

The Holmes Curve: A Ternary Peano–Morton Hybrid for Space-Filling Visualization

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Abstract

We present the Holmes Curve, a novel recursive mapping and visualization that extends the classical Peano and Morton (Z-order) curves into generalized base-N space. The Holmes Curve exhibits ternary interleaving, directional continuity, and a subtle skew symmetry that produces visually coherent wave-like filling patterns distinct from known fractal traversals. This mapping has applications in 2D/3D spatial indexing, data locality optimization, texture synthesis, and generative art.

Definition

```
Let  $n \in [0, N^{(2m)} - 1]$ 
be a linear index for order  $m$  and base  $N$ .
 $t_i = n \bmod N$ 

 $n \leftarrow \text{floor}(n / N)$ 

 $x_i = t_{(2i)}$ 
 $y_i = t_{(2i+1)}$ 

 $x = \sum (x_i * N^i)$ 
 $y = \sum (y_i * N^i)$ 

 $S(x, y) = (x + \alpha y, y)$ 
```

Implementation

Implemented in Python 3 using numpy and matplotlib, the viewer provides an interactive order slider, animation mode, and consistent viewport. Example CLI usage:

```
peano-viewer --order 4 --base 3 --interval0.2
from peano_morton_viewer import interactive_peano
interactive_peano(order=4)
```

Applications

• Spatial indexing and cache-efficient data layouts • Texture synthesis and fractal pattern generation • Procedural terrain and signal traversal algorithms • Educational visualization of radix-N interleaving • Aesthetic generative art and motion patterns

References

1. Peano, G. (1890). Sur une courbe, qui remplit toute une aire plane.
2. Morton, G. M. (1966). A Computer Oriented Geodetic Data Base and a New Technique in File Sequencing.
3. Holmes, J. (2025). The Holmes Curve: Ternary Skew Interleave for Space-Filling Visualization.

Citation

```
@article{holmes2025,  
  author = {Joshua Holmes},  
  title   = {The Holmes Curve: A Ternary Peano-Morton Hybrid for Space-Filling Visualization},  
  year    = {2025},  
  note    = {https://github.com/sleepyprogrammer1012/Peano-Morton-Interactive-Viewer-Holmes-  
  }       Curve}
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