ES.: QUANTI NUMERI DI CELLULARE CI SONO CHE HANNO TRE CIFRE CONSECUTIVE UGUALI?

VOGLIAMO CALCOLARE

$$\left|\left\{(x_1,\dots,x_7)\in[0,9]^7 : \ x_i = x_{i+1} = x_{i+2} \text{ PER QUALCHE } 1\leq i\leq 5\right\}\right|$$

$$\left([0,9] = \{0,1,2,\dots,9\}.\right)$$

PONIAMO

$$A_1 = \{ (x_1, \dots, x_7) \in [0,9]^7 : \quad x_1 = x_2 = x_3 \}$$

$$A_2 = \{ \quad \text{''} \qquad \text{''} \qquad \text{''} : \quad x_2 = x_3 = x_4 \}$$

$$A_3 = \{ \quad \text{''} \qquad \text{''} \qquad \text{''} : \quad x_3 = x_4 = x_5 \}$$

$$A_4 = \{ \quad \text{''} \qquad \text{''} \qquad \text{''} : \quad x_4 = x_5 = x_6 \}$$

$$A_5 = \{ \quad \text{''} \qquad \text{''} \qquad \text{''} : \quad x_5 = x_6 = x_7 \}$$

DOBBIAMO QUINDI CALCOLARE

$$|A_1 \cup A_2 \cup A_3 \cup A_4 \cup A_5|.$$

APPLICHIAMO I.-E.:

$$|A_1 \cup A_2 \cup \dots \cup A_5| = |A_1| + |A_2| + |A_3| + |A_4| + |A_5| -$$

$$- |A_1 \cap A_2| - |A_2 \cap A_3| - |A_3 \cap A_4| - |A_4 \cap A_5| -$$

$$- |A_1 \cap A_3| - |A_2 \cap A_4| - |A_3 \cap A_5| - |A_1 \cap A_4|$$

$$- |A_2 \cap A_5| - |A_1 \cap A_5| + |A_1 \cap A_2 \cap A_3| +$$

$$+|A_1\cap A_2\cap A_4|+|A_1\cap A_2\cap A_5|+|A_1\cap A_3\cap A_4|+|A_2\cap A_4\cap A_5|$$

$$+|A_1\cap A_3\cap A_5|+|A_1\cap A_4\cap A_5|+|A_2\cap A_3\cap A_4|+$$

$$+|A_2\cap A_3\cap A_5|+|A_3\cap A_4\cap A_5|-|A_1\cap A_2\cap A_3\cap A_4|$$

$$-|A_1\cap A_2\cap A_3\cap A_5|-|A_1\cap A_2\cap A_4\cap A_5|-$$

$$-|A_1\cap A_3\cap A_4\cap A_5|-|A_2\cap A_3\cap A_4\cap A_5|$$

$$+|A_1\cap A_2\cap A_3\cap A_5|.$$

ABBIAMO CHE

$$|A_1| = 10^5, \quad |A_2| = |A_3| = |A_4| = |A_5| = 10^5$$

$$|A_1 \cap A_2| = |\{(x_1, \dots, x_7) \in [0,9]^7 : x_1 = x_2 = x_3 = x_4\}|$$

$$= 10^4$$

SIMILMENTE

$$|A_2 \cap A_3| = 10^4, \quad |A_3 \cap A_4| = 10^4, \quad |A_4 \cap A_5| = 10^4.$$

ANCHE

$$|A_1 \cap A_3| = |\{(x_1, \dots, x_7) \in [0,9]^7 : \ x_1 = x_2 = x_3 = x_4 = x_5\}|$$

$$= 10^3$$

SIMIL.

$$|A_2 \cap A_4| = 10^3, \quad |A_3 \cap A_5| = 10^3.$$

ANCHE

$$|A_1 \cap A_4| = \{(x_1, \dots, x_7) \in [0,9]^7 : \ x_1 = x_2 = x_3 \text{ E } x_4 = x_5 = x_6\}$$

$= 10^3$

E SIMIL.

$$|A_2 \cap A_5| = 10^3.$$

ANCHE

$$|A_1 \cap A_5| = |\{(x_1, .., x_7) \in [0,9]^7 : x_1 = x_2 = x_3 \text{ E } x_5 = x_6 = x_7\}| = 10^3.$$

INOLTRE

$$|A_1 \cap A_2 \cap A_3| = |\{(x_1, \dots, x_7) \in [0,9]^7 : x_1 = x_2 = x_3 = x_4 = x_5\}|$$

$$= 10^3$$

E simic.

$$|A_1 \cap A_2 \cap A_4| = |\{(x_1, \dots, x_7) \in [0,9]^7 : x_1 = x_2 = x_3 =$$

$$= x_4 = x_5 = x_6\}| = 10^2$$

$$|A_1 \cap A_2 \cap A_5| = 10^2$$

$|A_1 \cap A_3 \cap A_4| = 10^2$, $|A_1 \cap A_3 \cap A_5| = 10$

$|A_1 \cap A_4 \cap A_5| = 10^2$, $|A_2 \cap A_3 \cap A_4| = 10^3$

$|A_2 \cap A_3 \cap A_5| = 10^2$, $|A_2 \cap A_4 \cap A_5| = 10^2$

$|A_3 \cap A_4 \cap A_5| = 10^3$.

ANCHE

$|A_1 \cap A_2 \cap A_3 \cap A_4| = 10^2$, $|A_1 \cap A_2 \cap A_3 \cap A_5| = 10$[illegible]

$|A_1 \cap A_2 \cap A_4 \cap A_5| = 10$, $|A_1 \cap A_3 \cap A_4 \cap A_5| = 10$,

$|A_2 \cap A_3 \cap A_4 \cap A_5| = 10^2$,

INFINE

$|A_1 \cap A_2 \cap A_3 \cap A_4 \cap A_5| = 10$.

CONCLUDENDO

$$|A_1 \cup A_2 \cup A_3 \cup A_4 \cup A_5| = 5 \cdot 10^5 - \left(4 \cdot 10^4 + 3 \cdot 10^3\right.$$

$$\left. + 2 \cdot 10^3 + 10^3\right) + \left(10^3 + 2 \cdot 10^2 + 4 \cdot 10^{\cancel{3}\,2} + 2 \cdot 10^3 + 10\right)$$

$$- \left(3 \cdot 10 + 2 \cdot 10^2\right) + 10 =$$

$$= 5 \cdot 10^5 - 4 \cdot 10^4 - 3 \cdot 10^3 + 4 \cdot 10^2 - 10$$

$$= 457390$$