Data 605 Homework 14 - Taylor Series

Jean Jimenez

2023-12-02

```r
library(ggplot2)
library(Deriv)
```

```
## Warning: package 'Deriv' was built under R version 4.3.2
```

Assignment

This week, we'll work out some Taylor Series expansions of popular functions:

1. $f(x) = \frac{1}{1-x}$
2. $f(x) = e^x$
3. $f(x) = \ln(1+x)$
4. $f(x) = x^{\frac{1}{2}}$

For each function, only consider its valid ranges as indicated in the notes when you are computing the Taylor Series expansion. Please submit your assignment as an R- Markdown document.

Taylor Series Expansion Formula:

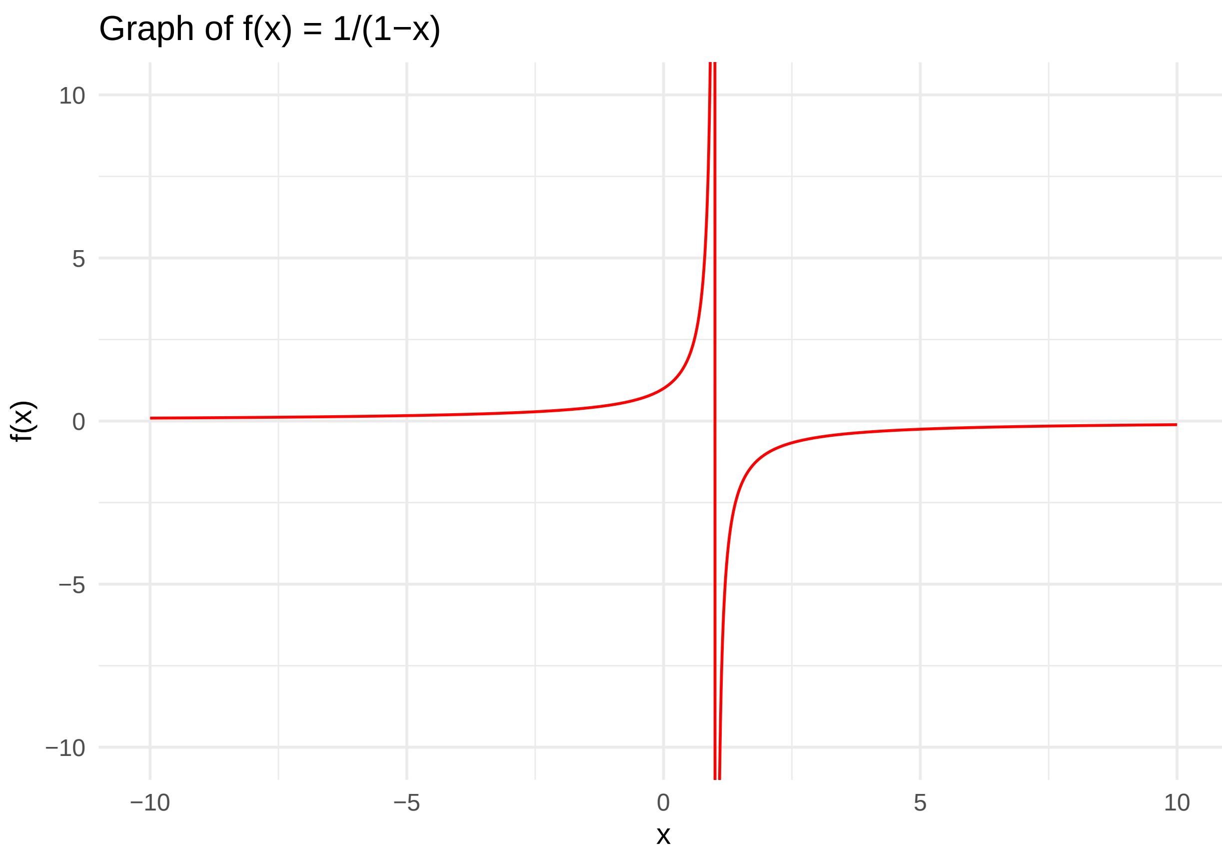
$$f(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f'''(a)}{3!}(x-a)^3 + \cdots$$

For this Homework, I will make a function that will evaluate and calculate the taylor series of each equation:

#1

$f(x) = \frac{1}{1-x}$

Visualization:



As you can see from the graph, the function $f(x) = \frac{1}{1-x}$ has a vertical asymptote at x=1. When x=1, f(x) is undefined.

