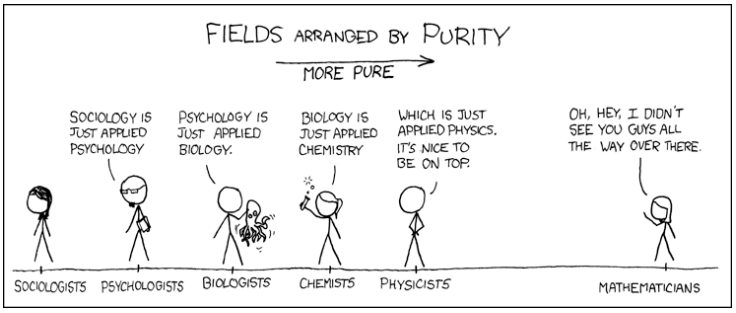
Course Description

MATH285 Honors Mathematics III
Functions of Several Variables

Summer 2021

Hello and welcome to MATH285 !

This is the second course in the *Honors Mathematics* sequence. We will take our investigation of continuity, differentiation and integration of functions into higher dimensions, where many exciting and wondrous phenomena await. For a better understanding of these higher-dimensional concepts, we will begin with an intensive summary of background from linear algebra.



R. Munroe, *Purity*, https://xkcd.com/435/

At the same time, we will start to study applications from engineering, physics and other natural sciences. For such applications where pencil-and-paper calculations become unwieldy, and to create professional visualizations, we will use a new software tool, the symbolic math processing program Mathematica® by Wolfram Research. There will also be a term project in the second half of the course. Many new and exciting things await!

Formally, our course is one of three parallel courses on functions of several variables available to you; Vv215 Calculus III is a less theoretically oriented course, while Vv255 Honors Calculus III has a greater emphasis on applications of theory and modeling.

Teaching team

Horst Hohberger has designed and given this course every year since 2007; it was never taught in exactly the same way twice! This is due to continuous student feedback, which is always very much welcomed. You can reach him at **horst@sjtu.edu.cn** or find him in his office, Room 441c of the Longbin Building.

Pingbang Hu is a Sophomore student majoring in ECE. He is interested in analysis, applied math, backend web developing and competitive programming. He can be reached at **ben102938@sjtu.edu.cn**.

Leyang Zhang is a Junior student majoring in ECE. He is interested in pure analysis and theoretical background of machine learning. He can be reached at **leyang_zhang2108@sjtu.edu.cn**.

Chengsong Zhang is a sophomore student majoring in ECE. He is currently interested in algebra, web programming and deep learning. You can contact him through email: **continue_revolution@sjtu.edu.cn** or QQ: 3357667574.

What you need (prerequisites)

The most essential prerequisite to understanding is to be able to admit when you don't understand something.

Richard Saul Wurman

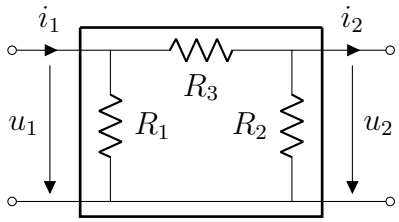
Our course continues directly from Vv186 Honors Mathematics II. We will use all of the concepts introduced there without further discussion, so it is advisable to review them at the start of the term. In particular, you should be familiar and comfortable with vector spaces, norms, convergence of sequences and series, continuity and differentiability a well as the definition of and differences between the regulated and Riemann integrals.

If you feel uncertain about any of the key concepts mentioned here, please do not hesitate to let me and/or the Teaching Assistants know! We will try to help as much as possible.

What you will gain from this course

The present course gives an introduction to linear algebra of finite-dimensional spaces and applies these results to the theory of functions of several variables. A strong emphasis is based on concepts that play an important role in physics and engineering applications, such as line and surface integrals and vector fields.

We start with an introduction to linear algebra, featuring the Gauß-Jordan algorithm for solving systems of equations, the theory of finite dimensional vector spaces, linear maps, matrices and determinants. These concepts are not only essential tools for the following calculus in $\mathbb{R}^n$ but (together with the topics covered in Honors Mathematics II and IV) comprise the content of an independent course in linear algebra.



