**Course Syllabus**

1. **Color**
2. **Camera models, camera calibration**
3. **Advanced image pre-processing**
   - **Line detection**
   - **Corner detection**
   - **Maximally stable extremal regions**
4. **Mathematical Morphology**
   - **binary**
   - **gray-scale**
   - **skeletonization**
   - **granulometry**
   - **morphological segmentation**
   - **Scale in image processing**
5. **Wavelet theory in image processing**
6. **Image Compression**
7. **Texture**
8. **Image Registration**
   - **rigid**
   - **non-rigid**
   - **RANSAC**

# References

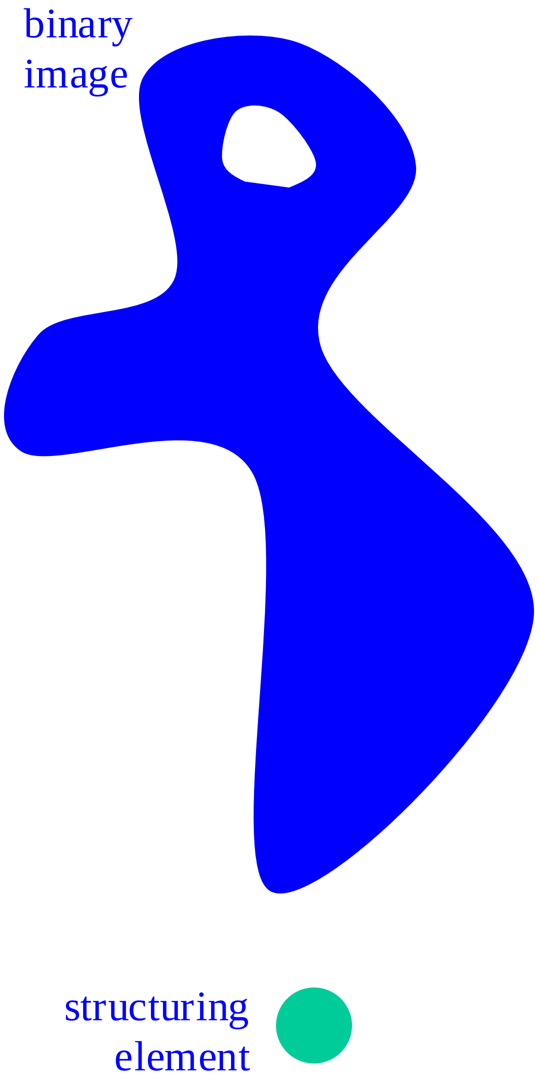
- Books:
  - Chapter 11, Image Processing, Analysis, and Machine Vision, Sonka et al
  - Chapter 9, Digital Image Processing, Gonzalez & Woods

## Topics

1. Basic Morphological concepts
2. Binary Morphological operations
   - Dilation & erosion
   - Hit-or-miss transformation
   - Opening & closing
3. Gray scale morphological operations
4. Some basic morphological operations
   - Boundary extraction
   - Region filling
   - Extraction of connected component
   - Convex hull
5. Skeletonization
6. Granularity
7. Morphological segmentation and watersheds

## Introduction

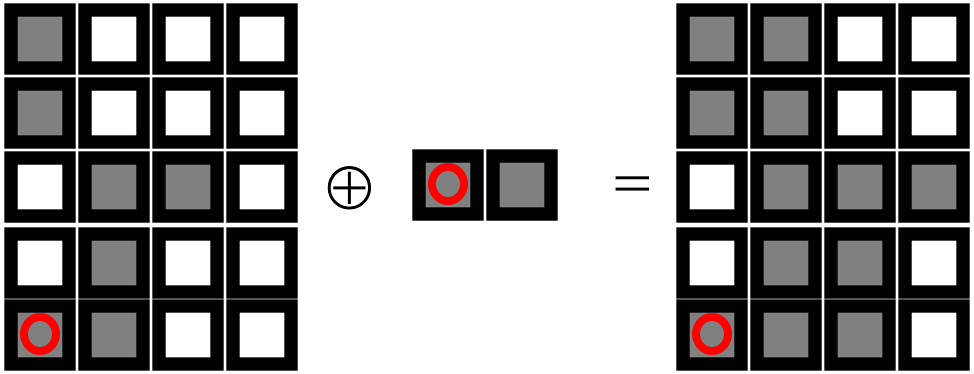
1. Morphological operators often take a binary image and a structuring element as input and combine them using a set operator (intersection, union, inclusion, complement).
2. The structuring element is shifted over the image and at each pixel of the image its elements are compared with the set of the underlying pixels.
3. If the two sets of elements match the condition defined by the set operator (e.g. if set of pixels in the structuring element is a subset of the underlying image pixels), the pixel underneath the origin of the structuring element is set to a pre-defined value (0 or 1 for binary images).
4. A morphological operator is therefore defined by its structuring element and the applied set operator.
5. Image pre-processing (noise filtering, shape simplification)
6. Enhancing object structures (skeletonization, thinning, convex hull, object marking)
7. Segmentation of the object from background
8. Quantitative descriptors of objects (area, perimeter, projection, Euler-Poincaré characteristics)



## Example: Morphological Operation

- Let '$\bigoplus$' denote a morphological operator

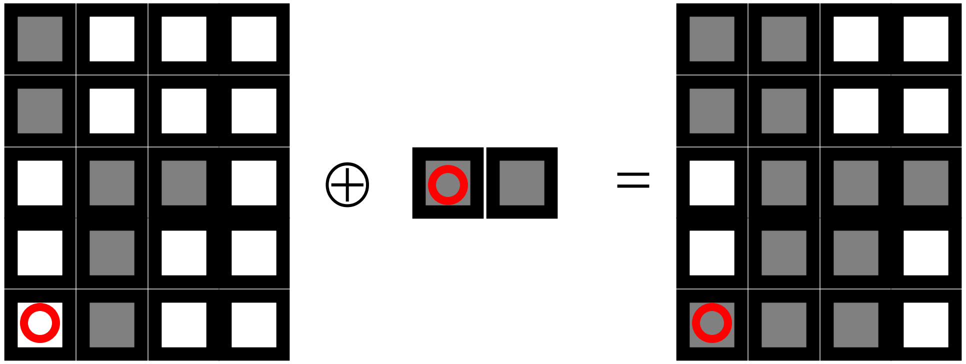
$$X \bigoplus B = \{p \in Z^2 | p = x + b, x \in X, b \in B\}$$



## Dilation

- Morphological dilation '$\oplus$' combines two sets using vector of set elements

$$X \oplus B = \{p \in Z^2 | p = x + b, x \in X, b \in B\}$$



Commutative: $X \oplus B = B \oplus X$

Associative: $X \oplus (B \oplus D) = (X \oplus B) \oplus D$

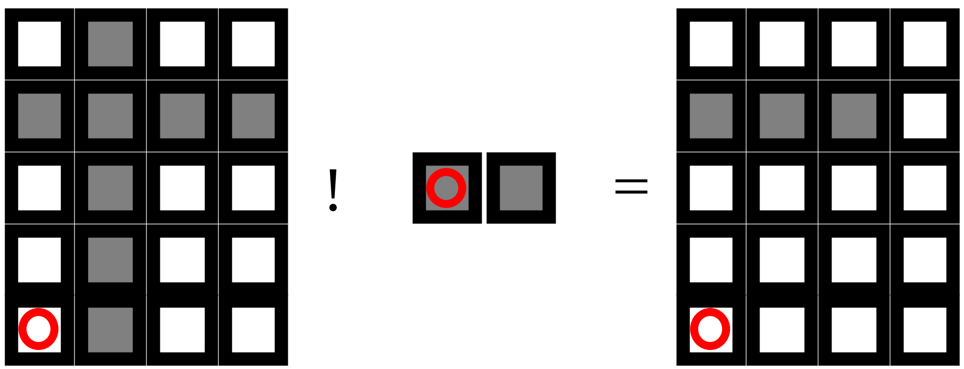
Invariant of translation: $X_h \oplus B = (X \oplus B)_h$

If $X \subseteq Y$ then $X \oplus B \subseteq Y \oplus B$

**Erosion**

1. Morphological erosion '$\ominus$' combines two sets using vector subtraction of set elements and is a dual operator of dilation

$$X \ominus B = \{p \in Z^2 | \forall b \in B, p + b \in X\}$$



Not Commutative: $X \ominus B \neq B \ominus X$

Not associative: $X \ominus (B \ominus D) = (X \ominus B) \ominus D$

Invariant of translation: $X_h \ominus B = (X \ominus B)_h$ and $X \ominus B_h = (X \ominus B)_{-h}$

