P 16.

A: $x \in P$ B: $x \in Q$.

pf:

$\forall x \in P$, since $P \subseteq Q$, then $x \in Q$.
$\forall x \notin P$, x can either be in Q or not in Q.

A	B
T	T
F	T / F

which is an implication relation. ※

P.18

pf: Since $\forall x \in \phi$, $x \in X$, where X is an arbitrary set. This vacuously true. ※.

P. 20

$M^c = \phi$ $\phi^c = M \setminus \phi = \{x : M(x) \wedge (\neg \phi(x))\} = M$.

P. 23.

pf:

A	B	C	$(A \vee B) \wedge C$	$(A \wedge C) \vee (B \wedge C)$
T	T	T	T	T
T	T	F	F	T
T	F	T	T	T
F	T	T	T	T
T	F	F	F	F
F	T	F	F	F
F	F	T	F	F
F	F	F	F	F

From the truth table, we know $(A \vee B) \wedge C \equiv (A \wedge C) \vee (B \wedge C)$. ※

P.24

$$(A \cup B) \cap C = (A \cap C) \cup (B \cap C)$$

pf:

We first prove $(A \cup B) \cap C \subseteq (A \cap C) \cup (B \cap C)$.

$\forall x \in (A \cup B) \cap C$, $x \in (A \cup B)$ and $x \in C$. Then consider these two cases:

$$\left.\begin{cases} \text{i) } x \in A \text{ and } x \in C \Rightarrow x \in (A \cap C) \Rightarrow x \in (A \cap C) \cup (B \cap C) \\ \text{ii) } x \in B \text{ and } x \in C \Rightarrow x \in (B \cap C) \Rightarrow x \in (A \cap C) \cup (B \cap C). \end{cases}\right\} (A \cup B) \cap C \subseteq (A \cap C) \cup (B \cap C)$$

($x \in A$ and $x \in B$ and $x \in C$ is not covered, think about why!)

Then we prove $(A \cup B) \cap C \supseteq (A \cap C) \cup (B \cap C)$.

$\forall x \in (A \cap C) \cup (B \cap C)$, we consider these two cases:

$$\left.\begin{cases} \text{i) } x \in A \cap C \Rightarrow x \in A \text{ and } x \in C \Rightarrow x \in (A \cup B) \text{ and } x \in C \Rightarrow x \in (A \cup B) \cap C \\ \text{ii) } x \in B \cap C \Rightarrow x \in B \text{ and } x \in C \Rightarrow x \in (A \cup B) \text{ and } x \in C \Rightarrow x \in (A \cup B) \cap C \end{cases}\right\} (A \cup B) \cap C \supseteq (A \cap C) \cup (B \cap C)$$

Since if $X \subseteq Y$ and $X \supseteq Y$, then $X = Y$, we deduce that

$$\boxed{(A \cup B) \cap C = (A \cap C) \cup (B \cap C)}$$ ※

※ On Horst's slide, page 44. all the rules can be varified by similar process.

P.25

1. F Take $x=0, y=-1$, then $0^2+(-1)^3=-1<0$.
2. F $f(0)=0^4=0$.
3. F If $a=0$, $a^4=0$.
4. X Since the word "big" is ambiguous, so we can't decide whether it is true or false.
5. F

A	B	$A \vee B$	$\neg(\neg A \wedge \neg B)$
T	T	T	T
T	F	T	T
F	T	T	T
F	F	F	F

$\Rightarrow A \vee B \Leftrightarrow \neg(\neg A \wedge \neg B)$.

$\boxed{\text{P. 26}}$ Let $(a_n)_{n\in\mathbb{N}}$ be a real sequence. If for some fixed $c \in \mathbb{R}$,

$\forall a>0 \ \exists N\in\mathbb{N} \ \forall n\geq N \ |a_n - c| < a$, then we say (a_n) converge. ※.