Electric Quadripole in Pi Configuration

The second part of the course starts with a discussion of convergence and continuity before embarking on differential and integral calculus in $\mathbb{R}^n$. The emphasis in this part is on curves (vector functions of a single variable) and on scalar functions of several variables. The basic concept of continuity in normed vector spaces, equivalence of norms in finite-dimensional spaces and the derivative as a linear map are introduced. We will see that the regulated integral lends itself to vector-valued functions of a single variable, while the Riemann integral is essential for functions of several variables. We will study line integrals, surface integrals and volume integrals of scalar functions. These integrals are essential for many problems in physics and engineering.

The third part of the course introduces vector fields and their properties in the context of their many physical applications: work in force fields; potential functions and gradient fields; rotation and presence of sources and sinks in fluids and electromagnetic fields; flux through surfaces...The links and relations between these properties become clear with the fundamental theorems of Green, Gauß and Stokes.

In this third part we will also look at higher-order derivatives and extrema of functions, both free and constrained.

In short, this course is chock-full of applications in physics and engineering. Just about all of the new techniques and concepts will be used in physics and engineering courses at JI (in fact, some of our exercises will be taken directly from the relevant textbooks). Understanding and internalizing the concepts in this course lays the groundwork for succeeding in all your future engineering courses.
Along the way, you will learn to use *Mathematica*® software for mathematical and physical applications and you will work in teams to complete your own term project and report.

Syllabus

We will meet for 30 lectures of 90 minutes each over the 12 weeks of summer term.

Lecture	Lecture Subject
1	Systems of Linear Equations
2	Finite-Dimensional Vector Spaces
3	Inner Product Spaces
4	Linear Maps
5	Matrices
6	Matrices
7	Theory of Systems of Linear Equations
8	Determinants
9	Determinants
10	**First Midterm Exam***
11	Sets and Equivalence of Norms
12	Convergence and Continuity
13	The First Derivative
14	The Regulated Integral for Vector-Valued Functions
15	Curves, Orientation, and Tangent Vectors
16	Curve Length, Normal Vectors, and Curvature
17	The Riemann Integral for Scalar-Valued Functions
18	Integration in Practice
19	Parametrized Surfaces and Tangent Spaces
20	Surface Integrals in $\mathbb{R}^n$
21	**Second Midterm Exam***
22	Potential Functions
23	Vector Fields and Line Integrals
24	Flux and Circulation
25	Theorems of Green, Gauß and Stokes
26	The Second Derivative
27	Free Extrema
28	The Implicit Function Theorem
29	Constrained Extrema
30	**Final Exam**

* The schedule of the Midterm Exams is approximate and subject to change; please see the syllabus tab on Canvas for the actual date.

Course outcomes

The *course outcomes* define a set of minimal skills or items of knowledge that you should feel confident in applying after completing the course. Naturally, we will cover much more than just the outcomes listed below. However, they represent a cross-section of basic skills and are a useful guide to how well you have mastered the contents of the course in general.

A quantitative measurement of these outcomes (except the last one) will be provided by *Course Outcome Quizzes*, which will also contribute to the course grade. You will be also asked to report your subjective impression on attaining the outcomes in the Course Evaluation survey at the end of the term.

After completing MATH285, students should be able to:

i) Work with orthonormal bases and projections.

ii) Express a linear map as a matrix in terms of a given basis.

iii) Analyze the properties of curves.

iv) Calculate volume, line and surface integrals of scalar functions.

v) Calculate line and surface integrals of vector fields.

vi) Analyze the properties (circulation, flux) and determine the potential (if it exists) of a vector field.

vii) Apply the theorems of Green, Gauß and Stokes.

viii) Determine free and constrained maxima and minima of potential functions.

Literature

What follows is a selection of literature for this course. Find out for yourselves which books are helpful and also do a little research by yourselves for other potentially useful books. You may click directly on the `doi` number to reach a web page for each work. From within the SJTU network, you should be able to freely download an electronic copy.

The first part of this course gives some necessary background in linear algebra. There are several books that might be helpful:

- Jim Hefferon, *Linear Algebra*, Online Book (St Michael's College, Colchester, Vermont, 2001), `http://joshua.smcvt.edu/linearalgebra/`

  A US-style textbook for a first course in linear algebra, with emphasis on calculations.

- Klaus Jänich, *Linear Algebra*, Undergraduate Texts in Mathematics (Springer-Verlag New York, 1994), doi:`10.1007/978-1-4612-4298-7`

  This is the translation of a classic German book that is used as a reference in nearly every course on linear algebra in Germany. Like Spivak's Calculus, it is written in a conversational style, eschewing the definition-theorem-proof-example chain that is so prevalent in serious textbooks.

