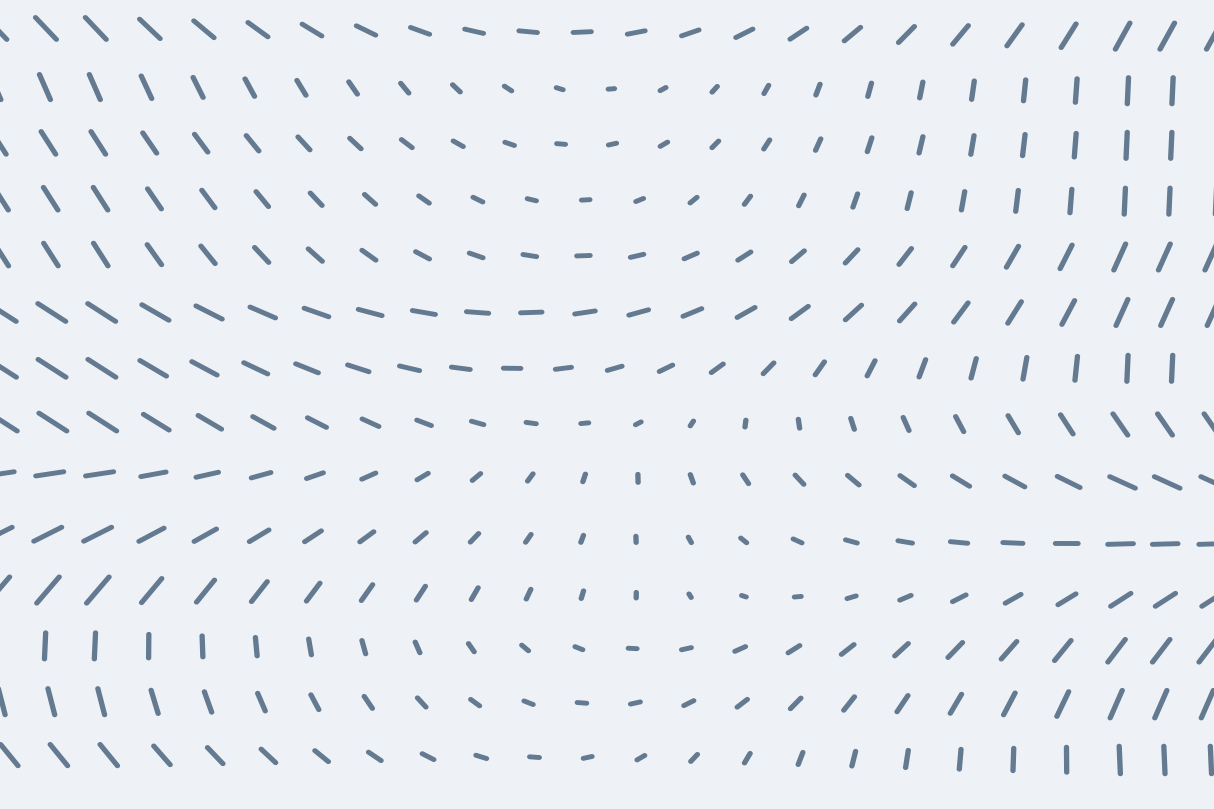

1) $F = \dfrac{SS_{Exp}/p}{SS_{Res}/(n-p-1)}$

$R^2 = 1 - \dfrac{SS_{Res}}{SS_{Total}}$

$R^2 = \dfrac{SS_{Exp}}{SS_{Total}}$

$\therefore F = \dfrac{R^2/p}{1-R^2/n-p-1} = \dfrac{R^2}{p} \times \dfrac{n-p-1}{1-R^2}$

$= \dfrac{R^2}{1-R^2} \cdot \underbrace{\dfrac{n-p-1}{p}}_{=c}$

$= \dfrac{R^2}{1-R^2} \cdot c$

$\therefore F = \dfrac{R^2}{1-R^2} \cdot c$

$F - F.R^2 = R^2 . c$

$F = R^2 (F + c)$

$\therefore R^2 = \dfrac{F}{F+c}$

2) $y = a + bx + \varepsilon$

$a, x, b, y \in R$

$\varepsilon \in N(0, \sigma^2)$

let $\theta = a + 3b$

$\hat{\theta} = \hat{a} + 3\hat{b}$

$Var[\hat{\theta}] = Var[\hat{a} + 3\hat{b}] = Var[\hat{a}] + 9\,Var[\hat{b}] + 6\,Cov(\hat{a}, \hat{b})$.

$S.E.[\hat{\theta}] = \sqrt{V[\hat{a}] + 9\,V[\hat{b}] + 6\,Cov(\hat{a}, \hat{b})}$

$\hat{b} = \frac{\sum x_i y_i}{\sum x_i^2}$

$\hat{a} = \bar{y}$

$Cov(\hat{a}, \hat{b}) = 0$

$\therefore Var[\hat{a} + 3\hat{b}] = Var[\hat{a}] + 9\,Var[\hat{b}]$

$Var[\hat{a}] = \frac{\sigma^2}{n}$ $\qquad s_x^2 = \frac{\sum x_i^2}{n}$

$Var[\hat{b}] = \frac{\sigma^2}{\sum x_i^2}$

$SE[a + 3b] = \sqrt{\frac{\sigma^2}{n}\left[1 + \frac{9}{s_x^2}\right]}$

$$\hat{\sigma}^2 = \frac{1}{n-2} \sum (y_i - \hat{a} - \hat{b} x_i)^2$$

$$1-\alpha \quad \text{c.I.} = \hat{a} + 3\hat{b} \pm t_{n-2,\frac{1-\alpha}{2}} \cdot \sqrt{\frac{\hat{\sigma}^2}{n} + \frac{9\,\hat{\sigma}^2}{\sum x_i^2}}$$

3) Let y^* be the prediction

$\therefore$ predicted value $\hat{y}^* = \hat{a} + \hat{b}x^*$

$$SE_{pred} = \sqrt{\hat{\sigma}^2\left[1 + \frac{1}{n} + \frac{x^{*2}}{\Sigma x_i^2}\right]}$$

The SE_{pred} is larger than SE confidence.

$\therefore$ the $1-\alpha$ prediction interval for $y^* =$

$$\hat{y}^* \pm t_{n-2,\frac{1-\alpha}{2}} \cdot SE_{pred}$$

The prediction interval is wider than the confidence interval.

$\therefore$ the prediction interval for y when $x = 3$ is

$$\left[\hat{a} + 3\hat{b} - t \cdot \sqrt{\hat{\sigma}^2\left(1 + \frac{1}{n} + \frac{9}{\Sigma x_i^2}\right)},\ \hat{a} + 3\hat{b} + t\sqrt{\hat{\sigma}^2\left(1 + \frac{1}{n} + \frac{9}{\Sigma x_i^2}\right)}\right]$$

Ani
Arn
aari
Mihir
Aarathi
Ishaan
Tanmay
Lucy
Navya
Raya

Devansh
Vedant
Reon
Anay

17.7ns

49 + 29.4 + 23.86

7.4