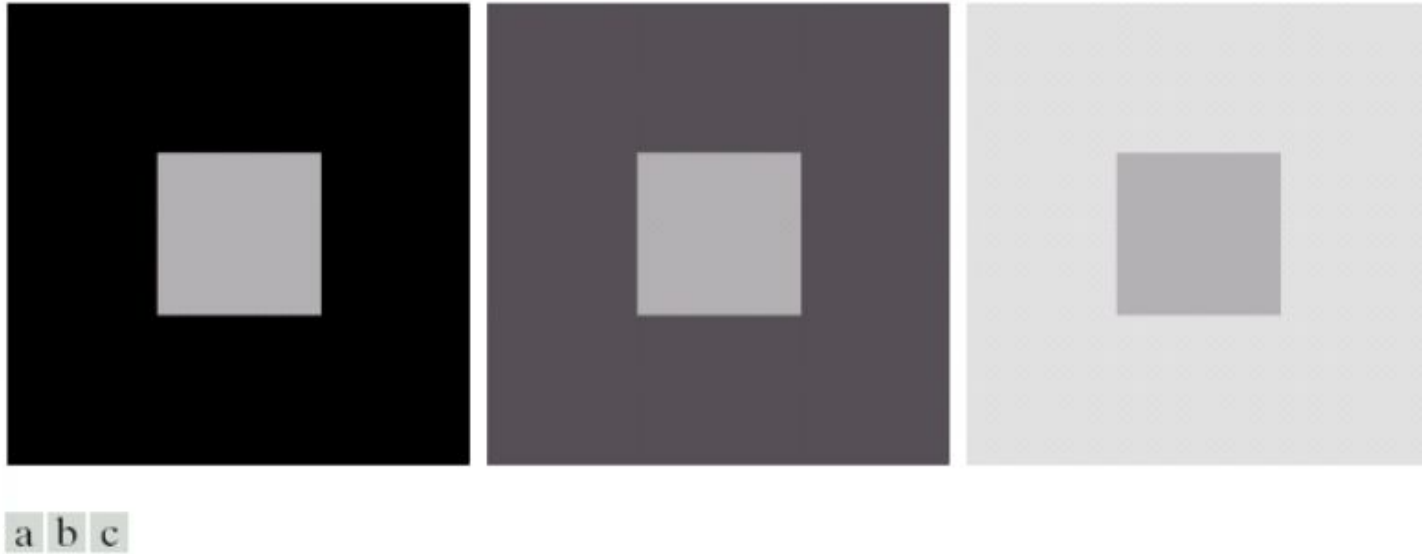


**FIGURE 2.7**

Illustration of the Mach band effect. Perceived intensity is not a simple function of actual intensity.

# Simultaneous Contrast

- Perceived Brightness is not a simple function of intensity



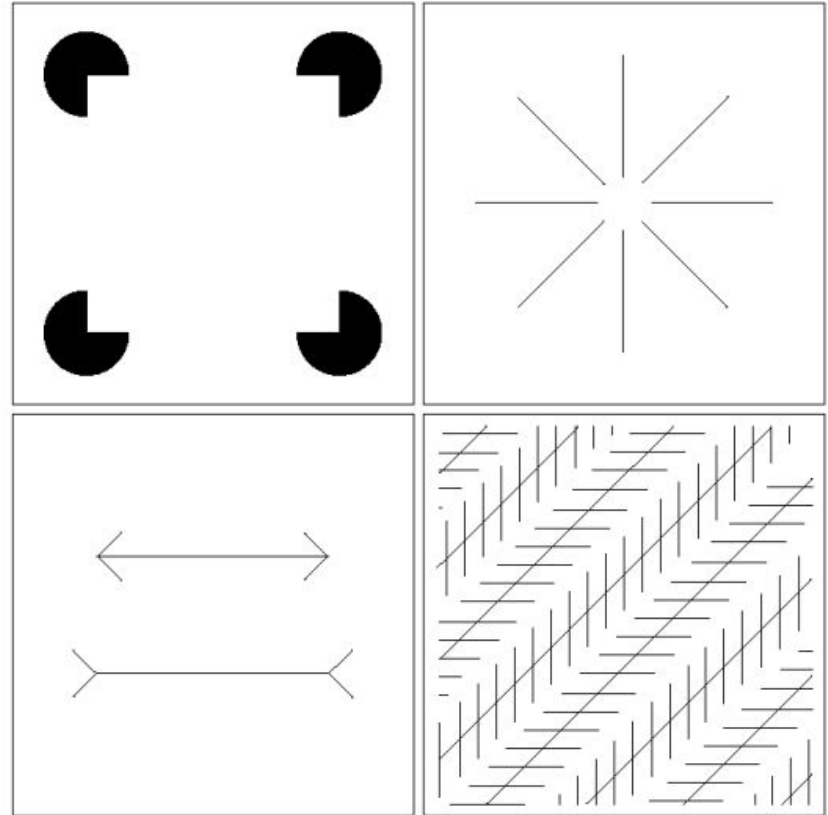
**FIGURE 2.8** Examples of simultaneous contrast. All the inner squares have the same intensity, but they appear progressively darker as the background becomes lighter.

# Optical Illusions

Not yet fully understood

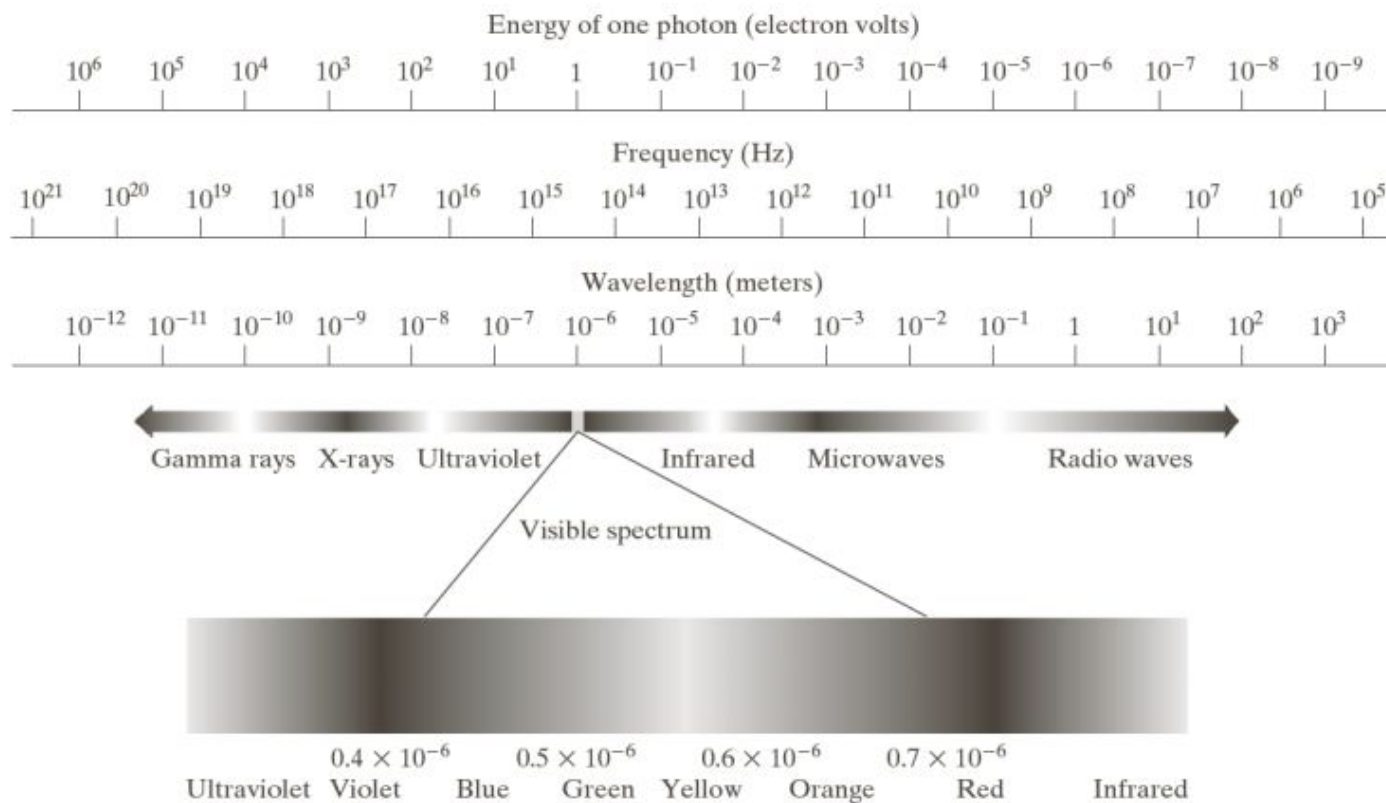
a b  
c d

**FIGURE 2.9** Some well-known optical illusions.



- Light and EM Spectrum
  - $\lambda = c/\nu$ 
    - $\lambda$  = Wavelength of light (m)
    - $c$  = Speed of light
    - $\nu$  = Frequency of light (Hz)
  - $E = h\nu$ 
    - $E$  = Energy
    - $h$  = Planck's Constant



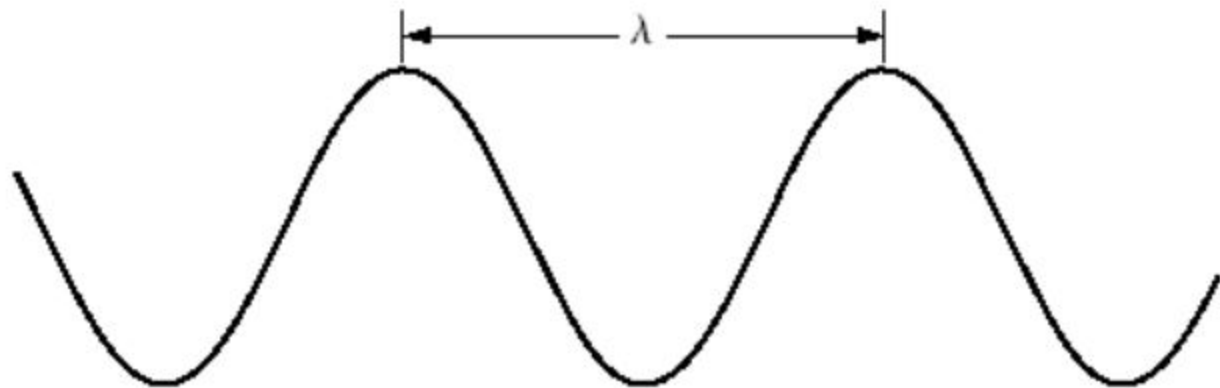


**FIGURE 2.10** The electromagnetic spectrum. The visible spectrum is shown zoomed to facilitate explanation, but note that the visible spectrum is a rather narrow portion of the EM spectrum.

