

Euclidean distance:

$$d(x, x^{(i)}) = \|x - x^{(i)}\|^2 = \sqrt{\sum_{i=1}^d (x - x^{(i)})^2}$$

define the function by its square:

$$d^2(x, x^{(i)}) = \sum_{i=1}^d (x - x^{(i)})^2$$

In terms of dot product:

$$d^2(x, x^{(i)}) = x^T x - 2x^T x^{(i)} + x^{(i)T} x^{(i)}$$

Because for Mercer's theorem, if K is positive definite, $k(x^{(i)}, x^{(j)})$ can be written as dot product $\phi(x^{(i)})^T \phi(x^{(j)})$

Thus, kernalize the classification rule using kernel functions:

$$d^2(x, x^{(i)}) = k(x, x) - 2k(x, x^{(i)}) + k(x^{(i)}, x^{(i)})$$