If $X \subseteq Y$ then $X \ominus B \subseteq Y \ominus B$

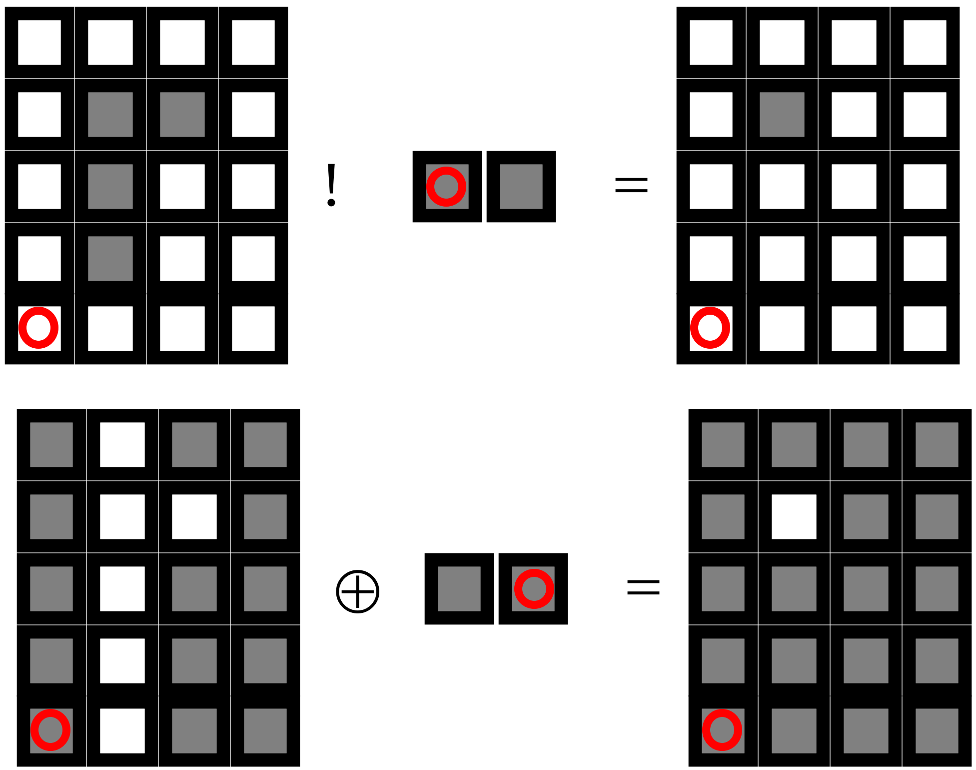
## Duality: Dilation and Erosion

- Transpose $\check{A}$ of a structuring element $A$ is defined as follows

$$\check{A} = \{-a | a \in A\}$$

- Duality between morphological dilation and erosion operators

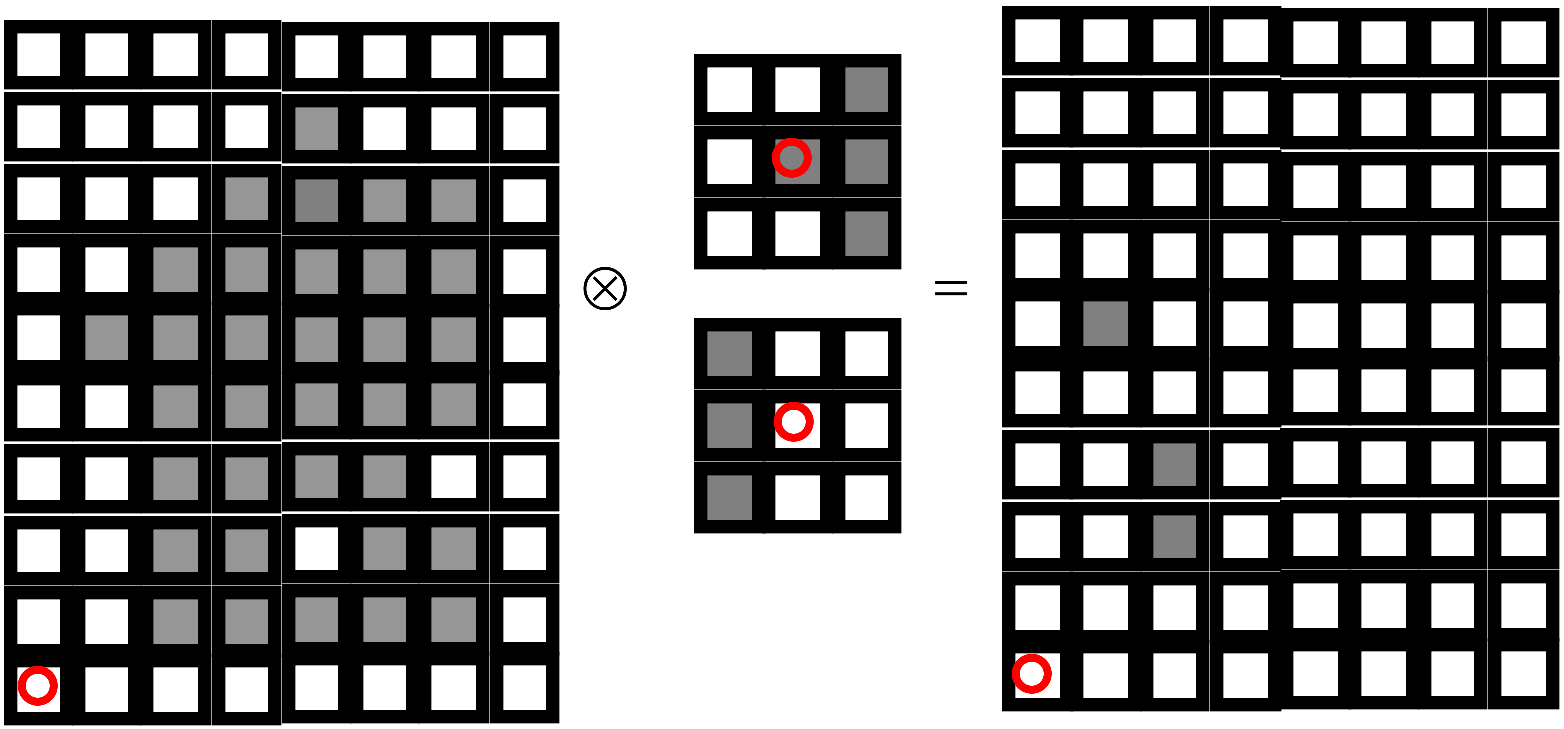
$$(X \ominus B)^c = X^c \oplus \check{B}$$



## Hit-Or-Miss transformation

- Hit-or-miss is a morphological operators for finding local patterns of pixels. Unlike dilation and erosion, this operation is defined using a composite structuring element $B = (B_1, B_2)$. The hit-or-miss operator is defined as follows

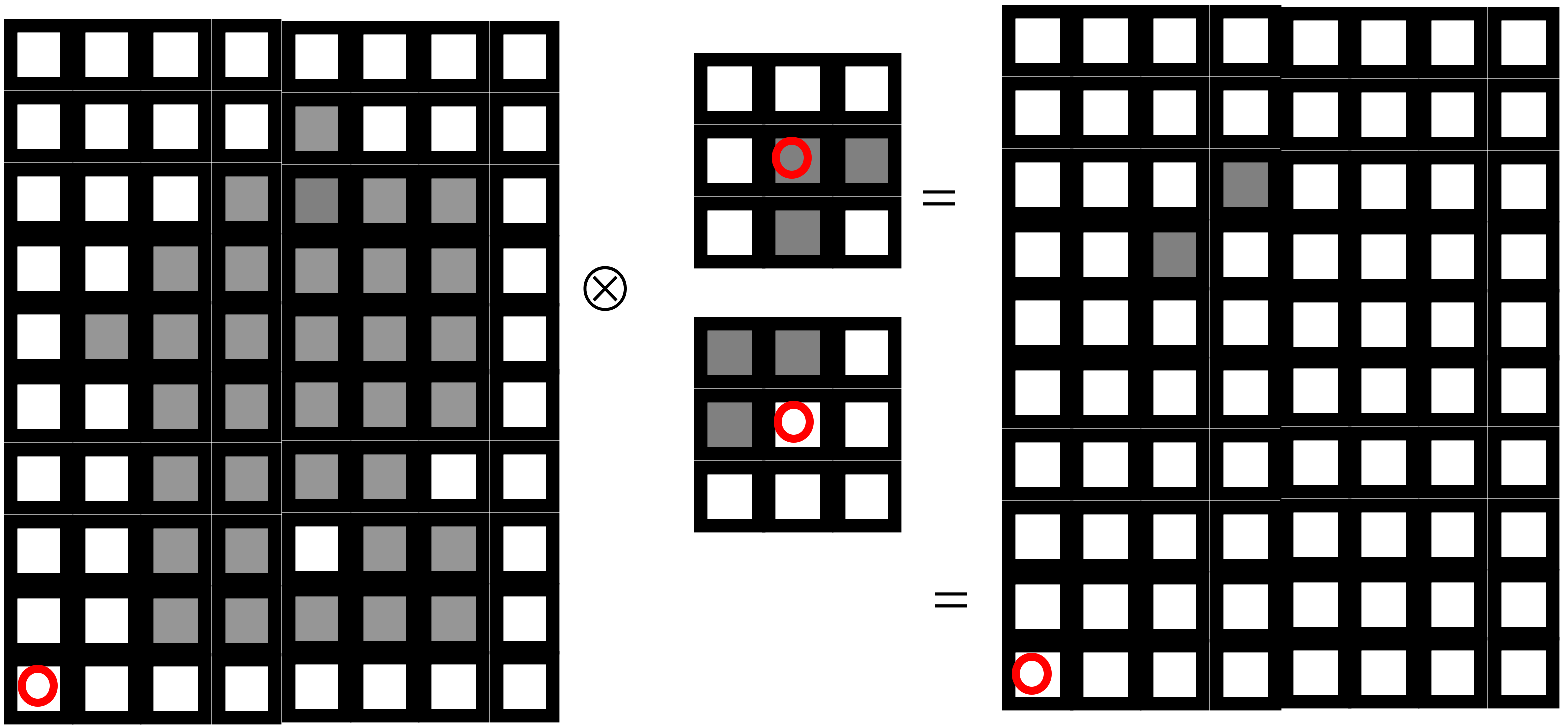
$$X \otimes B = \{x | B_1 \subset X \text{ and } B_2 \subset X^c\}$$



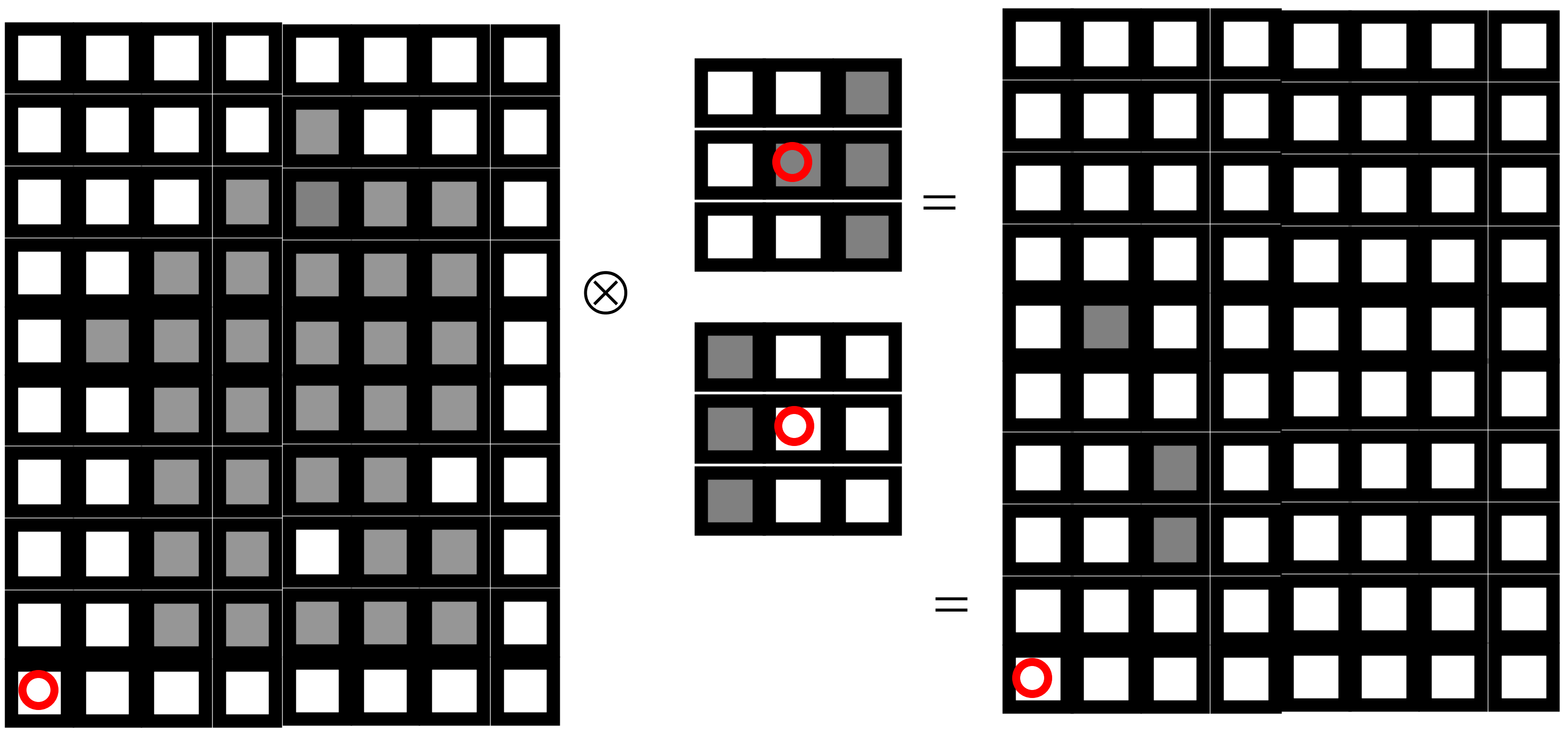
**Hit-Or-Miss transformation: another example**