- Visible Band: Violet (430nm ) ~ Red (790nm)

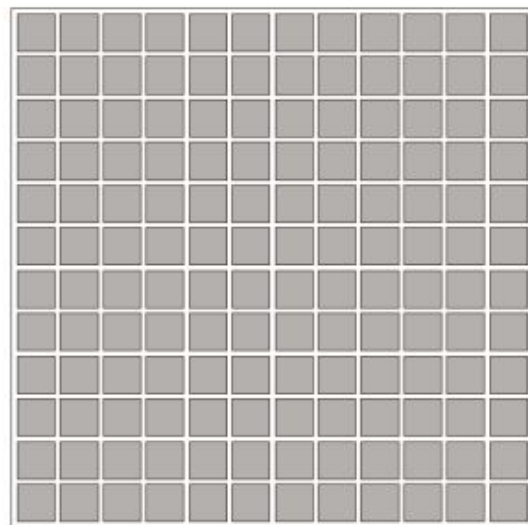
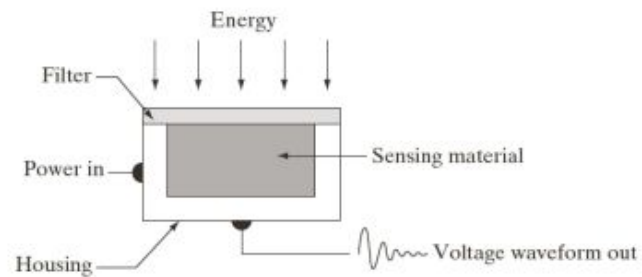
**FIGURE 2.11**

Graphical  
representation of  
one wavelength.



- Monochromatic / Achromatic Light Source
  - Intensity
- Chromatic Light Source
  - Radiance
    - Energy flowing from a light source (Watts)
  - Luminance
    - Energy Perceived by an observer (Lumens)
  - Brightness
    - Subjective descriptor of light perception

- Image Acquisition
  - Single Sensor
  - Sensor Strip
  - Sensor Array



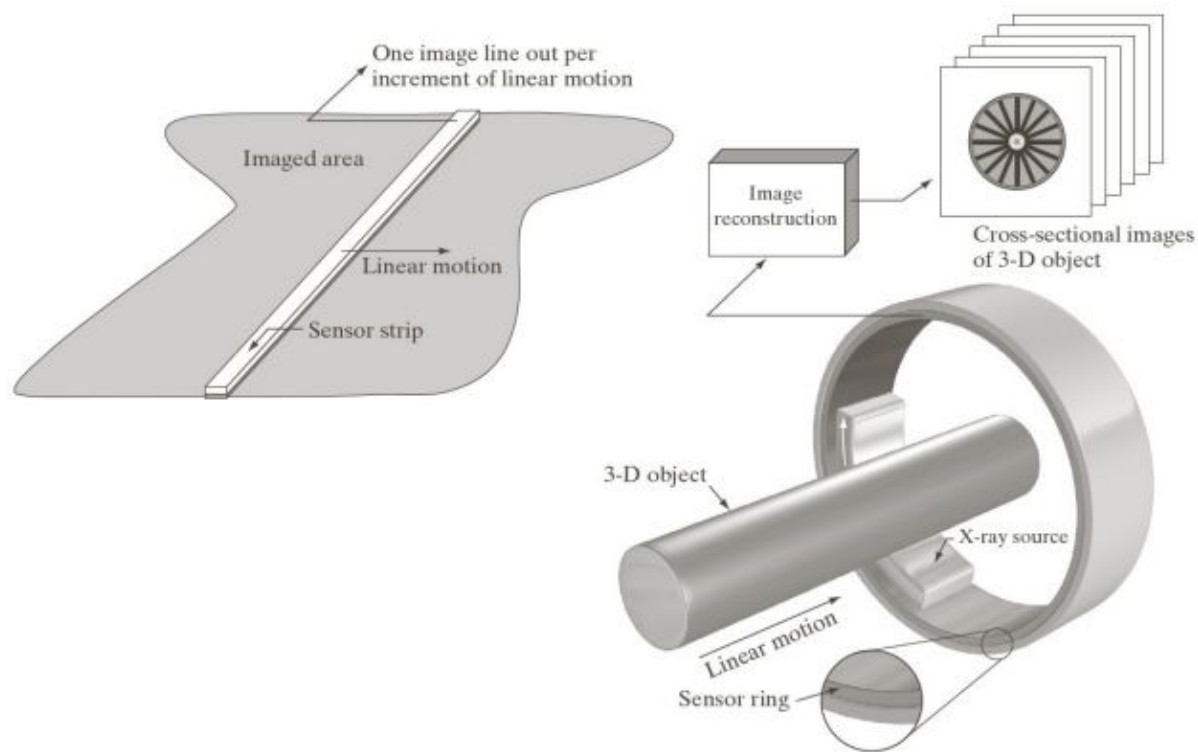
a  
b  
c

**FIGURE 2.12**

(a) Single imaging sensor.

(b) Line sensor.

(c) Array sensor.



a b

**FIGURE 2.14** (a) Image acquisition using a linear sensor strip. (b) Image acquisition using a circular sensor strip.

- Sensor Array Technology
- CCD: Charge-Coupled Device
  - Analog
  - Charge moves between capacitative bins
- CMOS APS: CMOS Active Pixel Sensor
  - Photo-detector + Amplifier @ Pixel
  - Used in Cell-Phone Camera / Webcam

- Image Formation
  - $f(x,y)$  is an image function where
    - $0 < f(x,y) < \infty$
  - $f(x,y) = i(x,y) * r(x,y)$  where
    - $0 < i(x,y) < \infty$  : Illumination
    - $0 < r(x,y) < 1$  : Reflectance / Transmittance

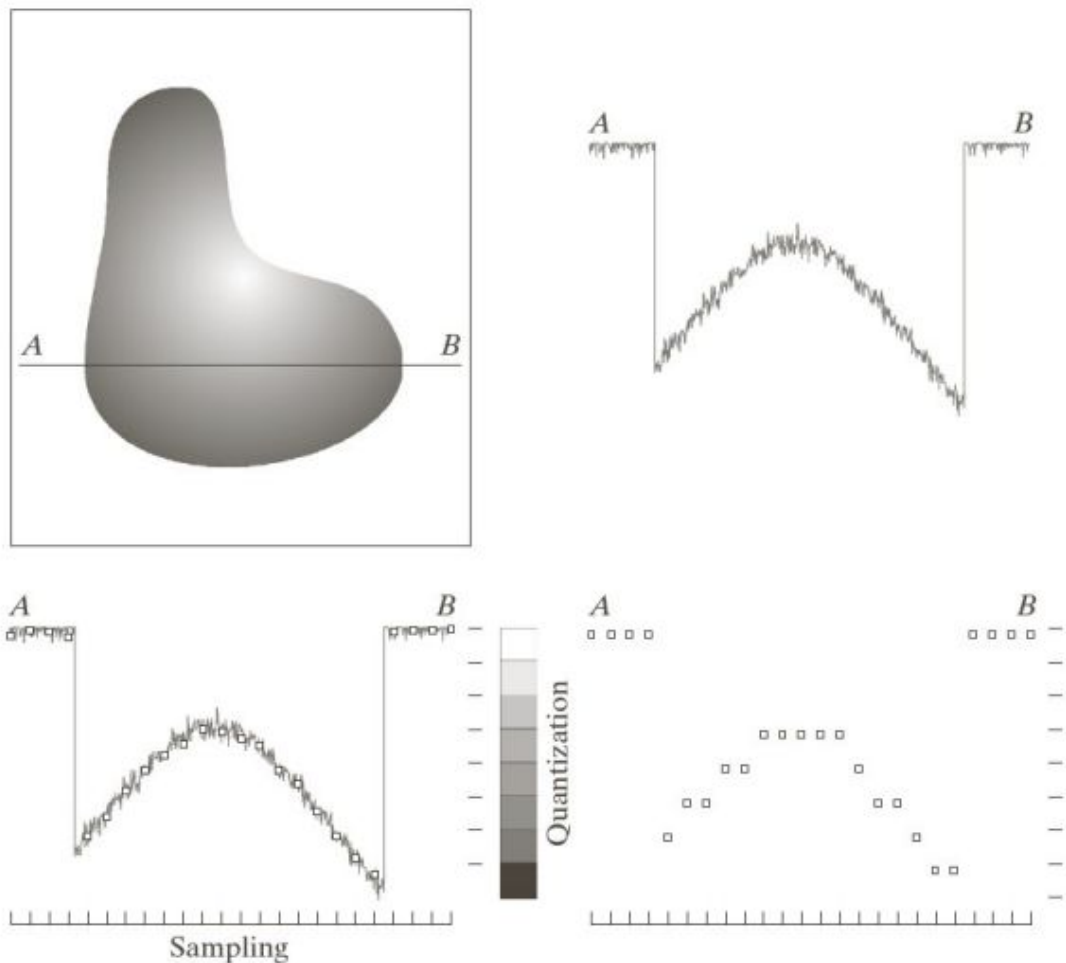


- Typical Illumination
  - Sun, Clear Sky: 90,000 lm/m<sup>2</sup>.
  - Sun, Cloudy Sky: 10,000 lm/m<sup>2</sup>.
  - Full Moon, Clear Sky: 0.1 lm/m<sup>2</sup>.
  - Commercial Office: 1,000 lm/m<sup>2</sup>.

