* Ray tracing

- it allow for the fluid simulation of lighting effects.
  - → an algorithm emit rays in the form of a 3D graphic, trace the rays' path and then calculates a realistic lighting model.

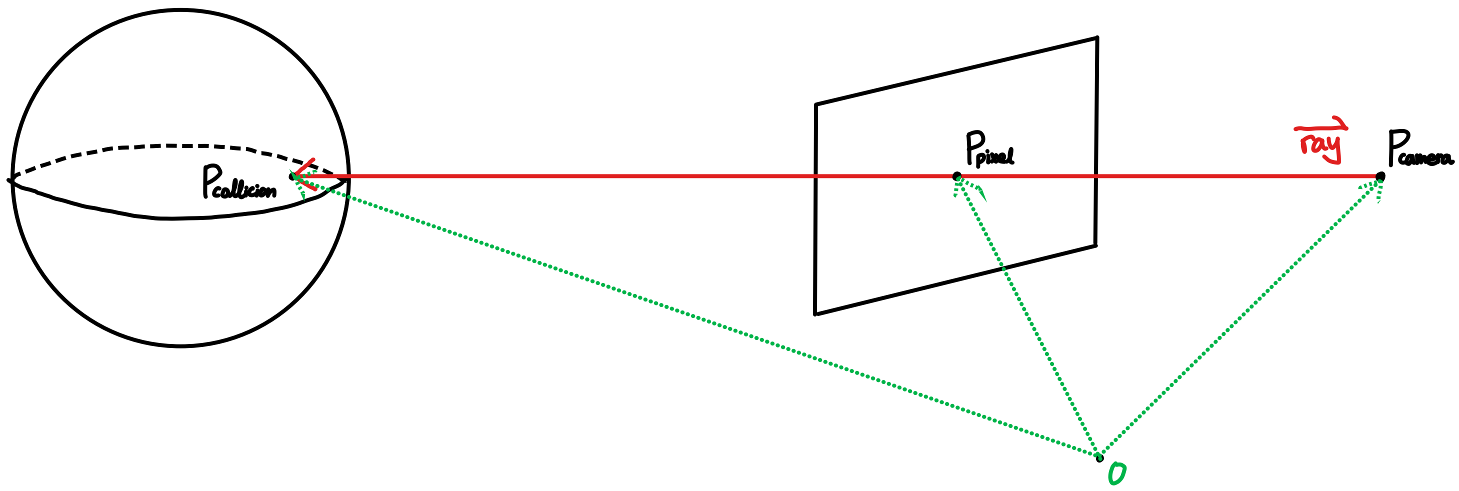
* Physically Based Rendering (Forward Ray Tracing)

- it follows the light particles (photons) from the light source to the object.
  - → it can most accurately determine the coloring of each object, but it is highly inefficient.
    (many rays from the light source never come through the vieuplane and into the eye.)

☆ * Backward Ray Tracing

- an eye ray is created at the eye, it passes through vieuplane and on to the world.
  - → the first object the eye ray hits is the object that will be visible from that point of the vieuplane.
  - → after the ray tracer allows that light ray to boune around, it figures out the exact coloring and shading of that point in the vieuplane and displays it on corrisponding pixel on the computer monitor screen.
  - → it assumes only the right rays that come through the vieuplane and on into the eye contribute to the final image of the scene.
  - → when an object is transparant or shiny, an error occurs : mix forward and backward ray tracing.

* Ray - Object Intersection



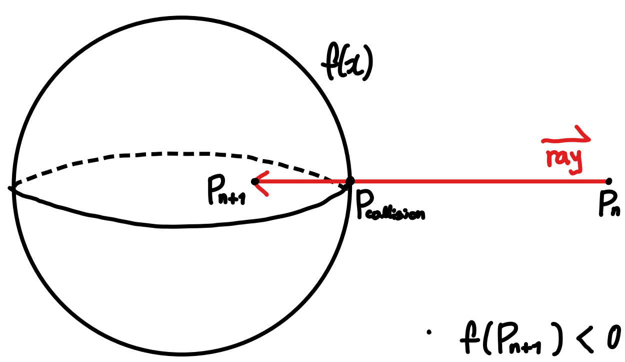
· direction of the pixel ray ($\overrightarrow{d_{ray}}$)

$$\overrightarrow{d_{ray}} = \frac{\overrightarrow{P_{pixel} - P_{camera}}}{\left|\overrightarrow{P_{pixel} - P_{camera}}\right|} \quad \left(= \frac{\overrightarrow{OP_{pixel}} - \overrightarrow{OP_{camera}}}{\left|\overrightarrow{OP_{pixel}} - \overrightarrow{OP_{camera}}\right|} \right)$$

· equation of the pixel ray

$$\overrightarrow{r_{pixel}} = \overrightarrow{P_{pixel}} + t \cdot \overrightarrow{d_{ray}} \quad \left(= \overrightarrow{ray} \right)$$

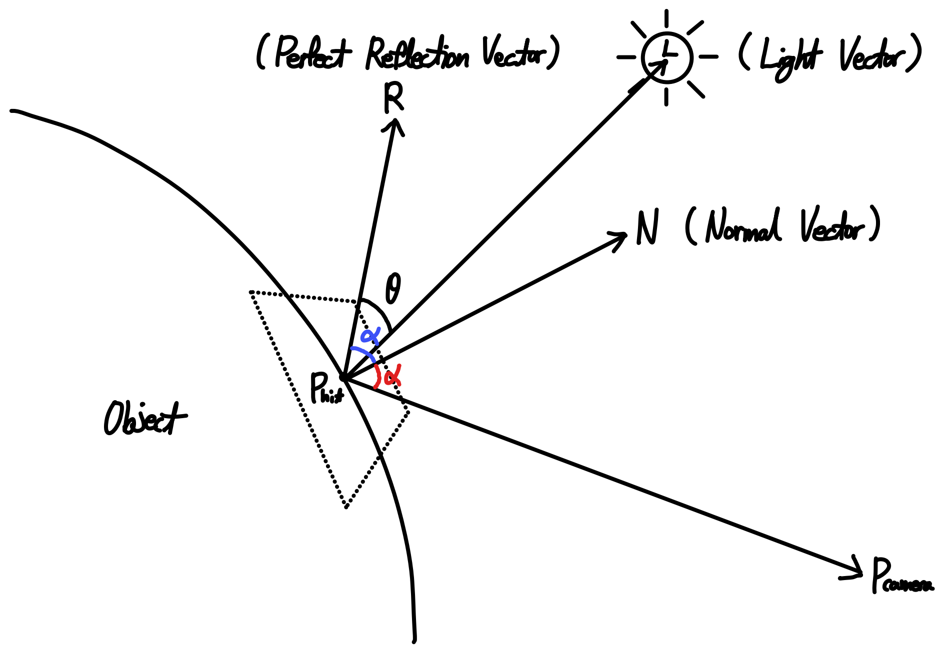
· ray - object intersection



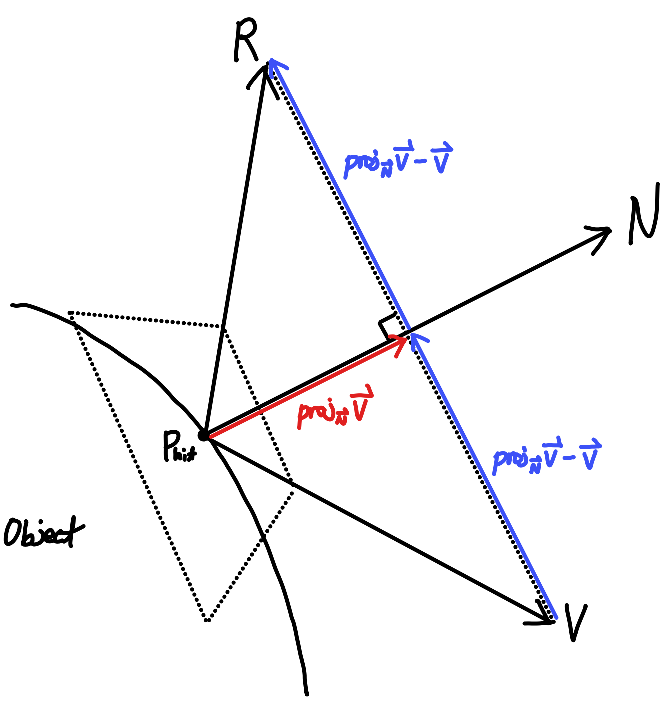
- $f(P_{n+1}) < 0$
- $f(P_{collision}) = 0$
- $f(P_n) > 0$

* Phong Reflection Model

· Perfect Reflection



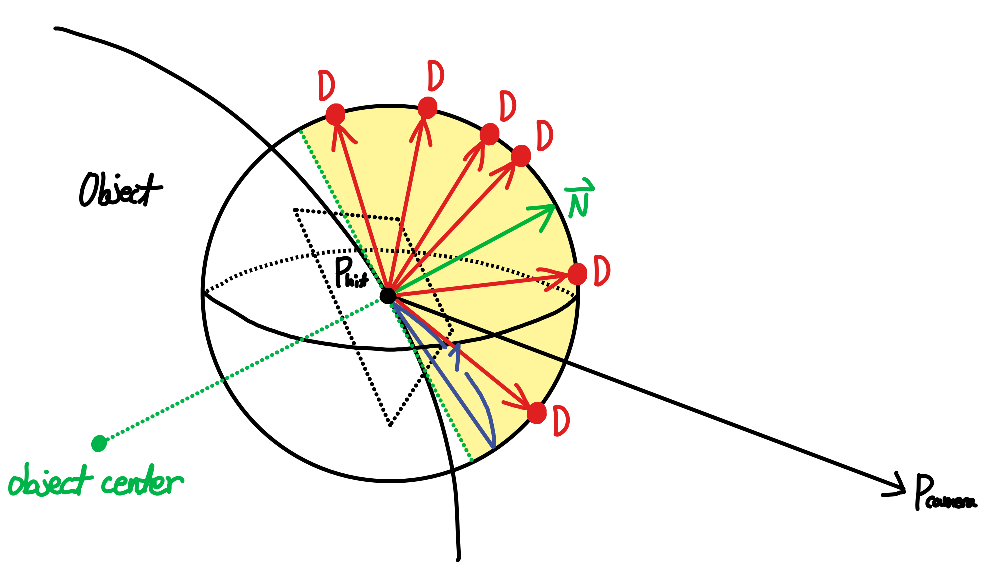
· More light energy will be reflected to $\overrightarrow{P_{hit}P_{camera}}$ direction when $\vec{L}$ and $\vec{R}$ are closer to each other.



$$\cdot\ \vec{R} = 2(proj_{\vec N}\vec V - \vec V)$$

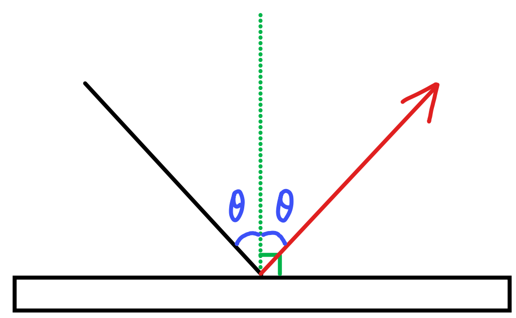
$$= 2\left(\underbrace{\frac{\vec N\cdot\vec V}{\vec N\cdot\vec N}}_{\because\ proj_{\vec N}\vec V = t\vec N}\cdot\vec N - \vec V\right)$$

· Diffuse Reflection

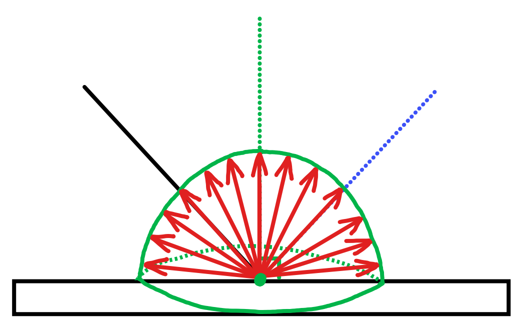


- perfect, diffuse, specular reflection

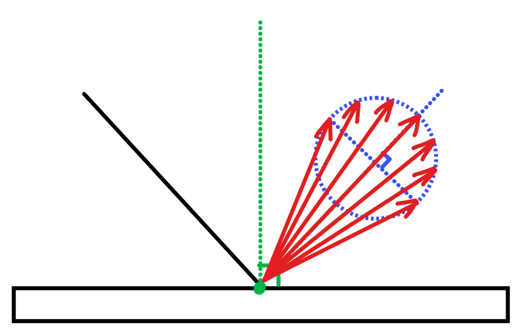
  - perfect reflection → metalic surface



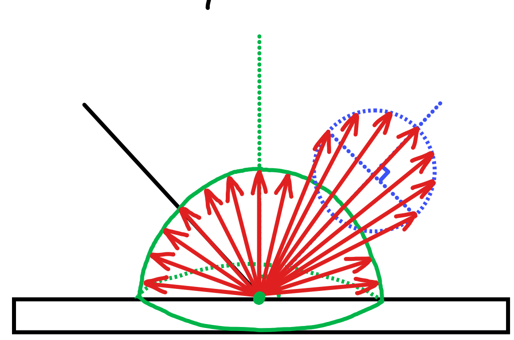
  - diffuse reflection



  - specular reflection



  - diffuse and specular reflection

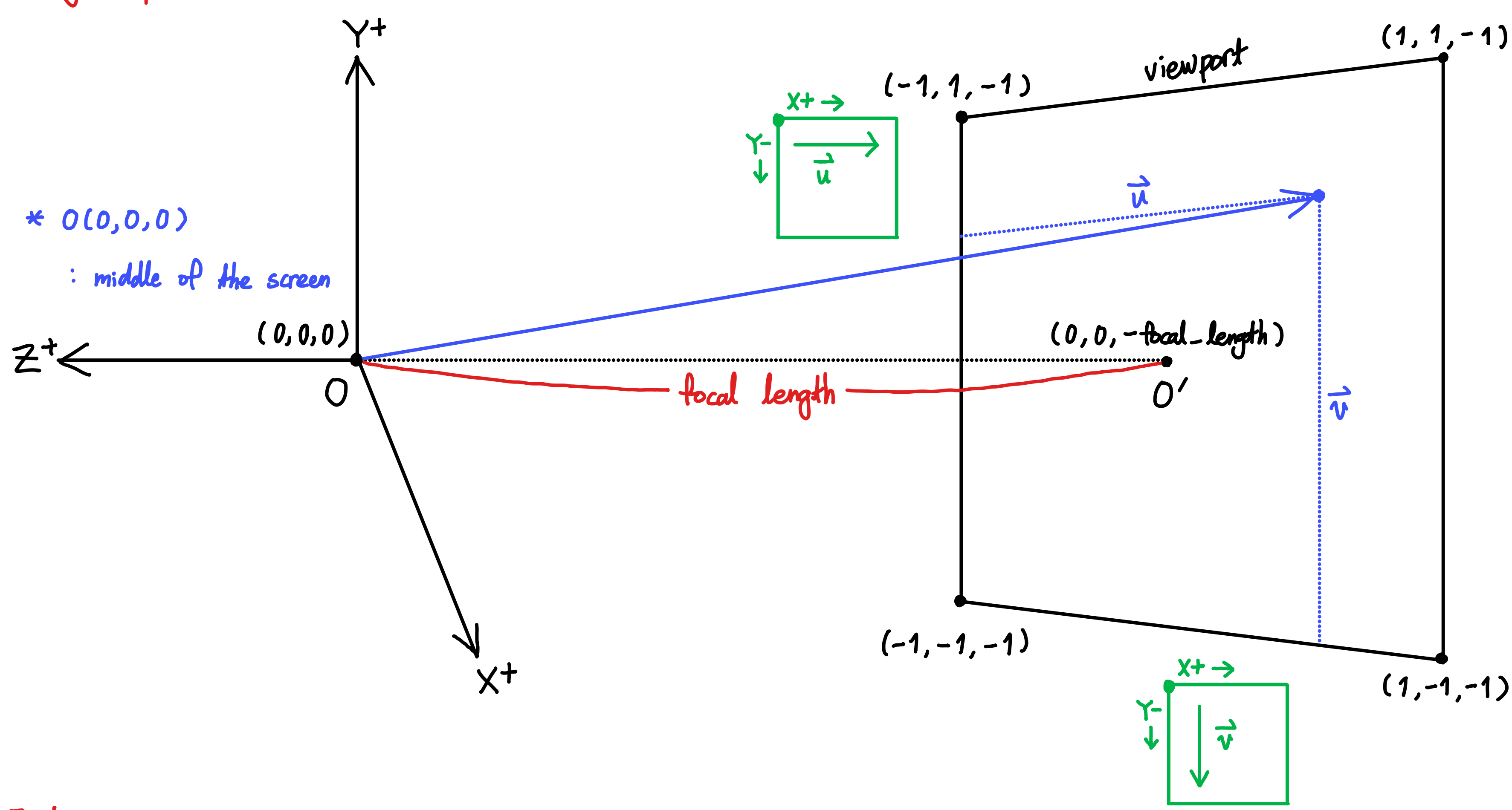


* Ray Tracer

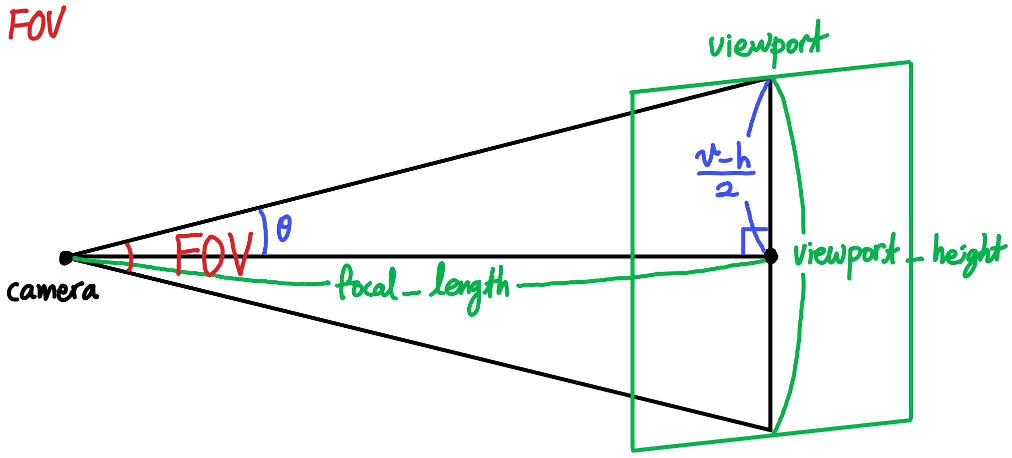
: sends ray through pixel, and calculates color corresponding pixel.

