X: m x d matrix

Y: m x 1 vector

W: diagonal matrix -> suppose W is $\begin{bmatrix} w_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & w_m \end{bmatrix}$

$$J(\theta) = \frac{1}{2}\sum_{i=1}^{m} \omega^{(i)}\left(\theta^T x^{(i)} - y^{(i)}\right)^2$$

$$let\ X\theta - y = \begin{bmatrix} z_1 \\ z_2 \\ \vdots \\ z_m \end{bmatrix}$$

$$(X\theta - y)^T W (X\theta - y)$$

$$= \begin{bmatrix} z_1 & z_2 & \dots & z_m \end{bmatrix} \begin{bmatrix} w_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & w_m \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \\ \vdots \\ z_m \end{bmatrix}$$

$$= \begin{bmatrix} z_1 w_1 & z_2 w_2 & \dots & z_m w_m \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \\ \vdots \\ z_m \end{bmatrix}$$

$$= z_1^2 w_1 + z_2^2 w_2 + \cdots + z_m^2 w_m$$

$$= \sum_{i=1}^{m} z_i^2 w_i$$

$$z_i = \sum_{j=1}^{d} x_i^{(j)} \theta^{(j)} - y_i = \theta^T x^{(i)} - y_i$$

$$(X\theta - y)^T W (X\theta - y) = \sum_{i=1}^{m} w_i \left(\theta^T x^{(i)} - y_i\right)^2 = \frac{1}{2}\sum_{i=1}^{m} 2w_i \left(\theta^T x^{(i)} - y_i\right)^2$$

$$w^{(i)} = 2w_i$$

$$\rightarrow diagonal\ matrix\ W = diag\left(\frac{w^{(1)}}{2}, \frac{w^{(2)}}{2}, \dots, \frac{w^{(m)}}{2}\right)$$

$$Then, (X\theta - y)^T W (X\theta - y) = \frac{1}{2}\sum_{i=1}^{m} w^{(i)} \left(\theta^T x^{(i)} - y_i\right)^2$$

$$Thus, J(\theta) = (X\theta - y)^T W (X\theta - y)$$