Relation with erosion:

$$X \otimes B = (X \ominus B_1) \cap (X^c \ominus B_2)$$



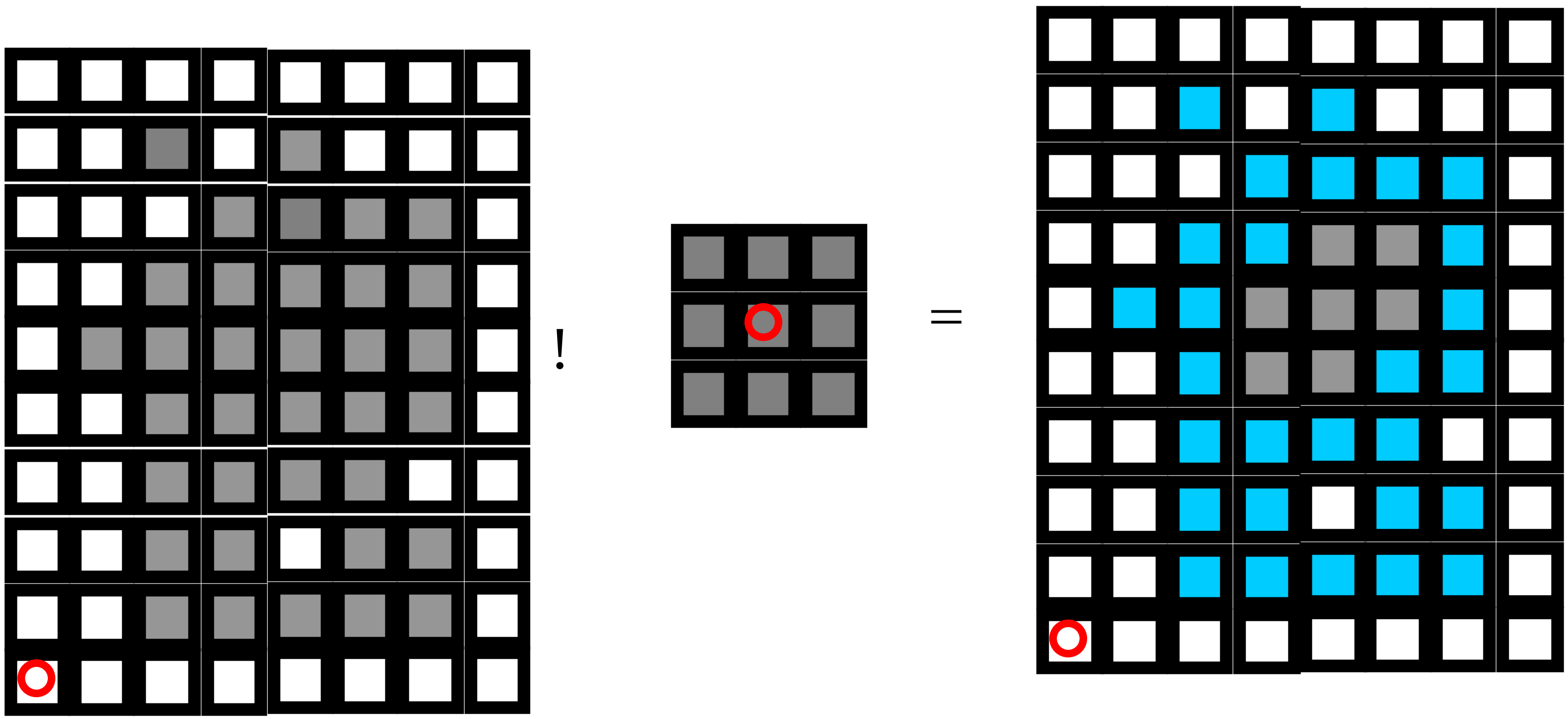
## Hit-Or-Miss transformation: yet another example



## Opening

- Erosion and dilation are not inverse transforms. An erosion followed by a dilation leads to an interesting morphological operation called **opening**

$$X \circ B = (X \ominus B) \oplus B$$



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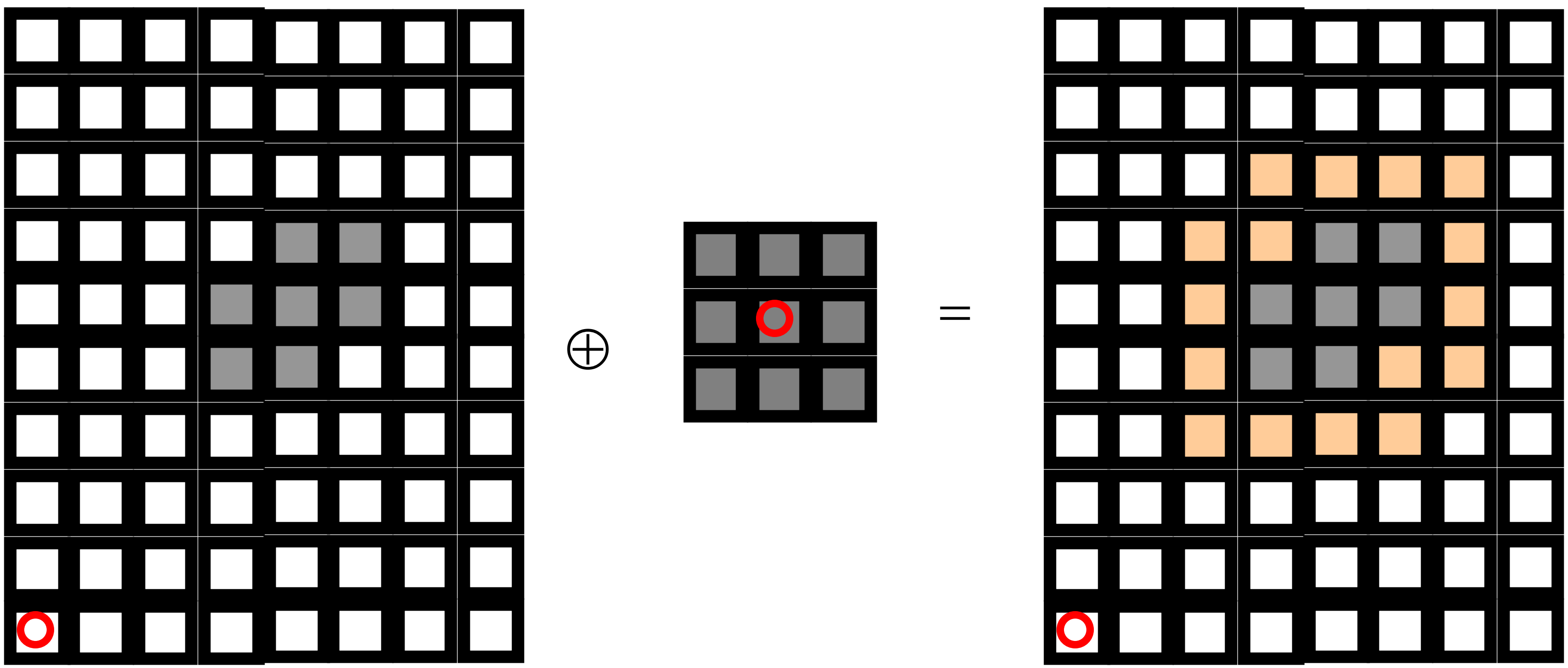

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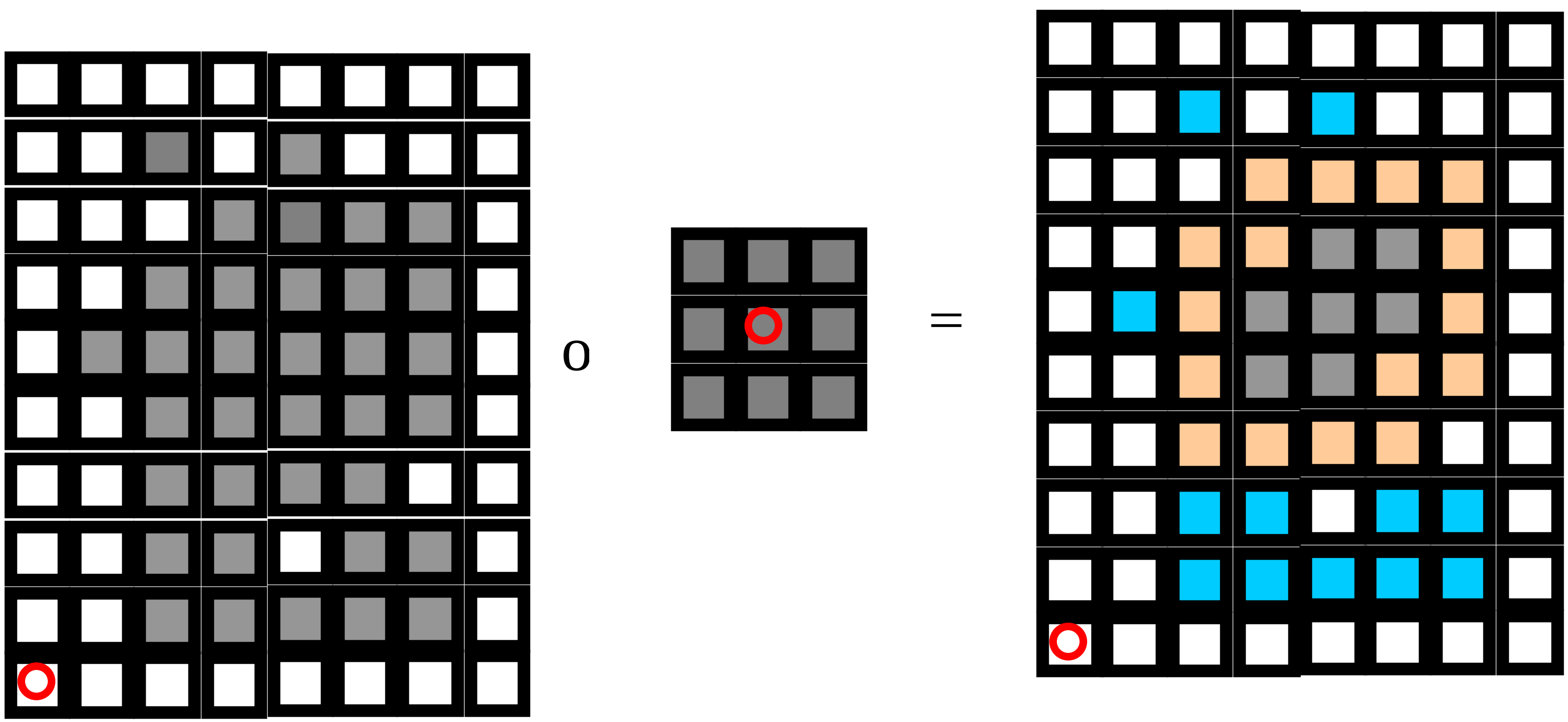