- step 1. calculate distance of camera to object
- step 2. recognize the hit point (on the object)
- step 3. calculate the color
- step 4. apply a light

· setting viewport



· FOV

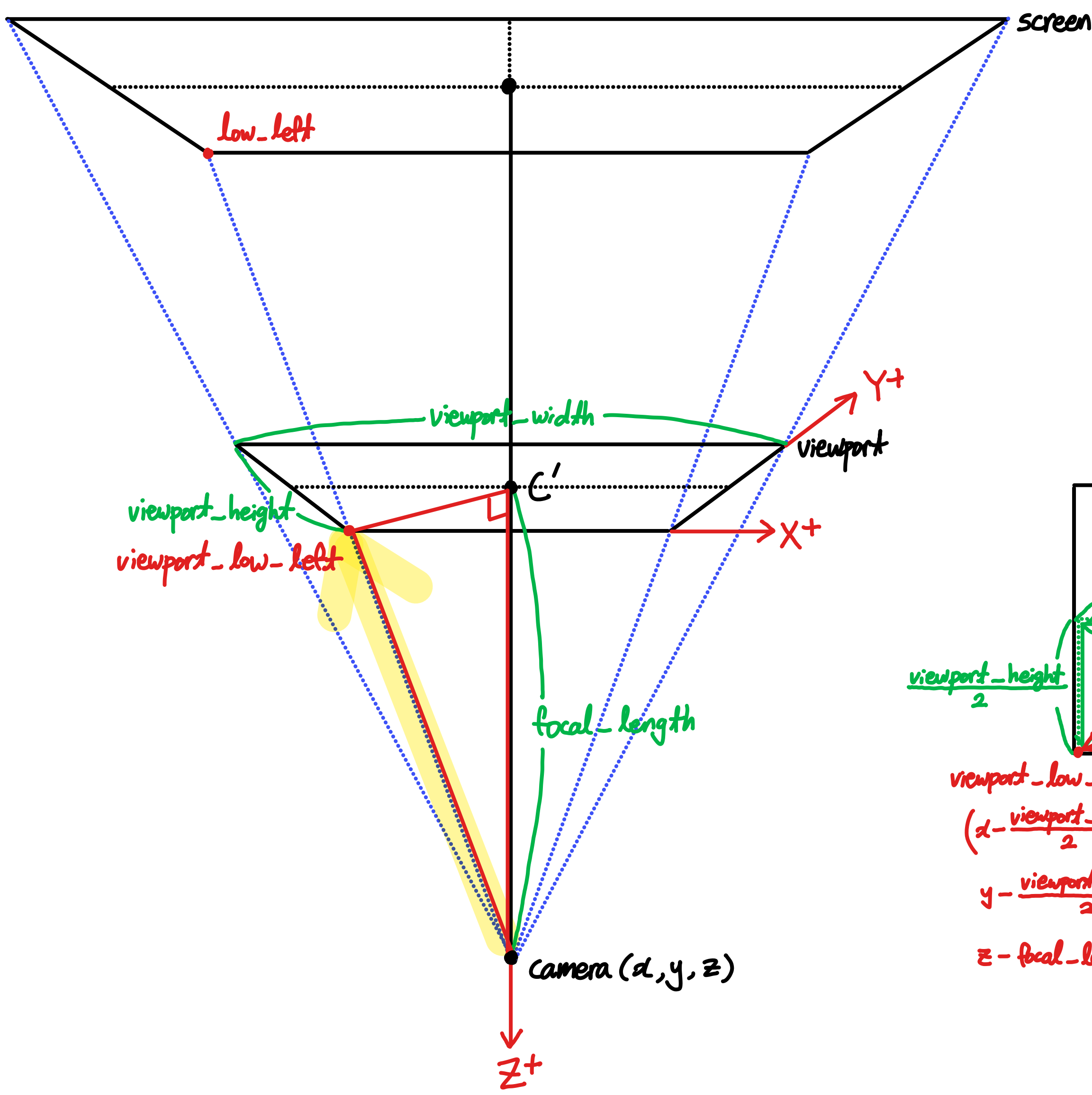


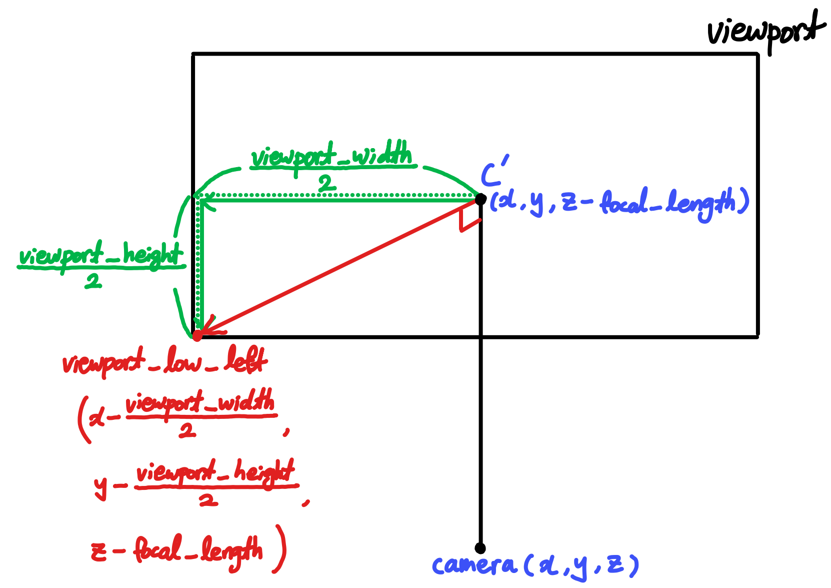
· $\tan\theta = \dfrac{\frac{viewport\_height}{2}}{focal\_length} \rightarrow viewport\_height = 2 \times focal\_length \times \tan\theta$

· also, $\theta\,(rad) = \dfrac{FOV(°)}{2} = \dfrac{\pi}{360} \times FOV$

$\therefore viewport\_height = 2 \times focal\_length \times \tan\left(\dfrac{\pi}{360} \times FOV\right)$

· setting ray



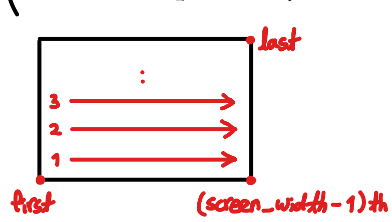


· viewport_low_left : a direction of the first ray.

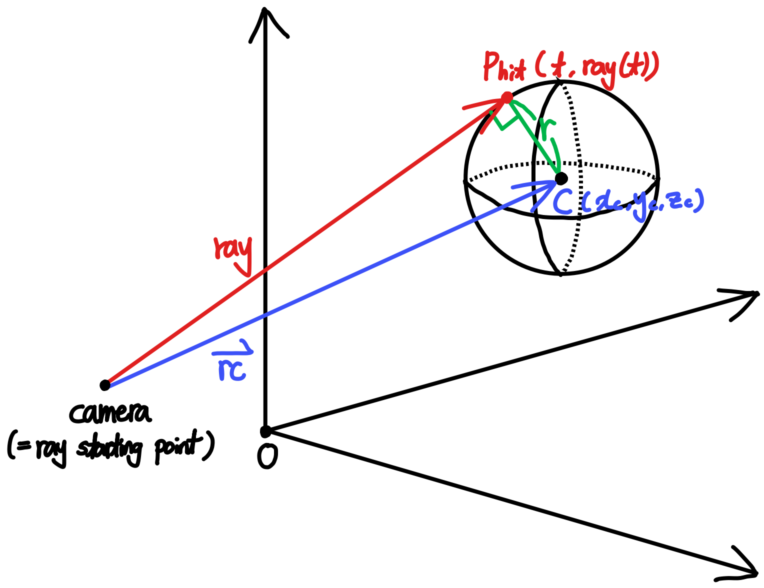
· ray

- start point : the position of camera (x, y, z)
- direction : $P(\text{viewport\_low\_left} + \text{viewport\_width} \times \vec{u} + \text{viewport\_height} \times \vec{v}) - P_{camera}(x, y, z)$

$$\left(\vec{u} = \frac{i}{\text{screen\_width} - 1},\ \vec{v} = \frac{j}{\text{screen\_height} - 1},\ 0 \le i \le \text{screen\_width} - 1,\ 0 \le j \le \text{screen\_height} - 1 \right)$$



- ray - sphere intersection



* equation of sphere

(center : $C(x_c, y_c, z_c)$, radius : r)

- $(x-x_c)^2 + (y-y_c)^2 + (z-z_c)^2 = r^2$
- $|\vec{p} - \vec{OC}| = r$
- $|\vec{p} - \vec{OC}| \cdot |\vec{p} - \vec{OC}| = r^2$

- $|ray(t) - \vec{rc}| = r$

  $\rightarrow |ray(t) - \vec{rc}| \cdot |ray(t) - \vec{rc}| = r^2$

  $\rightarrow ray(t)^2 - 2 \cdot \vec{rc} \cdot ray(t) + \vec{rc} \cdot \vec{rc} - r^2 = 0$

- $ray(t)$ is a point on the sphere and a point on the ray at the same time.

  on the ray's aspect, we can express it as $r_{point} + t \cdot \vec{d_{ray}}$.

- $ray(t)^2 - 2 \cdot \vec{rc} \cdot ray(t) + \vec{rc} \cdot \vec{rc} - r^2 = 0$

  $\rightarrow (r_{point} + t \cdot \vec{d_{ray}})^2 - 2 \cdot \vec{rc} \cdot (r_{point} + t \cdot \vec{d_{ray}}) + \vec{rc} \cdot \vec{rc} - r^2 = 0$

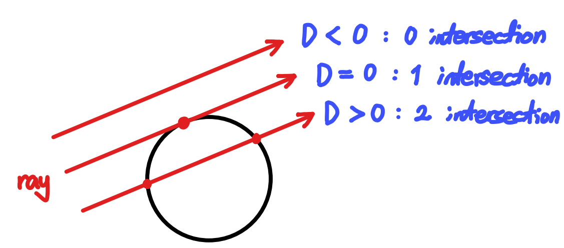
  $\rightarrow r_{point}^2 + 2 \cdot r_{point} \cdot \vec{d_{ray}} \cdot t + \vec{d_{ray}} \cdot \vec{d_{ray}} \cdot t^2 - 2 \cdot \vec{rc} \cdot r_{point} - 2 \cdot \vec{rc} \cdot \vec{d_{ray}} \cdot t + \vec{rc} \cdot \vec{rc} - r^2 = 0$

  $\rightarrow (\vec{d_{ray}} \cdot \vec{d_{ray}}) t^2 + (\cancel{2 \cdot r_{point} \cdot \vec{d_{ray}}} - 2 \cdot \vec{rc} \cdot \vec{d_{ray}}) t + (\cancel{r_{point}^2} - \cancel{2 \cdot \vec{rc} \cdot r_{point}} + \vec{rc} \cdot \vec{rc} - r^2) = 0$

  $\rightarrow \vec{d_{ray}} \cdot \vec{d_{ray}} \cdot t^2 - 2 \cdot \vec{rc} \cdot \vec{d_{ray}} \cdot t + \vec{rc} \cdot \vec{rc} - r^2 = 0$ (quadratic expression for t)

- t is also a point on the sphere, so there should be at least one intersection.

  $\rightarrow$ discriminant ≥ 0



$\therefore D = b^2 - 4ac = (-2 \cdot \vec{rc} \cdot \vec{d_{ray}})^2 - 4 \cdot \vec{d_{ray}} \cdot \vec{d_{ray}} \cdot (\vec{rc} \cdot \vec{rc} - r^2) \geq 0$,

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-1 \cdot (-2 \cdot \vec{rc} \cdot \vec{d_{ray}}) \pm \sqrt{(-2 \cdot \vec{rc} \cdot \vec{d_{ray}})^2 - 4 \cdot \vec{d_{ray}} \cdot \vec{d_{ray}} \cdot (\vec{rc} \cdot \vec{rc} - r^2)}}{2 \cdot \vec{d_{ray}} \cdot \vec{d_{ray}}}$$

- to reduce calculation, we can use $D/4$.

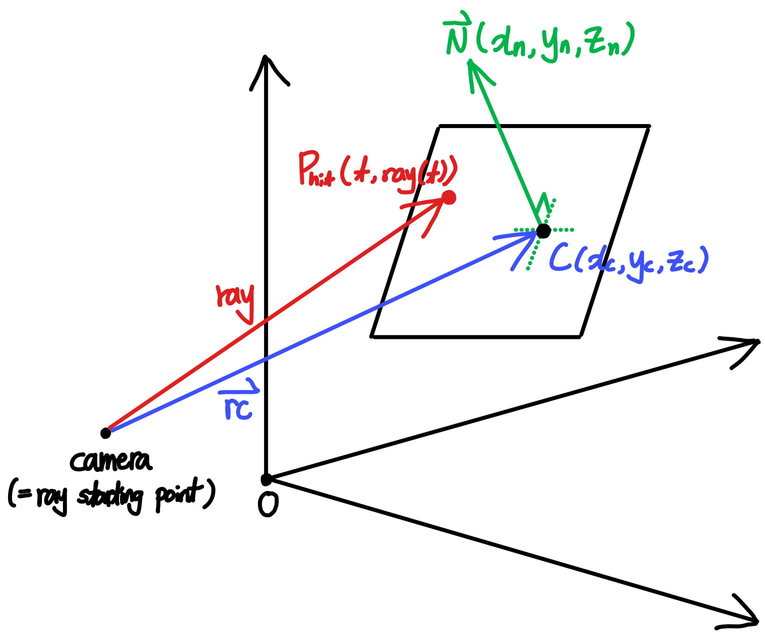
  $D/4 = (b/2)^2 - ac = (-1 \cdot \vec{rc} \cdot \vec{d_{ray}})^2 - \vec{d_{ray}} \cdot \vec{d_{ray}} \cdot (\vec{rc} \cdot \vec{rc} - r^2) \geq 0$,

$$t = \frac{-(b/2) \pm \sqrt{(b/2)^2 - ac}}{a} = \frac{-1 \cdot (-1 \cdot \vec{rc} \cdot \vec{d_{ray}})^2 \pm \sqrt{(-1 \cdot \vec{rc} \cdot \vec{d_{ray}})^2 - \vec{d_{ray}} \cdot \vec{d_{ray}} \cdot (\vec{rc} \cdot \vec{rc} - r^2)}}{\vec{d_{ray}} \cdot \vec{d_{ray}}}$$

- normal vector at P_{hit}

$$: \vec{N_{hit}} = \frac{\vec{CP_{hit}}}{|CP_{hit}|}$$

· ray – plane intersection



* equation of plane

· $ax + by + cz + d = 0$

(if $\vec{N}$ is (x_n, y_n, z_n),

$x_n \cdot x + y_n \cdot y + z_n \cdot z + d = 0$)

· $ax + by + cz + d = 0$

$\rightarrow x_n \cdot x + y_n \cdot y + z_n \cdot z + d = 0$

· we're checking at the hit point $P_{hit}(x_{hit}, y_{hit}, z_{hit})$.

$\rightarrow x_n \cdot x_{hit} + y_n \cdot y_{hit} + z_n \cdot z_{hit} + d = 0$

$\rightarrow \vec{N} \cdot P_{hit} + d = 0$

· the center of the plane $C(x_c, y_c, z_c)$ is also a point on the plane.

$\rightarrow x_n \cdot x_c + y_n \cdot y_c + z_n \cdot z_c + d = 0$

$\rightarrow \vec{N} \cdot P_c + d = 0$

$\rightarrow d = -\vec{N} \cdot P_c$

· P_{hit} is a point on the plane and a point on the ray at the same time.

on the ray's aspect, we can express it as $r_{point} + t \cdot \vec{d_{ray}}$.