- Typical Reflectance
  - Black Velvet: 0.01
  - Stainless Steel: 0.65
  - Flat White Wall Paint: 0.80
  - Silver-Plated Metal: 0.90
  - Snow: 0.93

- Intensity of a Monochrome Image at (u,v)
  - $L_{\min} \leq f(u,v) \leq L_{\max}$
  - $L_{\min} > 0$  and  $L_{\max}$  is Finite
  - $L_{\min} = i_{\min} * r_{\min}$  and  $L_{\max} = i_{\max} * r_{\max}$
  - Typical:  $L_{\min} \sim 10$  and  $L_{\max} \sim 1000$
  - $[L_{\min}, L_{\max}]$ : Grey / Intensity Scale
  - Translate to  $[0, L-1]$

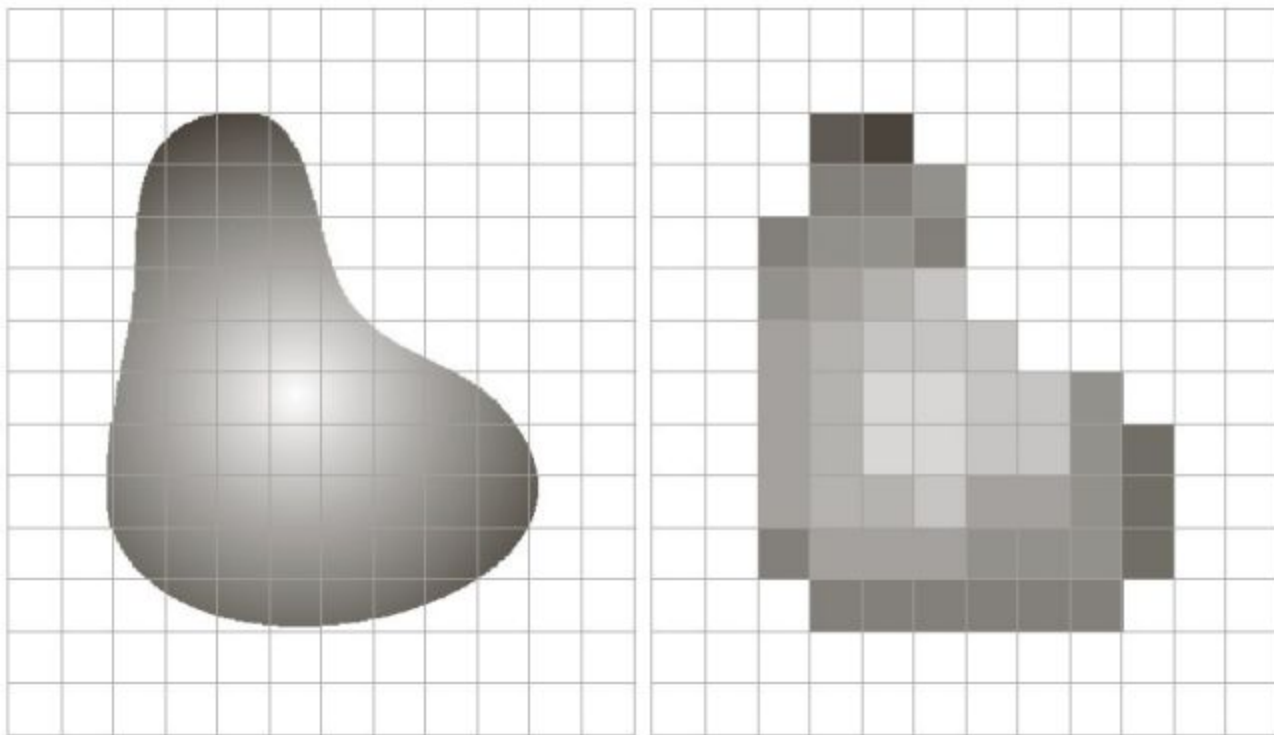
- Sampling for Image Formation
  - Single Sensor
    - Guided by mechanical motion – can be very accurate
    - Limited by optical system
  - Sensor Strip
    - No. of Sensor decide sampling in one dimension
    - Mechanical motions decides the other
  - Sensor Array
    - No. of Sensor decide sampling in both dimensions



a	b
c	d

**FIGURE 2.16**

Generating a digital image. (a) Continuous image. (b) A scan line from *A* to *B* in the continuous image, used to illustrate the concepts of sampling and quantization. (c) Sampling and quantization. (d) Digital scan line.

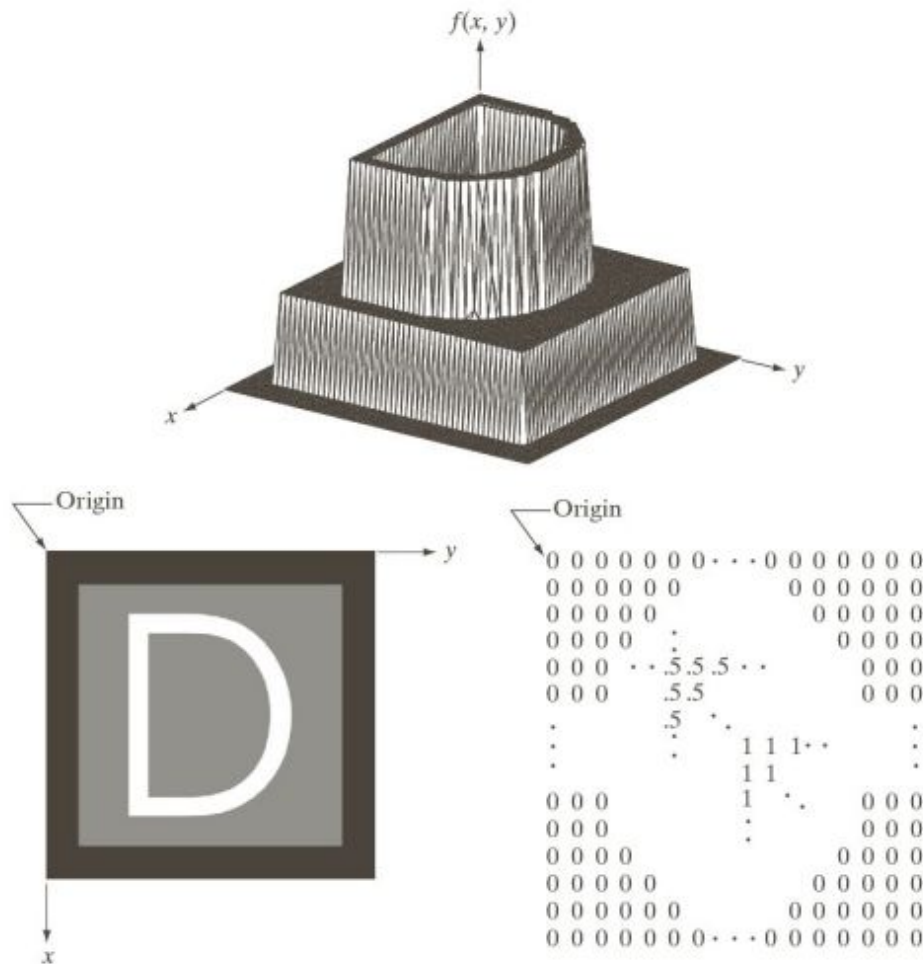


a b

**FIGURE 2.17** (a) Continuous image projected onto a sensor array. (b) Result of image sampling and quantization.

- Image Representation
  - 3-D Function Plot
    - Too detailed
    - Difficult for inference
  - 2-1/2-D Intensity Rendering
    - Good for visualization
    - Less amenable to computation
  - 2-D Numerical Array
    - Easy for computation
    - Large in data volume





a  
b c

**FIGURE 2.18**

(a) Image plotted as a surface.

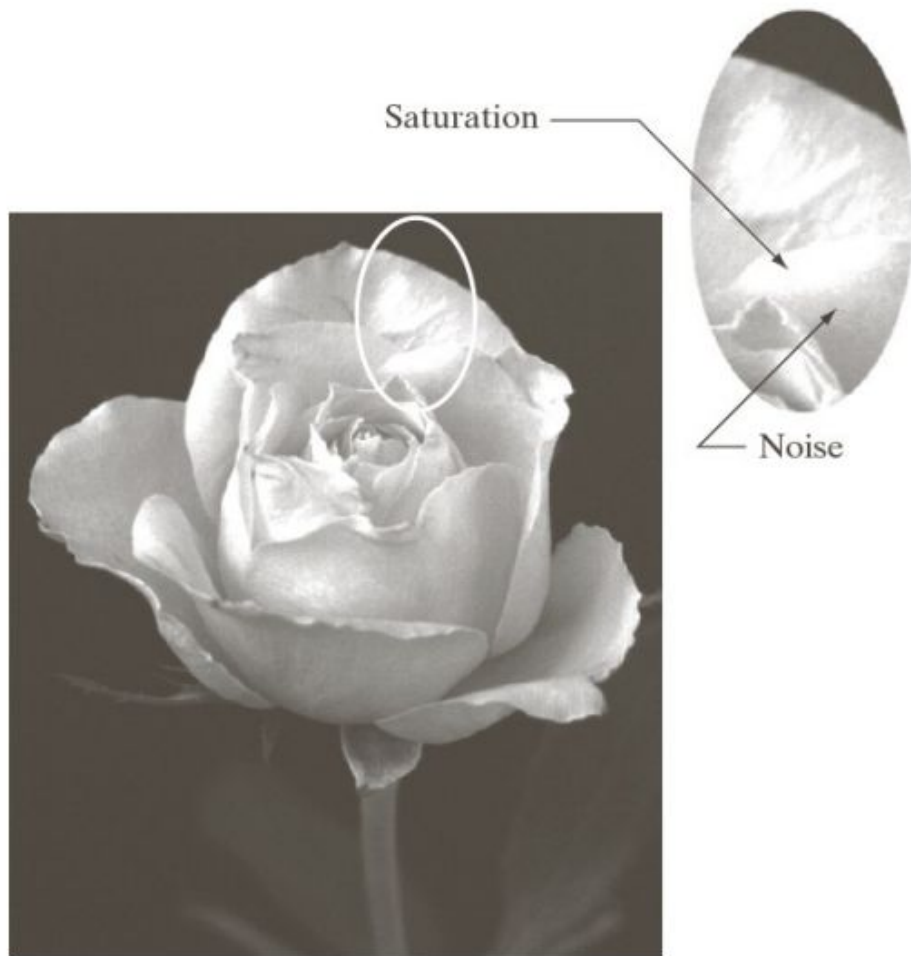
(b) Image displayed as a visual intensity array.

(c) Image shown as a 2-D numerical array (0, .5, and 1 represent black, gray, and white, respectively).



- Image Quantization

- Mapping Intensity values to a (small) range of integers  $[0, L - 1]$ ,  $L = 2^k$ .
- Image Size =  $N^2k$ .
- Dynamic Range of Grey Values
  - Ratio of *Maximum measureable intensity* to *Minimum detectable intensity*
  - Upper Limit: Decided by Saturation
  - Lower Limit: Decided by Noise
- Contrast
  - Maximum Intensity – Minimum Intensity



**FIGURE 2.19** An image exhibiting saturation and noise. Saturation is the highest value beyond which all intensity levels are clipped (note how the entire saturated area has a high, *constant* intensity level). Noise in this case appears as a grainy texture pattern. Noise, especially in the darker regions of an image (e.g., the stem of the rose) masks the lowest detectable true intensity level.