$\lim_{x \to 1^-} f(x) = \infty$

$\lim_{x \to 1^+} f(x) = -\infty$

Lets do Taylor expansion of f(x) at x=0. I will set a=0. I will be calculating up to the 5th derivative.

```
# Define the original function and its derivatives
f1 = expression(1 / (1 - x))
d1 = D(f1, "x")
d2 = D(d1, "x")
d3 = D(d2, "x")
d4 = D(d3, "x")
d5 = D(d4, "x")

# Evaluate each derivative at x = 0
f1_0 = eval(f1, list(x = 0))
d1_0 = eval(d1, list(x = 0))
d2_0 = eval(d2, list(x = 0))
d3_0 = eval(d3, list(x = 0))
d4_0 = eval(d4, list(x = 0))
d5_0 = eval(d5, list(x = 0))

#Taylor expansion function
taylor_expansion1 = function(x) {
  f1_0 + d1_0 * x + d2_0 * x^2 / factorial(2) + d3_0 * x^3 / factorial(3) +
```

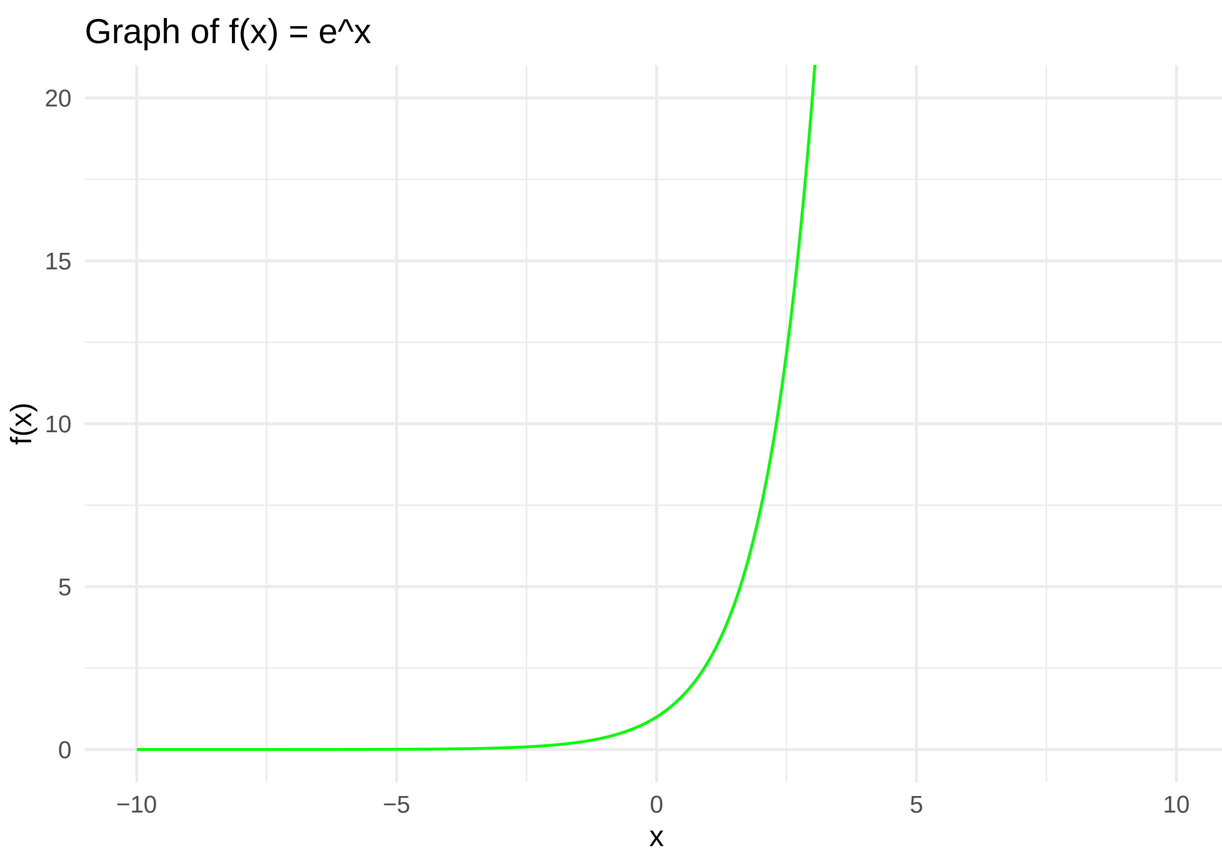
```
    d4_0 * x^4 / factorial(4) + d5_0 * x^5 / factorial(5)
}

# original function
f_original1 = function(x) {
  1 / (1 - x)
}

eval1=0

eval1_orig=eval(f_original1(eval1))

eval1_taylor=eval(taylor_expansion1(eval1))

cat("When centered at X=", eval1, "\n the original function outputs:",eval1_orig,". \n", "the taylor ex
```

```
## When centered at X= 0
##  the original function outputs: 1 .
##  the taylor expansion outputs: 1 .
```

