

$$k(x, x') = k_1(x, x') + k_2(x, x')$$

Let Gram matrix with $k_1(x, x')$ be K_1 , Gram matrix with $k_2(x, x')$ be K_2 .

K_1 and K_2 are positive semi-definite.

Let Gram matrix with $k(x, x')$ be K

$$K = K_1 + K_2$$

Since K_1 and K_2 are positive semi-definite, for any nonzero vector u ,

$$u^T K_1 u \geq 0, u^T K_2 u \geq 0$$

$$\rightarrow u^T K_1 u + u^T K_2 u \geq 0$$

$$\rightarrow u^T (K_1 + K_2) u \geq 0$$

$$\rightarrow u^T K u \geq 0$$

Thus, $k(x, x') = k_1(x, x') + k_2(x, x')$ is a valid kernel.