$\rightarrow \vec{N} \cdot P_{hit} + d = 0$

$\rightarrow \vec{N} \cdot P_{hit} - \vec{N} \cdot P_c = 0$

$\rightarrow \vec{N} \cdot (r_{point} + t \cdot \vec{d_{ray}}) - \vec{N} \cdot P_c = 0$

$\rightarrow \vec{N} \cdot \vec{d_{ray}} \cdot t + (\vec{N} \cdot r_{point} - \vec{N} \cdot P_c) = 0$

$\rightarrow \vec{N} \cdot \vec{d_{ray}} \cdot t + \vec{N} \cdot (r_{point} - P_c) = 0$

$\rightarrow \vec{N} \cdot \vec{d_{ray}} \cdot t - \vec{N} \cdot \vec{rc} = 0$

$\therefore t = \dfrac{\vec{N} \cdot \vec{rc}}{\vec{N} \cdot \vec{d_{ray}}} \quad (\vec{N} \cdot \vec{d_{ray}} \neq 0)$

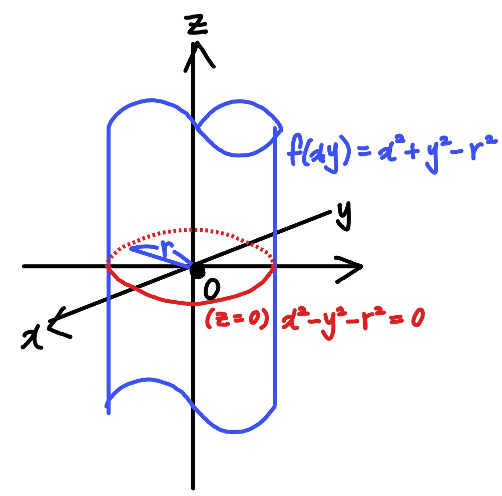
· normal vector at P_{hit}

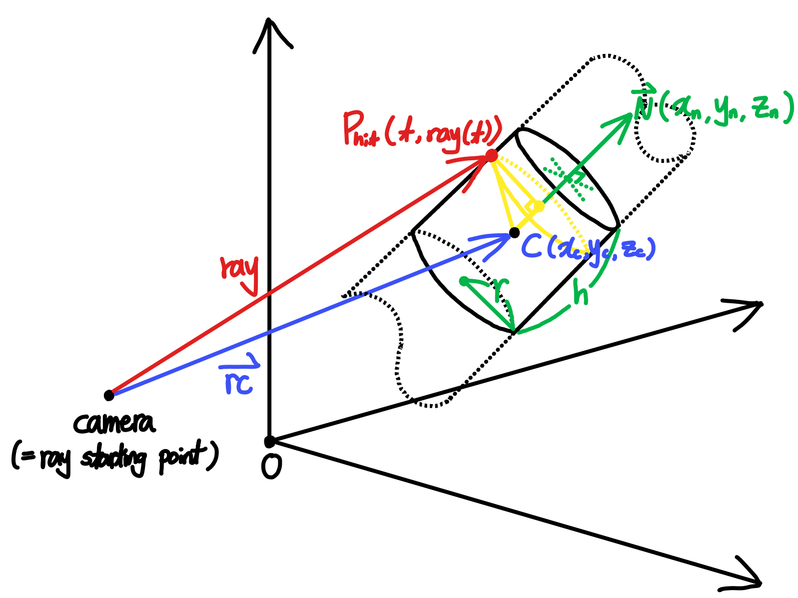
: $\vec{N_{hit}} = \vec{N}$

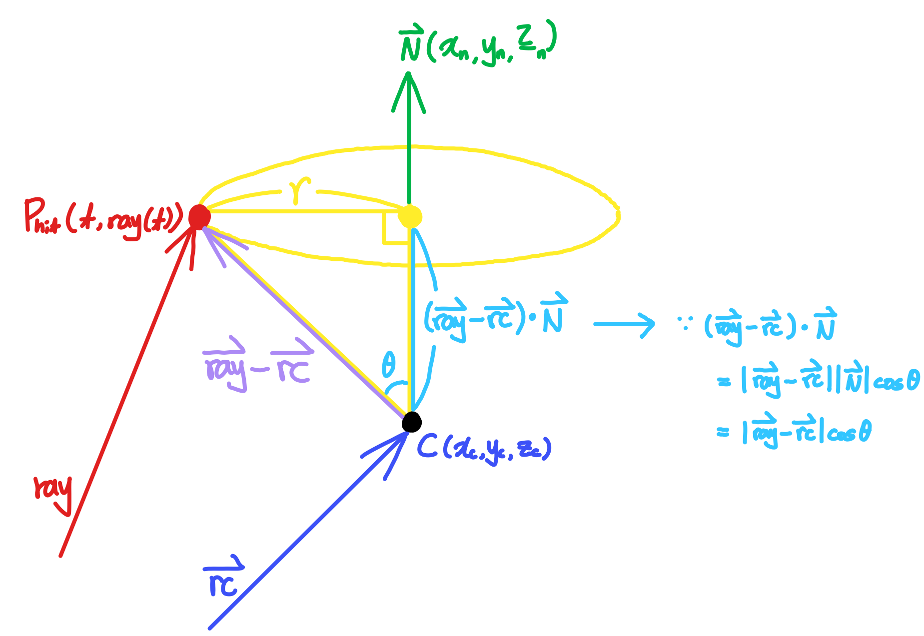
· ray-cylinder intersection

· step 1 : check hit point of the body of the cylinder.

· a basic form of cylinder is a cylinder that has infinite height without top and bottom circles.







· $|ray(t) - \vec{rc}|^2 = \{(ray(t) - \vec{rc})\cdot\vec{N}\}^2 + r^2$

$\rightarrow |ray(t) - \vec{rc}|\cdot|ray(t) - \vec{rc}| - \{(ray(t) - \vec{rc})\cdot\vec{N}\}\cdot\{(ray(t) - \vec{rc})\cdot\vec{N}\} - r^2 = 0$

$\rightarrow ray(t)^2 - 2\cdot\vec{rc}\cdot ray(t) + \vec{rc}\cdot\vec{rc} - (ray(t)\cdot\vec{N} - \vec{rc}\cdot\vec{N})^2 - r^2 = 0$

$\rightarrow ray(t)^2 - 2\cdot\vec{rc}\cdot ray(t) + \vec{rc}\cdot\vec{rc} - (ray(t)\cdot\vec{N})^2 + 2\cdot(ray(t)\cdot\vec{N})(\vec{rc}\cdot\vec{N}) + (\vec{rc}\cdot\vec{N})^2 - r^2 = 0$

· $ray(t)$ is a point on the sphere and a point on the ray at the same time. on the ray's aspect, we can express it as $r_{point} + t\cdot\vec{d_{ray}}$.

· $ray(t)^2 - 2\cdot\vec{rc}\cdot ray(t) + \vec{rc}\cdot\vec{rc} - (ray(t)\cdot\vec{N})^2 + 2\cdot(ray(t)\cdot\vec{N})(\vec{rc}\cdot\vec{N}) - (\vec{rc}\cdot\vec{N})^2 - r^2 = 0$

$\rightarrow (r_{point} + t\cdot\vec{d_{ray}})^2 - 2\cdot\vec{rc}\cdot(r_{point} + t\cdot\vec{d_{ray}}) + \vec{rc}\cdot\vec{rc} - \{(r_{point} + t\cdot\vec{d_{ray}})\cdot\vec{N}\}^2 + 2\cdot\{(r_{point} + t\cdot\vec{d_{ray}})\cdot\vec{N}\}(\vec{rc}\cdot\vec{N}) - (\vec{rc}\cdot\vec{N})^2 - r^2 = 0$

$\rightarrow r_{point}^2 + 2\cdot r_{point}\cdot\vec{d_{ray}}\cdot t + \vec{d_{ray}}\cdot\vec{d_{ray}}\cdot t^2 - 2\cdot\vec{rc}\cdot r_{point} - 2\cdot\vec{rc}\cdot\vec{d_{ray}}\cdot t + \vec{rc}\cdot\vec{rc}$
$\quad - (r_{point}\cdot\vec{N} + t\cdot\vec{d_{ray}}\cdot\vec{N})^2 + 2(r_{point}\cdot\vec{N} + t\cdot\vec{d_{ray}}\cdot\vec{N})(\vec{rc}\cdot\vec{N}) - (\vec{rc}\cdot\vec{N})^2 - r^2 = 0$

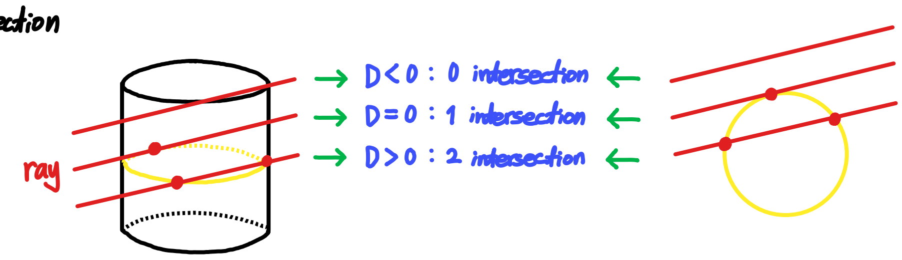
$\rightarrow r_{point}^2 + 2\cdot r_{point}\cdot\vec{d_{ray}}\cdot t + \vec{d_{ray}}\cdot\vec{d_{ray}}\cdot t^2 - 2\cdot\vec{rc}\cdot r_{point} - 2\cdot\vec{rc}\cdot\vec{d_{ray}}\cdot t + \vec{rc}\cdot\vec{rc}$
$\quad - (r_{point}\cdot\vec{N})^2 - 2\cdot(r_{point}\cdot\vec{N})(\vec{d_{ray}}\cdot\vec{N})t - (\vec{d_{ray}}\cdot\vec{N})^2\cdot t^2 + 2\cdot r_{point}\cdot\vec{N}\cdot(\vec{rc}\cdot\vec{N}) + 2\cdot(\vec{d_{ray}}\cdot\vec{N})(\vec{rc}\cdot\vec{N})\cdot t - (\vec{rc}\cdot\vec{N})^2 - r^2 = 0$

$\rightarrow \{\vec{d_{ray}}\cdot\vec{d_{ray}} - (\vec{d_{ray}}\cdot\vec{N})^2\}t^2 + \{\cancel{2\cdot r_{point}\cdot\vec{d_{ray}}} - 2\cdot\vec{rc}\cdot\vec{d_{ray}} \cancel{- 2\cdot(r_{point}\cdot\vec{N})(\vec{d_{ray}}\cdot\vec{N})} + 2\cdot(\vec{d_{ray}}\cdot\vec{N})(\vec{rc}\cdot\vec{N})\}t$
$\quad + \{\cancel{r_{point}^2} \cancel{- 2\cdot\vec{rc}\cdot r_{point}} + \vec{rc}\cdot\vec{rc} \cancel{- (r_{point}\cdot\vec{N})^2} \cancel{+ 2\cdot r_{point}\cdot\vec{N}\cdot(\vec{rc}\cdot\vec{N})} - (\vec{rc}\cdot\vec{N})^2 - r^2\} = 0$

$\rightarrow \{\vec{d_{ray}}\cdot\vec{d_{ray}} - (\vec{d_{ray}}\cdot\vec{N})^2\}t^2 - 2\{\vec{rc}\cdot\vec{d_{ray}} - (\vec{d_{ray}}\cdot\vec{N})(\vec{rc}\cdot\vec{N})\}t + \{\vec{rc}\cdot\vec{rc} - (\vec{rc}\cdot\vec{N})^2 - r^2\} = 0$

(quadratic expression for t)

- t is also a point on the body of the cylinder,
  so there should be at least one intersection
  → discriminant ≥ 0



$$\therefore D = b^2-4ac = [-2\cdot\{\vec{rc}\cdot\vec{d_{ray}} - (\vec{d_{ray}}\cdot\vec{N})(\vec{rc}\cdot\vec{N})\}]^2 - 4\cdot\{\vec{d_{ray}}\cdot\vec{d_{ray}} - (\vec{d_{ray}}\cdot\vec{N})^2\}\cdot\{\vec{rc}\cdot\vec{rc} - (\vec{rc}\cdot\vec{N})^2 - r^2\} \geq 0,$$

$$t = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$= \frac{-1\cdot -2\{\vec{rc}\cdot\vec{d_{ray}} - (\vec{d_{ray}}\cdot\vec{N})(\vec{rc}\cdot\vec{N})\} \pm \sqrt{[-2\{\vec{rc}\cdot\vec{d_{ray}} - (\vec{d_{ray}}\cdot\vec{N})(\vec{rc}\cdot\vec{N})\}]^2 - 4\cdot\{\vec{d_{ray}}\cdot\vec{d_{ray}} - (\vec{d_{ray}}\cdot\vec{N})^2\}\cdot\{\vec{rc}\cdot\vec{rc} - (\vec{rc}\cdot\vec{N})^2 - r^2\}}}{2\cdot\{\vec{d_{ray}}\cdot\vec{d_{ray}} - (\vec{d_{ray}}\cdot\vec{N})^2\}}$$

- to reduce calculation, we can use $D/4$.

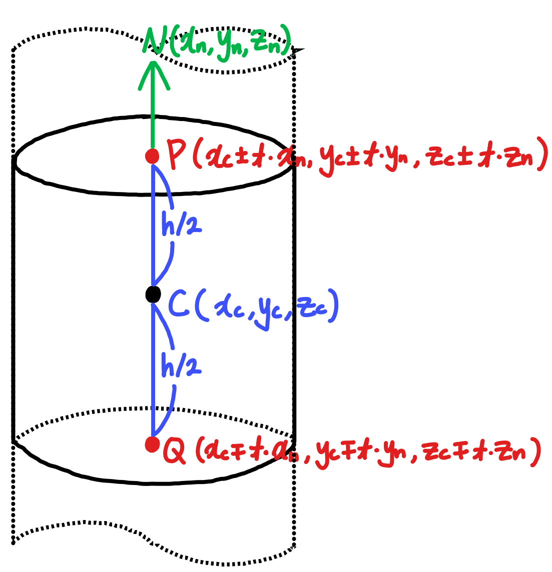
$$D/4 = (b/2)^2 - ac = [-1\cdot\{\vec{rc}\cdot\vec{d_{ray}} - (\vec{d_{ray}}\cdot\vec{N})(\vec{rc}\cdot\vec{N})\}]^2 - \{\vec{d_{ray}}\cdot\vec{d_{ray}} - (\vec{d_{ray}}\cdot\vec{N})^2\}\cdot\{\vec{rc}\cdot\vec{rc} - (\vec{rc}\cdot\vec{N})^2 - r^2\} \geq 0,$$

$$t = \frac{-(b/2)^2 \pm \sqrt{(b/2)^2 - ac}}{a}$$

$$= \frac{-1\cdot -1\{\vec{rc}\cdot\vec{d_{ray}} - (\vec{d_{ray}}\cdot\vec{N})(\vec{rc}\cdot\vec{N})\} \pm \sqrt{[-1\cdot\{\vec{rc}\cdot\vec{d_{ray}} - (\vec{d_{ray}}\cdot\vec{N})(\vec{rc}\cdot\vec{N})\}]^2 - \{\vec{d_{ray}}\cdot\vec{d_{ray}} - (\vec{d_{ray}}\cdot\vec{N})^2\}\cdot\{\vec{rc}\cdot\vec{rc} - (\vec{rc}\cdot\vec{N})^2 - r^2\}}}{\{\vec{d_{ray}}\cdot\vec{d_{ray}} - (\vec{d_{ray}}\cdot\vec{N})^2\}}$$

- step 2 : check height of the cylinder.

- to check the height, we need positions of the center of top and bottom circles.



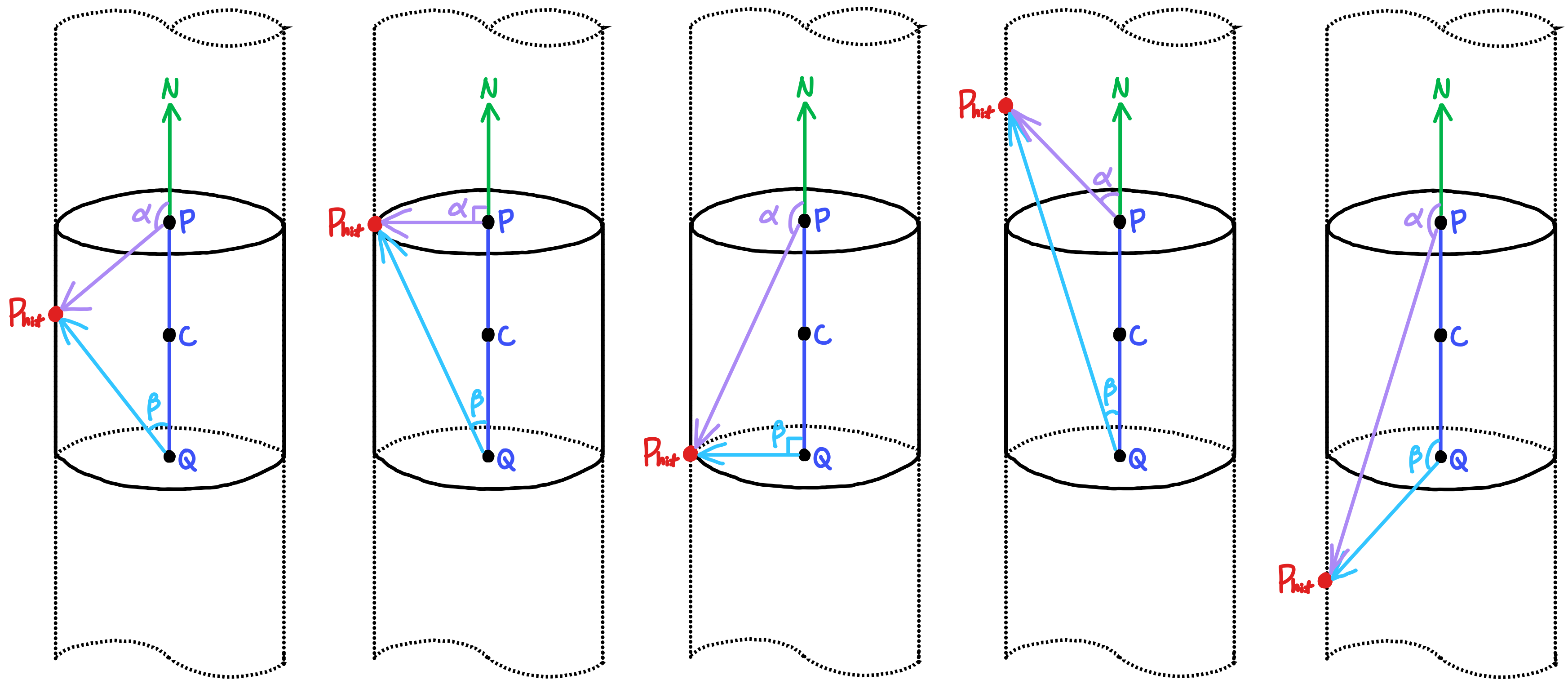
- P, C, Q are on the same line, start point is C and direction is $\vec{N}$ $(C + t\cdot\vec{N})$.
- P and Q are point symmeteric
  → if the position of P is $(x_c+t\cdot x_n,\ y_c+t\cdot y_n,\ z_c+t\cdot z_n)$, Q is $(x_c-t\cdot x_n,\ y_c-t\cdot y_n,\ z_c-t\cdot z_n)$.
- also, $\overline{CP} = \overline{CQ} = h/2$

$$\rightarrow \sqrt{(x_c+t\cdot x_n - x_c)^2 + (y_c+t\cdot y_n - y_c)^2 + (z_c+t\cdot z_n - z_c)^2} = \pm h/2$$

$$\rightarrow \sqrt{x_n^2\cdot t^2 + y_n^2\cdot t^2 + z_n^2\cdot t^2} = \pm h/2$$

$$\rightarrow \sqrt{(x_n^2+y_n^2+z_n^2)t^2} = \pm h/2$$

$$\therefore t = \pm h/2 \cdot \sqrt{1/(x_n^2+y_n^2+z_n^2)}$$



· we can check if P_{hit} is on cylinder or not by comparing angles between $\vec{N}$ and $\overrightarrow{PP_{hit}}$, and $\vec{N}$ and $\overrightarrow{QP_{hit}}$.