**TABLE 2.1**Number of storage bits for various values of  $N$  and  $k$ .

$N/k$	1 ( $L = 2$ )	2 ( $L = 4$ )	3 ( $L = 8$ )	4 ( $L = 16$ )	5 ( $L = 32$ )	6 ( $L = 64$ )	7 ( $L = 128$ )	8 ( $L = 256$ )
32	1,024	2,048	3,072	4,096	5,120	6,144	7,168	8,192
64	4,096	8,192	12,288	16,384	20,480	24,576	28,672	32,768
128	16,384	32,768	49,152	65,536	81,920	98,304	114,688	131,072
256	65,536	131,072	196,608	262,144	327,680	393,216	458,752	524,288
512	262,144	524,288	786,432	1,048,576	1,310,720	1,572,864	1,835,008	2,097,152
1024	1,048,576	2,097,152	3,145,728	4,194,304	5,242,880	6,291,456	7,340,032	8,388,608
2048	4,194,304	8,388,608	12,582,912	16,777,216	20,971,520	25,165,824	29,369,128	33,554,432
4096	16,777,216	33,554,432	50,331,648	67,108,864	83,886,080	100,663,296	117,440,512	134,217,728
8192	67,108,864	134,217,728	201,326,592	268,435,456	335,544,320	402,653,184	469,762,048	536,870,912

- Spatial Resolution
  - Line pairs per unit distance
    - Chart with alternating black and white lines
    - Largest number of discernible line pairs in unit distance
  - dpi: dots / inch
    - Used in printing / publishing
    - Newspaper: 75 dpi
    - Magazine: 133 dpi
    - Glossy Brochure: 133 dpi
    - High Quality Book: 2400 dpi

@1250 dpi: 3692 X 2812  
@72 dpi: 213 X 162



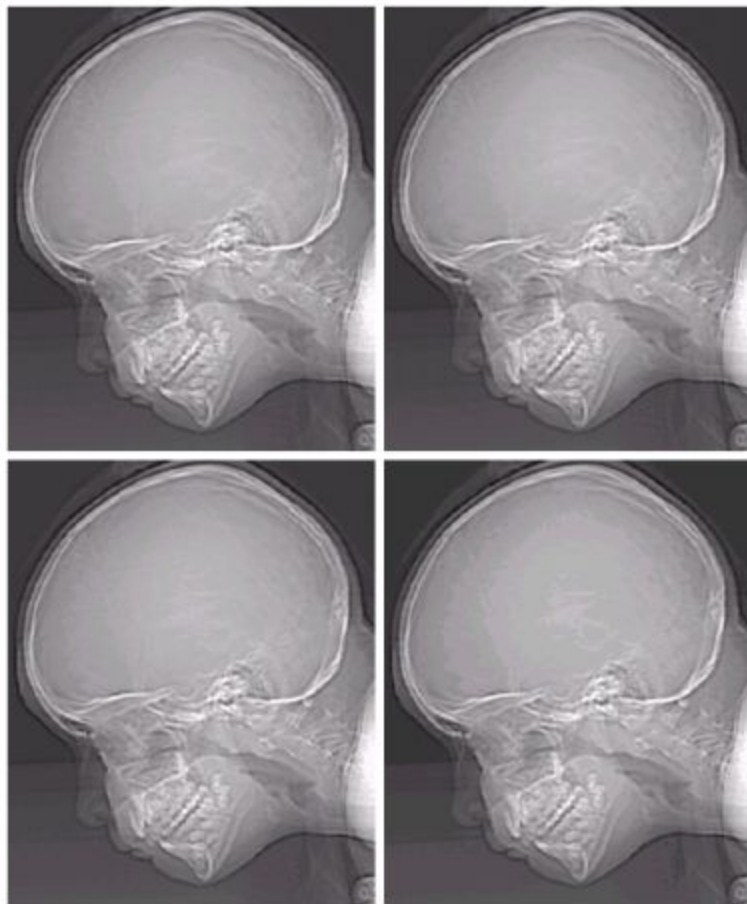
a b  
c d

**FIGURE 2.20** Typical effects of reducing spatial resolution. Images shown at: (a) 1250 dpi, (b) 300 dpi, (c) 150 dpi, and (d) 72 dpi. The thin black borders were added for clarity. They are not part of the data.



- Intensity Resolution
  - Smallest discernible change in intensity level
  - Fixed by hardware
    - Typically 8 bits / 16 bits
  - False Contouring: Low # of intensity levels





a b  
c d

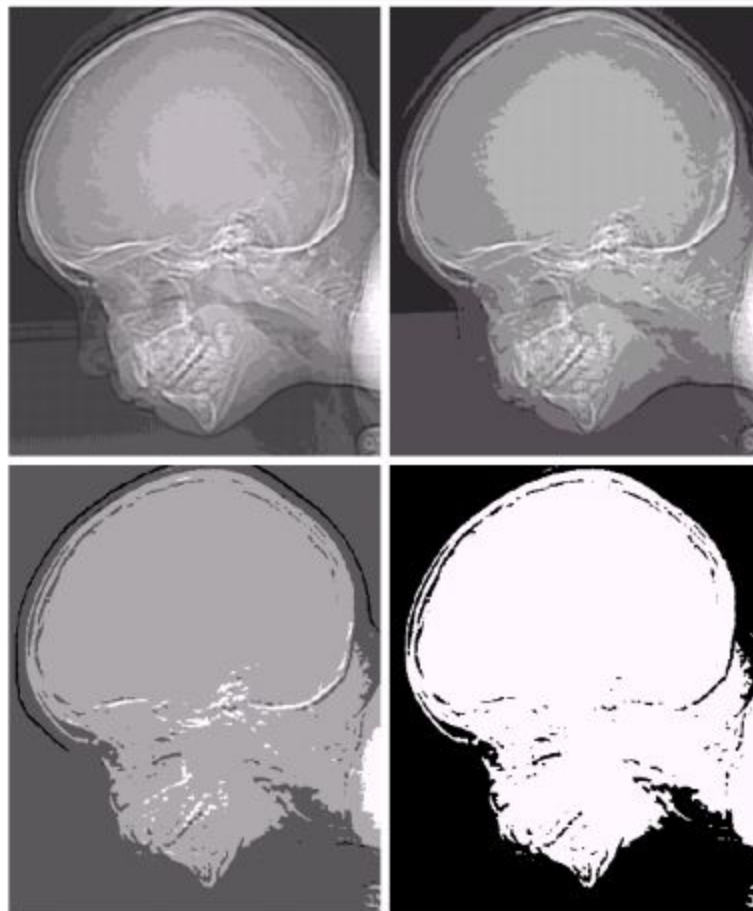
**FIGURE 2.21**

(a)  $452 \times 374$ ,  
256-level image.  
(b)–(d) Image  
displayed in 128,  
64, and 32 gray  
levels, while  
keeping the  
spatial resolution  
constant.



e f  
g h

**FIGURE 2.21**  
*(Continued)*  
(e)–(h) Image  
displayed in 16, 8,  
4, and 2 gray  
levels. (Original  
courtesy of  
Dr. David  
R. Pickens,  
Department of  
Radiology &  
Radiological  
Sciences,  
Vanderbilt  
University  
Medical Center.)



- Spatial & Intensity Resolutions: Together
  - Vary  $N$  (Spatial Resolution  $N \times N$ ) and  $k$  (Grayscale Resolution  $2^k$ )
  - Subjects rank images at different  $\langle N, k \rangle$  by their subjective quality
  - Experiments:
    - Nature depends on details in the image

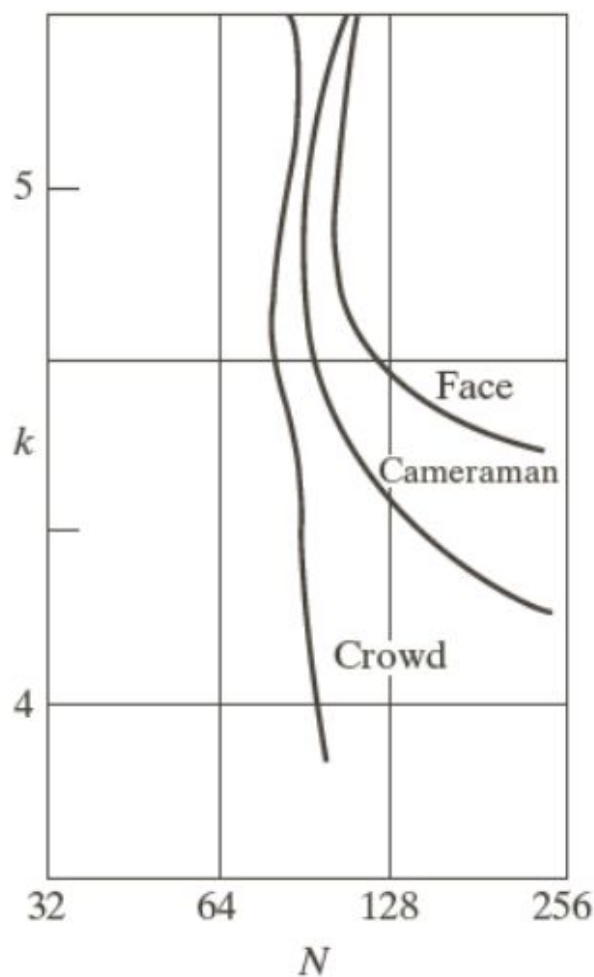


a b c

**FIGURE 2.22** (a) Image with a low level of detail. (b) Image with a medium level of detail. (c) Image with a relatively large amount of detail. (Image (b) courtesy of the Massachusetts Institute of Technology.)

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- Isopreference Curves on N-k Plane
  - Points having same subjective quality
  - I-p Curves shifts right & upward
  - Becomes more vertical to left with details in image



**FIGURE 2.23**

Typical isopreference curves for the three types of images in Fig. 2.22.

