Suppose $k|n$ for some $k, n \in \mathbb{Z}$, then

$$\sum_{i=0}^{n} f(i) = \sum_{i=0}^{\frac{n}{k}} f(ki) + \sum_{i=0}^{\frac{n}{k}-1} f(ki-1) + \sum_{i=0}^{\frac{n}{k}-1} f(ki-2) + \cdots + \sum_{i=0}^{\frac{n}{k}-1} f(ki-(k-1))$$

PROOF:

$$\sum_{i=0}^{n} f(i) =$$

$f(0) + f(1) + f(2) + \cdots + f(n)$

$= f(0) + f(1) + f(2) + \cdots + f(k-1)$

$+ f(k) + f(k+1) + f(k+2) + \cdots + f((k+(k-1)))$

$\ddots$

$+ f(k(\frac{n}{k}-1)) + f(k(\frac{n}{k}-1)+1) + f(k(\frac{n}{k}-1)+2) + \cdots + f(k(\frac{n}{k}-1)+(k-1))$

$+ f(k(\frac{n}{k}))$

It's easy to see that

$$k(\frac{n}{k}-1) = n-k,$$

$$k(\frac{n}{k}-1) + (k-1) = n-k+k-1 = n-1,$$

and of course

$$k\frac{n}{k} = n$$

Rearranging:

$$\sum_{i=0}^{n} f(i) =$$

$+ f(0) + f(k) + f(2k) + \cdots + f(k(\frac{n}{k}-1)) + f(k(\frac{n}{k}))$

$+ f(1) + f(k+1) + f(2k+1) + \cdots + f(k(\frac{n}{k}-1)+1)$

$+ f(2) + f(k+2) + f(2k+2) + \cdots + f(k(\frac{n}{k}-1)+2)$

$\ddots$

$+ f(k-1) + f(k+(k-1)) + f(2k+(k-1)) + \cdots + f(k(\frac{n}{k}-1)+(k-1))$

Now we can see that we have arranged the terms in such an order that they are in rows, with each row corresponding to each equivalence class modulo k.
There are $\frac{n}{k}$ terms in the first row, due to the extra $f(n)$ term, and there are $\frac{n}{k}-1$ terms in each of the other rows.
We may now simply collect the terms into sums to get

$$\sum_{i=0}^{n} f(i) = \sum_{i=0}^{\frac{n}{k}} f(ki) + \sum_{i=0}^{\frac{n}{k}-1} f(ki+1) + \sum_{i=0}^{\frac{n}{k}-1} f(ki+2) + \cdots + \sum_{i=0}^{\frac{n}{k}-1} f(ki+(k-1))$$

Additionally, we can see from the algebraic manipulations that the number of rows (not including the first row representing congruence to 0 mod k) is of course $k-1$. This allows us the more compact representation

$$\sum_{i=0}^{n} f(i) = \sum_{i=0}^{\frac{n}{k}} f(ki) + \sum_{q=1}^{k-1} \left[\sum_{i=0}^{\frac{n}{k}-1} f(ki+q) \right]$$