

Data 605 HW# 8

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#11

Question:

A company buys 100 light bulbs, each of which has an exponential lifetime of 1000 hours. What is the expected time for the first of these bulbs to burn out? (See Exercise 10.)

Work and Answer:

```
#num lights
lights = 100

#avg life per bulb
avg_life = 1000

#rate each bulb
single_rate = 1 / avg_life

#min
rate_min = lights * single_rate

exp_time= 1 / rate_min
exp_time

## [1] 10

cat("The expected time for the first of these bulbs to burn is about", exp_time,"hours \n")

## The expected time for the first of these bulbs to burn is about 10 hours
```

#14

Question:

Assume that X_1 and X_2 are independent random variables, each having an exponential density with parameter λ . Show that $Z = X_1 - X_2$ has density $f_Z(z) = \frac{1}{2}\lambda e^{-\lambda|z|}$.

Work and Answer:

```
library(ggplot2)

#prob density of Z = X1 - X2
```

```

prob_z = function(z, lambda) {
  0.5 * lambda * exp(-lambda * abs(z))
}

lambda = 1
z_values = seq(-10, 10, by = 0.1)

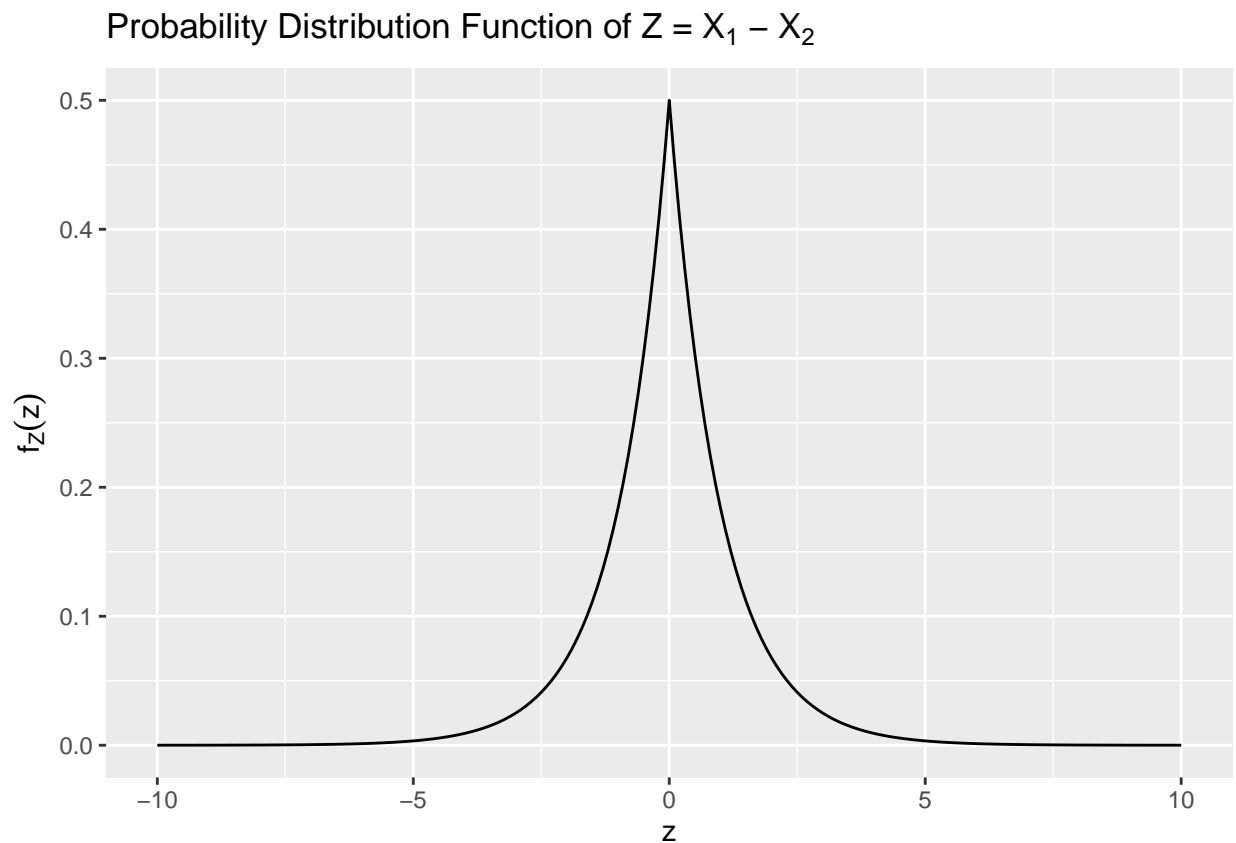
#prob density of z vals

prob_z_values = sapply(z_values, prob_z, lambda = lambda)
prob_z_values = data.frame(z = z_values, prob_z = prob_z_values)

#plot of probability dist func of Z

ggplot(prob_z_values, aes(x = z, y = prob_z)) +
  geom_line() +
  ggtitle(expression(paste("Probability Distribution Function of ", Z, " = ", X[1], " - ", X[2]))) +
  xlab("z") +
  ylab(expression(f[Z](z)))

```



#1

Question:

Let X be a continuous random variable with mean $\mu = 10$ and variance $\sigma^2 = \frac{100}{3}$. Using Chebyshev's Inequality, find an upper bound for the following probabilities:

- (a) $P(|X - 10| \geq 2)$
- (b) $P(|X - 10| \geq 5)$
- (c) $P(|X - 10| \geq 9)$
- (d) $P(|X - 10| \geq 20)$

Work and Answer:

For any random variable X , with mean μ and variance σ^2 :

$$P(|X - \mu| \geq k\sigma) \leq \frac{1}{k^2}$$

```
#given
mu = 10
variance = 100 / 3

#stddev

std = sqrt(variance)

#k values
k_values = c(2, 5, 9, 20) / std

#calc upper bounds
upper_bounds = 1 / (k_values^2)

#result for each condition dataframe
result = data.frame(
  Condition = c("|X - 10| >= 2", "|X - 10| >= 5", "|X - 10| >= 9", "|X - 10| >= 20"),
  Upper_Bound = upper_bounds
)

print(result)
```

```
##           Condition Upper_Bound
## 1  |X - 10| >= 2  8.33333333
## 2  |X - 10| >= 5  1.33333333
## 3  |X - 10| >= 9  0.41152263
## 4 |X - 10| >= 20  0.08333333
```