Jean Jimenez Data 605 HW#2

Problem Set 1

(1) Show that $A^TA \neq AA^T$

Let

$$A = \begin{bmatrix} 7 & 1 & 4 \\ 2 & 5 & 3 \end{bmatrix}$$

$$A^T = \begin{bmatrix} 7 & 2 \\ 1 & 5 \\ 4 & 3 \end{bmatrix}$$

$$A^T \times A = \begin{bmatrix} 7 & 2 \\ 1 & 5 \\ 4 & 3 \end{bmatrix} \begin{bmatrix} 7 & 1 & 4 \\ 2 & 5 & 3 \end{bmatrix} = \begin{bmatrix} 53 & 39 & 46 \\ 39 & 26 & 23 \\ 46 & 23 & 25 \end{bmatrix}$$

$$A \times A^T = \begin{bmatrix} 7 & 1 & 4 \\ 2 & 5 & 3 \end{bmatrix} \begin{bmatrix} 7 & 2 \\ 1 & 5 \\ 4 & 3 \end{bmatrix} = \begin{bmatrix} 66 & 38 \\ 38 & 38 \end{bmatrix}$$

$A^T \times A$ produces a 3x3 Matrix
$A \times A^T$ produces a 2x2 Matrix
Therefore,

$$A^TA \neq AA^T$$

(2) When does $A^TA = AA^T$?

Let $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$A^T = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$A^T \times A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$A \times A^T = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Since $A^T = A$ in this Case, $A^TA = AA^T$

A is an identity Matrix. When A is an identity Matrix or symetrical,

$$A^TA = AA^T$$