- Sheldon Axler, *Linear Algebra Done Right*, 3rd ed., Undergraduate Texts in Mathematics (Springer International Publishing, 2015), doi:`10.1007/978-3-319-11080-6`

  This very readable book by a US-American author is the reaction to the prevalent style of linear algebra textbook in the US (see Hefferon above for an example). It is very much in the continental-European tradition of putting structures and relationships before calculations. (You will notice similarities with Jänich's book above.)

- Serge Lang, *Introduction to Linear Algebra*, 2nd ed., Undergraduate Texts in Mathematics (Springer New York, 1986), doi:10.1007/978-1-4612-1070-2

  Another serious book on linear algebra, written by an eminent french-born mathematician (now deceased). Although first published in 1986 and in much the same vein as Axler's book, it never became widely popular as an undergraduate textbook in the US, perhaps because it was considered too difficult. But it is actually very readable.

- Serge Lang, *Linear Algebra*, 3rd ed., Undergraduate Texts in Mathematics (Springer New York, 1987), doi:10.1007/978-1-4757-1949-9

  A more advanced version of the above book, treating many topics that we do not have time for in this course. Read this for a deeper understanding of linear algebra.

- Rami Shakarchi, *Solutions Manual for Lang's Linear Algebra* (Springer New York, 1996), doi:10.1007/978-1-4612-0755-9

  You may find this useful :-)

For the remainder of the course, focussing on multidimensional calculus, there are several books that can be read alongside the lecture notes. However, none of these books covers the course material in exactly the same way as we do and most of them also include much additional material. I am teaching the material in the style of German textbooks, few of which have been translated into English. In the anglo-saxon literature, one is often faced either with (many!) easy calculus books that ignore general concepts or graduate books that are much too advanced.

- Serge Lang, *Calculus of Several Variables*, 3rd ed., Undergraduate Texts in Mathematics (Springer New York, 1987), doi:10.1007/978-1-4612-1068-9

  Yes, Serge Lang wrote a lot of undergraduate and graduate textbooks. This one overlaps with some of the course, but takes a different approach to some topics. It is perhaps less abstract than what we do.

- Wendell Fleming, *Calculus of Several Variables*, Undergraduate Texts in Mathematics (Springer New York, 1977), doi:10.1007/978-1-4684-9461-7

  Like Lang's book, this one also overlaps with part of our course. However, the part that does not overlap is more abstract than what we do, so it is useful for further reading.

- J. J. Duistermaat and J. A. C. Kolk, *Multidimensional Real Analysis I* (Cambridge University Press, 2004), doi:10.1017/CBO9780511616716

  J. J. Duistermaat and J. A. C. Kolk, *Multidimensional Real Analysis II* (Cambridge University Press, 2004), doi:10.1017/CBO9780511616723

  This is for the ambitious student. Like Lang, Duistermaat was a very well-known and respected mathematician, and he does everything rigorously and leaves nothing unproven.

Mathematica®

In modern applications, the use of computer software to perform complex and/or tedious calculations is indispensable. Our course will continually refer to the Mathematica® software package, published by Wolfram Research.

This program has been chosen because (i) it is capable of symbolic manipulation of mathematical expressions and thereby complements other software such as MatLab; (ii) it is easy to learn; (iii) it has very good graphics capabilities; (iv) it is easy to mix programming features (loops, conditionals, etc.) with mathematical evaluations; (v) it is the "swiss army knife" of mathematical software and you should at some point in your studies become familiar with it.

Mathematical software, ideally Mathematica, *will be required for some of the assignments* If you wish, you may of course use a different software package, but it is up to you to verify that the necessary statistical functions can be performed. Hence, while you are strongly encouraged to use Mathematica (and the lecture slides provide support), it is not compulsory.

Obtaining Mathematica

i) Visit

https://user.wolfram.com/portal/registration.html

and create a Wolfram ID. You must use an @sjtu.edu.cn email address and give your first and last names in pinyin (example: Xu Baishen enters last name: Xu and first name: Baishen).