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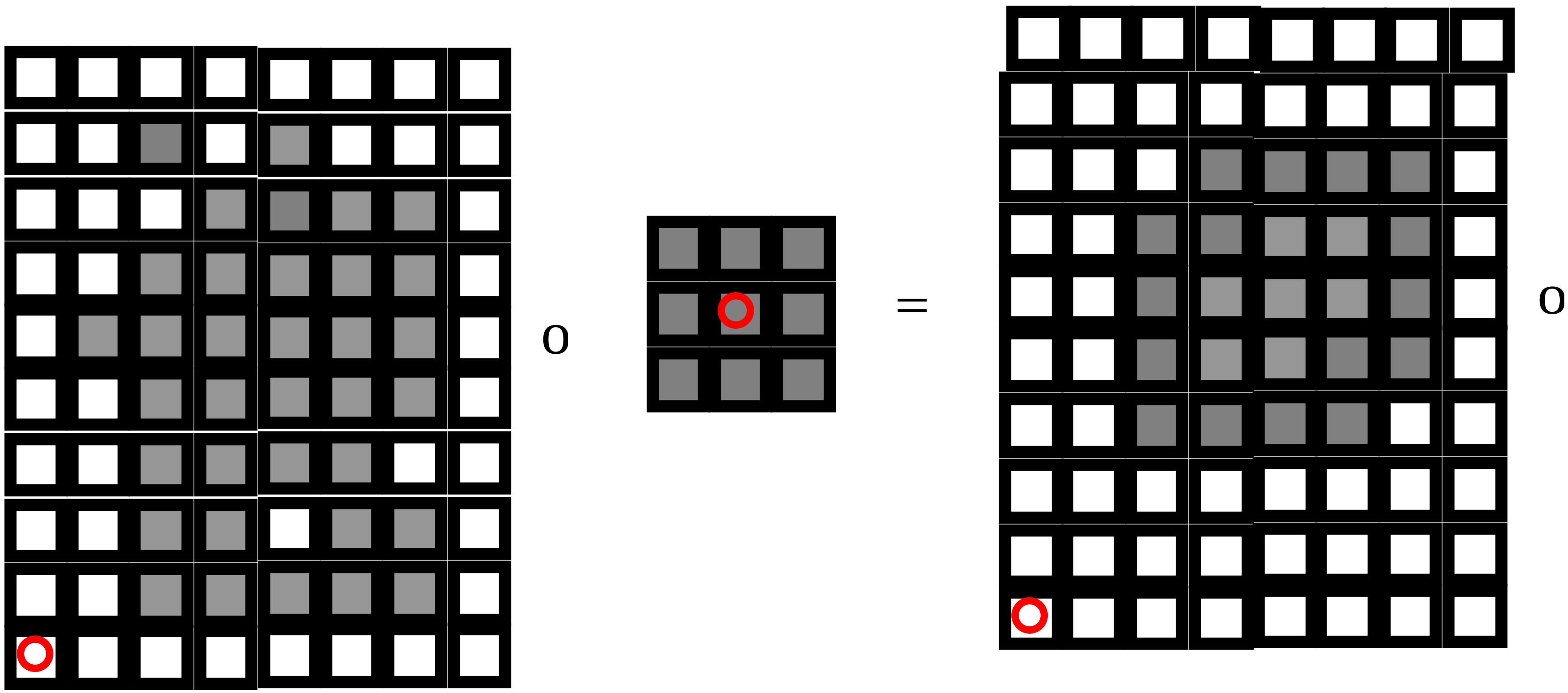
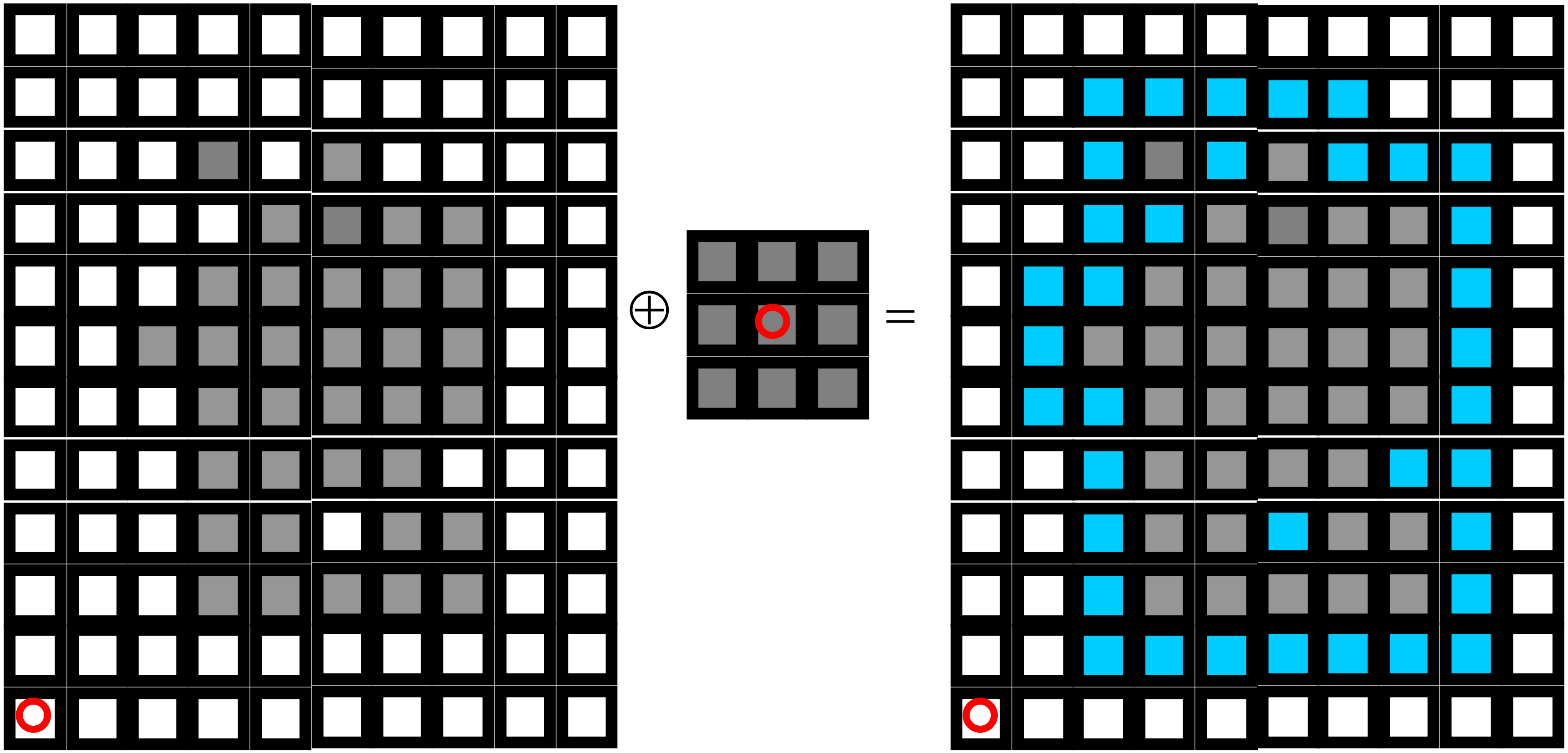


## Closing

- A dilation followed by an erosion leads to the interesting morphological operation called **closing**

$$X \bullet B = (X \oplus B) \ominus B$$



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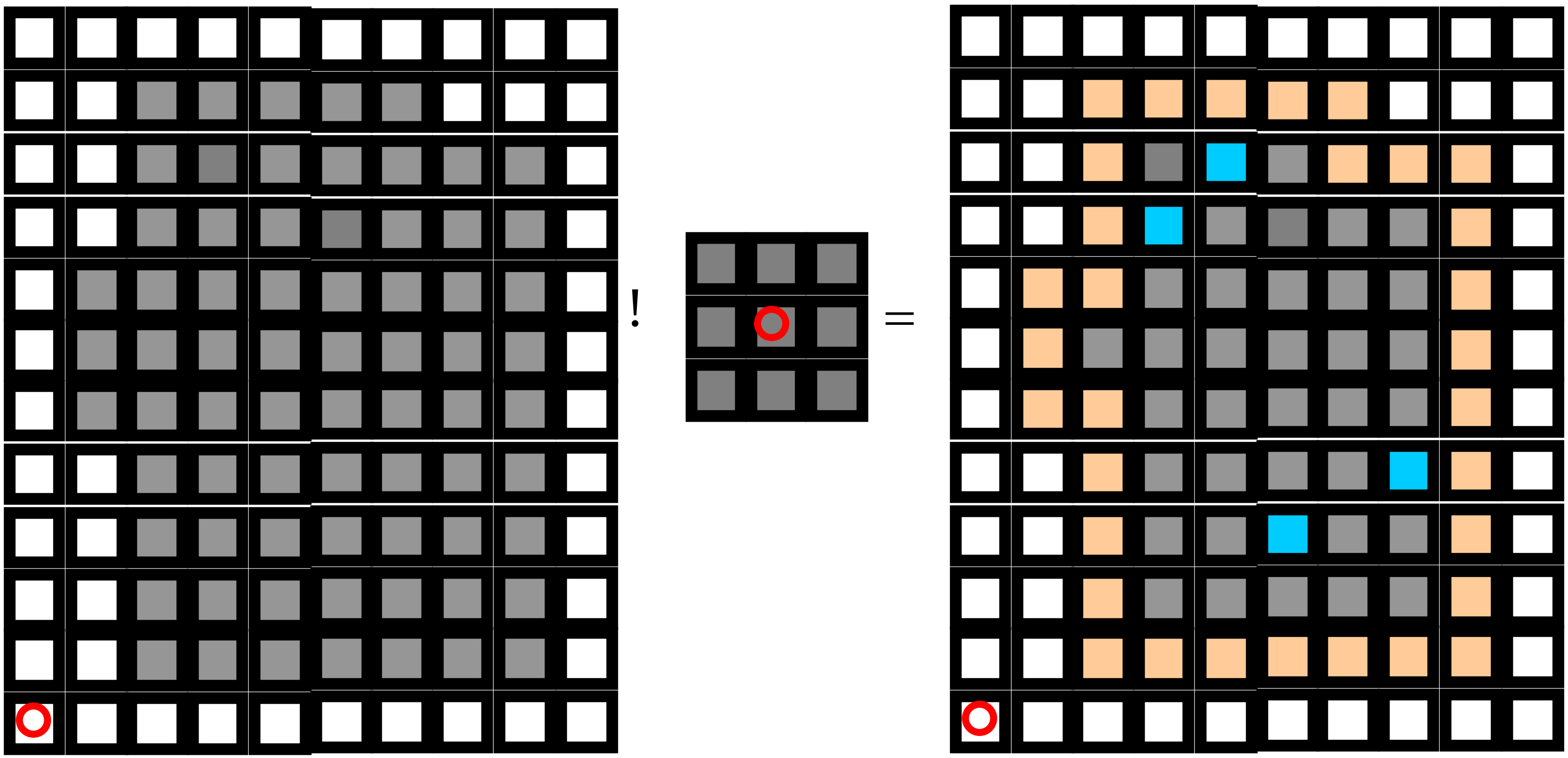