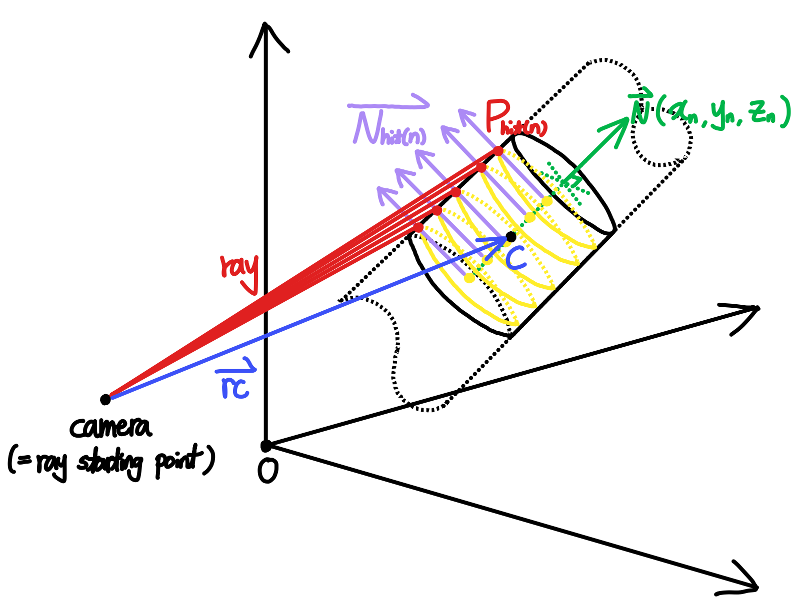
→ $\pi/2 \le \alpha < \pi$, $0 < \beta \le \pi/2$

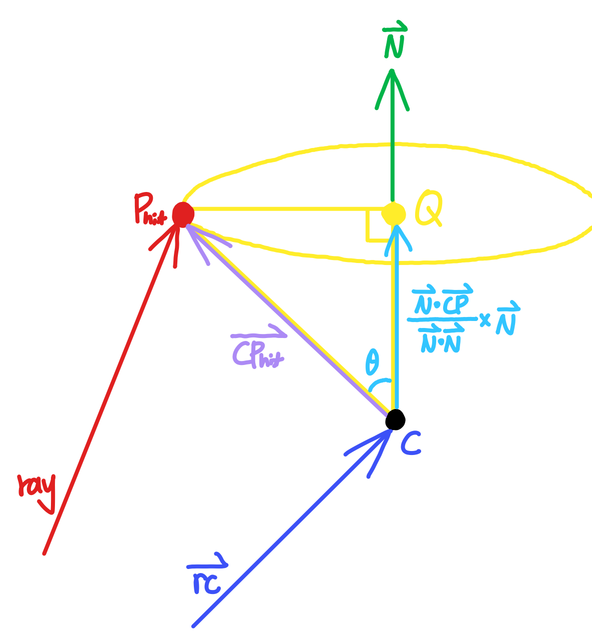
(α : angle between $\vec{N}$ and $\overrightarrow{PP_{hit}}$, β : angle between $\vec{N}$ and $\overrightarrow{QP_{hit}}$)

→ $\vec{N} \cdot \overrightarrow{PP_{hit}} \le 0$, $\vec{N} \cdot \overrightarrow{QP_{hit}} \ge 0$

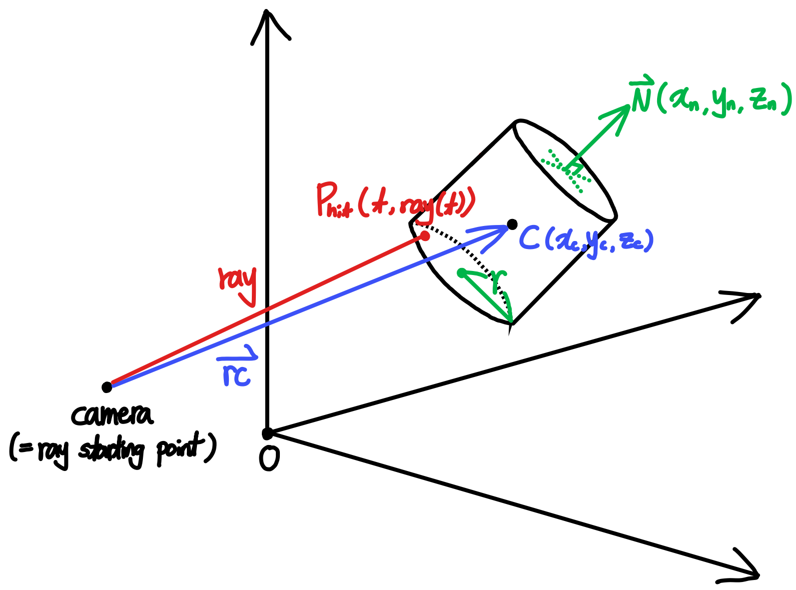
· normal vector at P_{hit}

$$: \overrightarrow{N_{hit}} = \frac{\overrightarrow{QP_{hit}}}{|\overrightarrow{QP_{hit}}|} = \frac{\overrightarrow{P_{hit} - (C + \frac{\vec{N}\cdot\vec{CP}}{\vec{N}\cdot\vec{N}} \times \vec{N})}}{\left|\overrightarrow{P_{hit} - (C + \frac{\vec{N}\cdot\vec{CP}}{\vec{N}\cdot\vec{N}} \times \vec{N})}\right|}$$



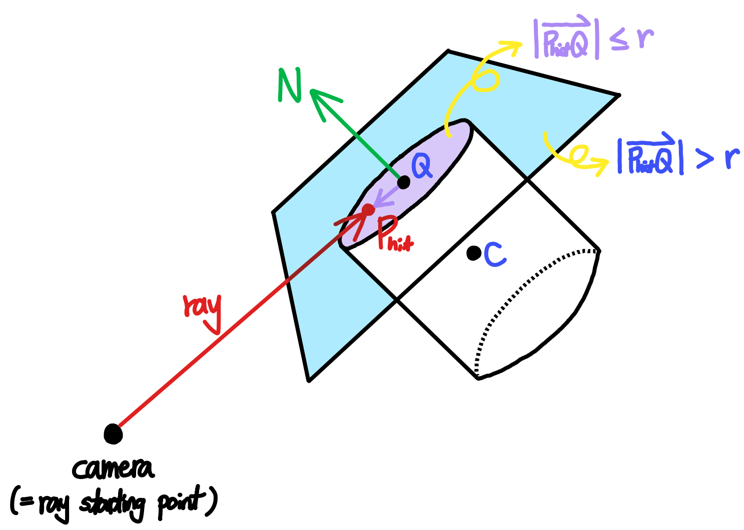


· step 3 : check hit point of the top and the bottom circles of the cylinder.



* top circle of the cylinder

· think as ray-plane intersection, and then limit the range of $\overrightarrow{P_{hit}Q}$.



1) $Q\left(x_c + h/2 \cdot \sqrt{(1/x_n^2+y_n^2+z_n^2)} \cdot x_n,\ y_c + h/2 \cdot \sqrt{(1/x_n^2+y_n^2+z_n^2)} \cdot y_n,\ z_c + h/2 \cdot \sqrt{(1/x_n^2+y_n^2+z_n^2)} \cdot z_n\right)$,

plane hit point $t = \dfrac{\vec{N} \cdot \vec{rQ}}{\vec{N} \cdot \vec{d_{ray}}}$ $(\vec{N} \cdot \vec{d_{ray}} \neq 0)$

2) $|\overrightarrow{P_{hit}Q}| \le r$

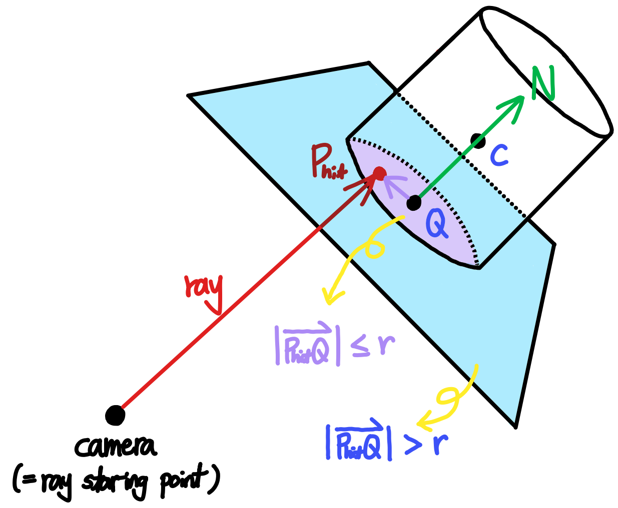
→ 1) ∩ 2) is a set of hit point on the top circle of the cylinder.

· normal vector at P_{hit}

: $\overrightarrow{N_{hit}} = \vec{N}$

* bottom circle of the cylinder

· think as ray-plane intersection, and then limit the range of $\overrightarrow{P_{hit}Q}$.



1) $Q\left(x_c - h/2 \cdot \sqrt{(1/x_n^2+ n^2+z_n^2)} \cdot x_n,\ y_c - h/2 \cdot \sqrt{(1/x_n^2+y_n^2+z_n^2)} \cdot y_n,\ z_c - h/2 \cdot \sqrt{(1/x_n^2+y_n^2+z_n^2)} \cdot z_n\right)$,

plane hit point $t = \dfrac{\vec{N} \cdot \overrightarrow{rQ}}{\vec{N} \cdot \overrightarrow{d_{ray}}}$ $(\vec{N} \cdot \overrightarrow{d_{ray}} \neq 0)$

2) $|\overrightarrow{P_{hit}Q}| \le r$

→ 1) ∩ 2) is a set of hit point on the top circle of the cylinder.

· normal vector at P_{hit}

: $\overrightarrow{N_{hit}} = \vec{N}$

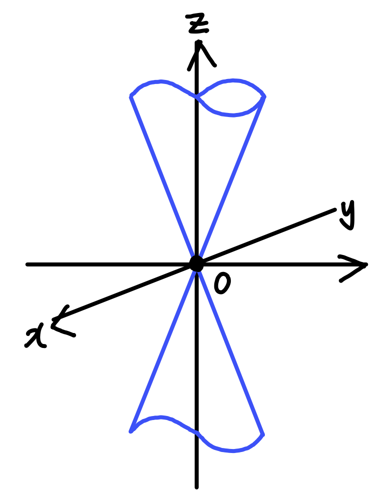
· if a hit point of the top circle t_top is less than a hit point of the bottom circle t_bottom, the top circle of the cylinder will be drawn. (we'll ignore the bottom circle)

· if a hit point of the bottom circle t_bottom is less than a hit point of the top circle t_top, the bottom circle of the cylinder will be drawn. (we'll ignore the top circle)

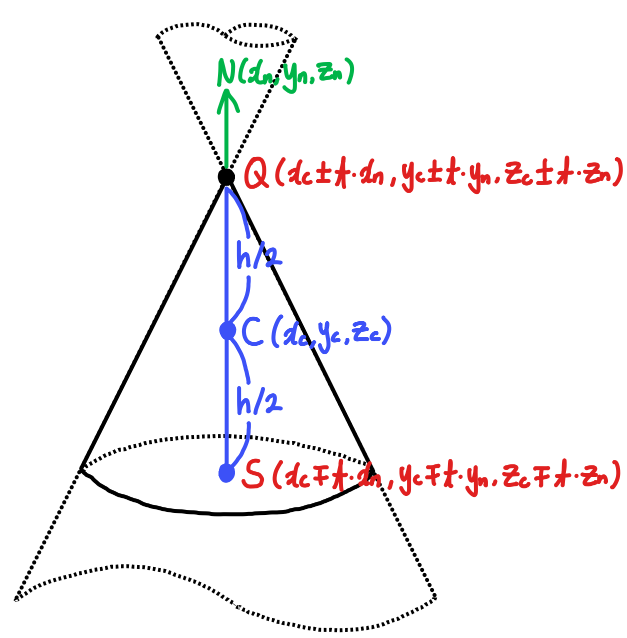
· ray - cone intersection

· step 1 : check hit point of the body of the cone.

· a basic form of cylinder is a cone that has infinite height without top and bottom circles.



· check a center of the bottom circle and a point of the top.



- Q, C, S are on the same line, start point is C and direction is $\vec{N}$ $(C + t \cdot \vec{N})$.

- Q and S are point symmeteric

  → if the position of Q is $(x_c + t \cdot x_n,\ y_c + t \cdot y_n,\ z_c + t \cdot z_n)$, S is $(x_c - t \cdot x_n,\ y_c - t \cdot y_n,\ z_c - t \cdot z_n)$.

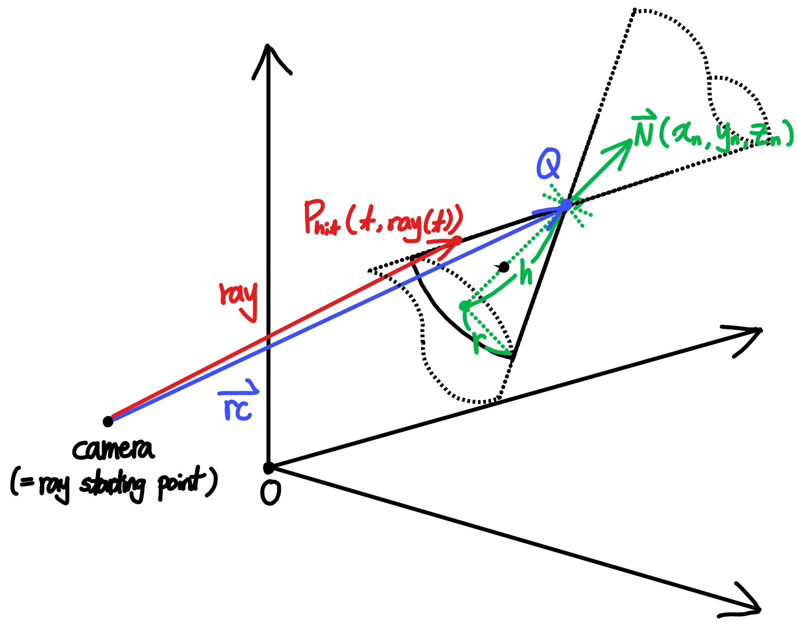
- also, $\overline{CQ} = \overline{CS} = h/2$

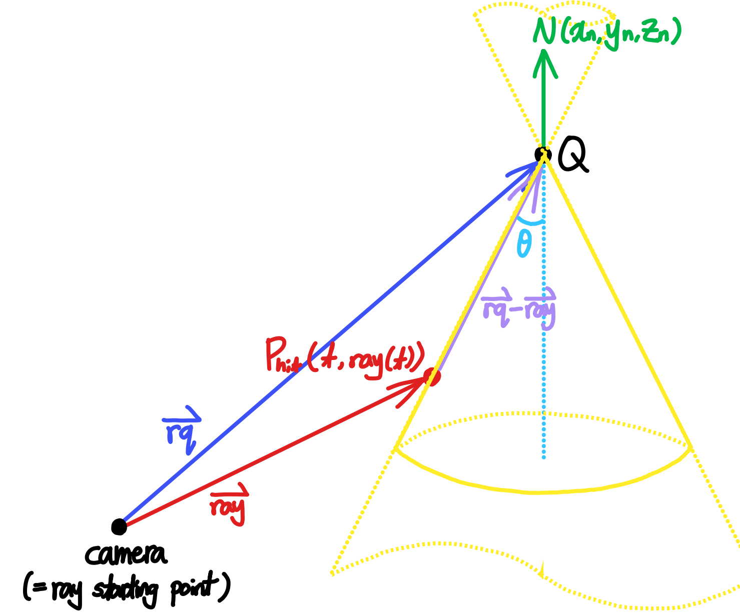
  → $\sqrt{(x_c + t \cdot x_n - x_c)^2 + (y_c + t \cdot y_n - y_c)^2 + (z_c + t \cdot z_n - z_c)^2} = \pm h/2$

  → $\sqrt{x_n^2 \cdot t^2 + y_n^2 \cdot t^2 + z_n^2 \cdot t^2} = \pm h/2$

  → $\sqrt{(x_n^2 + y_n^2 + z_n^2) t^2} = \pm h/2$

  $\therefore t = \pm h/2 \cdot \sqrt{1 / (x_n^2 + y_n^2 + z_n^2)}$





- $(\vec{rq} - ray(t)) \cdot \vec{N} = |\vec{rq} - ray(t)||\vec{N}| \cdot \cos\theta$

  $\rightarrow$ since $|\vec{N}| = 1$, $(\vec{rq} - ray(t)) \cdot \vec{N} = |\vec{rq} - ray(t)| \cdot \cos\theta$

  $\rightarrow (\vec{rq}\cdot\vec{N} - ray(t)\cdot\vec{N})^2 = (\vec{rq} - ray(t))^2 \cdot \cos^2\theta$

  $\rightarrow \{(\vec{rq}\cdot\vec{N})^2 - 2\cdot(\vec{rq}\cdot\vec{N})\cdot ray(t)\cdot\vec{N} + ray(t)^2\cdot(\vec{N}\cdot\vec{N})\} - \{\vec{rq}\cdot\vec{rq} - 2\cdot\vec{rq}\cdot ray(t) + ray(t)^2\}\cdot\cos^2\theta = 0$

- $ray(t)$ is a point on the sphere and a point on the ray at the same time.
  on the ray's aspect, we can express it as $r_{point} + t\cdot\vec{d_{ray}}$.