#2

$f(x) = e^x$

Visualization:



As you can see from the graph, the function $f(x) = e^x$ has a horizontal asymptote at y=0. When f(x)=0, x is undefined.

$\lim_{x \to -\infty} f(x) = 0$

Lets do Taylor expansion of f(x) at x=0. I will set a=0. I will be calculating up to the 5th derivative.

```r
# Define the original function and its derivatives
f2 = expression(exp(x))
d21 = D(f2, "x")
d22 = D(d21, "x")
d23 = D(d22, "x")
d24 = D(d23, "x")
d25 = D(d24, "x")

# Evaluate each derivative at x = 0
f2_0 = eval(f2, list(x = 0))
d21_0 = eval(d21, list(x = 0))
d22_0 = eval(d22, list(x = 0))
d23_0 = eval(d23, list(x = 0))
d24_0 = eval(d24, list(x = 0))
d25_0 = eval(d25, list(x = 0))

#Taylor expansion function
taylor_expansion2 = function(x) {
  f2_0 + d21_0 * x + d22_0 * x^2 / factorial(2) + d23_0 * x^3 / factorial(3) +
    d24_0 * x^4 / factorial(4) + d25_0 * x^5 / factorial(5)
}

# original function
f_original2 = function(x) {
  exp(x)
}

eval2=0

eval2_orig=eval(f_original2(eval2))

eval2_taylor=eval(taylor_expansion2(eval2))

cat("When centered at X=", eval2, "\n the original function outputs:",eval2_orig,". \n", "the taylor ex
```

```
## When centered at X= 0
##  the original function outputs: 1 .
##  the taylor expansion outputs: 1 .
```

