

$k(x, x') = x^T \text{rev}(x')$ where $\text{rev}(x)$ reverses the order of the components in vector x'

Counterexample:

Let Gram matrix with k be K , nonzero vector $v = (1, 0, 0, \dots, 0, 0)^T$ ($m - 1$ '0's')

$$v^T K v$$

$$= \begin{pmatrix} 1 \\ \vdots \\ 0 \end{pmatrix} \begin{pmatrix} x_1^T \text{rev}(x_1) & \dots & x_1^T \text{rev}(x_m) \\ \vdots & \ddots & \vdots \\ x_m^T \text{rev}(x_1) & \dots & x_m^T \text{rev}(x_m) \end{pmatrix} (1, 0, \dots, 0)$$

$$= x_1^T \text{rev}(x_1)$$

$$= x_1^T (-x_1)$$

$$= -1 < 0$$

Thus, $k(x, x') = x^T \text{rev}(x')$ is not a valid kernel.