- $\{(\vec{rq}\cdot\vec{N})^2 - 2\cdot(\vec{rq}\cdot\vec{N})\cdot ray(t)\cdot\vec{N} + ray(t)^2\cdot(\vec{N}\cdot\vec{N})\} - \{\vec{rq}\cdot\vec{rq} - 2\cdot\vec{rq}\cdot ray(t) + ray(t)^2\}\cdot\cos^2\theta = 0$

  $\rightarrow \{(\vec{rq}\cdot\vec{N})(\vec{rq}\cdot\vec{N}) - 2\cdot(\vec{rq}\cdot\vec{N})\cdot\vec{N}\cdot(r_p + t\cdot\vec{d_{ray}}) + (r_p\cdot\vec{N} + t\cdot\vec{d_{ray}}\cdot\vec{N})^2$

  $\quad - (\vec{rq}\cdot\vec{rq})\cdot\cos^2\theta + 2\cdot\vec{rq}\cdot(r_p + t\cdot\vec{d_{ray}})\cdot\cos^2\theta - (r_p + t\cdot\vec{d_{ray}})^2\cdot\cos^2\theta = 0$

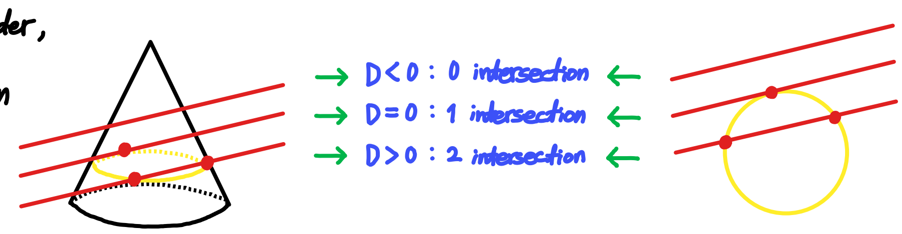
  $\rightarrow (\vec{rq}\cdot\vec{N})(\vec{rq}\cdot\vec{N}) \cancel{- 2\cdot(\vec{rq}\cdot\vec{N})\cdot\vec{N}\cdot r_p} - 2\cdot(\vec{rq}\cdot\vec{N})\cdot(\vec{N}\cdot\vec{d_{ray}})t + \cancel{r_p^2\cdot(\vec{N}\cdot\vec{N})} + \cancel{2\cdot r_p\cdot\vec{N}\cdot(\vec{d_{ray}}\cdot\vec{N})t} + (\vec{d_{ray}}\cdot\vec{N})(\vec{d_{ray}}\cdot\vec{N})t^2$

  $\quad - (\vec{rq}\cdot\vec{rq})\cdot\cos^2\theta + \cancel{2\cdot\vec{rq}\cdot r_p\cdot\cos^2\theta} + 2\cdot(\vec{rq}\cdot\vec{d_{ray}})\cdot\cos^2\theta\, t \cancel{- r_p^2\cos^2\theta} \cancel{- 2\cdot r_p\cdot\vec{d_{ray}}\cos^2\theta\, t} - (\vec{d_{ray}}\cdot\vec{d_{ray}})\cdot\cos^2\theta\, t^2 = 0$

  $\rightarrow \{(\vec{d_{ray}}\cdot\vec{N})(\vec{d_{ray}}\cdot\vec{N}) - (\vec{d_{ray}}\cdot\vec{d_{ray}})\cdot\cos^2\theta\}t^2 - 2\{(\vec{rq}\cdot\vec{N})(\vec{N}\cdot\vec{d_{ray}}) - (\vec{rq}\cdot\vec{d_{ray}})\cdot\cos^2\theta\}t + \{(\vec{rq}\cdot\vec{N})(\vec{rq}\cdot\vec{N}) - (\vec{rq}\cdot\vec{rq})\cdot\cos^2\theta\} = 0$

  (quadratic expression for t)

- t is also a point on the body of the cylinder,
  so there should be at least one intersection
  $\rightarrow$ discriminant ≥ 0



$\therefore D = b^2 - 4ac = [-2\{(\vec{rq}\cdot\vec{N})(\vec{N}\cdot\vec{d_{ray}}) - (\vec{rq}\cdot\vec{d_{ray}})\cdot\cos^2\theta\}]^2$

$\quad - 4\cdot\{(\vec{d_{ray}}\cdot\vec{N})(\vec{d_{ray}}\cdot\vec{N}) - (\vec{d_{ray}}\cdot\vec{d_{ray}})\cdot\cos^2\theta\}\cdot\{(\vec{rq}\cdot\vec{N})(\vec{rq}\cdot\vec{N}) - (\vec{rq}\cdot\vec{rq})\cdot\cos^2\theta\} \geq 0$,

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-1\cdot -2\{(\vec{rq}\cdot\vec{N})(\vec{N}\cdot\vec{d_{ray}}) - (\vec{rq}\cdot\vec{d_{ray}})\cdot\cos^2\theta\} \pm \sqrt{[-2\{(\vec{rq}\cdot\vec{N})(\vec{N}\cdot\vec{d_{ray}}) - (\vec{rq}\cdot\vec{d_{ray}})\cdot\cos^2\theta\}]^2 - 4\cdot\{(\vec{d_{ray}}\cdot\vec{N})(\vec{d_{ray}}\cdot\vec{N}) - (\vec{d_{ray}}\cdot\vec{d_{ray}})\cdot\cos^2\theta\}\cdot\{(\vec{rq}\cdot\vec{N})(\vec{rq}\cdot\vec{N}) - (\vec{rq}\cdot\vec{rq})\cdot\cos^2\theta\}}}{2\cdot\{(\vec{d_{ray}}\cdot\vec{N})(\vec{d_{ray}}\cdot\vec{N}) - (\vec{d_{ray}}\cdot\vec{d_{ray}})\cdot\cos^2\theta\}}$$

- to reduce calculation, we can use D/4.

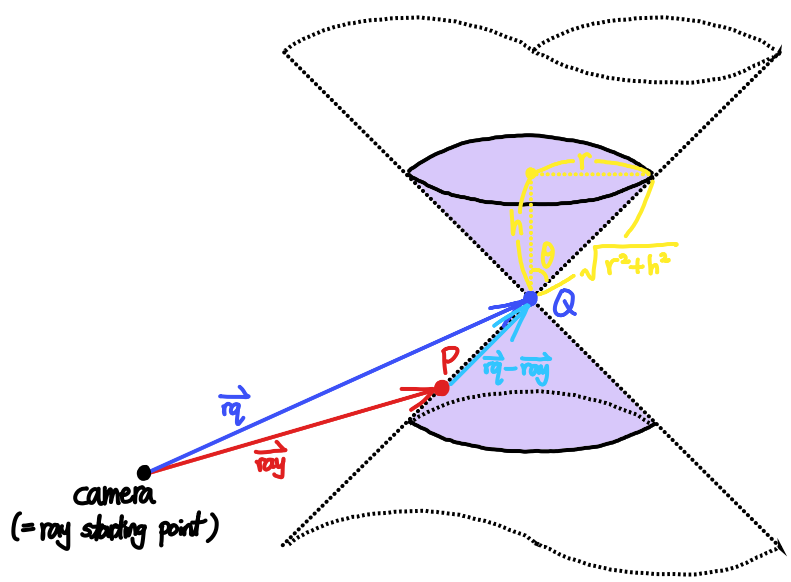
$$D/4 = (b/2)^2 - ac = \left[-1\cdot\{(\vec{rq}\cdot\vec{N})(\vec{N}\cdot\vec{dray}) - (\vec{rq}\cdot\vec{dray})\cdot\cos^2\theta\}\right]^2$$
$$-1\cdot\{(\vec{dray}\cdot\vec{N})(\vec{dray}\cdot\vec{N}) - (\vec{dray}\cdot\vec{dray})\cdot\cos^2\theta\}\cdot\{(\vec{rq}\cdot\vec{N})(\vec{rq}\cdot\vec{N}) - (\vec{rq}\cdot\vec{rq})\cdot\cos^2\theta\} \geq 0,$$

$$t = \frac{-(b/2)^2 \pm \sqrt{(b/2)^2 - ac}}{a}$$

$$= \frac{-1\cdot -1\cdot\{(\vec{rq}\cdot\vec{N})(\vec{N}\cdot\vec{dray}) - (\vec{rq}\cdot\vec{dray})\cdot\cos^2\theta\} \pm \sqrt{\left[-1\cdot\{(\vec{rq}\cdot\vec{N})(\vec{N}\cdot\vec{dray}) - (\vec{rq}\cdot\vec{dray})\cdot\cos^2\theta\}\right]^2 - \{(\vec{dray}\cdot\vec{N})(\vec{dray}\cdot\vec{N}) - (\vec{dray}\cdot\vec{dray})\cdot\cos^2\theta\}\cdot\{(\vec{rq}\cdot\vec{N})(\vec{rq}\cdot\vec{N}) - (\vec{rq}\cdot\vec{rq})\cdot\cos^2\theta\}}}{\{(\vec{dray}\cdot\vec{N})(\vec{dray}\cdot\vec{N}) - (\vec{dray}\cdot\vec{dray})\cdot\cos^2\theta\}}$$

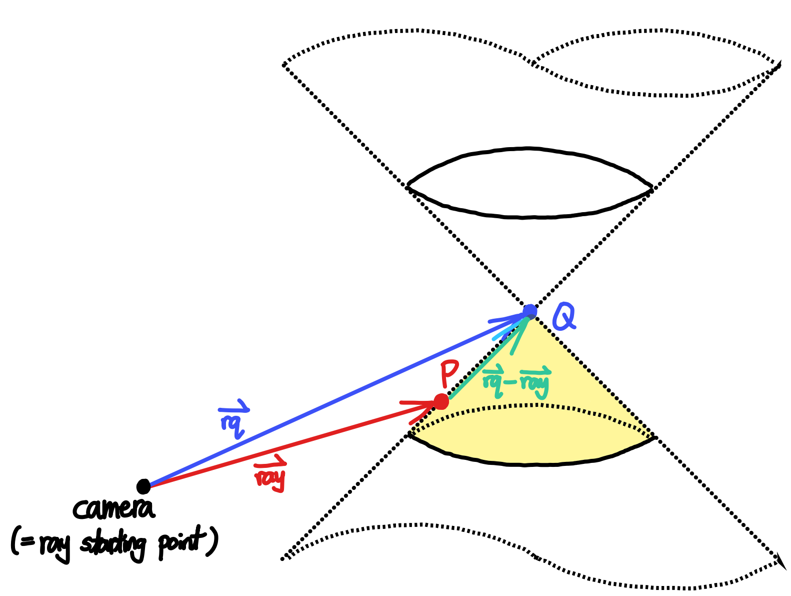
- step 2 : check height of the cone.

- limit the range of height



$*\ |\vec{PQ}| = |\vec{rq} - ray(t)| \leq \sqrt{r^2+h^2}$

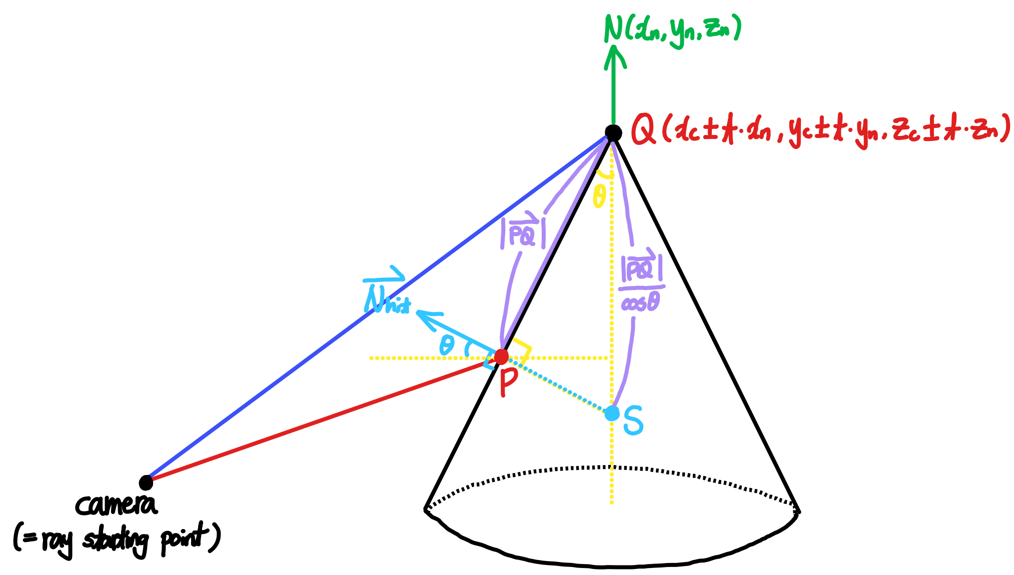
- take the bottom side



$*\ \vec{rq}\cdot\vec{PQ} = \vec{rq}\cdot(\vec{rq} - \vec{ray}) \geq 0$

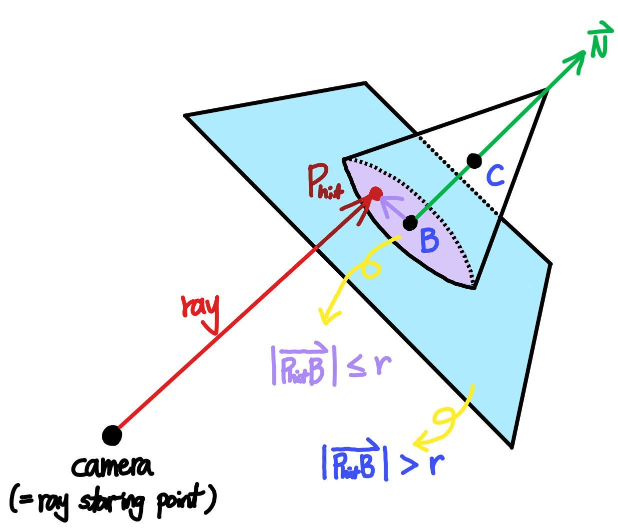
· normal vector at P_{hit}

$$: \overrightarrow{N_{hit}} = \frac{\overrightarrow{SP_{hit}}}{|\overrightarrow{SP_{hit}}|} = \frac{\overrightarrow{Q + \frac{\overrightarrow{PQ}}{\cos\theta} \times \vec{N}}}{\left|\overrightarrow{Q + \frac{\overrightarrow{PQ}}{\cos\theta} \times \vec{N}}\right|}$$



· step3 : check hit point of the top circle of the cone.

· think as ray-plane intersection, and then limit the range of $\overrightarrow{P_{hit}B}$.



1) $B\left(x_c - h/2 \cdot \sqrt{(1/x_n^2 + \ n^2 + z_n^2)} \cdot x_n,\ y_c - h/2 \cdot \sqrt{(1/x_n^2 + y_n^2 + z_n^2)} \cdot y_n,\ z_c - h/2 \cdot \sqrt{(1/x_n^2 + y_n^2 + z_n^2)} \cdot z_n\right)$,

plane hit point $t = \dfrac{\vec{N} \cdot \overrightarrow{rB}}{\vec{N} \cdot \overrightarrow{d_{ray}}}$ $(\vec{N} \cdot \overrightarrow{d_{ray}} \neq 0)$

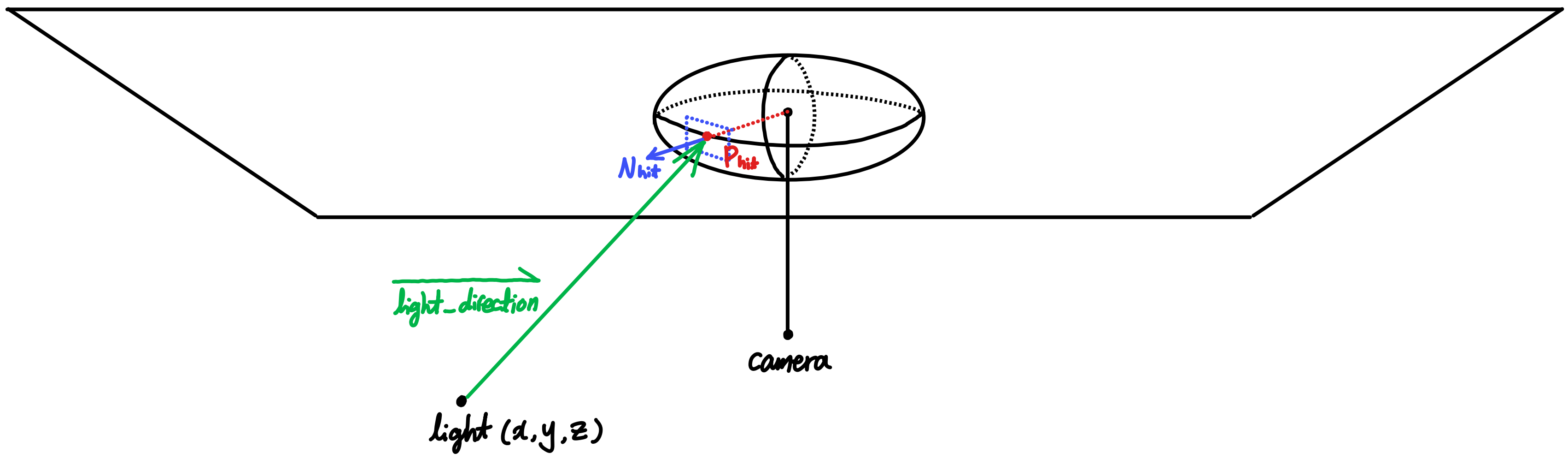
2) $|\overrightarrow{P_{hit}B}| \leq r$

→ 1) ∩ 2) is a set of hit point on the top circle of the cylinder.