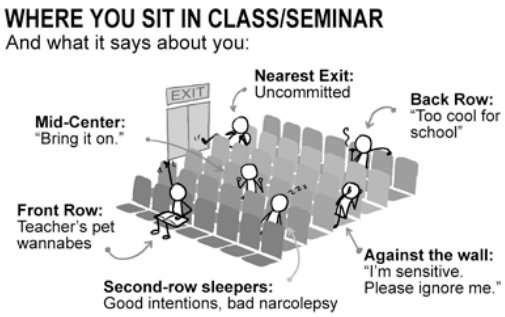
ii) Next, visit

https://user.wolfram.com/portal/requestAK/c51e79e5334a3600a4f740a2b3720961216dbc17

and request an Activation Number. Make a note of the activation number. You will be directed to a page where you can download the installation binaries for the most current version of Mathematica. The software binaries are available for Windows, Linux or macOS.

iii) After downloading, you can install the software. You will be asked to enter the Activation Number you noted above and you will need internet access. Mathematica will then run on a temporary two-week license. Your name will be checked against a list, and if successful, the license will automatically be extended for one year. *Therefore, it is very important that you enter your name properly when you request the Wolfram ID.*

Lecture attendance and participation



J. Cham, *Where do you sit?*, phdcomics.com/comics/archive.php?comicid=1017

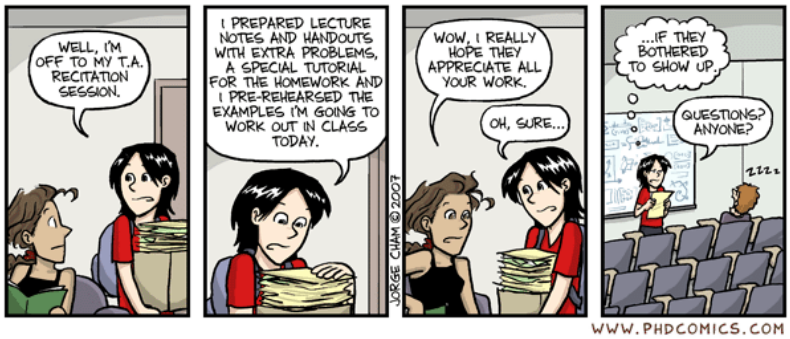
Our course will attempt to blend online and in-class teaching. The classroom is E-2-206 in Dongzhongyuan 东中院)) and online access will be implemented via Zhumu (see Canvas for details). You may participate either by coming to the classroom or by logging into the live video link.

If you participate by joining online, please observe the rules of online etiquette: you should log in using your real name (in pinyin transliteration as appropriate), you should have your camera turned on and your microphone turned off, but be ready to answer questions, etc. Details will be given in class.

An effort will be made to provide recordings of the classes, but there is no guarantee regarding the sound and video quality or even that the recordings will be available for every session.

Recitation classes, office hours and Piazza

There will be recitation classes and office hours offered by the teaching assistants, as well as office hours offered by the instructor. None of these are mandatory but you may find them very helpful. The detailed times will be published on Canvas.



J. Cham, *Appreciation*, http://phdcomics.com/comics/archive.php?comicid=922

In addition, our class will use *Piazza* for you to pose questions online. Piazza provides a much better venue than traditional office hours, since questions and answers are visible to everyone and any student can participate in the discussion on a given topic. You can sign up for Piazza here:

https://piazza.com/sjtu.org/summer2021/math285

Course evaluation

Furthermore, at the end of the term there will be a course evaluation survey administered by the Undergraduate Education Office (UEO). The survey is completely confidential and only the collated comments and score summaries will be communicated to the instructor. Filling out the survey makes a small contribution to the course grade, so the whether or not you filled out the survey will also be communicated. Your honest and thoughtful opinions are very much appreciated!