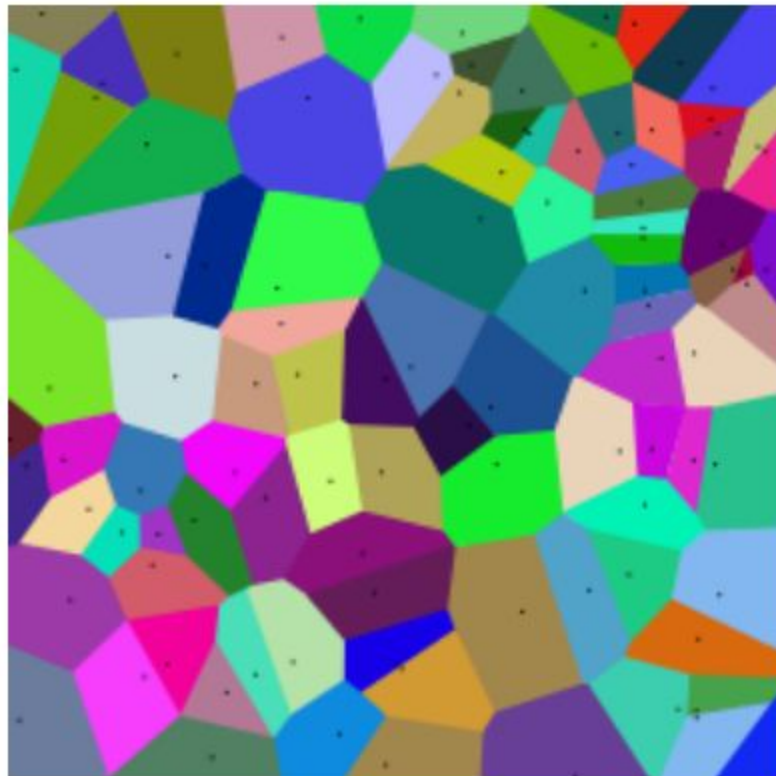
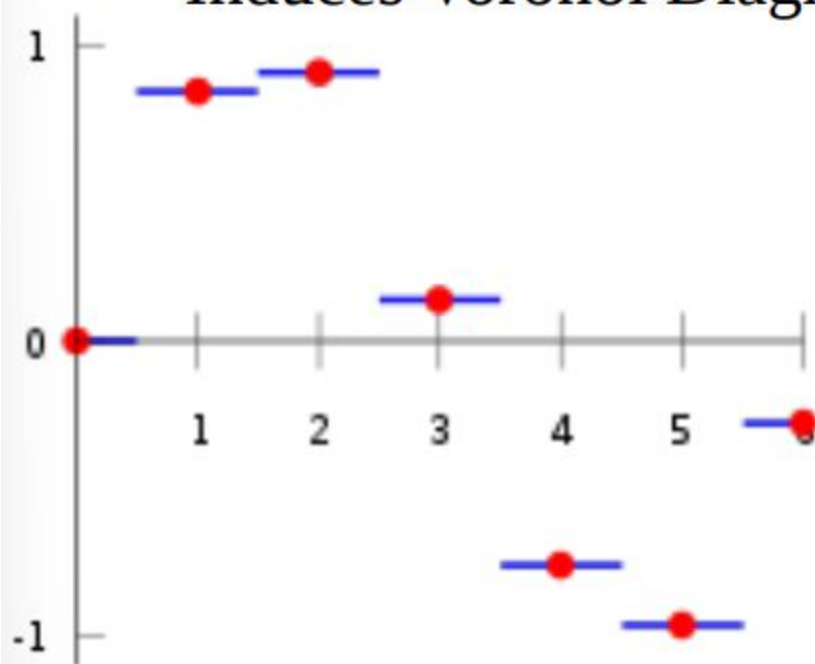
- Image Interpolation (Tool)
  - Zooming
  - Shrinking
  - Rotating
  - Geometric Correction
- Zooming & Shrinking
  - Re-sampling

- Enlargement by 1.5 times
  - Image is 500 X 500
  - Imaginary Array: 750 X 750 (same pixel spacing)
  - Shrink to fit the original image (reduced pixel spacing)
  - Set value from the closest pixel in original image
  - Expand it to the original expanded size
  - This is **Nearest Neighbor Interpolation**
    - Produces artifacts (distortion of straight edge)



- **Nearest Neighbor Interpolation**

- Induces Voronoi Diagram



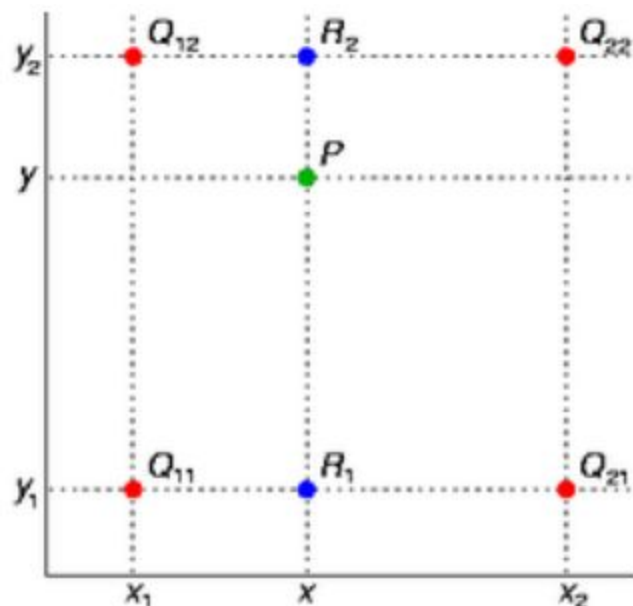


- **Bilinear Interpolation**
  - Use 4 nearest neighbor
  - Solve for 4 coefficients
  - Product of 2 linear functions

$$f(R_1) \approx \frac{x_2 - x}{x_2 - x_1} f(Q_{11}) + \frac{x - x_1}{x_2 - x_1} f(Q_{21})$$

$$f(R_2) \approx \frac{x_2 - x}{x_2 - x_1} f(Q_{12}) + \frac{x - x_1}{x_2 - x_1} f(Q_{22})$$

$$f(P) \approx \frac{y_2 - y}{y_2 - y_1} f(R_1) + \frac{y - y_1}{y_2 - y_1} f(R_2).$$

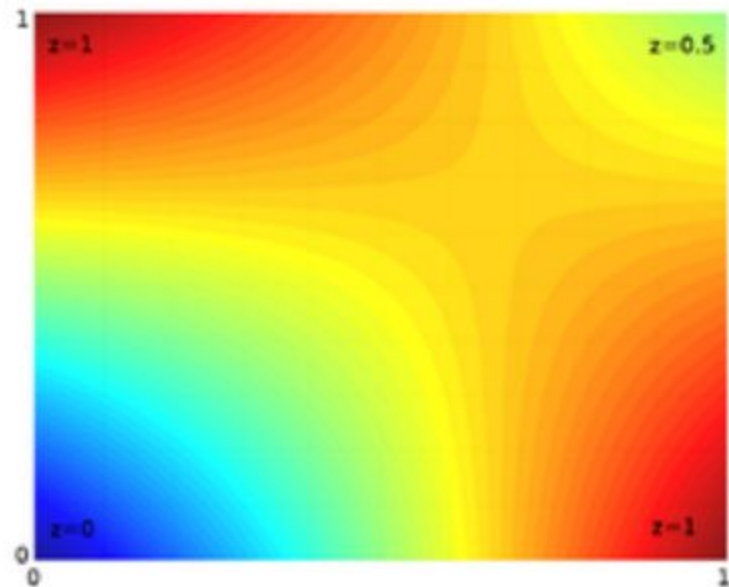
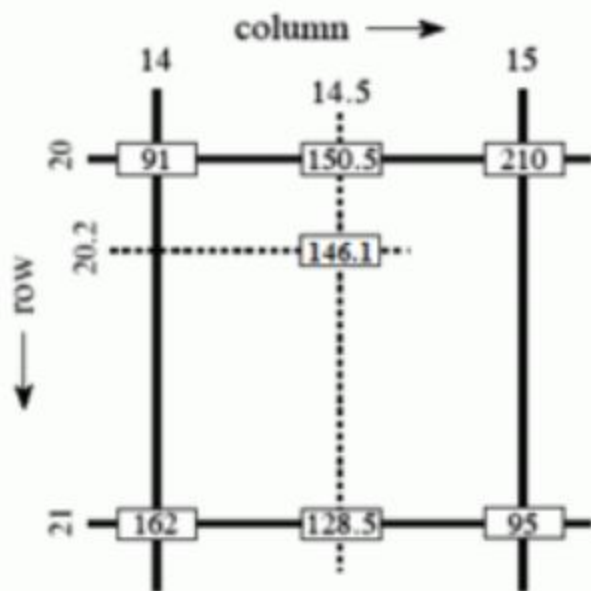


$$\begin{aligned} f(x, y) \approx & \frac{f(Q_{11})}{(x_2 - x_1)(y_2 - y_1)} (x_2 - x)(y_2 - y) \\ & + \frac{f(Q_{21})}{(x_2 - x_1)(y_2 - y_1)} (x - x_1)(y_2 - y) \\ & + \frac{f(Q_{12})}{(x_2 - x_1)(y_2 - y_1)} (x_2 - x)(y - y_1) \\ & + \frac{f(Q_{22})}{(x_2 - x_1)(y_2 - y_1)} (x - x_1)(y - y_1). \end{aligned}$$

# • Bilinear Interpolation

$$f(x, y) \approx f(0, 0)(1-x)(1-y) + f(1, 0)x(1-y) + f(0, 1)(1-x)y + f(1, 1)xy.$$

$$f(x, y) \approx \begin{bmatrix} 1-x & x \end{bmatrix} \begin{bmatrix} f(0, 0) & f(0, 1) \\ f(1, 0) & f(1, 1) \end{bmatrix} \begin{bmatrix} 1-y \\ y \end{bmatrix}.$$



