H3.1

$$\theta = (X^T X)^{-1} X^T Y$$

$$E[\theta] = E[(X^T X)^{-1} X^T Y]$$

Since X is fixed, so equation becomes:

$$\begin{aligned} E[\theta] &= (X^T X)^{-1} X^T E[Y] \\ &= (X^T X)^{-1} X^T X \theta^* \\ &= \theta^* \end{aligned}$$

3.2

$$Var(\theta) = Var((X^T X)^{-1} X^T Y)$$

Since X is fixed, so equation becomes:

$$\begin{aligned} Var(\theta) &= (X^T X)^{-1} X^T Var(Y)((X^T X)^{-1} X^T)^T \\ &= (X^T X)^{-1} X^T \sigma^2 I((X^T X)^{-1} X^T)^T \\ &= (X^T X)^{-1} X^T ((X^T X)^{-1} X^T)^T \sigma^2 \\ &= (X^T X)^{-1} X^T X((X^T X)^{-1})^T \sigma^2 \\ &= (X^T X)^{-1} X^T X (X^T X)^{-1} \sigma^2 \\ &= (X^T X)^{-1} \sigma^2 \end{aligned}$$