· normal vector at P_{hit}

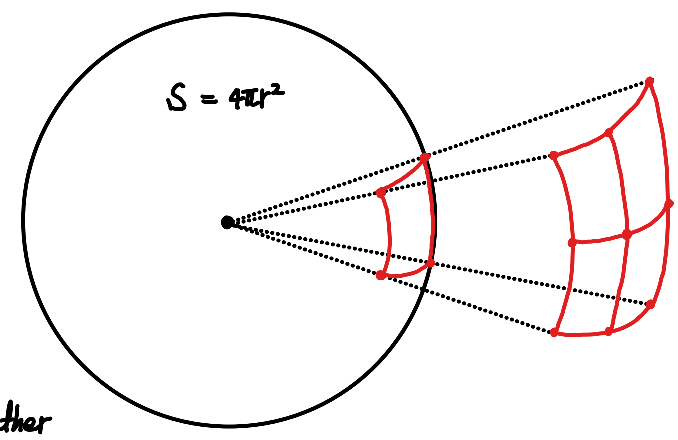
$: \overrightarrow{N_{hit}} = \vec{N}$

· apply light



- $\overrightarrow{light\_direction} = P_{hit} - P_{light}(x, y, z)$
- $distance = |\overrightarrow{light\_direction}|$
- by inverse square law of light, $light \propto \frac{1}{4\pi d^2}$.

  also, we should consider the angle of the light_direction and the normal vector of each hit point (= light_intensity)



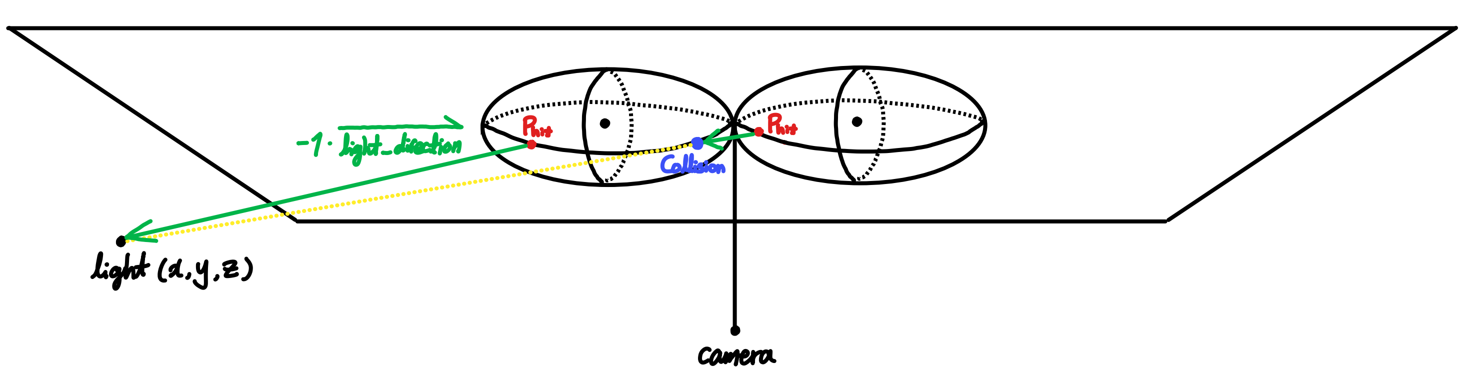
- more light will reflect when $\overrightarrow{light\_direction}$ and $\overrightarrow{N_{hit}}$ are closer to each other

  $\rightarrow light\_intensity = -\frac{\overrightarrow{light\_direction}}{|\overrightarrow{light\_direction}|} \cdot \overrightarrow{N_{hit}}$ (if $light\_intensity < 0$, $light\_intensity = 0$)

$\therefore light\_color = \frac{light\_rgb \times light\_brightness}{4 \times \pi \times distance \times distance} \times light\_intensity$

- light should apply when a line that starts from P_{hit} and direction is $-\overrightarrow{light\_direction}$ can reach the light without hitting other objects.

  (when it can't reach, light_color is 0)



* set camera vector

- for me, it was hard to change camera vector when I set viewport. so decieded not to change it.
  instead, I moved every objects including light, and then change camera position properly.
  with fixing camera vector as (0,0,-1).
- by moving every object and camera position, we can see the same result as we expected.

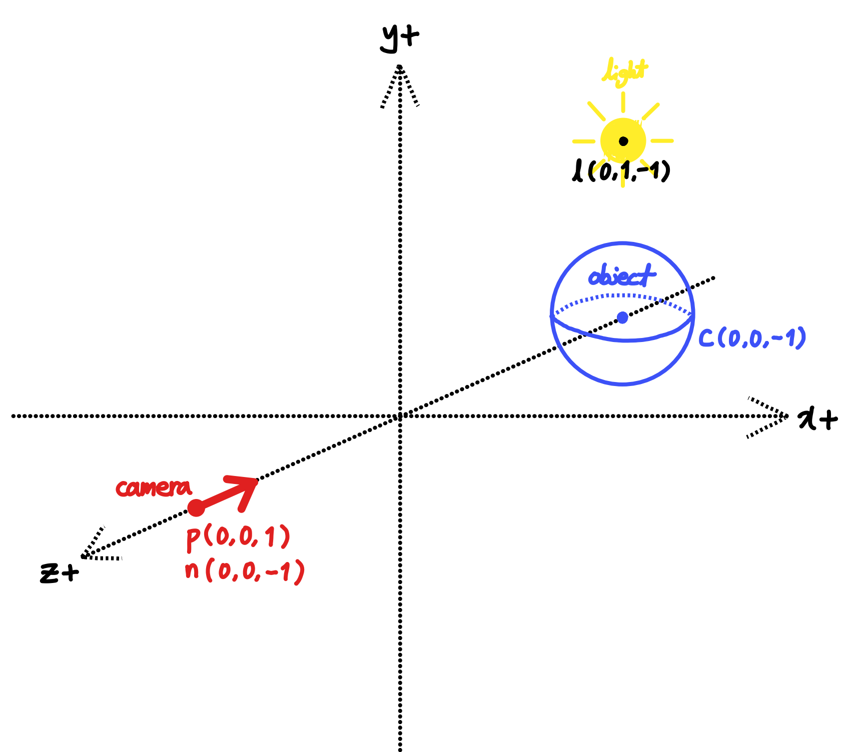
· formular for rotation

- $R_x \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$

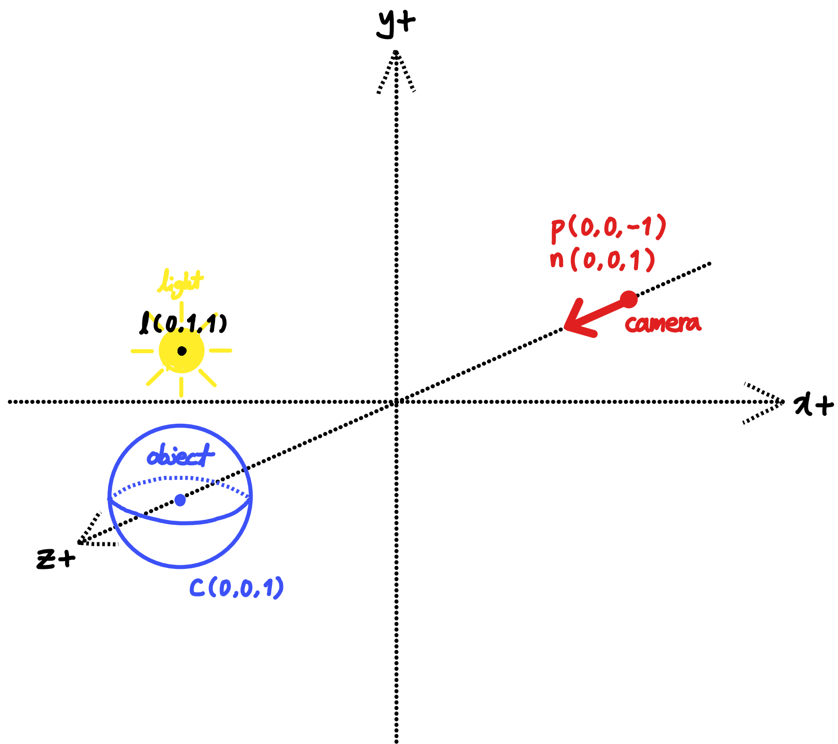
- $R_y \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$

- $R_z \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$

1. camera vector (0,0,-1) : basic form

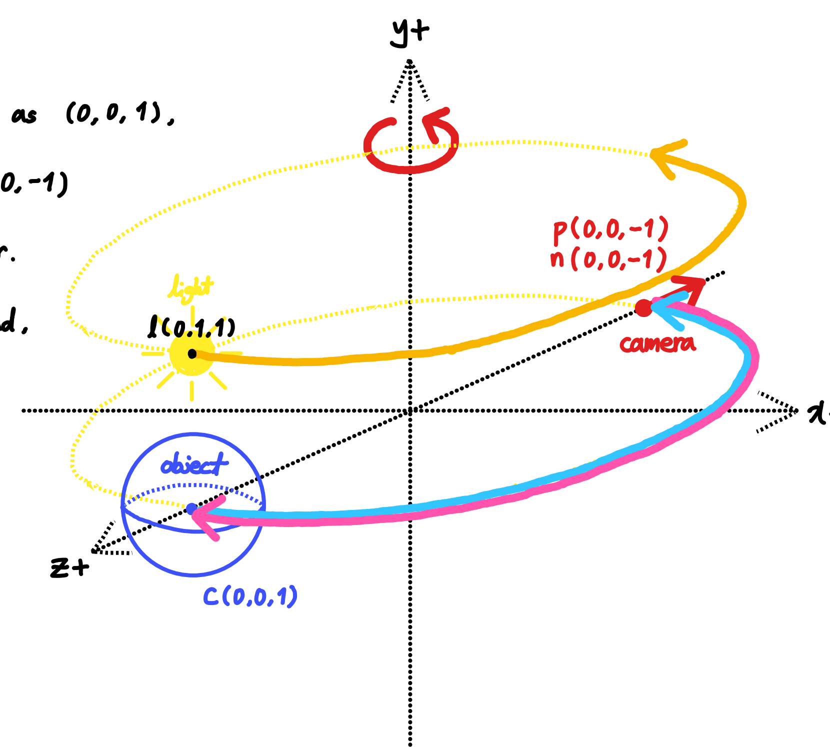


2. camera vector (0,0,1)

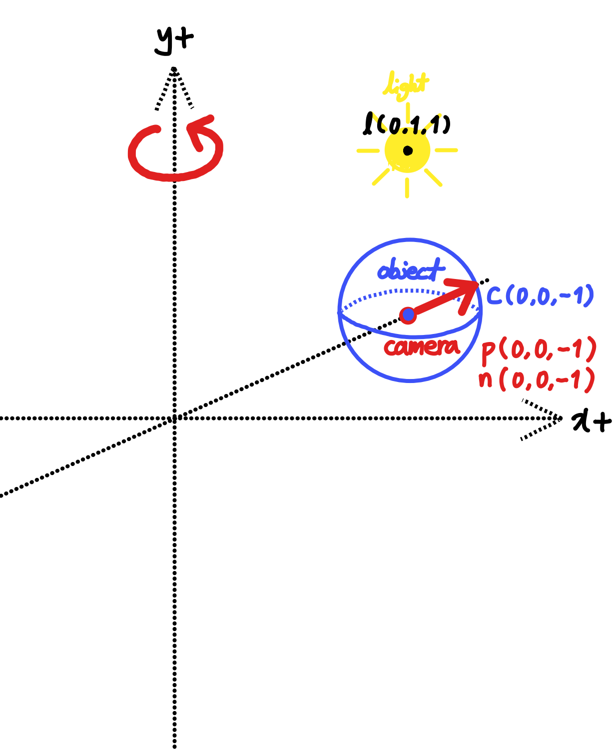


⇒ expect

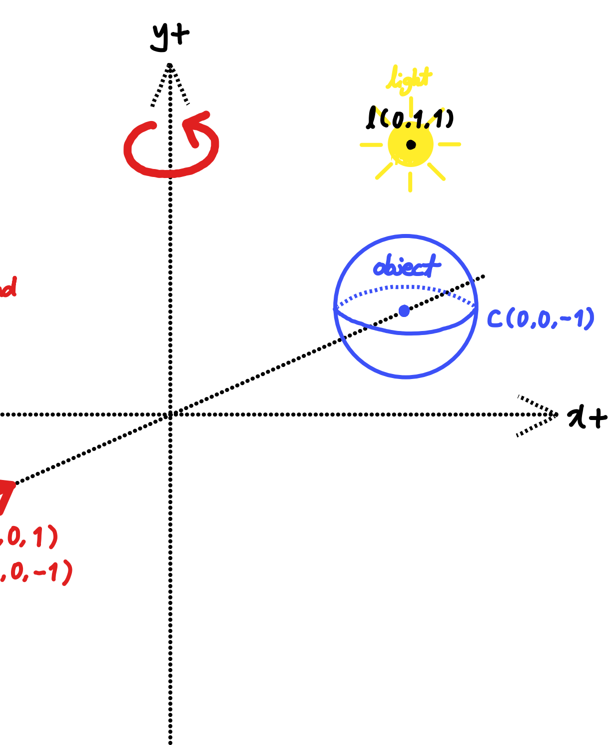
- even if I set camera position as (0,0,-1) and camera vector as (0,0,1), camera vector won't be changed and it will still be (0,0,-1) because I didn't make any function for setting camera vector.
- so, I'm gonna rotate every object and light to y axis : $+\pi$ rad, and rotate camera to y axis : $-\pi$ rad.



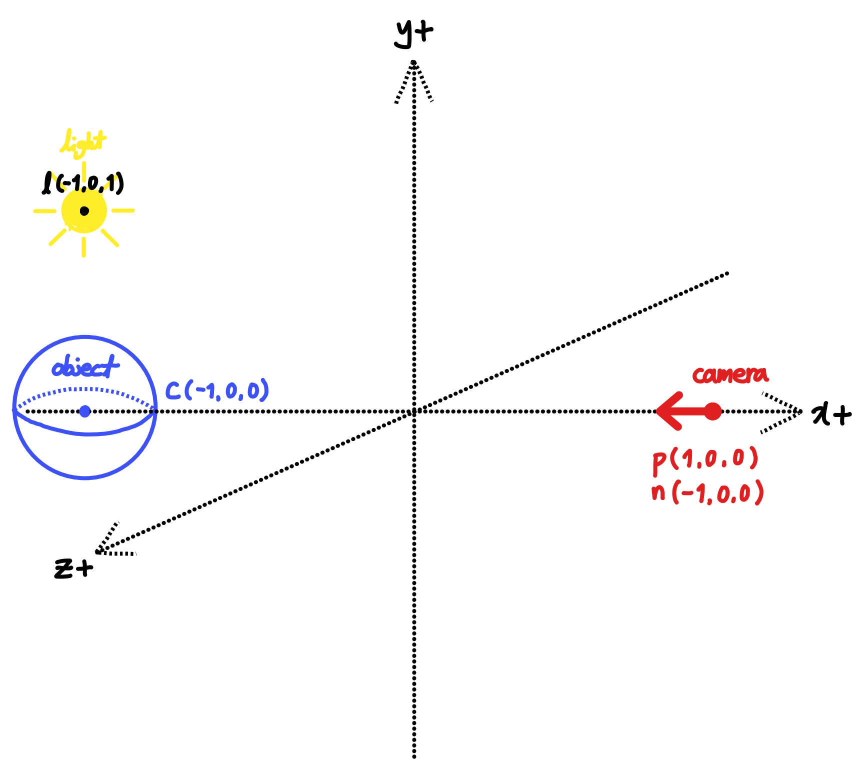
rotate objs
y axis : $+\pi$ rad



rotate cam
y axis : $-\pi$ rad

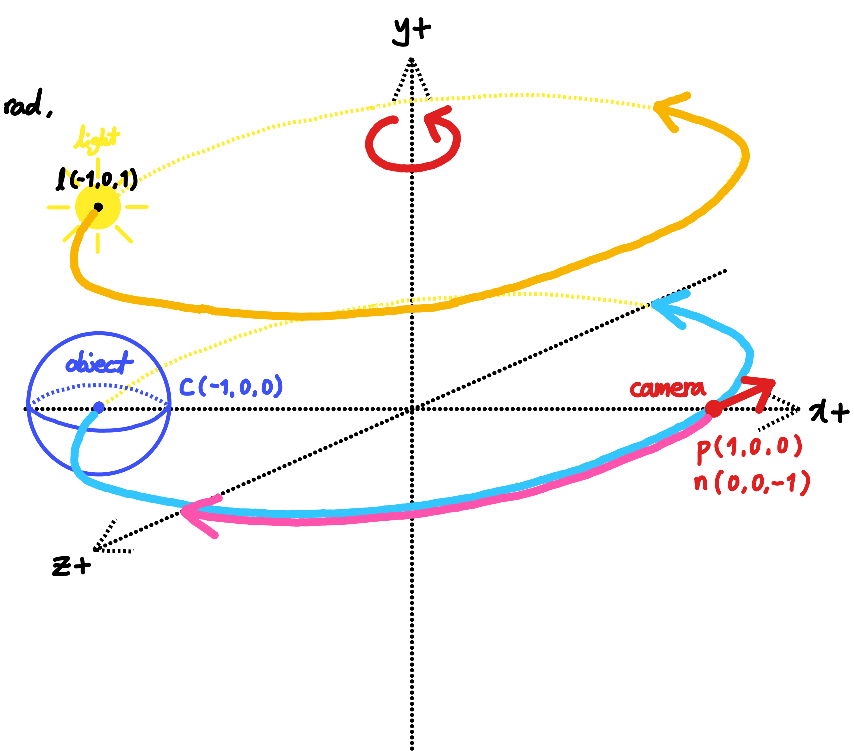


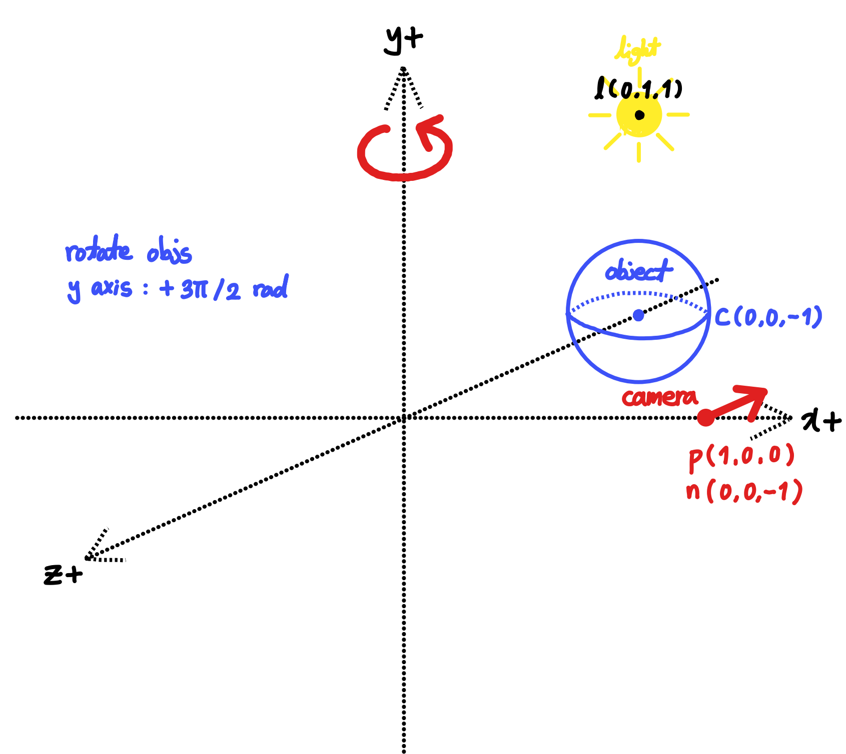
3. camera vector (-1,0,0)

