

class 1 and class 0 are equally likely $\rightarrow \gamma = \frac{1}{2}$

$P(y = 1|x)$

$$\begin{aligned}
 & \gamma \prod_{j=1}^d \frac{1}{\sqrt{2\pi}\sigma_j} e^{-\frac{(x_j - \mu_j^1)^2}{2\sigma_j^2}} \\
 = & \frac{\gamma \prod_{j=1}^d \frac{1}{\sqrt{2\pi}\sigma_j} e^{-\frac{(x_j - \mu_j^1)^2}{2\sigma_j^2}}}{(1 - \gamma) \prod_{j=1}^d \frac{1}{\sqrt{2\pi}\sigma_j} e^{-\frac{(x_j - \mu_j^0)^2}{2\sigma_j^2}} + \gamma \prod_{j=1}^d \frac{1}{\sqrt{2\pi}\sigma_j} e^{-\frac{(x_j - \mu_j^1)^2}{2\sigma_j^2}}} \\
 = & \frac{\prod_{j=1}^d \frac{1}{\sqrt{2\pi}\sigma_j} e^{-\frac{(x_j - \mu_j^1)^2}{2\sigma_j^2}}}{\prod_{j=1}^d \frac{1}{\sqrt{2\pi}\sigma_j} e^{-\frac{(x_j - \mu_j^0)^2}{2\sigma_j^2}} + \prod_{j=1}^d \frac{1}{\sqrt{2\pi}\sigma_j} e^{-\frac{(x_j - \mu_j^1)^2}{2\sigma_j^2}}} \\
 = & \frac{\frac{1}{(\sqrt{2\pi}\sigma_j)^d} e^{\sum_{j=1}^d -\frac{(x_j - \mu_j^1)^2}{2\sigma_j^2}}}{\frac{1}{(\sqrt{2\pi}\sigma_j)^d} e^{\sum_{j=1}^d -\frac{(x_j - \mu_j^0)^2}{2\sigma_j^2}} + \frac{1}{(\sqrt{2\pi}\sigma_j)^d} e^{\sum_{j=1}^d -\frac{(x_j - \mu_j^1)^2}{2\sigma_j^2}}} \\
 = & \frac{1}{e^{\sum_{j=1}^d \frac{(x_j - \mu_j^1)^2}{2\sigma_j^2} - \sum_{j=1}^d \frac{(x_j - \mu_j^0)^2}{2\sigma_j^2}} + 1} \\
 = & \frac{1}{1 + e^{\sum_{j=1}^d \frac{\mu_j^{1^2} - \mu_j^{0^2}}{2\sigma_j^2} + \sum_{j=1}^d \frac{\mu_j^0 - \mu_j^1}{\sigma_j^2} x_j}}
 \end{aligned}$$

$P(y = 1|x)$ for logistic regression:

$P(y = 1|x)$

$$\begin{aligned}
 & \frac{1}{1 + e^{-\theta^T x}} \\
 = & \frac{1}{1 + e^{-\theta_0 - \sum_{j=1}^d \theta_j x_j}}
 \end{aligned}$$

$$\text{let } \theta_0 = \sum_{j=1}^d \frac{\mu_j^{0^2} - \mu_j^{1^2}}{2\sigma_j^2}, \theta_j = \frac{\mu_j^0 - \mu_j^1}{\sigma_j^2} \quad (j = 1, 2, \dots, d)$$

$$\rightarrow P(y = 1|x) = \frac{1}{1 + e^{\sum_{j=1}^d \frac{\mu_j^{1^2} - \mu_j^{0^2}}{2\sigma_j^2} + \sum_{j=1}^d \frac{\mu_j^0 - \mu_j^1}{\sigma_j^2} x_j}}$$

$$\text{Thus, with } \theta_0 = \sum_{j=1}^d \frac{\mu_j^{1^2} - \mu_j^{0^2}}{2\sigma_j^2}, \theta_j = \frac{\mu_j^0 - \mu_j^1}{\sigma_j^2} \quad (j = 1, 2, \dots, d), P(y = 1|x) \text{ for Gaussian}$$

Naive Bayes with uniform priors is equivalent to $P(y = 1|x)$ for logistic regression.