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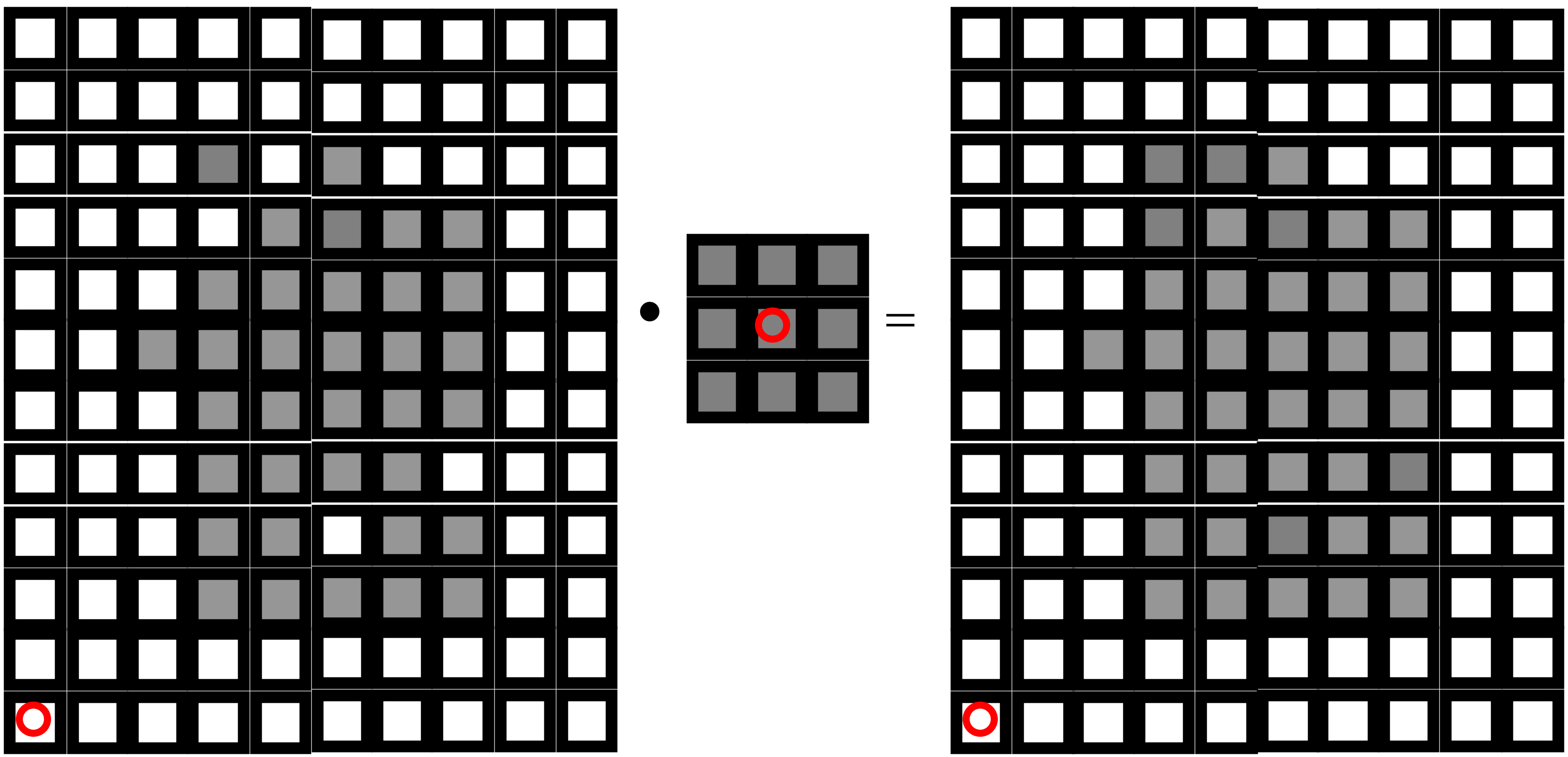
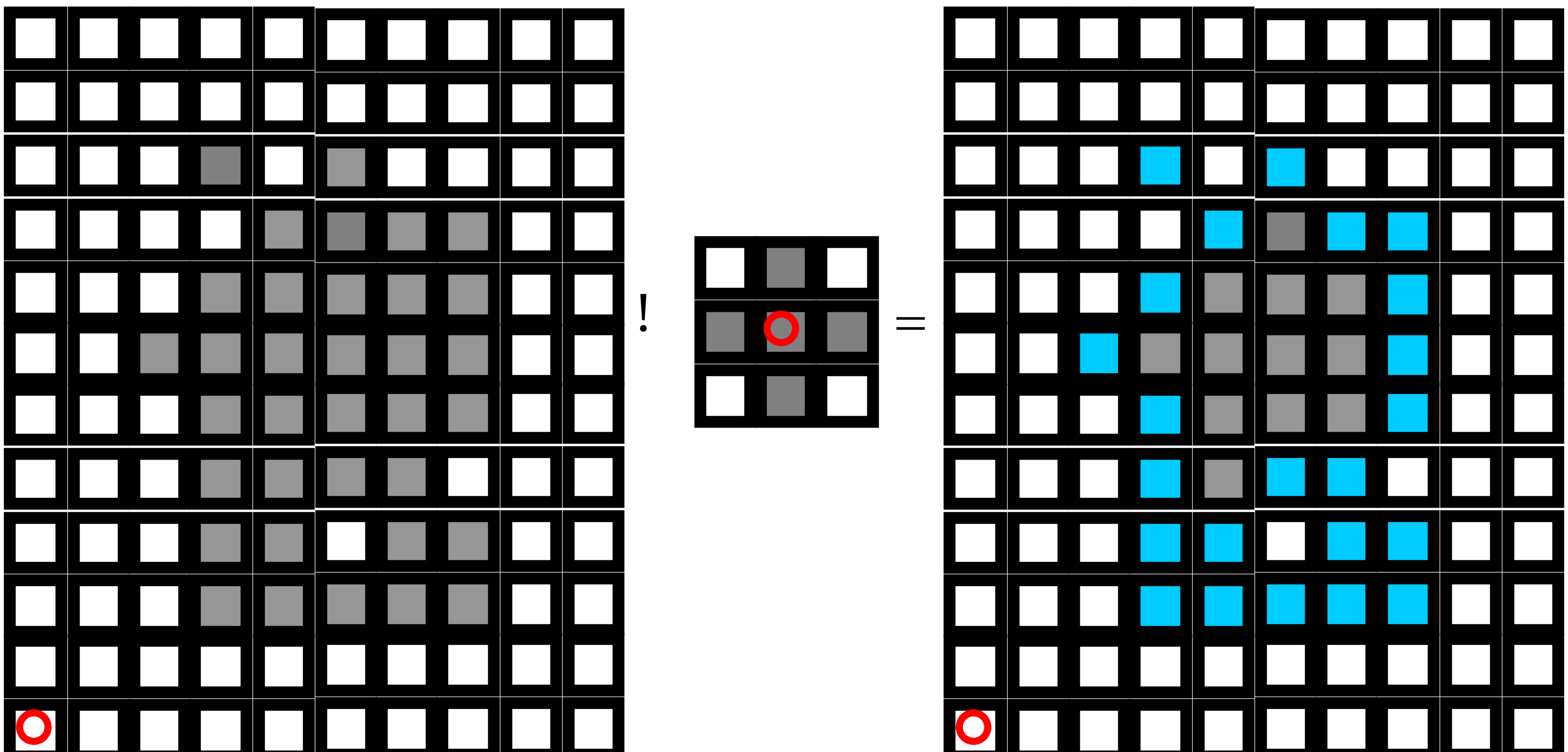


**Morphological Boundary Extraction**

- The boundary of an object A denoted by $\delta(A)$ can be obtained by first eroding the object and then subtracting the eroded image from the original image.