- **Bilinear Interpolation**

1	3	6	1
3	3	1	7
8	1	8	7
6	1	1	1

**Raster dataset**

1	3	6	1
3	3	1	7
8	1	8	7
6	1	1	1

**Nearest Neighbor  
resampling**

3	3	4	4
4	3	6	6
4	3	4	4
4	2	3	4

**Bilinear  
Interpolation**

- **Bicubic Interpolation**

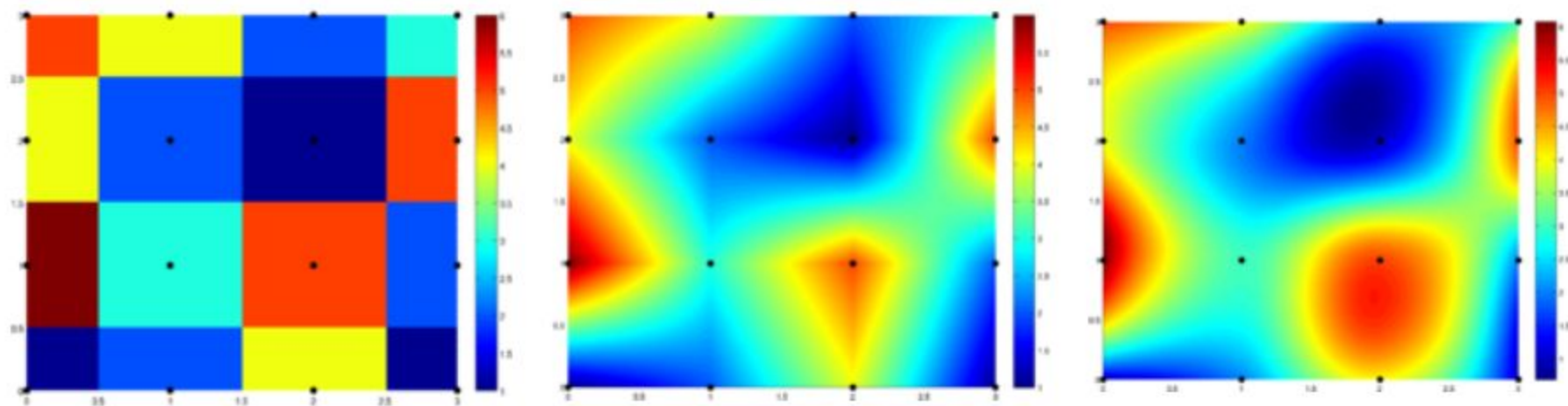
- The function values  $f$  and the derivatives  $f_x$ ,  $f_y$  and  $f_{xy}$  are known at the four corners  $(0,0)$ ,  $(1,0)$ ,  $(0,1)$ , and  $(1,1)$  of the unit square. The interpolated surface can then be written

$$p(x, y) = \sum_{i=0}^3 \sum_{j=0}^3 a_{ij} x^i y^j.$$

- Yields a surface  $p(x,y)$  on the unit square which is continuous and with continuous derivatives

- **Bicubic Interpolation (Discrete)**
  - Use 16 nearest neighbor
  - Solve for 16 coefficients
  - Used in Adobe Photoshop / Corel Photopaint

- **Interpolation Comparative**

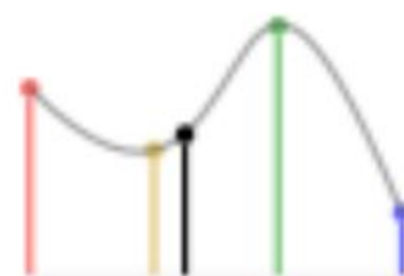




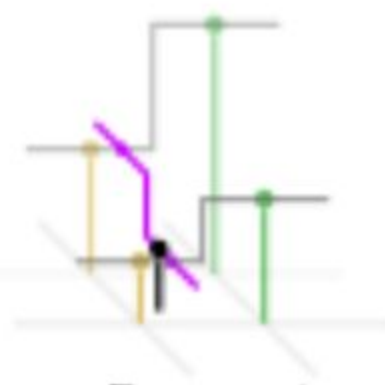
1D nearest-  
neighbour



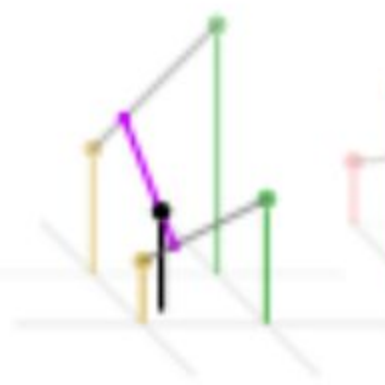
Linear



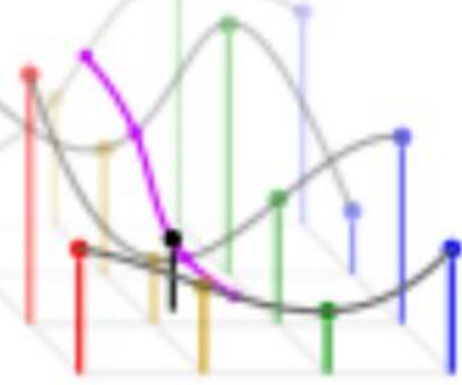
Cubic



2D nearest-  
neighbour



Bilinear



Bicubic



# Mathematical Tools used in DIP

Array versus Matrix operations

Linear vs Non-linear operations

Arithmetic operations

Set and Logical operations

Spatial operations

Vector and Matrix operations

Image transforms

Probabilistic methods

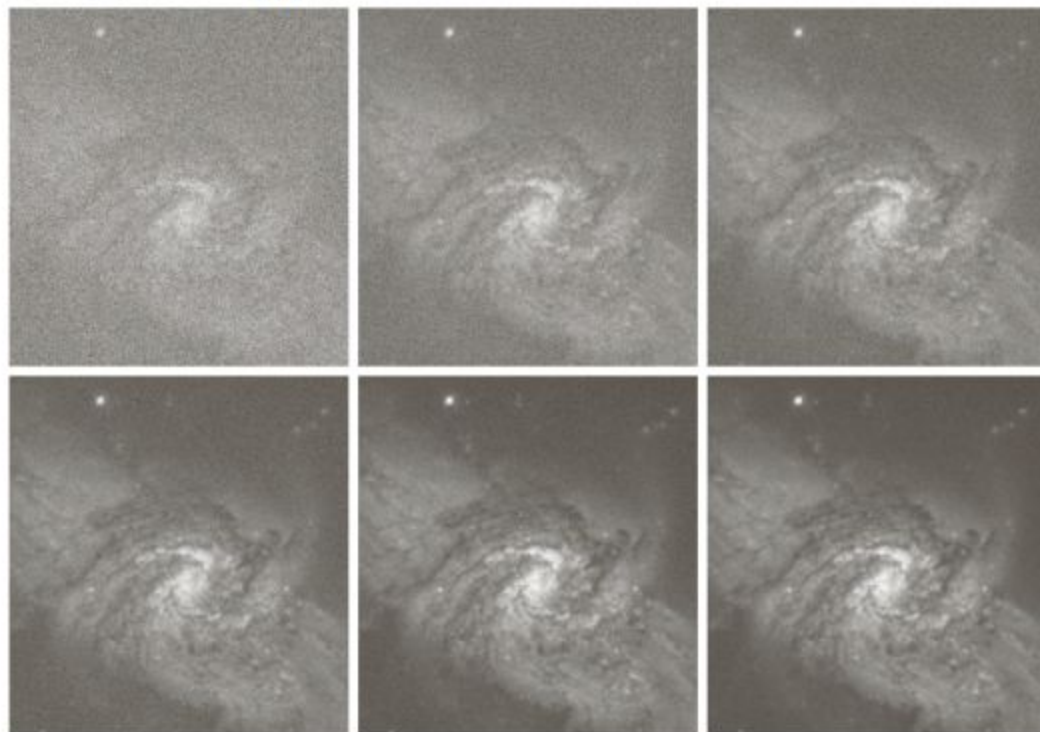


Linear vs Non-linear operations:

Linear operation: Sum operation over images

Non-linear operation: Max operation over images

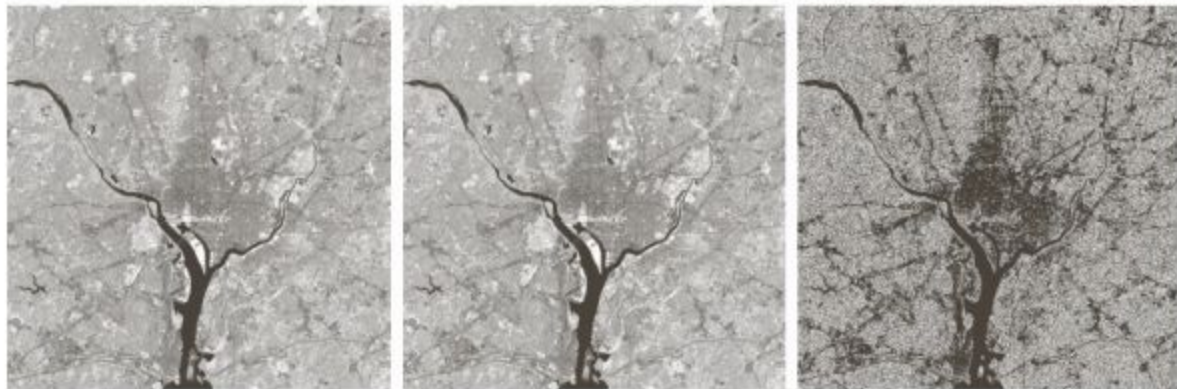
## Addition operation over Images



a b c  
d e f

**FIGURE 2.26** (a) Image of Galaxy Pair NGC 3314 corrupted by additive Gaussian noise. (b)–(f) Results of averaging 5, 10, 20, 50, and 100 noisy images, respectively. (Original image courtesy of NASA.)

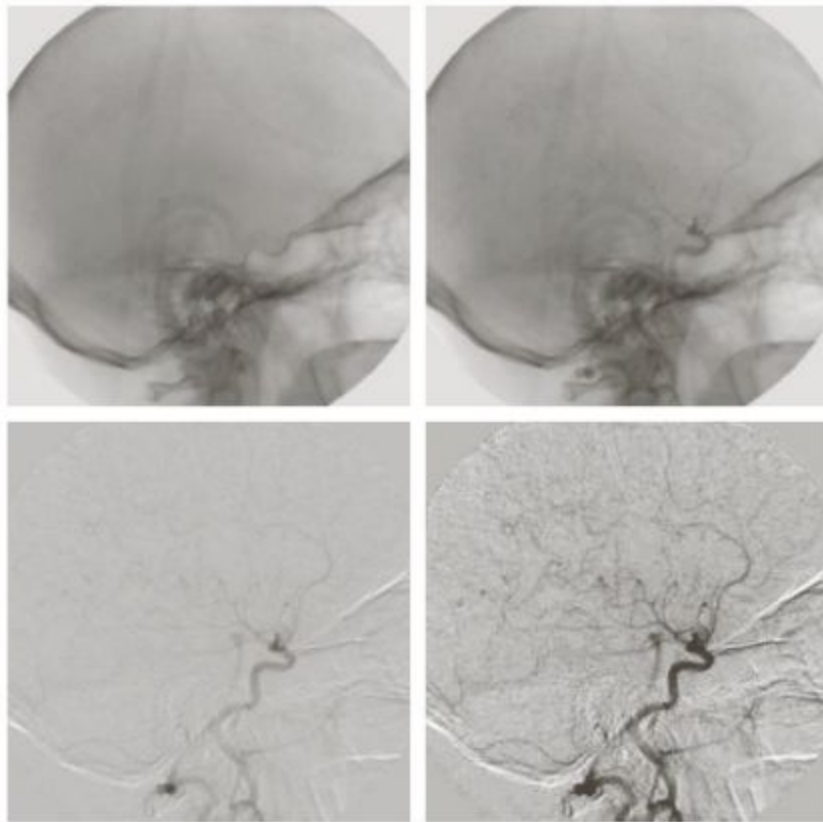
## Difference operation over Images



a b c

**FIGURE 2.27** (a) Infrared image of the Washington, D.C. area. (b) Image obtained by setting to zero the least significant bit of every pixel in (a). (c) Difference of the two images, scaled to the range  $[0, 255]$  for clarity.