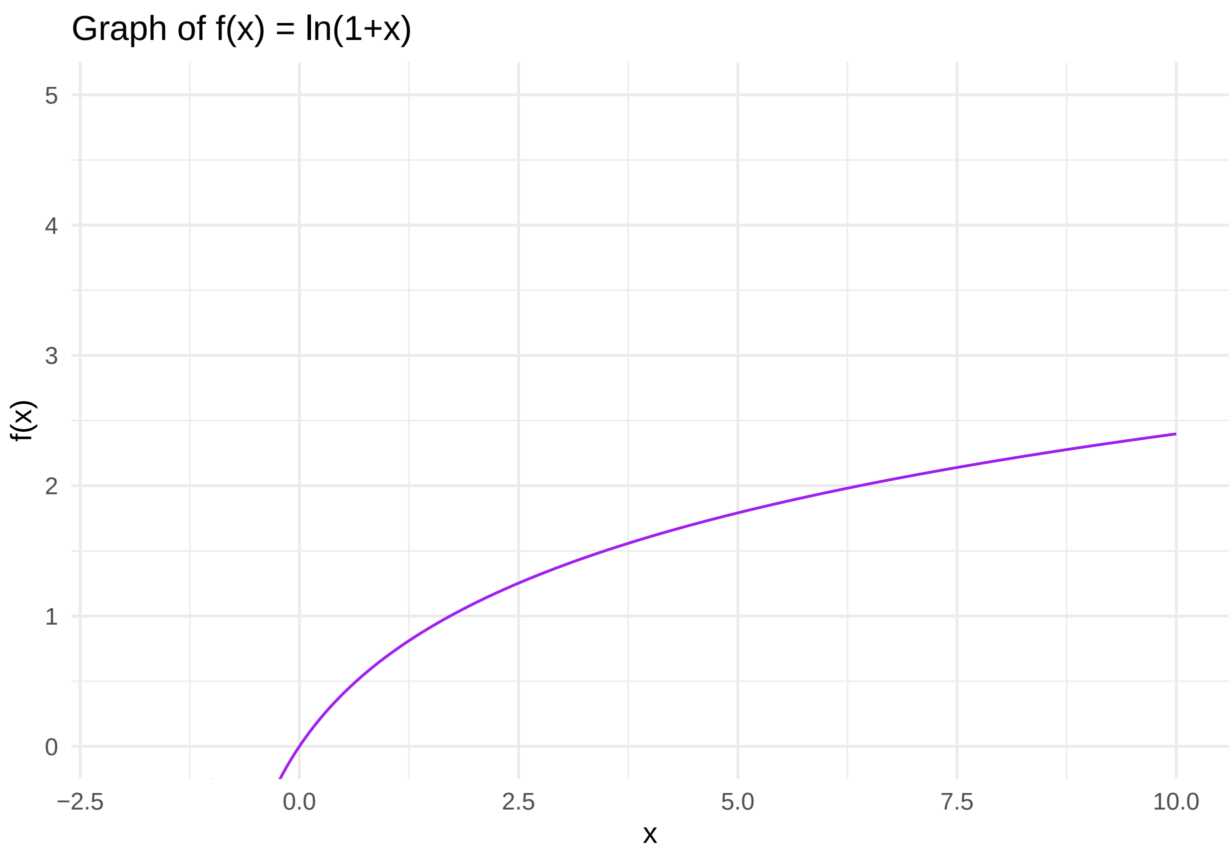
#3

$f(x) = \ln(1 + x)$

Visualization:

```
## Warning in log(1 + x3): NaNs produced
```

```
## Warning: Removed 900 rows containing missing values (`geom_line()`).
```



As you can see from the graph, the function $f(x) = \ln(1 + x)$ has a verticle asymptote at x=-1. When x=-1, f(x) is undefined.

$\lim_{x \to -1^+} f(x) = -\infty$

Lets do Taylor expansion of f(x) at x=0. I will set a=0. I will be calculating up to the 5th derivative.

```
# Define the original function and its derivatives
f3 = expression(log(1+x))
d31 = D(f3, "x")
d32 = D(d31, "x")
d33 = D(d32, "x")
d34 = D(d33, "x")
d35 = D(d34, "x")

# Evaluate each derivative at x = 0
f3_0 = eval(f3, list(x = 0))
d31_0 = eval(d31, list(x = 0))
d32_0 = eval(d32, list(x = 0))
d33_0 = eval(d33, list(x = 0))
d34_0 = eval(d34, list(x = 0))
d35_0 = eval(d35, list(x = 0))

#Taylor expansion function
taylor_expansion3 = function(x) {
  f3_0 + d31_0 * x + d32_0 * x^2 / factorial(2) + d33_0 * x^3 / factorial(3) +
    d34_0 * x^4 / factorial(4) + d35_0 * x^5 / factorial(5)
}
```