$$\delta(A) = A - A \ominus B$$



# Quiz

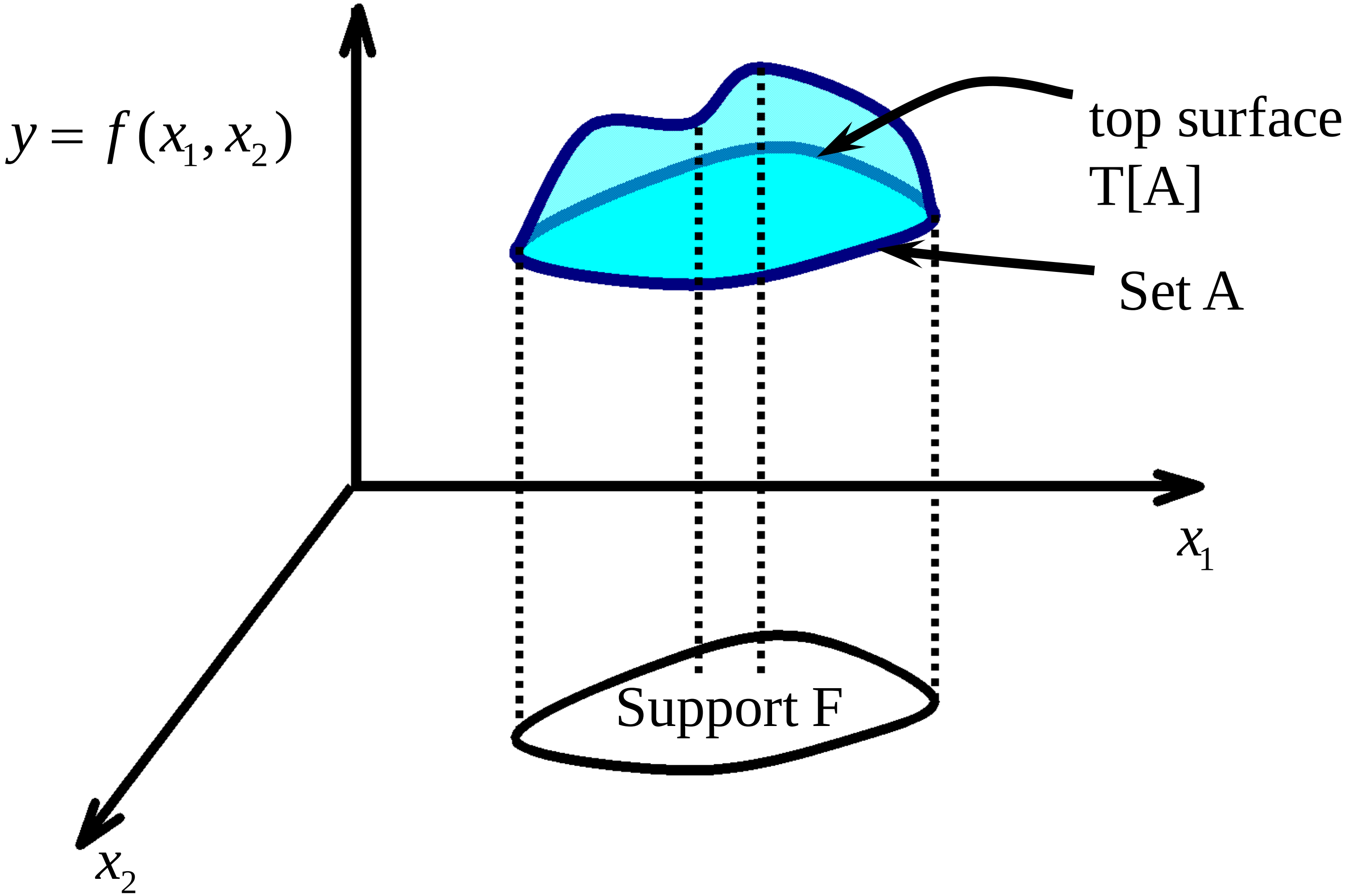
- How to extract edges along a given orientation using morphological operations?

- An opening followed by a closing
- Or, a closing followed by an opening

$$(X \circ B) \bullet B$$
$$(X \bullet B) \circ B$$

# Gray Scale Morphological Operation



# Gray Scale Morphological Operation

- $A$: a subset of n-dimensional Euclidean space, $A \subset R^n$
- $F$: support of $A$

$$F = \{x \in R^{n-1} \mid \exists\, y \in R \text{ s.t. } (x, y) \in A\}$$

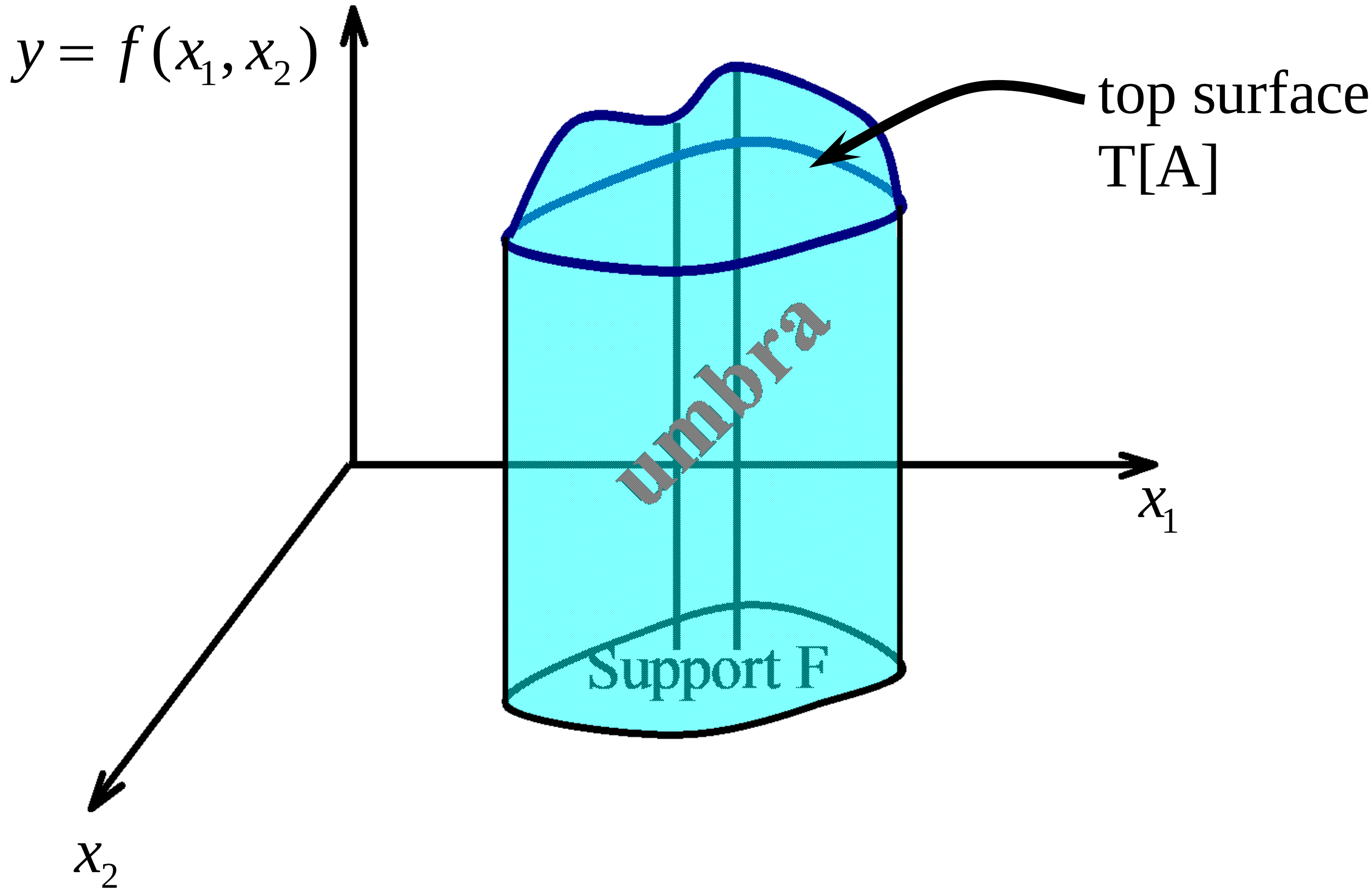
- Top hat or surface $T(A): F \to R^n$

$$T(A)(x) = \max\{y \mid (x, y) \in A\}$$

- A top surface is essentially a gray scale image $f: F \to R$
- An umbra $U(f)$ of a gray scale image $f: F \to R$ is the whole subspace below the top surface representing the gray scale image $f$. Thus,

$$U(f) = \{(x, y) \in F \times R, y \le f(x)\}$$

# Gray Scale Morphological Operation



# Gray Scale Morphological Operation

- The gray scale dilation between two functions may be defined as the top surface of the dilation of their umbras

$$f \oplus k = T(U(f) \oplus U(k))$$

- More computation-friendly definitions

$$f \oplus k = \max_{z \in k}\{ f(x-z) + k(z)\}$$

$$f \ominus k = \min_{z \in k}\{ f(x+z) - k(z)\}$$

- Commonly, we consider the structure element k as a binary set. Then the definitions of gray-scale morphological operations simplifies to

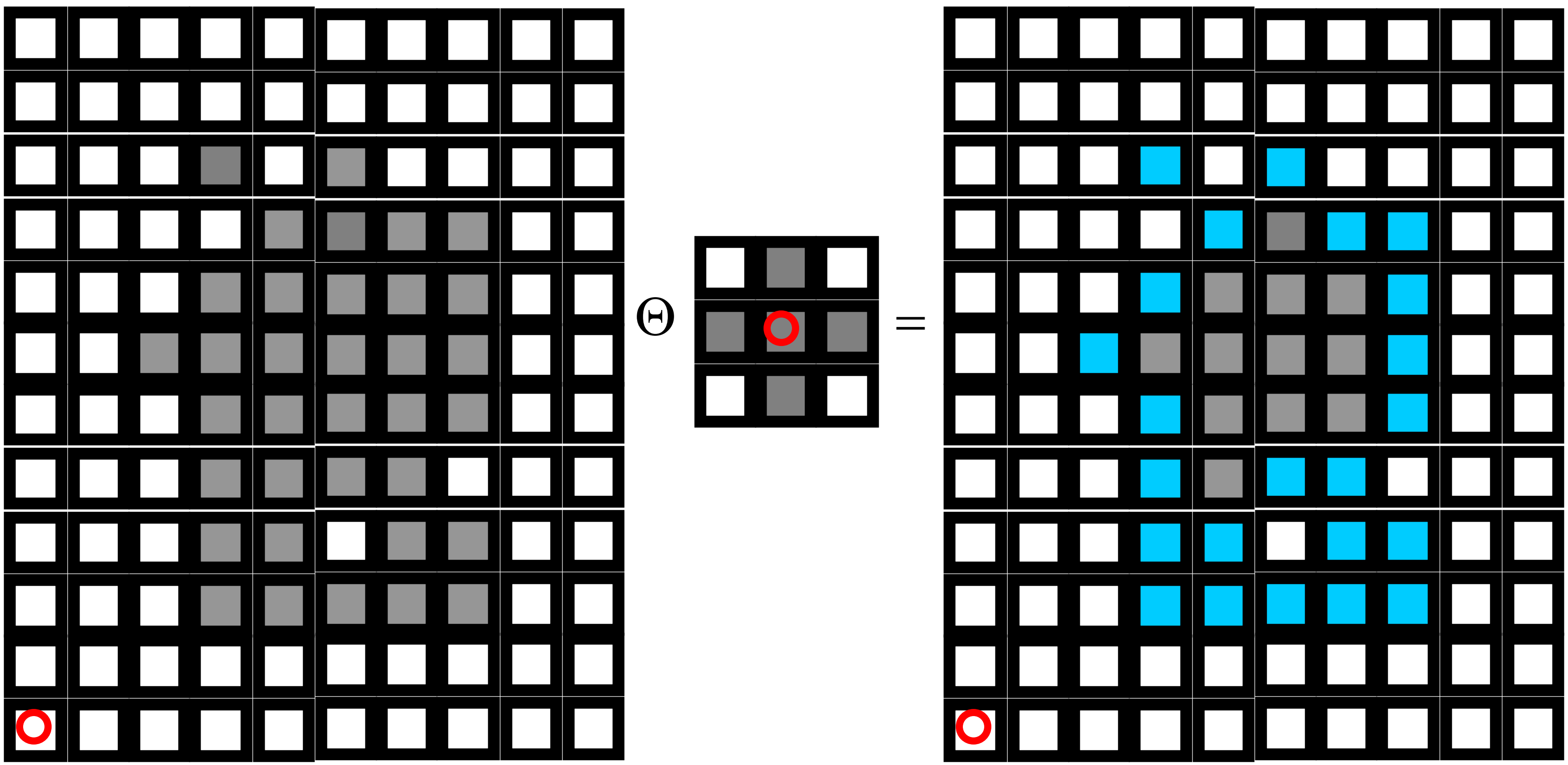
$$f \oplus k = \max_{z \in k}\{ f(x-z)\}$$

$$f \ominus k = \min_{z \in k}\{ f(x+z)\}$$

# Morphological Boundary Extraction

- The boundary of an object A denoted by δ(A) can be obtained by first eroding the object and then subtracting the eroded image from the original image.

$$\delta(A) = A - A \Theta B$$



# Quiz

- How to extract edges along a given orientation using morphological operations?

