

if decide $y = j$

$Risk(\alpha_i|y = j)$

$$= \sum_{k=1}^c L(\alpha_k|y = j)p(y = k|x)$$

$$= 0 * p(y = j|x) + \sum_{k=1, k \neq j}^c \lambda_s p(y = k|x)$$

$$= \lambda_s(1 - p(y = j|x))$$

j is the most probable class $\rightarrow p(y = j|x) \geq p(y = k|x)$

if $p(y = j|x) \geq 1 - \frac{\lambda_r}{\lambda_s}, 1 - p(y = j|x) \leq \frac{\lambda_r}{\lambda_s}$

*$Risk(\alpha_i|y = j) = \lambda_s(1 - p(y = j|x)) \leq \lambda_s * \frac{\lambda_r}{\lambda_s} = \lambda_r$, which is smaller than the loss of reject*

Thus, the minimum risk is obtained if we decide $y = j$ if $p(y = j|x) \geq p(y = k|x)$ for

all k and if $p(y = j|x) \geq 1 - \frac{\lambda_r}{\lambda_s}$