Coursework

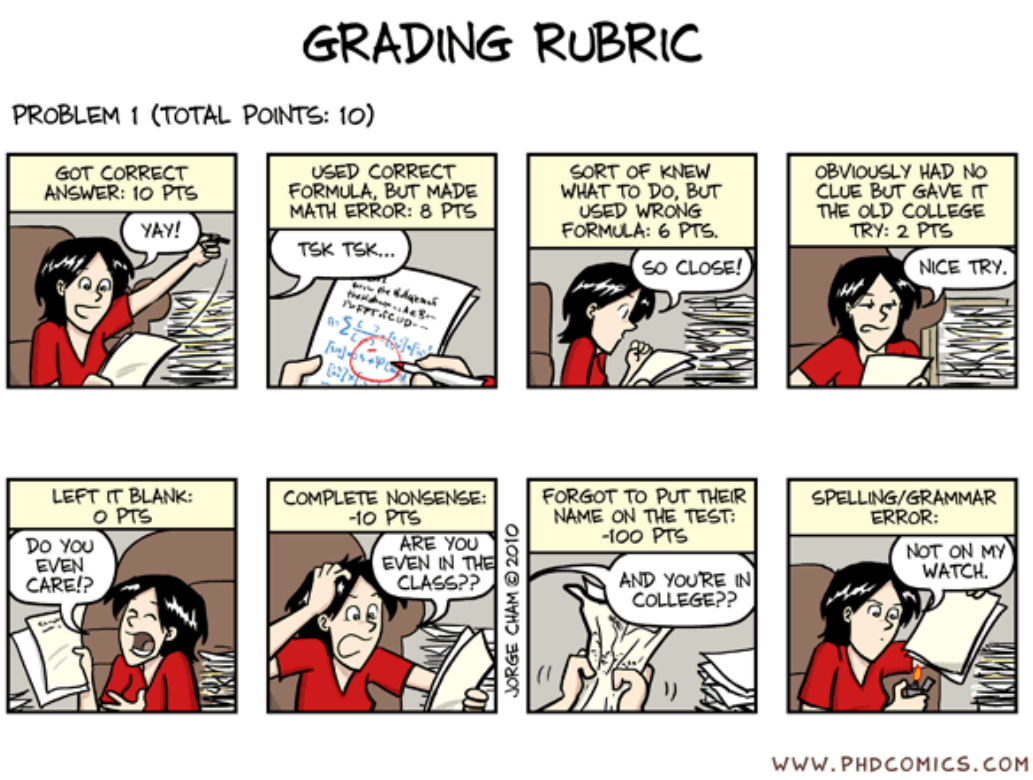
There will be weekly assignments. The goal of these assignments is for you to practice the lecture material and investigate some concepts by yourselves. the goal is *not* to evaluate you or to put undue pressure on you. Therefore, the coursework will not quantitatively contribute to the course grade.

However, you do need to attain at least 60% of the points of the coursework over the whole term. If you do not, you will **automatically fail** the course. On the other hand, it does not matter at all what percentage above 60 you attain. As mentioned, the course grade is not directly affected.

You are asked to hand in coursework on time. The due time is usually tied to a lecture period. If you hand it in later the same day, you may do so at the convenience of the teaching assistants (who may well refer you to the instructor if they are not able to accommodate you). After that, you must hand it in to the instructor personally and be prepared to explain the reasons for the delay.

You will be randomly assigned into groups of three students each; each group will jointly work on each assignment and hand in a single submission. To ensure that all members of each group contribute to the assignments, there will be a peer evaluation process. Details will be announced on Canvas. As for the course evaluation, participating in the peer evaluation process will make a small contribution to the total course grade.

In general, every member of an assignment group will receive credit for the assignments; however, if the peer evaluation process indicates that any members are not contributing to the assignments, the corresponding points for the affected assignments may be voided for them.

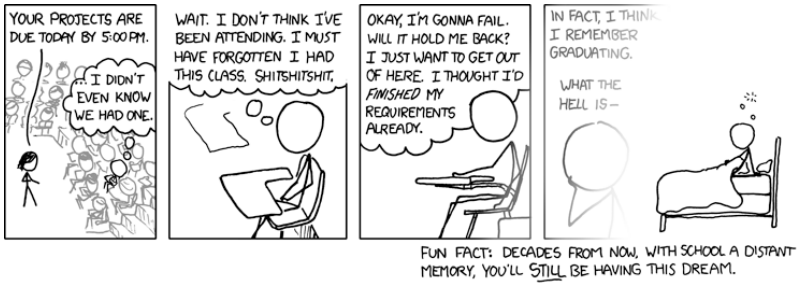


J. Cham, *Grading Rubric*, http://phdcomics.com/comics/archive.php?comicid=1319

Term project

The term project forms important part of the course. You will be randomly assigned into groups of 4-5 members each to work on these over the course of 3-5 weeks. Details will be provided in the project description.

Just as for the coursework groups, there will be a peer evaluation process. The results will have the potential to positively or negatively affect the individual grades for the term project. Participating in the peer evaluation process will make a small contribution to the total course grade.



R. Munroe, *Students*, https://xkcd.com/557/

Examinations

There will be three exams, two midterms and a final. These will be given in-class and **you must personally attend on-campus** if it is possible for you to do so. If it is not, perhaps because you are situated outside of Shanghai or there is a medical reason preventing you from attending, please contact the instructor beforehand. The arrangements described in the framed box below will then need to be made.