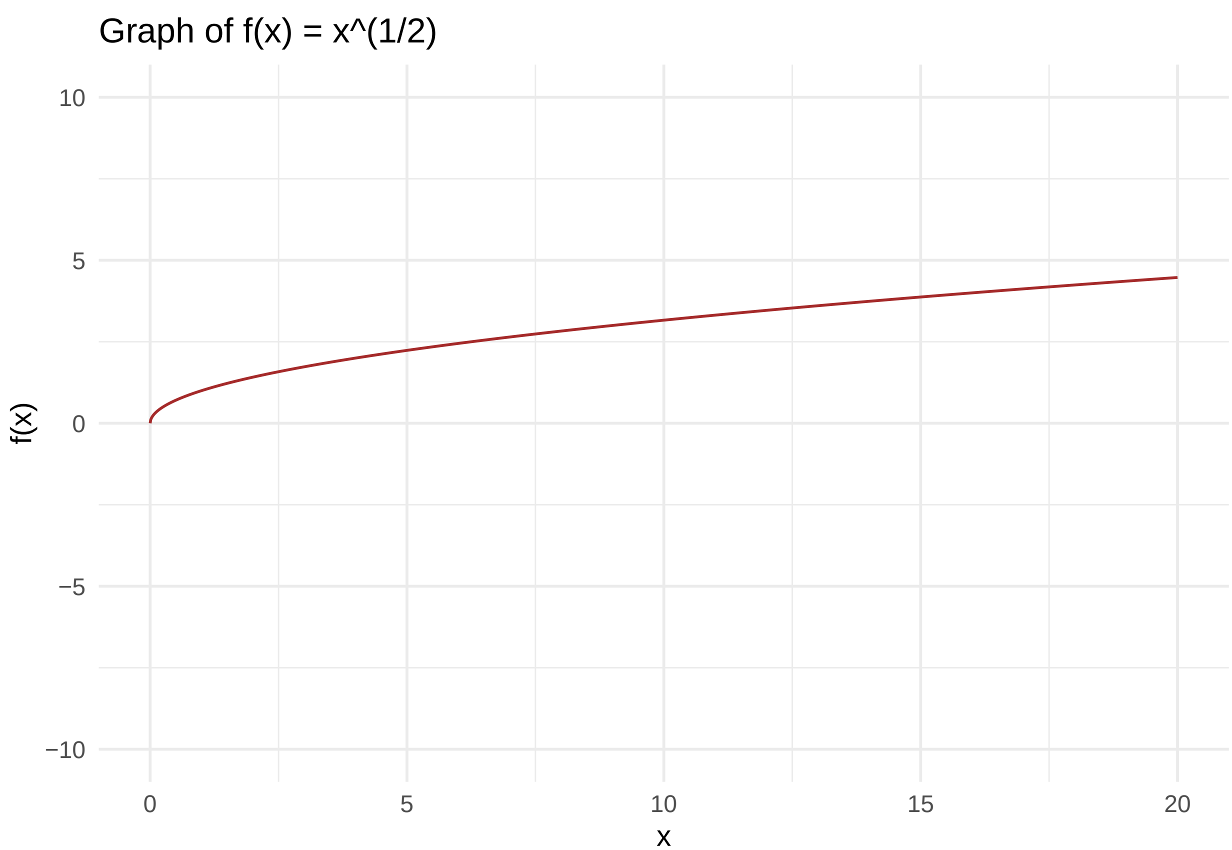
```
# original function
f_original3 = function(x) {
  log(1-x)
}

eval3=0

eval3_orig=eval(f_original3(eval3))

eval3_taylor=eval(taylor_expansion3(eval3))

cat("When centered at X=", eval3, "\n the original function outputs:",eval3_orig,". \n", "the taylor exp
```

```
## When centered at X= 0
##  the original function outputs: 0 .
##  the taylor expansion outputs: 0 .
```

#4

$f(x) = x^{\frac{1}{2}}$

Visualization:



As you can see from the graph, the function $f(x) = x^{\frac{1}{2}}$ has a vertical asymptote at x=0. When x is negative, f(x) is undefined. This is because the square root of a negative number is imaginary. Function only exists from $[0, \infty)$.

Lets do Taylor expansion of f(x) at x=2. I will be calculating up to the 5th derivative.

```r
f_original4 = function(x) x^0.5


d41 = Deriv(f_original4, "x")
d42 = Deriv(d41, "x")
d43 = Deriv(d42, "x")
d44 = Deriv(d43, "x")
d45 = Deriv(d44, "x")
d46 = Deriv(d45, "x")

# Evaluate each derivative at x = 1
eval_point = 1
d41_0 = d41(eval_point)
d42_0 = d42(eval_point)
d43_0 = d43(eval_point)
d44_0 = d44(eval_point)
d45_0 = d45(eval_point)
d46_0 = d46(eval_point)


taylor_expansion4 = function(x) {
  f_original4(eval_point) +
  d41_0 * (x - eval_point) +
  d42_0 / factorial(2) * (x - eval_point)^2 +
  d43_0 / factorial(3) * (x - eval_point)^3 +
  d44_0 / factorial(4) * (x - eval_point)^4 +
  d45_0 / factorial(5) * (x - eval_point)^5 +
  d46_0 / factorial(6) * (x - eval_point)^6
}

# Evaluate at x = 2
eval_point2 = 2
eval_orig4 = f_original4(eval_point2)
eval_taylor4 = taylor_expansion4(eval_point2)

# Output results
cat("When centered at X =", eval_point2, "\nthe original function outputs:", eval_orig4, ".\n", "The Ta
```

```
## When centered at X = 2
## the original function outputs: 1.414214 .
##  The Taylor expansion outputs: 1.405273 .
```