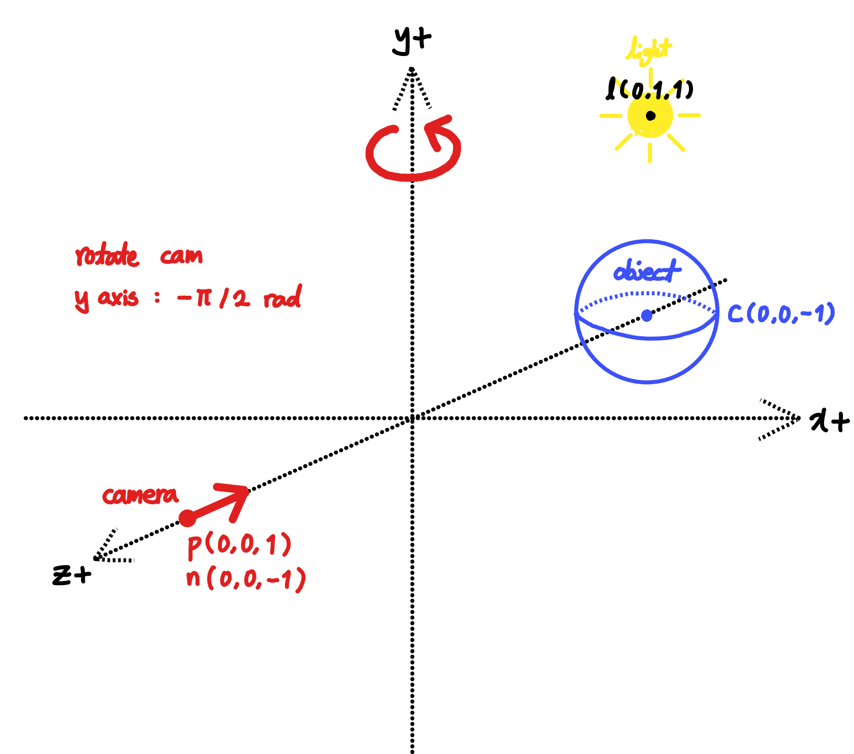


⇒ expect

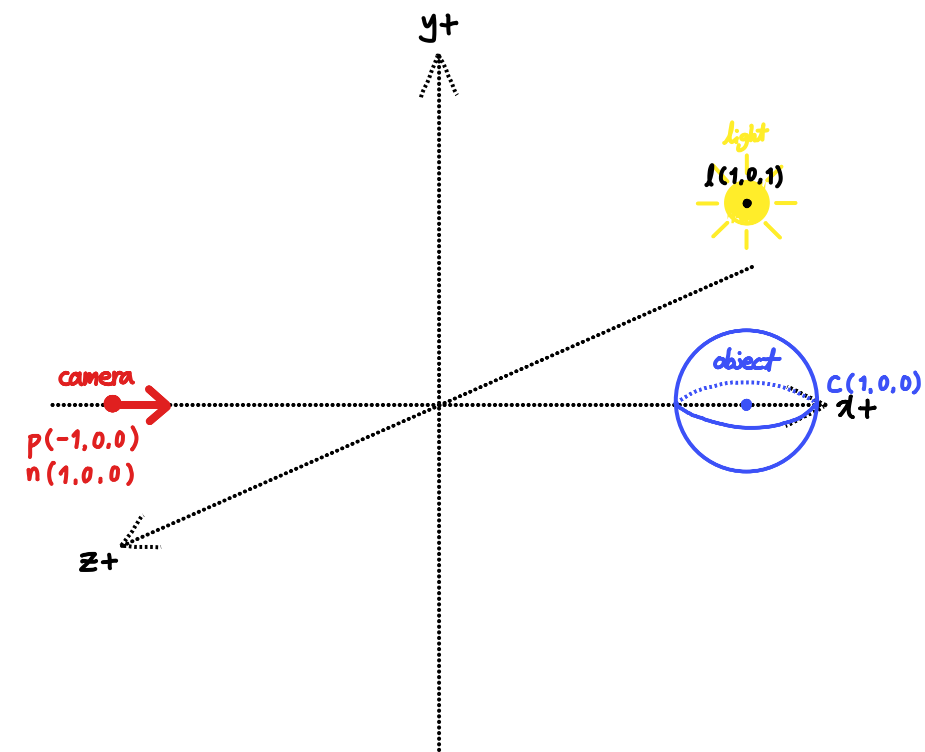
· I'm gonna rotate every object and light to y axis : $+3\pi/2$ rad,
and rotate camera to y axis : $-\pi/2$ rad.





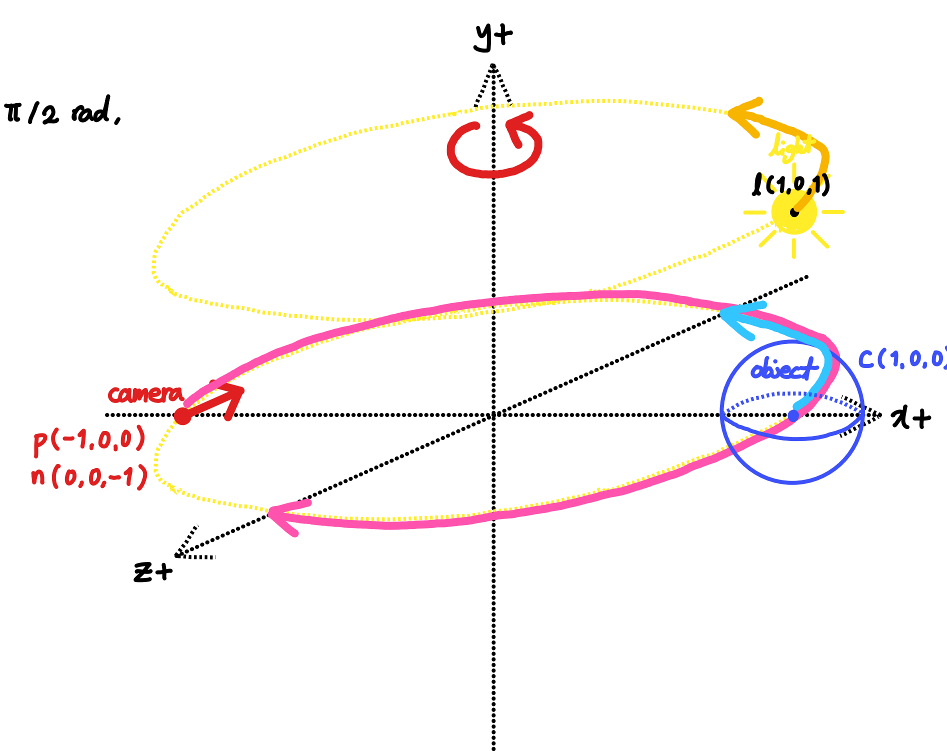


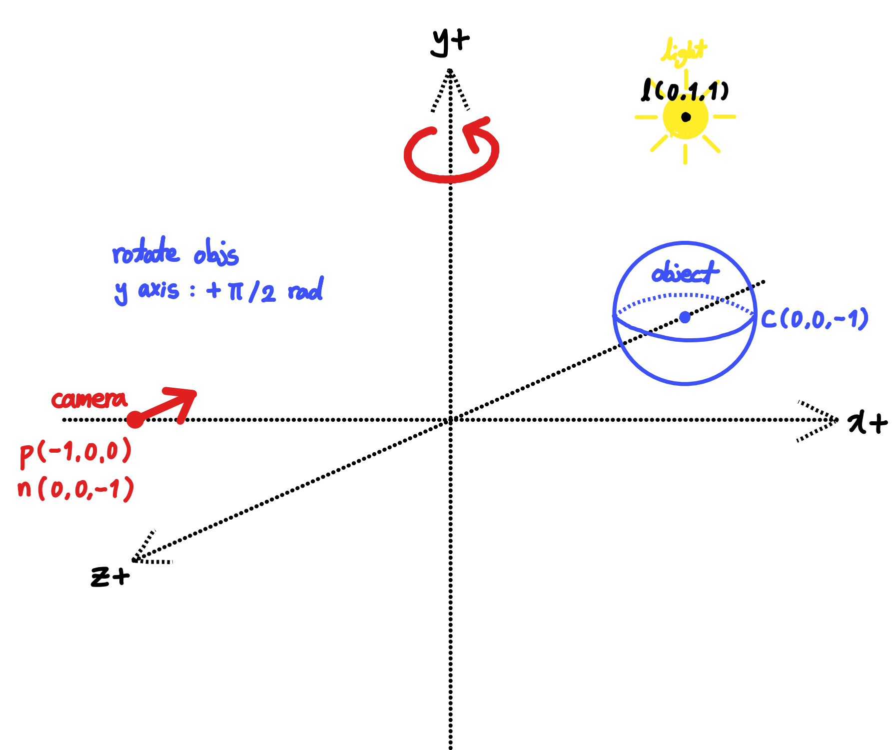
4. camera vector (1,0,0)

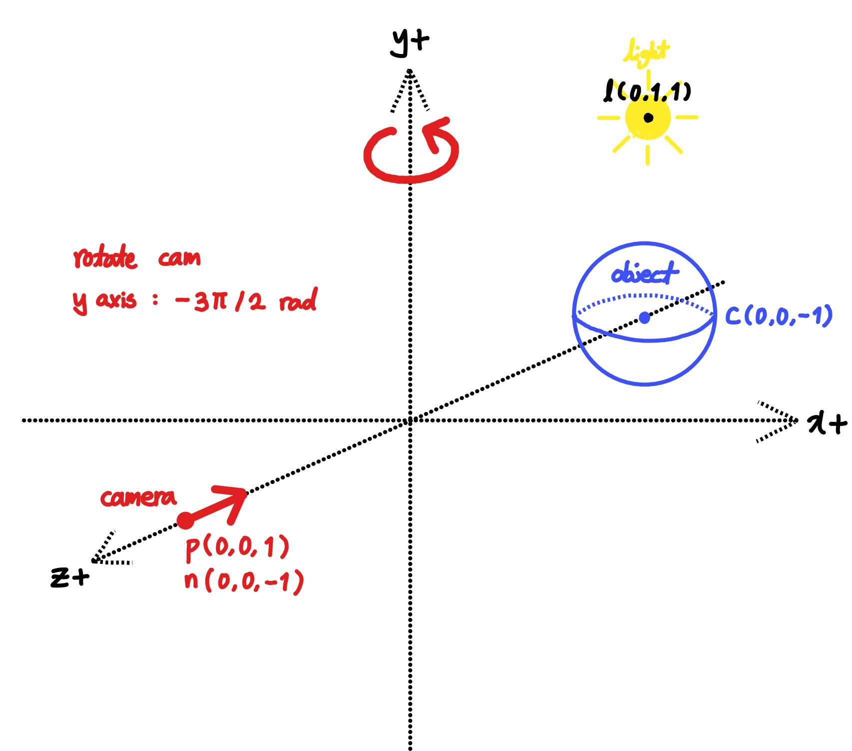


⇒ expect

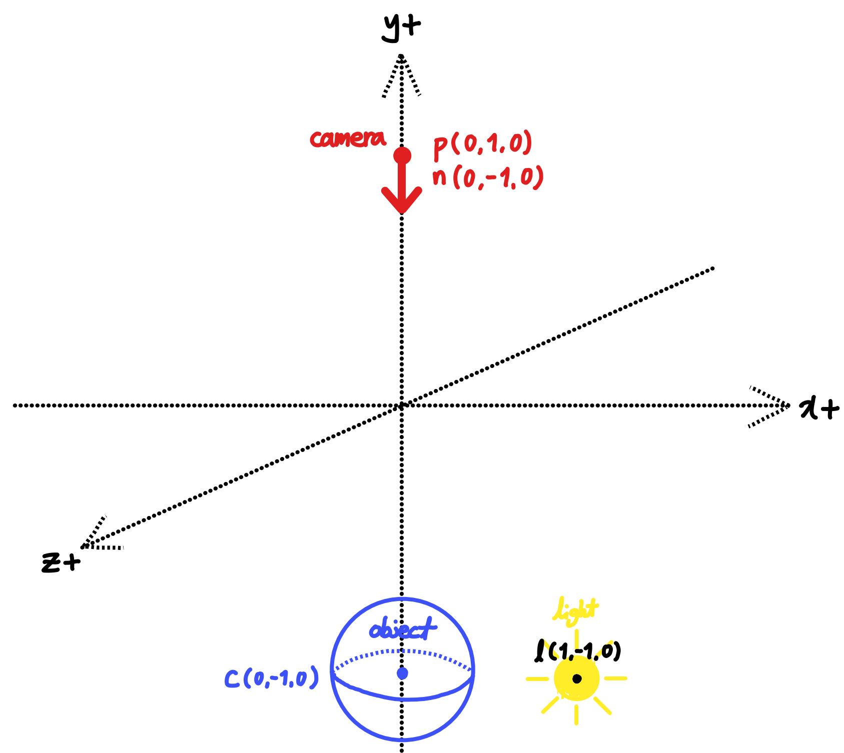
· I'm gonna rotate every object and light to y axis : + π/2 rad,
and rotate camera to y axis : - 3π/2 rad.