For all exams you may use an english, monolingual dictionary in book form. Cell phones and electronic devices are not permitted.

Paper for your exam answers and for notes will be provided for on-campus exams. You may not bring your own paper. You must, however, bring your own pens, rulers and other necessary writing materials. A pen and a ruler will be sufficient, there is no need for other stationery, though you can bring more if you like.

It is allowed to bring drinks and snacks to the exams as long as no disturbance of others is created.

No item not mentioned above (esp. cell phones, tablet computers, backpacks) may be at your desk unless you are explicitly granted permission by the supervisors.

Taking Examinations Remotely

If you are unable to attend an exam in-person, but able to take the exam remotely via video link, you must ensure the following:

- Your exam location must be an enclosed space that is not entered by anyone else during the exam. The space must be well-lit and quiet.
- You must be seated at a desk that is empty apart from a computer and any permitted material for the exam. A cell phone may be at hand for emergency communication and picture taking (see below), but may not normally be used during the exam.
- You must have set up a camera that clearly shows your desk, your computer screen, and yourself. The video feed of this camera needs to be visible throughout the exam and may not be turned off during the exam time. The camera may be either a second camera or the only camera used in the video feed. A camera embedded in a laptop screen (which does not show the screen, desk or your hands) is not sufficient by itself.
- You may mute your microphone by default, but be ready to unmute it when asked.
- Your computer sound should be turned on, in case there is an urgent message from the instructor.
- You will receive the exam paper electronically, e.g., via the chat function of Zoom. You may view the paper on your computer and write solutions on your own paper. These solution should then be photographed and the image files emailed to the instructor, horst@sjtu.edu.cn.
- The photographs of the answers must be taken before the end of the exam and should be emailed as soon as feasible. Allowances for network connections will be made as necessary. However, no solutions may be photographed after the exam time has passed.

Honor Code policy

Please familiarize yourself with JI's Honor Code, found at

https://www.ji.sjtu.edu.cn/academics/academic-integrity/honor-code/.

In addition to the general rules for examinations etc., the following specific policy applies to the coursework:

Coursework and Term Projects: External Sources, Collaboration and Piazza

Students within each assignment group, are allowed and supposed to collaborate in a completely unrestricted manner. However, it is prohibited to discuss assignment problems or to share solutions with members of other groups in any way.

Instead, if you encounter problems in solving an exercise, you are explicitly encouraged to post a question on Piazza. **Posting a reasonable question on Piazza is never a violation of the Honor Code.** However, do not share full and complete solutions to current assignments on Piazza (or on any other public medium). Such sharing will very likely constitute a violation of the Honor Code. Please use reasonable judgment!

The above comments regarding collaboration and the use of Piazza in coursework also apply, *mutatis mutandis*, to term projects.

It may happen that you find the solution of a homework problem in some outside source (book, internet site, etc.). In that case you are not allowed to just copy the solution; this is considered a violation of the Honor Code.

The correct way of using external sources for homework problems is to understand the contents of your source and then to write down the solution in your own words and without referring back to the source the solution of the problem. Your solution should differ in style significantly from the published solution. If in doubt, cite the source that you used.

On the other hand, for term projects, you should **always** cite all sources that you use, both in-text and in the bibliography.

Lastly, Section 5 of the Honor Code is fully enforced: any violation of the Honor Code by a term project or an assignment group will cause all group members to be sanctioned equally.

All members of the Teaching Team will be happy to answer any any questions regarding the application of the Honor Code.

Course grade components

Below is a summary of the components of the course grade:

Grade Component	Points
Completing the Peer Evaluation for the Term Project	1
Completing the Peer Evaluation for the Coursework	1
Completing the Course Evaluation	1
Course Outcome Quizzes	7
Term Project	15
First Midterm Exam	25
Second Midterm Exam	25
Final Exam	25
Total	100

The course will be graded on a letter scale (A-F), with a certain number of points corresponding to each letter grade.

The grading scale will *usually* be based on the top approximately 6-12% of students receiving a grade of A+, with the following grades determined by (mostly) fixed point increments.

Apart from this normalization, the grade distribution is up to you! If (for example) all students obtain many points in the exams, I am happy to see everyone receive a grade of A. Students are primarily evaluated with respect to a fixed point scale, not with respect to each other.

And now, enjoy the course...

Please don't hesitate to let any member of the teaching team know if you have any questions about course policies or content. Also, don't hesitate to contact us if you need help or support, either academically or otherwise. We are there to help!