# Data 605 Homework #3

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## Assignment 3

### Problem Set 1

#### #1

What is the rank of the matrix A?

[1 2 3 4]

[-1 0 1 3]

A=

[0 1 -2 1]

[5 4 -2 -3]

To solve this problem, I inserted matrix A into R and used the rankMatrix() function

```
#what is the rank of matrix A?

library(Matrix)
A=matrix(c(1,2,3,4,-1,0,1,3),nrow=4,ncol=4)

rank_A=rankMatrix(A)
rank_A
```

```
## [1] 2
## attr(,"method")
## [1] "tolNorm2"
## attr(,"useGrad")
## [1] FALSE
## attr(,"tol")
## [1] 8.881784e-16
```

**Answer The rank of matrix A is 2**

#### #2

The maximum rank of a mxn matrix is n because rank is defined by the dimensions of column and row space. It cannot be more than n. The minimum rank of a non-zero matrix is 1 because it will at least have 1 row.

#### #3

What is the rank of matrix B?

[1 2 1]

B= [3 6 3 ]

[2 4 2]

To solve this problem, I inserted matrix B into R and again used the rankMatrix() function.

```
B=matrix(c(1,2,1,3,6,3,2,4,2),ncol=3, nrow=3)
rank_B=rankMatrix(B)
rank_B
```

```
## [1] 1
## attr(,"method")
## [1] "tolNorm2"
## attr(,"useGrad")
## [1] FALSE
## attr(,"tol")
## [1] 6.661338e-16
```

**Answer** The rank of matrix B is 1.

## Problem Set 2

Compute the convectors and eigenvalues of the given matrix.

For this question, I did the computation manually. Below is my work:

Find Eigenvalue & Eigenvector

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 0 & 0 & 6 \end{bmatrix}$$

$$\begin{bmatrix} 1-\lambda & 2 & 3 \\ 0 & 4-\lambda & 5 \\ 0 & 0 & 6-\lambda \end{bmatrix}$$

$$\det \begin{vmatrix} 1-\lambda & 2 & 3 \\ 0 & 4-\lambda & 5 \\ 0 & 0 & 6-\lambda \end{vmatrix} = (1-\lambda)\det \begin{vmatrix} 4-\lambda & 5 \\ 0 & 6-\lambda \end{vmatrix} - (2)\det \begin{vmatrix} 0 & 5 \\ 0 & 6-\lambda \end{vmatrix} + (3)\det \begin{vmatrix} 0 & 4-\lambda \\ 0 & 0 \end{vmatrix}$$

$$= (1-\lambda)\left[(4-\lambda)(6-\lambda)\right] + \cancel{(-2)(0)} + \cancel{(3)(0)}$$

$= (1-\lambda)(4-\lambda)(6-\lambda)$

$0 = (1-\lambda)(4-\lambda)(6-\lambda)$

Eigenvalues → $\boxed{\lambda_1 = 1}\ \boxed{\lambda_2 = 4}\ \boxed{\lambda_3 = 6}$

Finding Eigenvector

$\lambda_1 = 1$

$$\begin{bmatrix} 1-\lambda & 2 & 3 \\ 0 & 4-\lambda & 5 \\ 0 & 0 & 6-\lambda \end{bmatrix} \Rightarrow \begin{bmatrix} 0 & 2 & 3 \\ 0 & 3 & 5 \\ 0 & 0 & 5 \end{bmatrix} \xrightarrow{R_1 = \frac{R_1}{2}} \begin{bmatrix} 0 & 1 & \frac{3}{2} \\ 0 & 3 & 5 \\ 0 & 0 & 5 \end{bmatrix}$$

$$\xrightarrow{R_2 = R_2 - 3R_1} \begin{bmatrix} 0 & 1 & \frac{3}{2} \\ 0 & 0 & -\frac{1}{2} \\ 0 & 0 & 5 \end{bmatrix} \xrightarrow{R_2 = 2R_2} \begin{bmatrix} 0 & 1 & \frac{3}{2} \\ 0 & 0 & 1 \\ 0 & 0 & 5 \end{bmatrix} \xrightarrow{R_1 = R_1 - \frac{3R_2}{2}} \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 5 \end{bmatrix}$$

$$\xrightarrow{R_3 = R_3 - 5R_2} \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ is the Eigenvector when $\lambda_1 = 1$

$\lambda_2 = 4$

$$\begin{bmatrix} 1-\lambda & 2 & 3 \\ 0 & 4-\lambda & 5 \\ 0 & 5 & 6-\lambda \end{bmatrix} \Rightarrow \begin{bmatrix} -3 & 2 & 3 \\ 0 & 0 & 5 \\ 0 & 5 & 2 \end{bmatrix} \xrightarrow{R_1 = -\frac{R_1}{3}} \begin{bmatrix} 1 & -\frac{2}{3} & -1 \\ 0 & 0 & 5 \\ 0 & 5 & 2 \end{bmatrix} \xrightarrow{R_2 = \frac{R_2}{5}} \begin{bmatrix} 1 & -\frac{2}{3} & -1 \\ 0 & 0 & 1 \\ 0 & 0 & 2 \end{bmatrix}$$

$$\xrightarrow{R_1 = R_1 + R_2} \begin{bmatrix} 1 & -\frac{2}{3} & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 2 \end{bmatrix} \xrightarrow{R_3 = R_3 - 2R_2} \begin{bmatrix} 1 & -\frac{2}{3} & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$\begin{bmatrix} \frac{2}{3} \\ 1 \\ 0 \end{bmatrix}$ is the Eigenvect when $\lambda_2 = 4$

$\lambda_3 = 6$

$$\begin{bmatrix}1-\lambda & 2 & 3\\ 0 & 4-\lambda & 5\\ 0 & 5 & 6-\lambda\end{bmatrix} \Rightarrow \begin{bmatrix}-5 & 2 & 3\\ 0 & -2 & 5\\ 0 & 5 & 0\end{bmatrix} \xrightarrow{R_1=-\frac{R_1}{5}} \begin{bmatrix}1 & -\frac{2}{5} & -\frac{3}{5}\\ 0 & -2 & 5\\ 0 & 0 & 0\end{bmatrix} \xrightarrow{R_2=-\frac{R_2}{2}} \begin{bmatrix}1 & -\frac{2}{5} & -\frac{3}{5}\\ 0 & 1 & -\frac{5}{2}\\ 0 & 0 & 0\end{bmatrix}$$

$$\xrightarrow{R_1=R_1+\frac{2R_2}{5}} \begin{bmatrix}1 & 0 & -\frac{8}{5}\\ 0 & 1 & -\frac{5}{2}\\ 0 & 0 & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix} = \begin{bmatrix}0\\ 0\\ 0\end{bmatrix} \qquad \begin{bmatrix}+\frac{8}{5}\\ \frac{5}{2}\\ 1\end{bmatrix}$$

$\boxed{\text{is the eigen vector when } \lambda_3=6}$

I will check my work with R below:

```r
D=matrix(c(1,2,3,0,4,5,0,0,6), nrow=3, ncol=3)

eigen(D)
```

```
## eigen() decomposition
## $values
## [1] 6 4 1
##
## $vectors
##      [,1]       [,2]        [,3]
## [1,]    0  0.0000000  0.83077316
## [2,]    0  0.3713907 -0.55384878
## [3,]    1 -0.9284767  0.05538488
```

My eigenvalues and eigenvectors match.