$$Beta(\theta|a,b) \propto \theta^{a-1}(1-\theta)^{b-1}$$

$$P(\theta) = \theta^{\alpha-1}(1-\theta)^{b-1}$$

$$f(\theta|D) = \frac{P(D|\theta)P(\theta)}{P(D)} = \theta^{\sum_{i=1}^{m} x^{(i)}+a-1}(1-\theta)^{m-\sum_{i=1}^{m} x^{(i)}+b-1}$$

$$logf(\theta) = \left(\sum_{i=1}^{m} x^{(i)} + a - 1\right) log\,\theta + \left(m - \sum_{i=1}^{m} x^{(i)} + b - 1\right) log(1-\theta)$$

$$\theta_{MAP} = argmax_{\theta} logf(\theta)$$

$$\frac{\partial \log f(\theta)}{\partial \theta} = \frac{\sum_{i=1}^{m} x^{(i)} + a - 1}{\theta} - \frac{m - \sum_{i=1}^{m} x^{(i)} + b - 1}{1-\theta} = 0$$

$$(1-\theta)\left(\sum_{i=1}^{m} x^{(i)} + a - 1\right) - \theta\left(m - \sum_{i=1}^{m} x^{(i)} + b - 1\right) = 0$$

$$\sum_{i=1}^{m} x^{(i)} + a - 1 - \theta\left(\sum_{i=1}^{m} x^{(i)} + a - 1\right) - m\theta + \theta\left(\sum_{i=1}^{m} x^{(i)} - b + 1\right) = 0$$

$$\sum_{i=1}^{m} x^{(i)} + a - 1 + \theta(-a - b - m + 2) = 0$$

$$\theta_{MAP} = \frac{\sum_{i=1}^{m} x^{(i)} + a - 1}{m + a + b - 2}$$