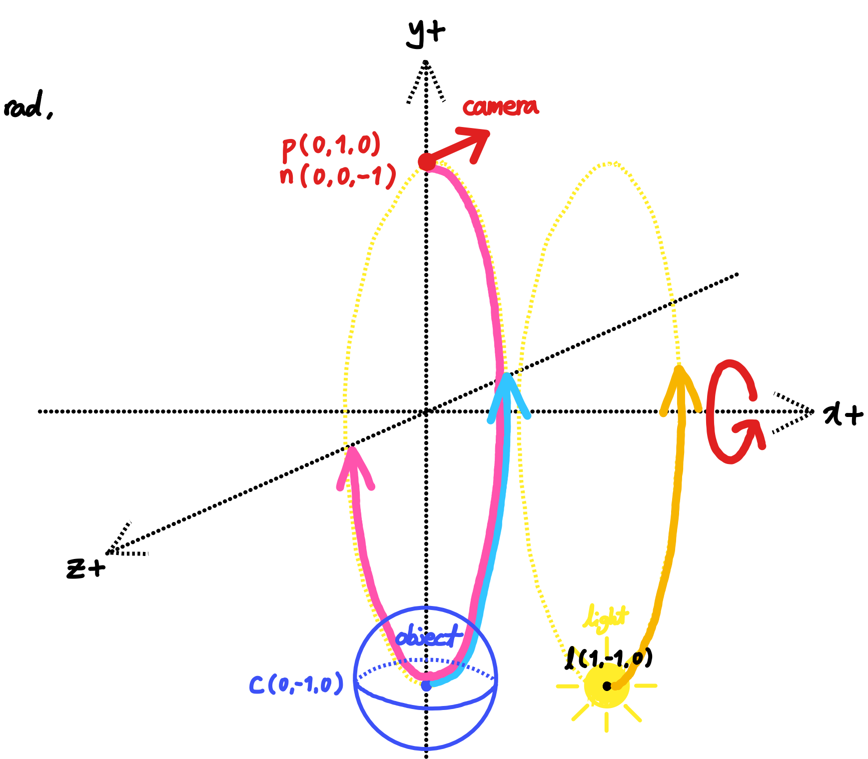
5. camera vector (0,-1,0)

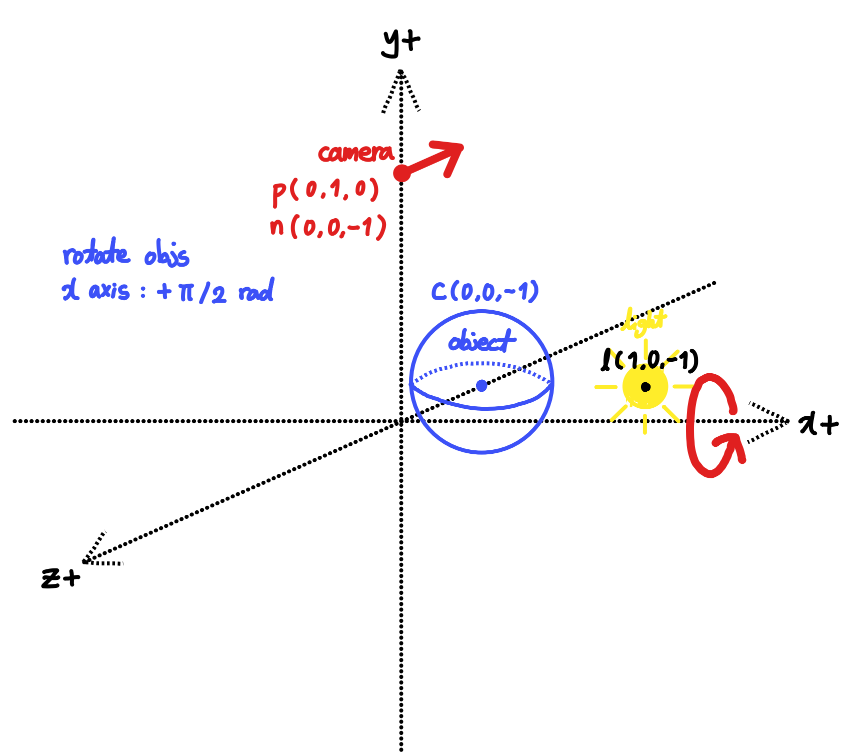


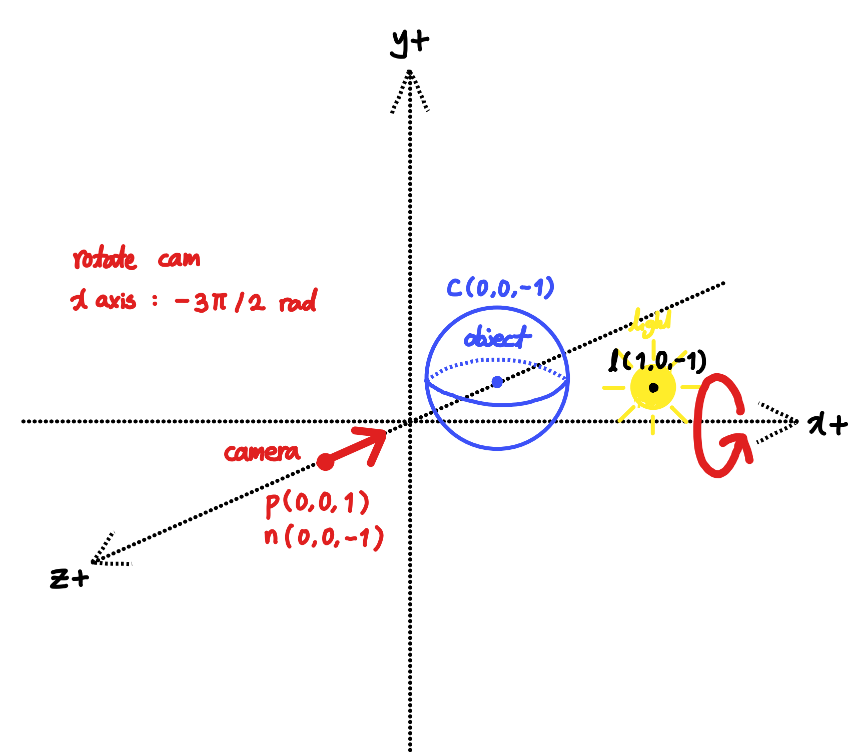
⇒ expect

· I'm gonna rotate every object and light to x axis : $+\pi/2$ rad,

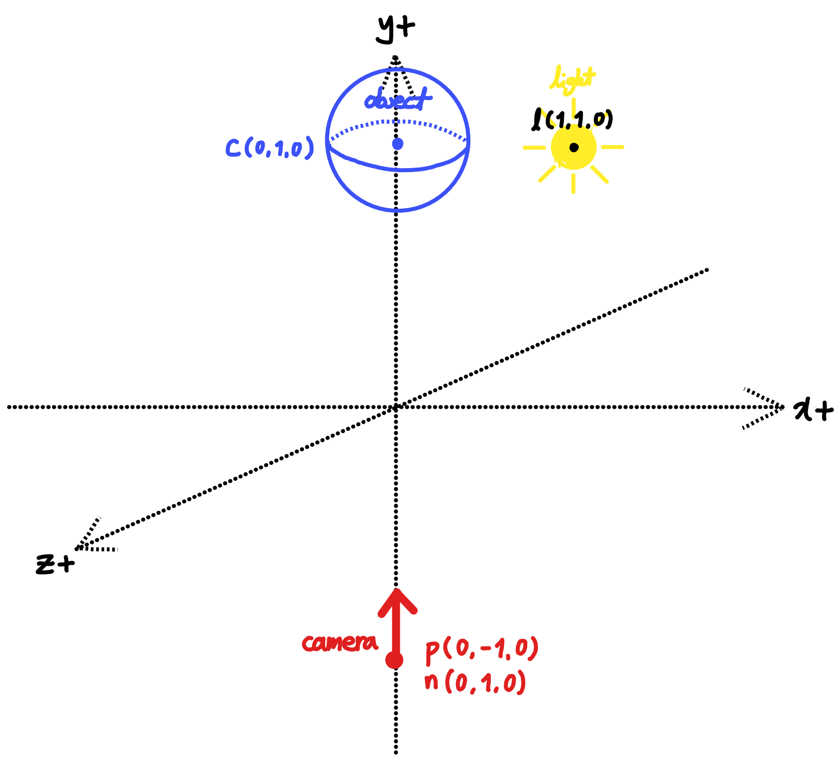
and rotate camera to x axis : $-3\pi/2$ rad.



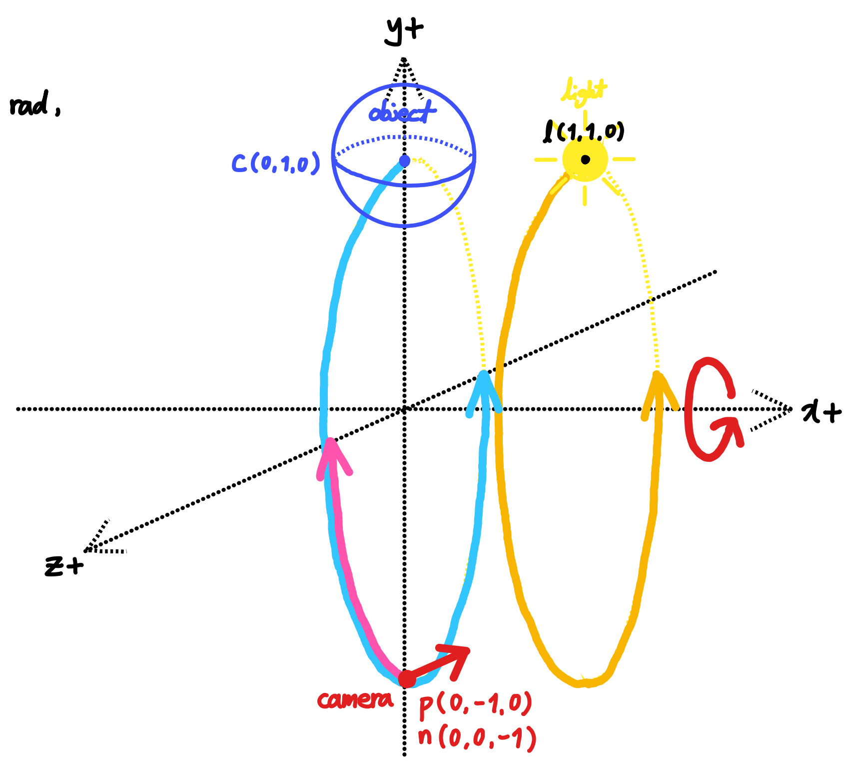




6. camera vector (0,1,0)

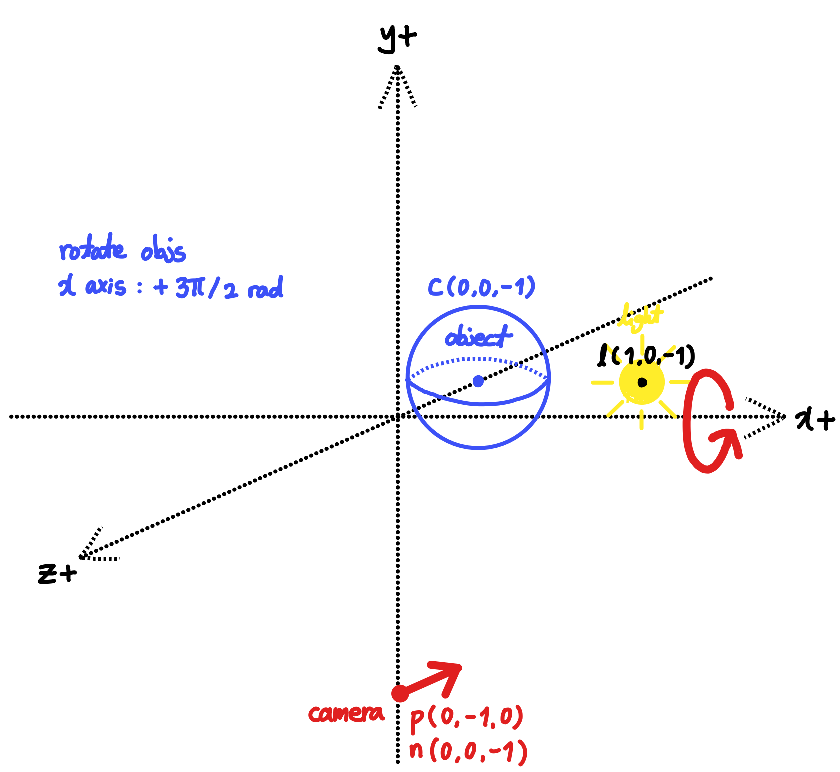


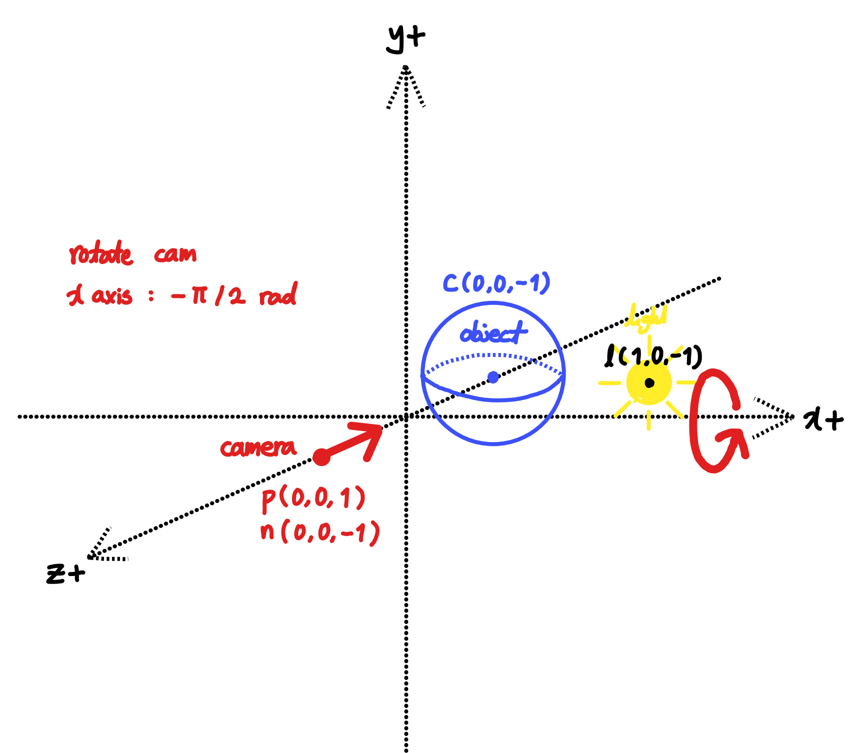
⇒ expect



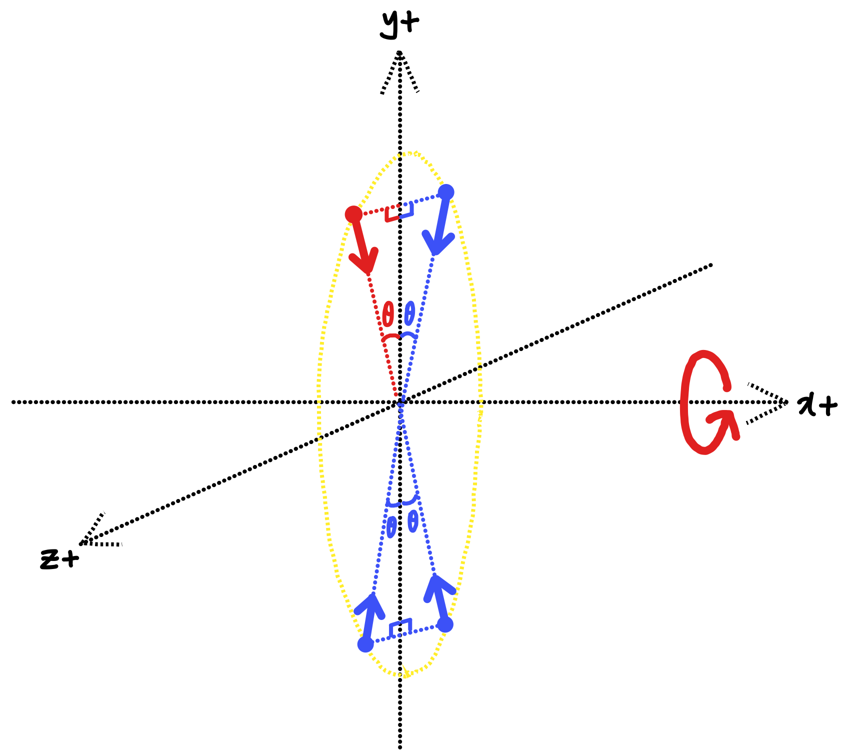
· I'm gonna rotate every object and light to x axis : $+3\pi/2$ rad,

and rotate camera to x axis : $-\pi/2$ rad.





7. camera vector $(0, y, z)$



- rotate x axis
- $\cos\theta = \dfrac{y}{\sqrt{y^2+z^2}}$, $\sin\theta = \sqrt{1-\cos^2\theta}$
- cam_vec.y < 0 && cam_vec.z < 0

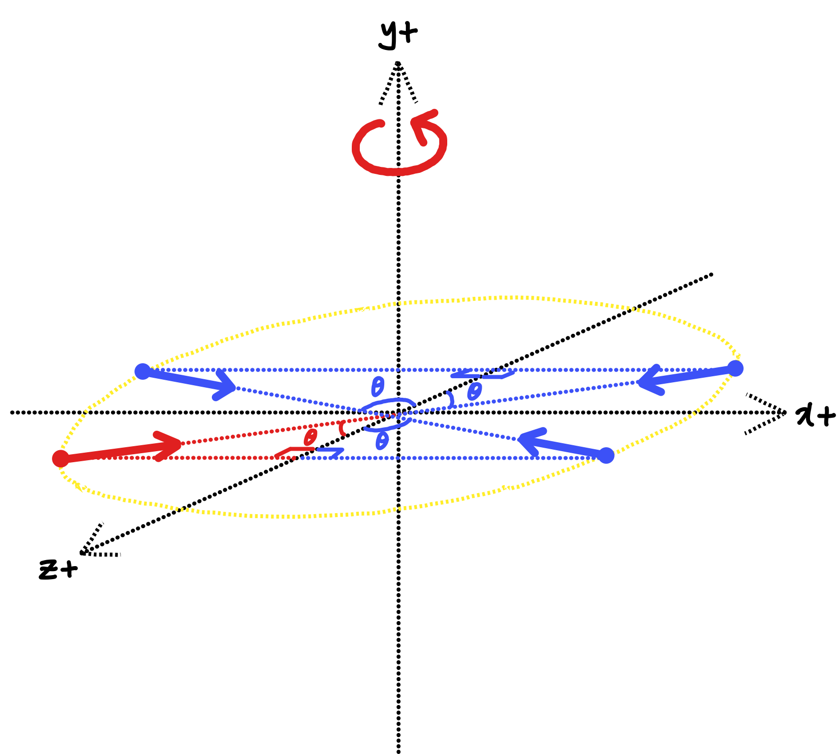
  : $\cos\theta$, $\sin\theta$
- cam_vec.y < 0 && cam_vec.z > 0

  : $-\cos\theta$, $\sin\theta$
- cam_vec.y > 0 && cam_vec.z > 0

  : $-\cos\theta$, $-\sin\theta$
- cam_vec.y > 0 && cam_vec.z < 0

  : $\cos\theta$, $-\sin\theta$

8. camera vector $(x, 0, z)$



- rotate y axis
- $\cos\theta = \dfrac{z}{\sqrt{x^2+z^2}}$, $\sin\theta = \sqrt{1-\cos^2\theta}$
- cam_vec.x > 0 && cam_vec.z < 0

  : $\cos\theta$, $\sin\theta$
- cam_vec.x > 0 && cam_vec.z > 0

  : $\cos\theta$, $-\sin\theta$
- cam_vec.x < 0 && cam_vec.z > 0

  : $-\cos\theta$, $-\sin\theta$
- cam_vec.x < 0 && cam_vec.z < 0

  : $\cos\theta$, $-\sin\theta$