Let X and Y be two independent Poisson random variables.

$\lambda 1$ and $\lambda 2$ are parameters for X and Y

$$P(X = x) = \frac{e^{-\lambda 1}\lambda 1^{x}}{x!}, P(Y = y) = \frac{e^{-\lambda 2}\lambda 2^{y}}{y!}$$

Let Z = X + Y

$$P(Z = z)$$

$$= P(X = x \,\&\, Y = z - x)$$

$$= \sum_{x=0}^{z} P(X = x)P(Y = z - x)$$

$$= \sum_{x=0}^{z} \frac{e^{-\lambda 1}\lambda 1^{x}}{x!} * \frac{e^{-\lambda 2}\lambda 2^{z-x}}{(z-x)!}$$

$$= \sum_{x=0}^{z} \frac{1}{x!\,(z-x)!} * e^{-\lambda 1-\lambda 2} * \lambda 1^{x} * \lambda 2^{z-x}$$

$$= \sum_{x=0}^{z} \frac{z!}{x!\,(z-x)!} * \frac{1}{z!} e^{-\lambda 1-\lambda 2} * \lambda 1^{x} * \lambda 2^{z-x}$$

$$= \sum_{x=0}^{z} \binom{z}{x} * \frac{\lambda 1^{x} * \lambda 2^{z-x}}{z!} * e^{-(\lambda 1+\lambda 2)}$$

$$= e^{-(\lambda 1+\lambda 2)} \sum_{x=0}^{z} \binom{z}{x} * \frac{\lambda 1^{x} * \lambda 2^{z-x}}{z!}$$

$$= \frac{e^{-(\lambda 1+\lambda 2)}(\lambda 1 + \lambda 2)^{z}}{z!}$$

Thus, the sum of two independent Poisson random variable is also a Poisson random variable.