## Difference operation over Images

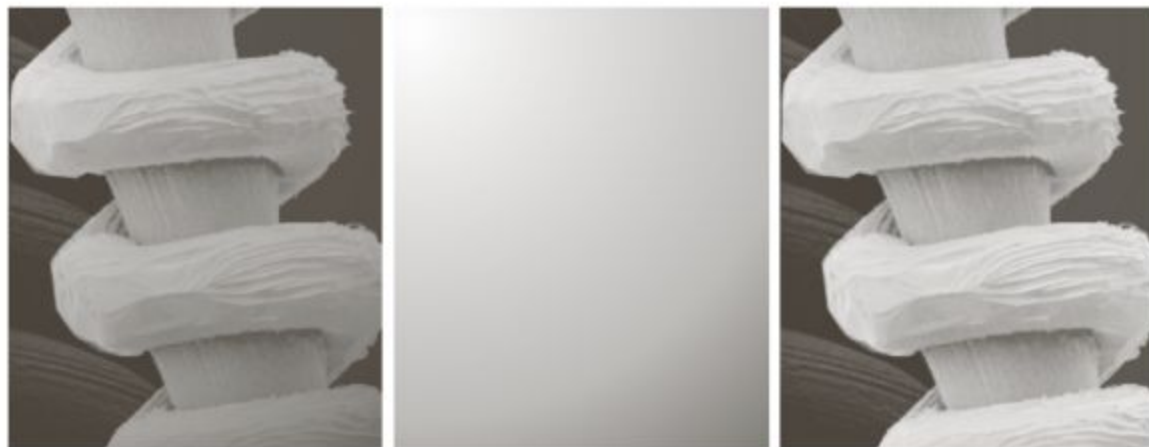


a b  
c d

**FIGURE 2.28**

Digital subtraction angiography. (a) Mask image. (b) A live image. (c) Difference between (a) and (b). (d) Enhanced difference image. (Figures (a) and (b) courtesy of The Image Sciences Institute, University Medical Center, Utrecht, The Netherlands.)

## Division operation over Images



a b c

**FIGURE 2.29** Shading correction. (a) Shaded SEM image of a tungsten filament and support, magnified approximately 130 times. (b) The shading pattern. (c) Product of (a) by the reciprocal of (b). (Original image courtesy of Mr. Michael Shaffer, Department of Geological Sciences, University of Oregon, Eugene.)

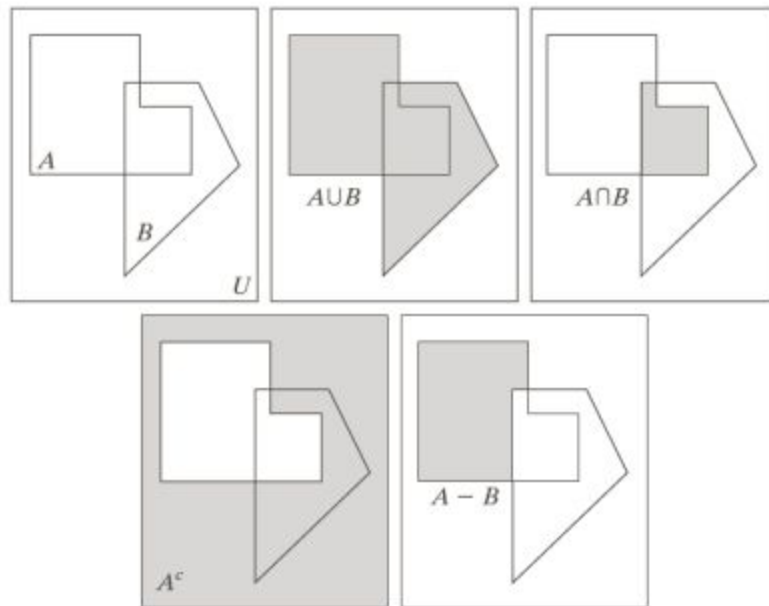
## Multiplication operation over Images



a b c

**FIGURE 2.30** (a) Digital dental X-ray image. (b) ROI mask for isolating teeth with fillings (white corresponds to 1 and black corresponds to 0). (c) Product of (a) and (b).

## Set operations over binary images

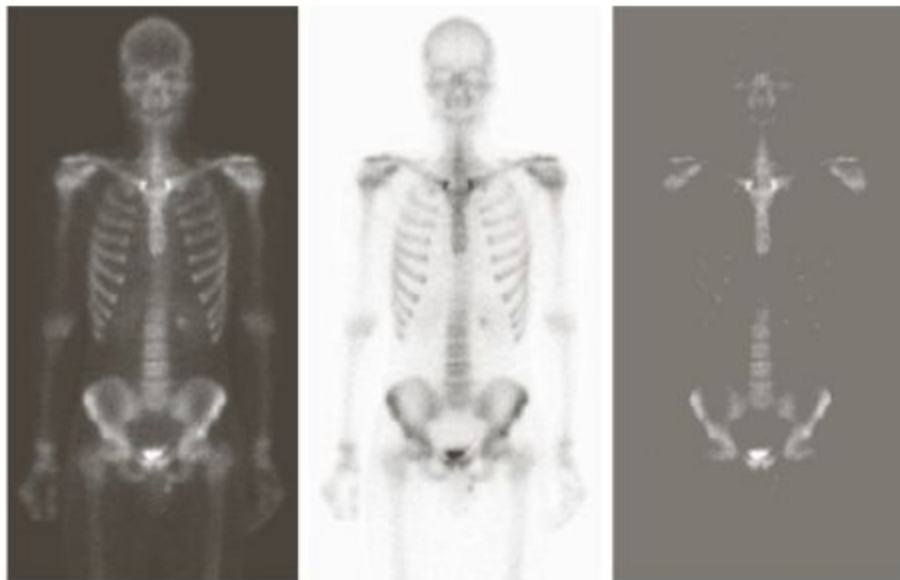


a	b	c
d	e	

**FIGURE 2.31**

(a) Two sets of coordinates,  $A$  and  $B$ , in 2-D space. (b) The union of  $A$  and  $B$ . (c) The intersection of  $A$  and  $B$ . (d) The complement of  $A$ . (e) The difference between  $A$  and  $B$ . In (b)–(e) the shaded areas represent the member of the set operation indicated.

## Set operations over Gray scale Images

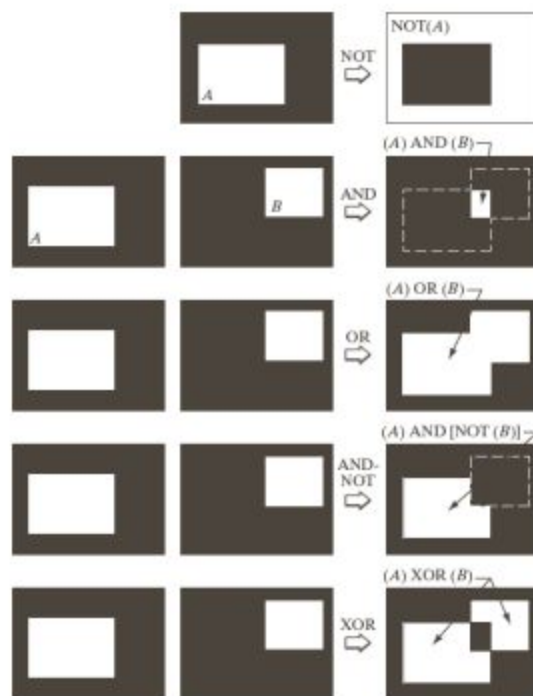


a b c

**FIGURE 2.32** Set operations involving gray-scale images. (a) Original image. (b) Image negative obtained using set complementation. (c) The union of (a) and a constant image. (Original image courtesy of G.E. Medical Systems.)

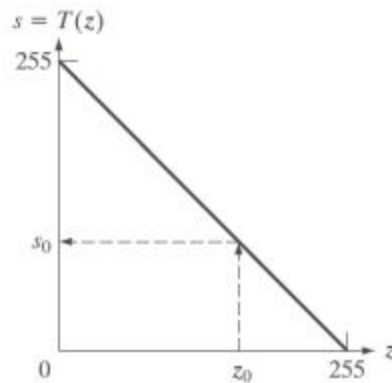


## Logical operations over Binary Images



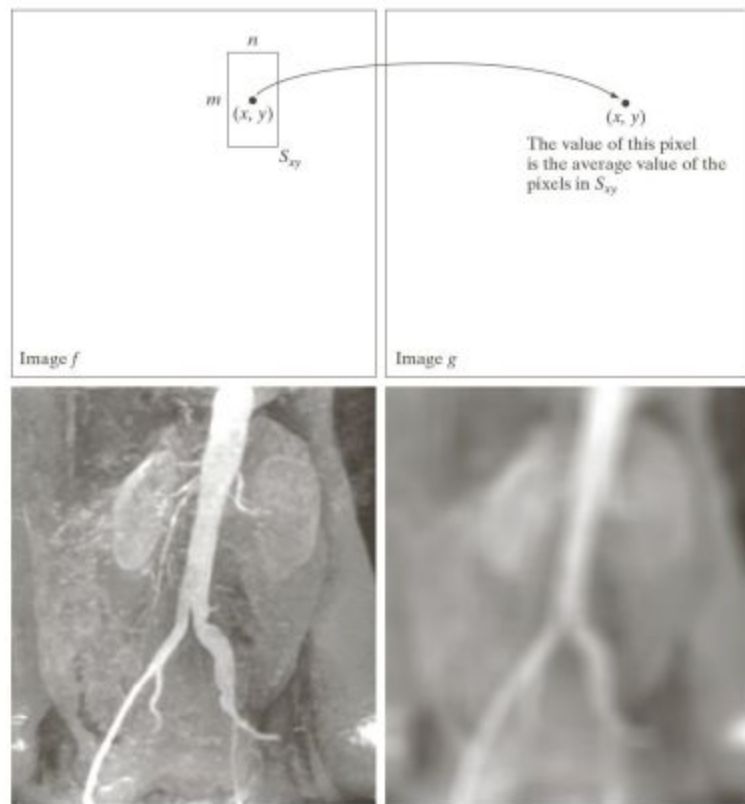
**FIGURE 2.33**  
Illustration of logical operations involving foreground (white) pixels. Black represents binary 0s and white binary 1s. The dashed lines are shown for reference only. They are not part of the result.