# Morphological noise filtering

- An opening followed by a closing
- Or, a closing followed by an opening

$$(X \circ B) \bullet B$$

$$(X \bullet B) \circ B$$

# Morphological noise filtering

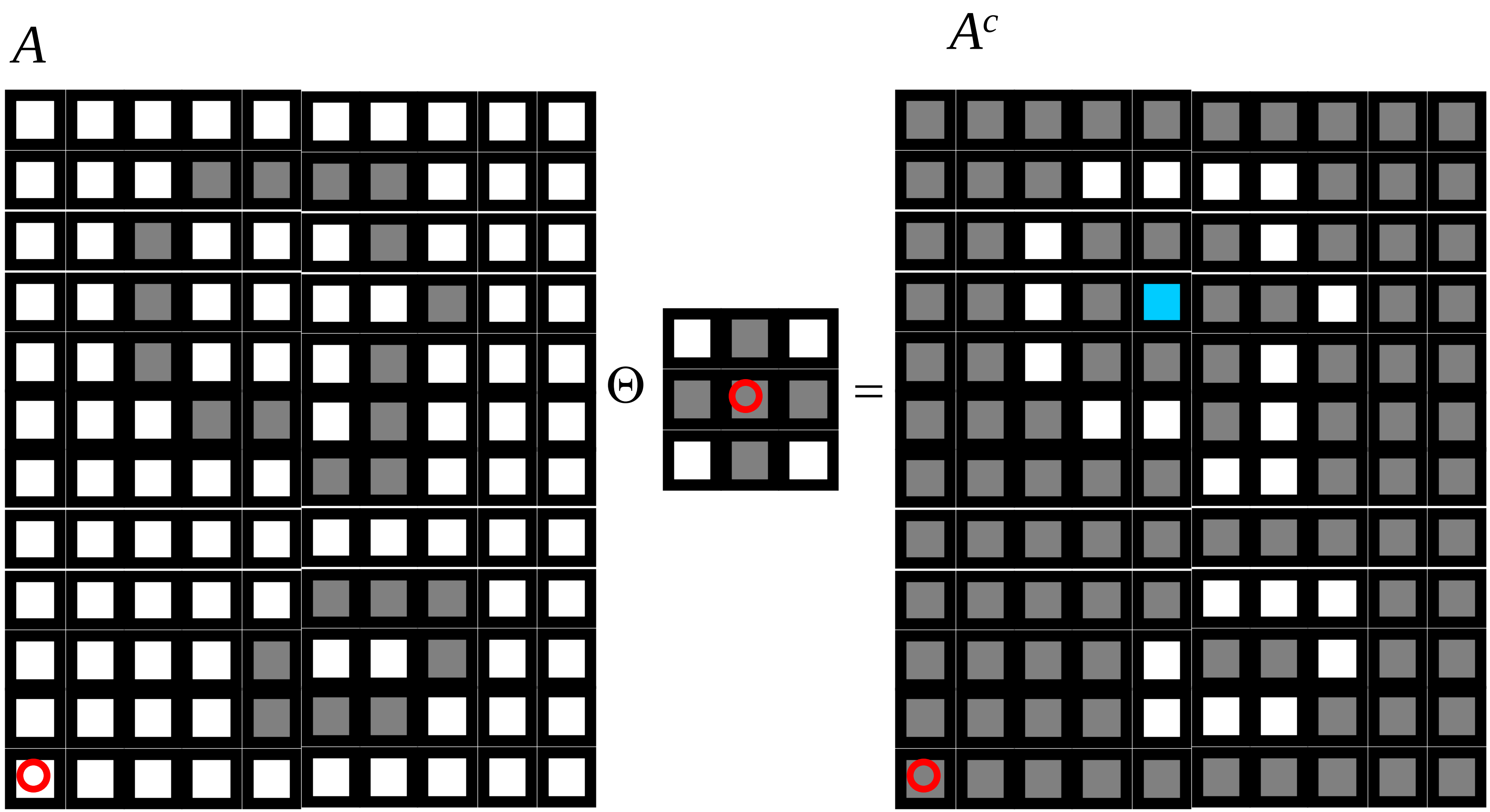
MATLAB DEMO

# Morphological Region Filling

- Task: Given a binary image *X* and a (seed) point *p*, fill the region surrounded by the pixels of *X* and contains *p*.
- A: An image where only the boundary pixels are labeled 1 and others are labeled 0
- $A^c$: The Complement of A
- We start with an image $X_0$ where only the seed point *p* is 1 and others are 0. Then we repeat the following steps until it converges

$$X_k = (X_{k-1} \oplus B) \cap A^c \qquad k = 1, 2, 3, \ldots$$

# Morphological Region Filling



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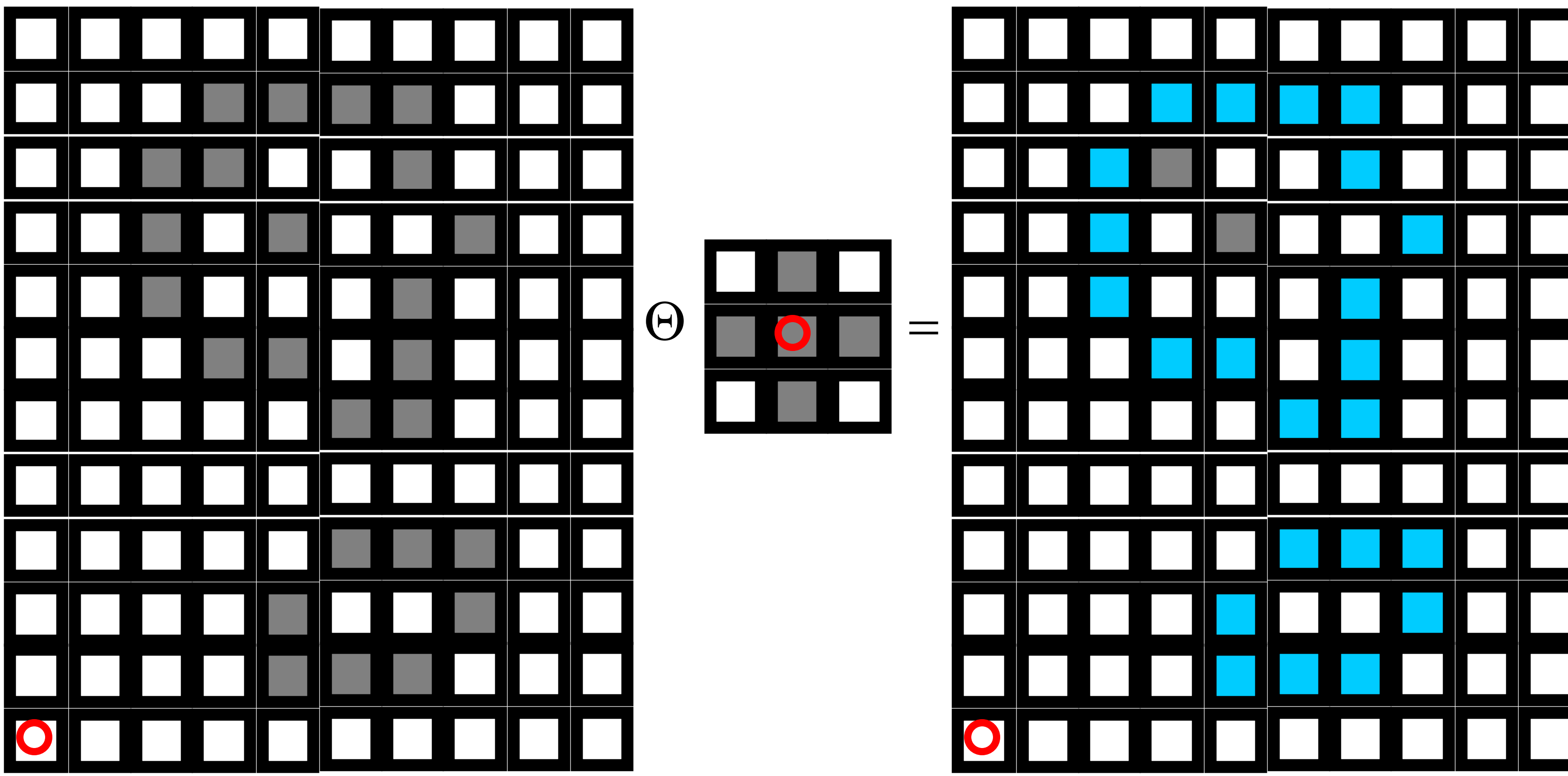
A



