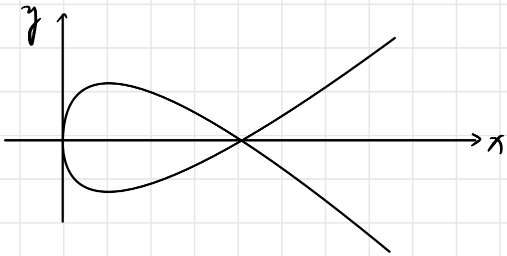
$\mathcal{C}$: $\gamma : [-2,2] \to \mathbb{R}^2$ $\quad\quad$ $\gamma(t) = \begin{pmatrix} 2t^2 \\ \frac{2}{3}t^3 - 2t \end{pmatrix}$



①

For $t \in [-2,2]$, $\gamma'(t) = \begin{pmatrix} 4t \\ 2t^2-2 \end{pmatrix} \Rightarrow \|\gamma'(t)\| = \sqrt{(4t)^2+(2t^2-2)^2} = 2(t^2+1)$

$l(\mathcal{C}) = \int_{-2}^{2} \|\gamma(t)\| \, dt = \frac{56}{3}$

② $\gamma_2(t) = 0 \Rightarrow t = 0 \vee \pm\sqrt{3}$

$$\Rightarrow \gamma(+\sqrt{3}) = \begin{pmatrix} 6 \\ 0 \end{pmatrix} = \gamma(-\sqrt{3})$$

$$T \circ \gamma(t) = \frac{\gamma'(t)}{\|\gamma'(t)\|} = \frac{1}{2(t^2+1)} \begin{pmatrix} 4t \\ 2t^2-2 \end{pmatrix}$$

$$T \circ \gamma(\sqrt{3}) = \frac{1}{2}\begin{pmatrix} \sqrt{3} \\ 1 \end{pmatrix} \qquad T \circ \gamma(-\sqrt{3}) = \frac{1}{2}\begin{pmatrix} -\sqrt{3} \\ 1 \end{pmatrix}$$

$$\Rightarrow \measuredangle\left(T(\gamma(\sqrt{3})), T(\gamma(-\sqrt{3}))\right) = \arccos\left\langle T\circ\gamma(\sqrt{3}), T\circ\gamma(-\sqrt{3})\right\rangle = \frac{2\pi}{3}$$

$x \in \mathbb{R}^n$, find the derivative of

$$\Phi : \mathrm{Mat}(n \times n; \mathbb{R}) \to \mathbb{R} \qquad A \mapsto \|Ax\|^2$$

Sol:

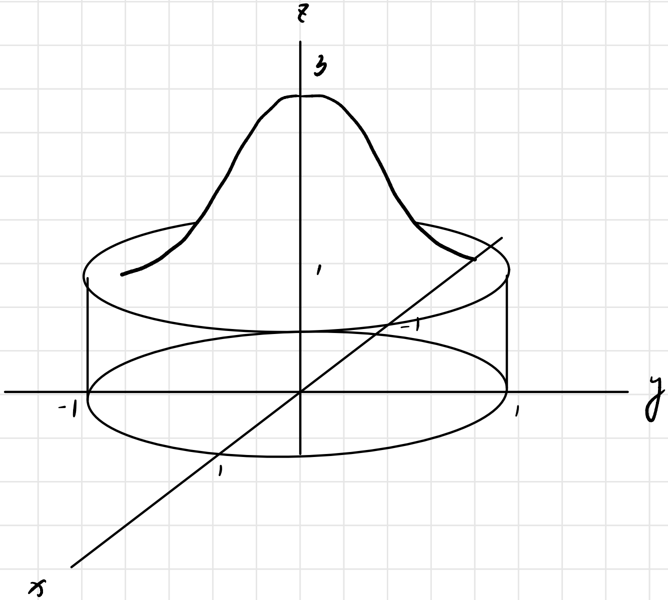
$\|Ax\| = \langle Ax, Ax \rangle$ and the map $x : A \mapsto Ax$ is linear with $Dx|_A H = Hx$. Then by product rule:

$$\begin{aligned} D\Phi|_A H &= \langle Dx|_A H, Ax \rangle + \langle Ax, Dx|_A H \rangle \\ &= \langle Hx, Ax \rangle + \langle Ax, Hx \rangle \\ &= 2 \langle Ax, Hx \rangle \end{aligned}$$

Let $B \subset \mathbb{R}^3$ given by

$$B = \left\{ (x,y,z) \in \mathbb{R}^3 : x^2+y^2 \le 1,\ 0 \le z \le 2 + \cos\sqrt{x^2+y^2} \right\}$$

1. sketch B



2. $|B|$

cylindrical : $\Phi\left(\begin{pmatrix} x \\ y \\ z \end{pmatrix}\right) = \begin{pmatrix} r\cos\theta \\ r\sin\theta \\ z \end{pmatrix} \Rightarrow \cos(\pi\sqrt{x^2+y^2}) = \cos(\pi r)$

$$|B| = \iiint_B dx\,dy\,dz = \iiint_B r\,dr\,d\theta\,dz = \int_0^{2\pi}\int_0^1\int_0^{2+\cos(\pi r)} r\,dz\,dr\,d\theta$$

	D	I
+	r	$\cos\pi r$
-	1	$\frac{1}{\pi}\sin\pi r$

$$= 2\pi \cdot \int_0^1 (2+\cos(\pi r))\,r\,dr = 2\pi + 2\pi\int_0^1 r\cos(\pi r)\,dr$$

$$= 2\pi + 2\pi\left(\frac{r}{\pi}\sin(\pi r)\Big|_0^1 - \frac{1}{\pi}\int_0^1 \sin(\pi r)\,dr\right)$$

$$= 2\pi - 2\pi\left(\frac{1}{\pi} \cdot \frac{-\cos(\pi r)}{\pi}\Big|_0^1\right) = 2\pi - 2\pi\left(\frac{2}{\pi^2}\right) = 2\pi - \frac{4}{\pi}.\ ※$$