## Spatial operation over Images (Single pixel operations)



**FIGURE 2.34** Intensity transformation function used to obtain the negative of an 8-bit image. The dashed arrows show transformation of an arbitrary input intensity value  $z_0$  into its corresponding output value  $s_0$ .

## Spatial operation over Images: Neighbourhood operations



a b  
c d

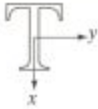




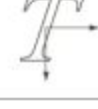
**FIGURE 2.35**

Local averaging using neighborhood processing. The procedure is illustrated in (a) and (b) for a rectangular neighborhood. (c) The aortic angiogram discussed in Section 1.3.2. (d) The result of using Eq. (2.6-21) with  $m = n = 41$ . The images are of size  $790 \times 686$  pixels.

# Geometric Spatial Transformation

**TABLE 2.2**

Affine transformations based on Eq. (2.6.–23).

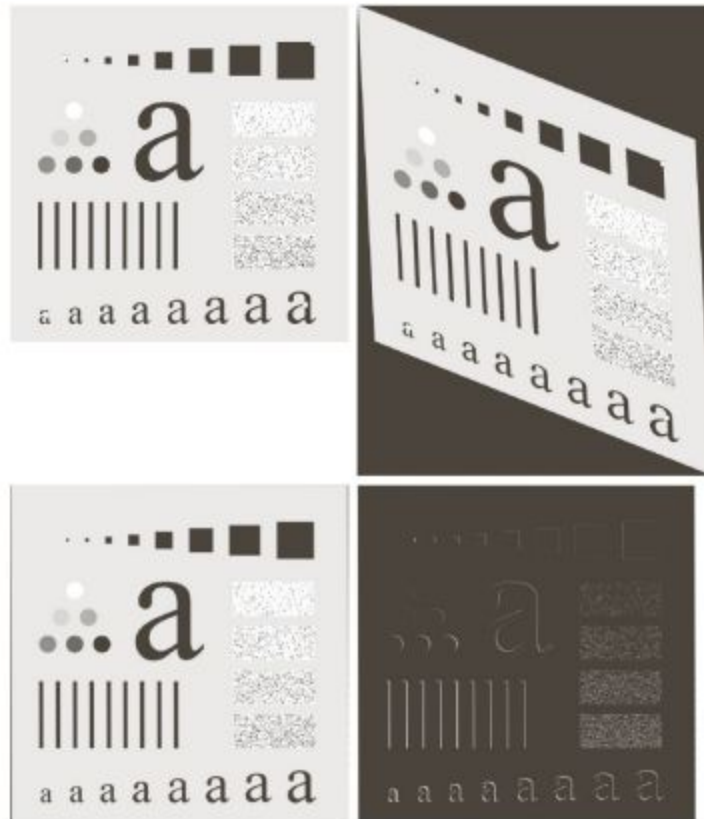
Transformation Name	Affine Matrix, T	Coordinate Equations	Example
Identity	$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$\begin{aligned} x &= v \\ y &= w \end{aligned}$	
Scaling	$\begin{bmatrix} c_x & 0 & 0 \\ 0 & c_y & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$\begin{aligned} x &= c_x v \\ y &= c_y w \end{aligned}$	
Rotation	$\begin{bmatrix} \cos \theta & \sin \theta & 0 \\ -\sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$\begin{aligned} x &= v \cos \theta - w \sin \theta \\ y &= v \sin \theta + w \cos \theta \end{aligned}$	
Translation	$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ t_x & t_y & 1 \end{bmatrix}$	$\begin{aligned} x &= v + t_x \\ y &= w + t_y \end{aligned}$	
Shear (vertical)	$\begin{bmatrix} 1 & 0 & 0 \\ s_v & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$\begin{aligned} x &= v + s_v w \\ y &= w \end{aligned}$	
Shear (horizontal)	$\begin{bmatrix} 1 & s_h & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$\begin{aligned} x &= v \\ y &= s_h v + w \end{aligned}$	

## Image Rotation



**FIGURE 2.36** (a) A 300 dpi image of the letter T. (b) Image rotated  $21^\circ$  clockwise using nearest neighbor interpolation to assign intensity values to the spatially transformed pixels. (c) Image rotated  $21^\circ$  using bilinear interpolation. (d) Image rotated  $21^\circ$  using bicubic interpolation. The enlarged sections show edge detail for the three interpolation approaches.

# Image Registration

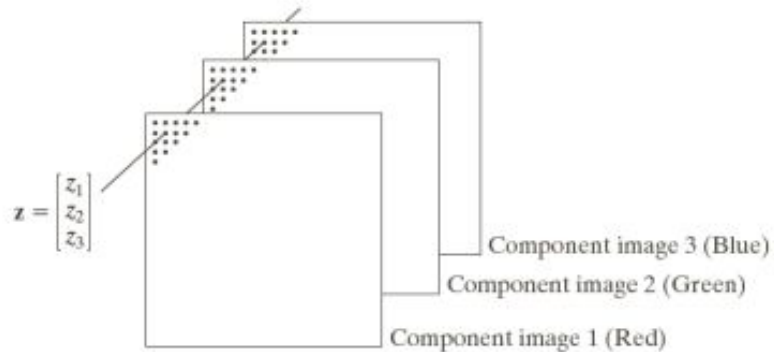


a	b
c	d

**FIGURE 2.37**

Image registration. (a) Reference image. (b) Input (geometrically distorted image). Corresponding tie points are shown as small white squares near the corners. (c) Registered image (note the errors in the borders). (d) Difference between (a) and (c), showing more registration errors.

# Matrix & Vector Operations



**FIGURE 2.38**

Formation of a vector from corresponding pixel values in three RGB component images.

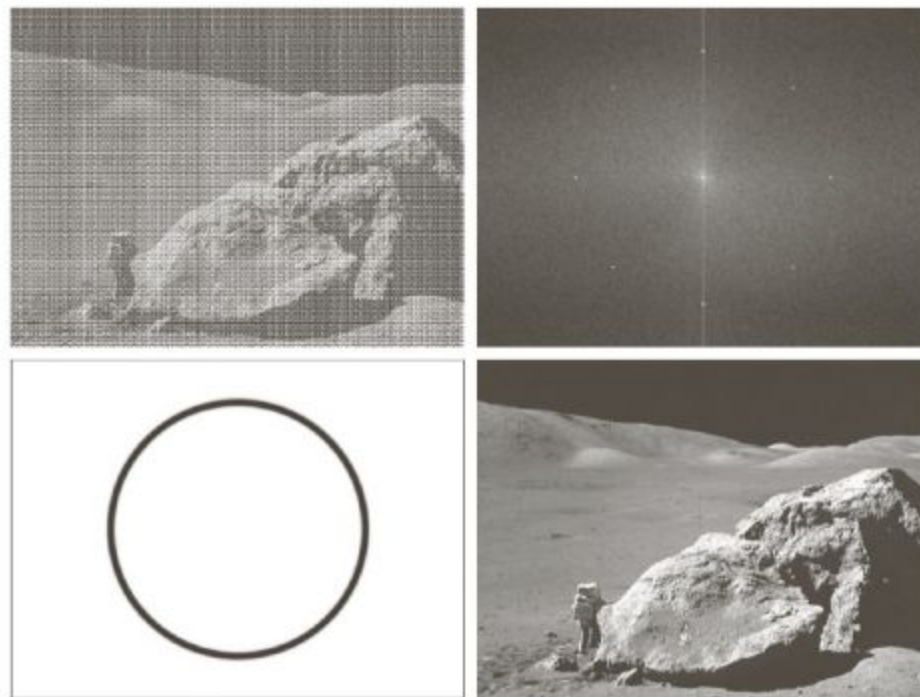
# Image Transforms



**FIGURE 2.39**  
General approach  
for operating in  
the linear  
transform  
domain.



# Image enhancement through Fourier Transformation



a b  
c d

**FIGURE 2.40**

(a) Image corrupted by sinusoidal interference. (b) Magnitude of the Fourier transform showing the bursts of energy responsible for the interference. (c) Mask used to eliminate the energy bursts. (d) Result of computing the inverse of the modified Fourier transform. (Original image courtesy of NASA.)

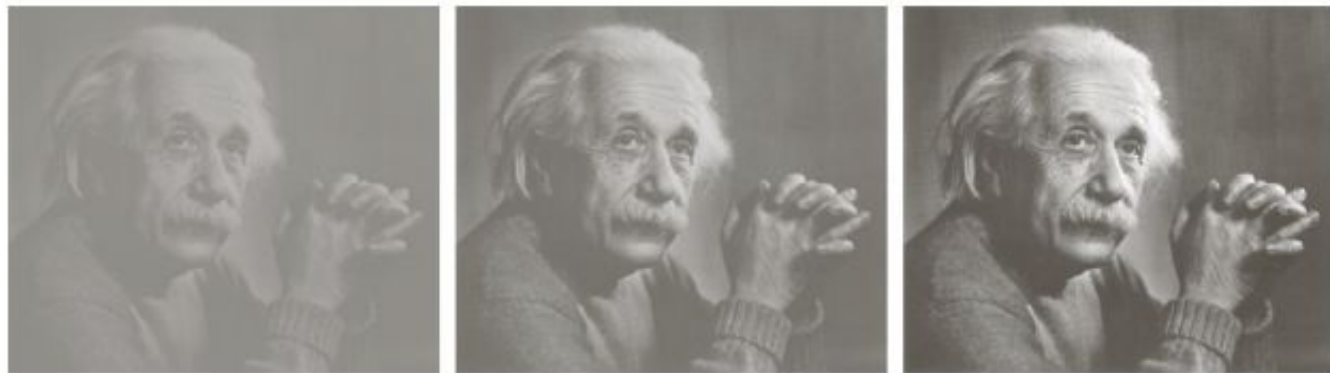
# Probabilistic models:

Probability of specific intensity level

Mean intensity, variance & SD of intensity, higher order moments of intensity

Stochastic Image Processing: Sequence of images & probability of intensity values

Random fields: Entire image to be a spatial random event



a b c

**FIGURE 2.41**

Images exhibiting (a) low contrast, (b) medium contrast, and (c) high contrast.