HW401

1) $y = \hat{\alpha}_0 + \hat{\alpha}_1 \cdot x + \hat{\alpha}_2 \frac{(3x^2-1)}{2} + \hat{\alpha}_3 \left(\frac{5x^3-3x}{2}\right)$

$= \hat{\alpha}_0 + \hat{\alpha}_1 x + \frac{3x^2\hat{\alpha}_2}{2} - \frac{\hat{\alpha}_2}{2} + \frac{5x^3\hat{\alpha}_3}{2} - \frac{3x\hat{\alpha}_3}{2}$

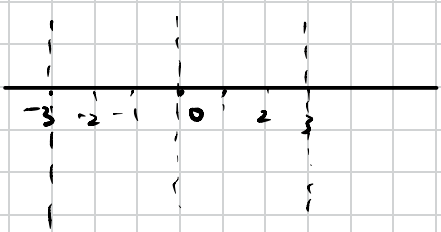
$= \hat{\alpha}_0 - \frac{\hat{\alpha}_2}{2} + \hat{\alpha}_1 x - \frac{3x\hat{\alpha}_3}{2} + \frac{3x^2\hat{\alpha}_2}{2} + \frac{5x^3\hat{\alpha}_3}{2}$

$y = \hat{\beta}_0 + \hat{\beta}_1 x + \hat{\beta}_2 x^2 + \hat{\beta}_3 x^3$

$\hat{\beta}_0 = \hat{\alpha}_0 - \frac{\hat{\alpha}_2}{2}$ $\quad$ $\hat{\beta}_2 = \frac{3\hat{\alpha}_2}{2}$

$\hat{\beta}_1 = \hat{\alpha}_1 - \frac{3}{2}\hat{\alpha}_3$ $\quad$ $\hat{\beta}_3 = \frac{5\hat{\alpha}_3}{2}$

2)



$y = \beta_{0,1}[x \geq -3] + \beta_{0,2}[x \geq 0] + \beta_{1,1}(x+3)_+$

$+ \beta_{1,2}[x] + \beta_{0,3}[x \geq 3] + \beta_{1,3}(x-3)_+$

$= \beta_{0,1}\mathbb{1}(x \geq -3)$

3) $y = e^a x^b e^\varepsilon$

$$\begin{bmatrix} \hat{a} \\ \hat{b} \end{bmatrix} = (X^T X^{-1}) X^T Y = \begin{bmatrix} n & \Sigma x_i \\ \Sigma x_i & \Sigma x_i^2 \end{bmatrix}$$

$\log y = a + b \log x + \varepsilon$

So, $\begin{bmatrix} \hat{a} \\ \hat{b} \end{bmatrix} = (U^T U)^{-1} U^T V = \begin{bmatrix} n & \Sigma u_i \\ \Sigma u_i & \Sigma u_i^2 \end{bmatrix}^{-1} = \begin{bmatrix} 1/n & 0 \\ 0 & (\Sigma u_i^2)^{-1} \end{bmatrix}$

$= \begin{bmatrix} 1 & 1 \\ u_{2,1} & u_{2,2} \end{bmatrix} \begin{bmatrix} v_{1,1} \\ v_{1,2} \end{bmatrix} = \begin{bmatrix} \Sigma v_i \\ \Sigma u_i v_i \end{bmatrix} =$

$= \begin{bmatrix} 1/n & 0 \\ 0 & (\Sigma u_i^2)^{-1} \end{bmatrix} \begin{bmatrix} \Sigma v_i \\ \Sigma u_i v_i \end{bmatrix} = \begin{bmatrix} \Sigma v_i / n \\ \frac{\Sigma u_i v_i}{\Sigma u_i^2} \end{bmatrix}$