

$k(x, x') = x^T \text{rev}(x')$ where $\text{rev}(x)$ reverses the order of the components in vector x'

Counterexample:

Let Gram matrix with k be K ,

$$X = \begin{bmatrix} -1 & 1 \\ -1 & 1 \end{bmatrix}$$

nonzero vector $v = [1, 0]$

$$v^T K v$$

$$= (1, 0) \begin{pmatrix} k(x_1, x_1) & k(x_1, x_2) \\ k(x_2, x_1) & k(x_2, x_2) \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$= (1, 0) \begin{pmatrix} [-1, 1] \begin{bmatrix} 1 \\ -1 \end{bmatrix} & [-1, 1] \begin{bmatrix} 1 \\ -1 \end{bmatrix} \\ [-1, 1] \begin{bmatrix} 1 \\ -1 \end{bmatrix} & [-1, 1] \begin{bmatrix} 1 \\ -1 \end{bmatrix} \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$= (1, 0) \begin{pmatrix} -2 & -2 \\ -2 & -2 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$= -2 < 0$$

Thus, $k(x, x') = x^T \text{rev}(x')$ is not a valid kernel.