1) $X \sim R$

$X \sim N(\mu, \sigma^2)$

$\mu \in R$

$\sigma^2 > 0$

$$f(x) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{\frac{-1}{2\sigma^2}(x-\mu)^2}$$

$$\hat{\mu}_{MLE} = \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi}\,\sigma} e^{\frac{-1}{2\sigma^2}\cdot(x_i-\mu)^2}$$

$$= \log\left[\prod_{i=1}^{n} \frac{1}{\sqrt{2\pi}\,\sigma} e^{\frac{-1}{2\sigma^2}(x_i-\mu)^2}\right]$$

$$= \sum_{i=1}^{n} \log\left[\frac{1}{\sqrt{2\pi}\,\sigma} e^{\frac{-1}{2\sigma^2}\cdot(x_i-\mu)^2}\right]$$

$$= \sum_{i=1}^{n} \left[\log\left[\frac{1}{\sqrt{2\pi}\,\sigma}\right] + \log\left\{e^{\frac{-1}{2\sigma^2}(x_i-\mu)^2}\right\}\right]$$

$$= \sum_{i=1}^{n} \left[\log(1) - \frac{1}{2}\log(2\pi) - \log(\sigma) - \frac{1}{2\sigma^2}(x_i-\mu)^2\right]$$

$$= \sum_{i=1}^{n} \left[0 - \frac{1}{2}\log(2\pi) - \log(\sigma) - \frac{1}{2\sigma^2}(x_i-\mu)^2\right]$$

$$= -\frac{n}{2}\log(2\pi) - n\log(\sigma) - \sum_{i=1}^{n} \frac{1}{2\sigma^2}(x_i - \mu)^2$$

$$= -\frac{n}{2}\log(2\pi) - n\log(\sigma) \frac{-1}{2\sigma^2} \sum_{i=1}^{n}(x_i - \mu)^2$$

$$\hat{\mu}_{MLE} = \frac{d}{d\mu}\left[-\frac{n}{2}\log(2\pi) - n\log(\sigma) \frac{-1}{2\sigma^2} \sum_{i=1}^{n}(x_i - \mu)^2\right]$$

$$= \left[0 - 0 + \frac{d}{d\mu}\left(\frac{-1}{2\sigma^2} \sum_{i=1}^{n}(x_i - \mu)^2\right)\right]$$

$$= \frac{-1}{2\sigma^2} \cdot 2 \cdot \sum_{i=1}^{n}(x_i - \mu)(-1) = 0$$

$$= -2 \cdot \sum_{i=1}^{n}(x_i - \mu) = 0$$

$$= \sum_{i=1}^{n}(x_i - \mu) = 0$$

$$= \sum_{i=1}^{n} x_i - n\mu = 0$$

$$= \sum x_i = n\mu$$

$$\hat{\mu}_{MLE} = \frac{\sum x_i}{n} = \bar{x}$$

$$\therefore \hat{\mu}_{MLE} = \bar{x}$$

$$Var[\bar{x}] = Var\left[\frac{1}{n} \cdot \sum x_i\right]$$

$$= \frac{1}{n^2} Var\left[\sum x_i\right]$$

$$= \frac{1}{n^2} \cdot \sum Var[x_i]$$

$$= \frac{1}{n^2} \cdot n \cdot \sigma^2 = \underline{\underline{\frac{\sigma^2}{n}}}$$

Amoon – 10.5
Faijoz – 10.5
Miwa – 10.5 + 1.9 = 12.4
Tamsh – 10.5 + 1.9 = 12.4

2)

$X \in R^{n \times p}$

To show

$$P_X = X(X^TX)^{-1}X^T$$

Proof

$$P_X^2 = P_X$$

$$P_X^2 = P_X . P_X = \left[X(X^TX)^{-1}X^T\right]\left[X(X^TX)^{-1}X^T\right]$$

$$= X(X^TX)^{-1}\cancel{X^TX}(\cancel{X^TX})^{-1}X^T$$

$$= X(X^TX)^{-1} I X^T = X(X^TX)^{-1}X^T (I)$$

$= P_X$ $\therefore$ P_X is idempotent.

$$P_X u = (X(X^TX)^{-1}X^T)u$$

$$= X(X^TX)^{-1}X^T . X\beta$$

$$= X . I . \beta = X\beta = u$$

$$\therefore P_X u = u$$

3) $$SS_{Tot} = \sum_{i=1}^{n} (y_i - \bar{y})^2$$

$$= \sum_{i=1}^{n} (y_i - \bar{y} + \hat{y}_i - \hat{y}_i)^2$$

$$= \sum_{i=1}^{n} (\underbrace{y_i - \hat{y}_i} + \underbrace{\hat{y}_i - \bar{y}})^2$$

$$= \sum_{i=1}^{n} \left[(y_i - \hat{y}_i)^2 + (\hat{y}_i - \bar{y})^2 + 2(y_i - \hat{y}_i)(\hat{y}_i - \bar{y}) \right]$$

$$= \underbrace{\sum_{i=1}^{n} (y_i - \hat{y}_i)^2}_{SS_{Res}} + \underbrace{\sum_{i=1}^{n} (\hat{y}_i - \bar{y})^2}_{SS_{Exp}} + 2\sum_{i=1}^{n} (y_i - \hat{y}_i)(\hat{y}_i - \bar{y})$$

To show : $\sum_{i=1}^{n} (y_i - \hat{y}_i)(\hat{y}_i - \bar{y}) = 0$

$$= \underbrace{\sum_{i=1}^{n} \hat{u}_i \hat{y}_i}_{=0} - \sum_{i=1}^{n} \hat{u}_i \bar{y}$$

$$= \bar{y} \underbrace{\sum_{i=1}^{n} \hat{u}_i}_{0} = 0$$

$$\therefore 2\sum_{i=1}^{n} (y_i - \hat{y}_i)(\hat{y}_i - \bar{y}) = 0$$

$\therefore SS_{Total} = SS_{Exp} + SS_{Res}.$

1) $X \sim N(\mu, \sigma^2)$

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{1}{2\sigma^2}(x-\mu)^2\right)$$

$$L = \prod_{i=1}^{n} \left[\frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(\frac{-1}{2\sigma^2}(x_i-\mu)^2\right)\right]$$

$$= \left[\frac{1}{\sqrt{2\pi\sigma^2}}\right]^n \cdot \left[\exp\left(\frac{-1}{2\sigma^2}(x_i-\mu)^2\right)\right]^n$$

$$= (2\pi\sigma^2)^{-n/2}$$

05962314 ×4

65585454

05962314 ×12

71547768

02222 ×6

13332

02004 ×6

12024

04232 ×6

25392

04748 ×6

28488

02906 ×6

17436

05244 ×6

31464

03465 ×6

20790

04111 ×6

24666

0 2 0 5 9 2 9 8 × 12

2 4 7 1 1 5 7 6

0 4 2 3 2 × 11

4 6 5 5 2

0 4 7 4 9 2 × 11

4 1 2

11 - add number + neighbor

12 - double num + neighbor

6 - num + half the neigh + 5 → odd

num + half the neig → even