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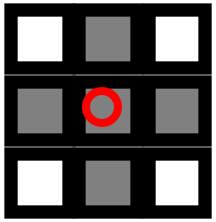
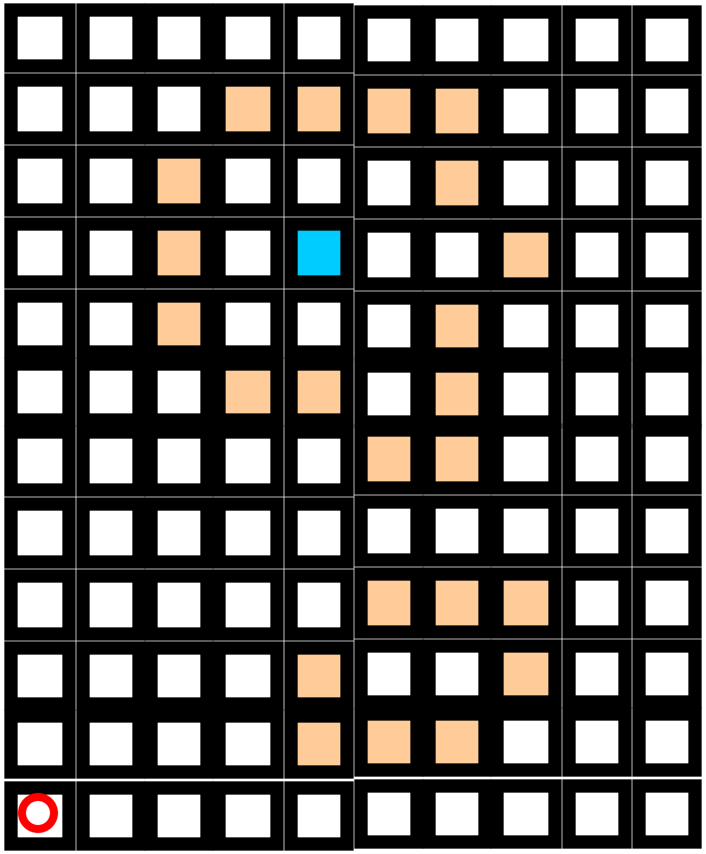
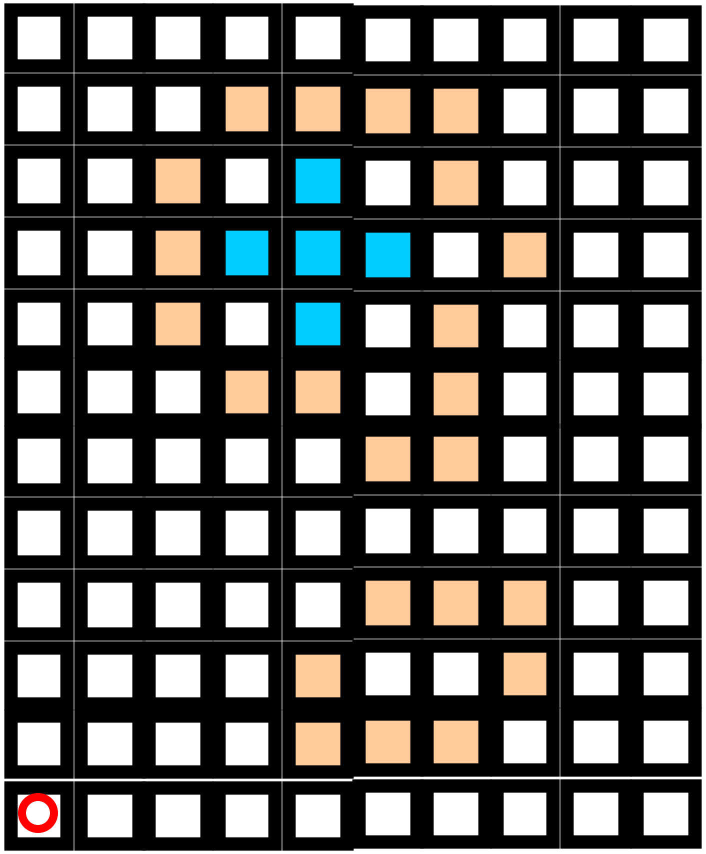
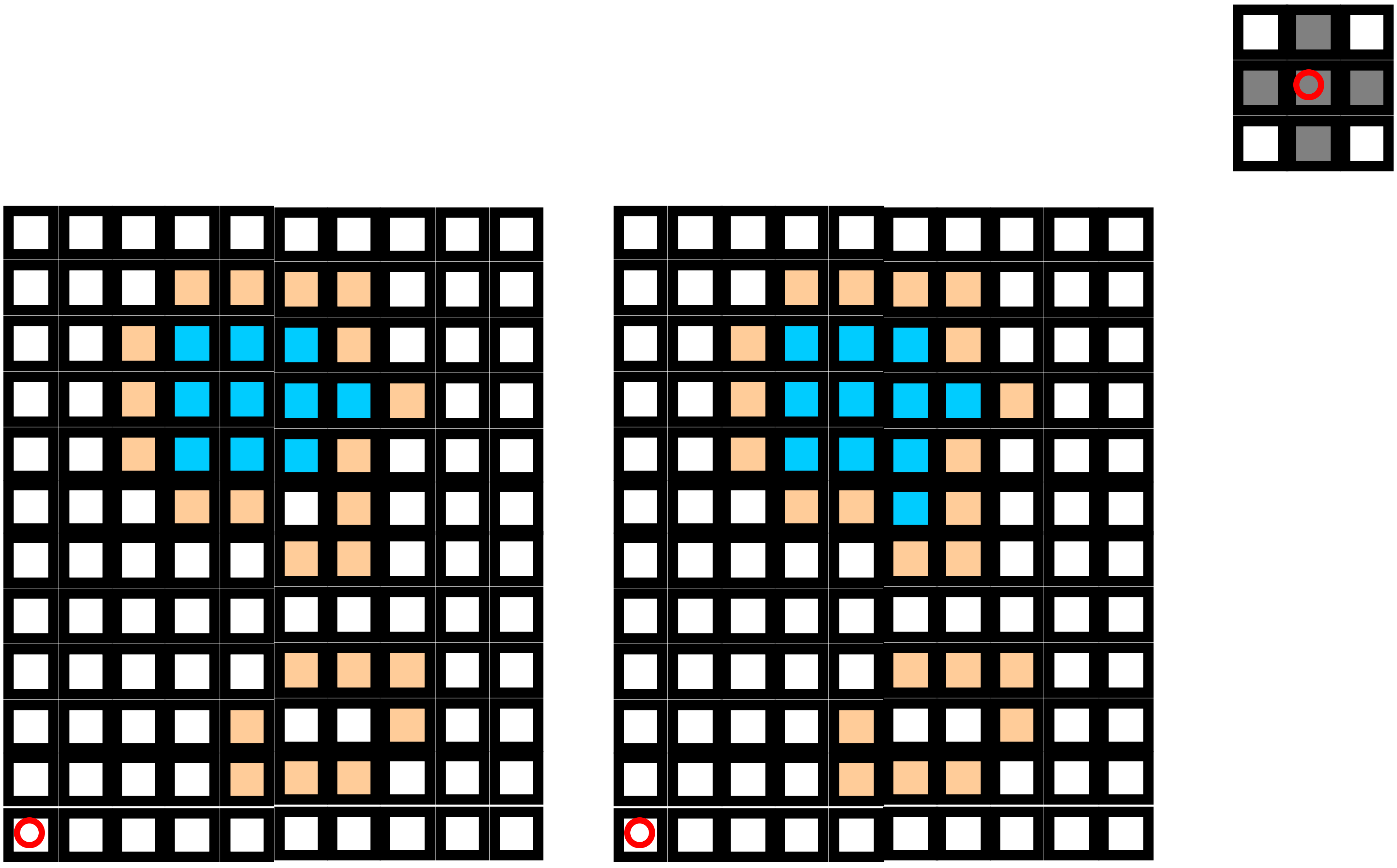


$$X_k = (X_{k-1} \oplus B) \cap \delta(A)^c \qquad k = 1, 2, 3, \ldots$$



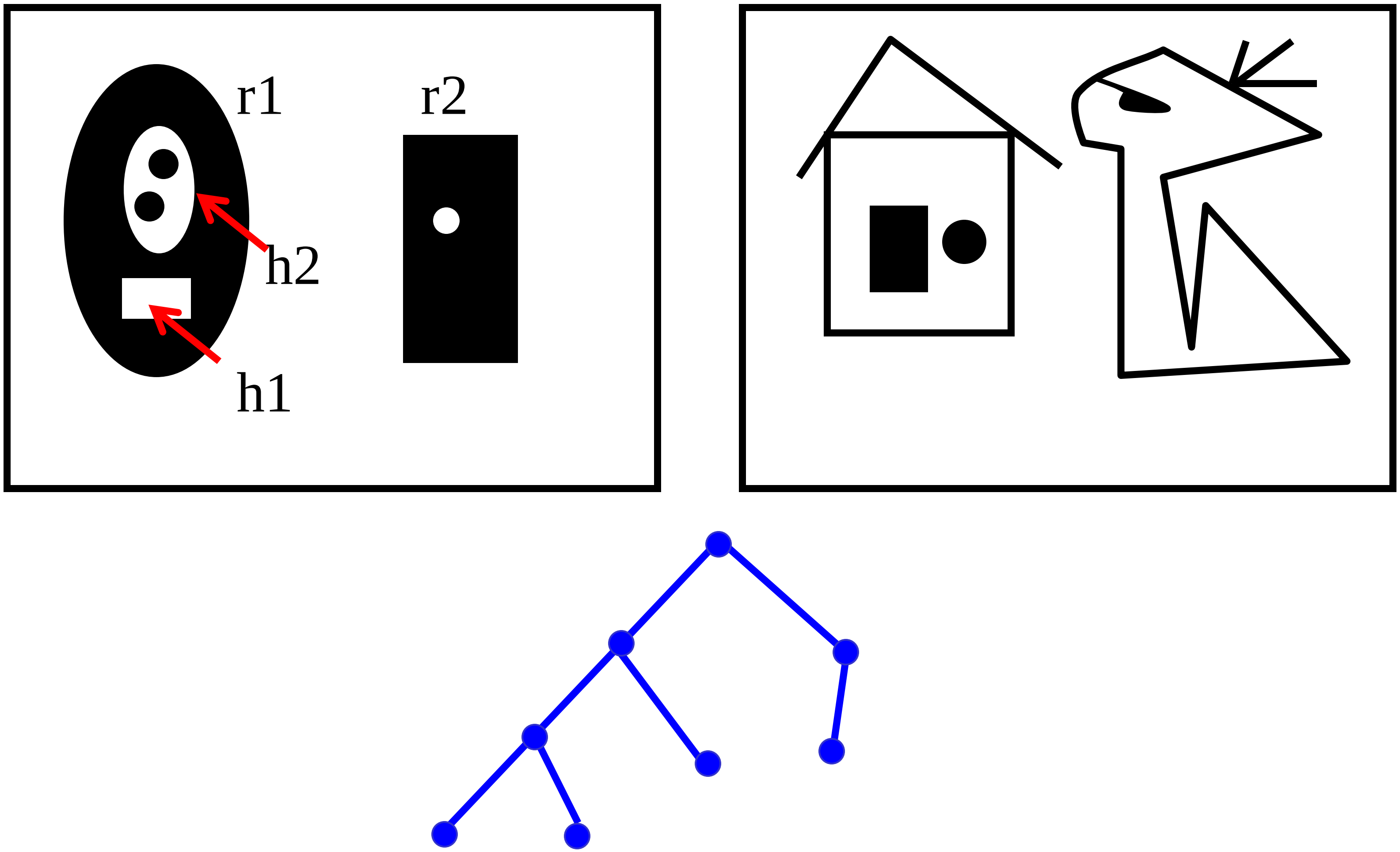


# Morphological Region Filling



# Homotopic Transformation

- Homotopic tree



# Quitz: Homotopic Transformation

- What is the relation between an element in the ith and i+1th levels?