$$P(X_1 = x_1) = \alpha e^{-\frac{(x_1-\mu_1)^2}{2\sigma_1^2}}$$

$$= \int P(X_1 = x_1 \mid X_0 = x_0) P(X_0 = x_0) dx_0$$

$$= \int \alpha_1 e^{-\frac{(x_1-x_0)^2}{2\sigma^2}} * \alpha_0 e^{-\frac{(x_0-\mu_0)^2}{2\sigma_0^2}} dx_0$$

$$= \alpha_0 \alpha_1 \int e^{-\frac{(x_1-x_0)^2}{2\sigma^2} - \frac{(x_0-\mu_0)^2}{2\sigma_0^2}} dx_0$$

$$= \alpha_0 \alpha_1 \int e^{-\frac{1}{2}\left(\frac{(x_1-x_0)^2}{\sigma^2} + \frac{(x_0-\mu_0)^2}{\sigma_0^2}\right)} dx_0$$

$$= \alpha_0 \alpha_1 \int e^{-\frac{1}{2} * \frac{\sigma_0^2 (x_1-x_0)^2 + \sigma^2 (x_0-\mu_0)^2}{\sigma^2 \sigma_0^2}} dx_0$$

$$= \alpha_0 \alpha_1 \int e^{-\frac{1}{2} * \frac{\sigma_0^2 x_1^2 - 2\sigma_0^2 x_1 x_0 + \sigma_0^2 x_0^2 + \sigma^2 x_0^2 - 2\sigma^2 x_0 \mu_0 + \sigma^2 \mu_0^2}{\sigma^2 \sigma_0^2}} dx_0$$

$$= \alpha_0 \alpha_1 \int e^{-\frac{1}{2\sigma^2\sigma_0^2} * \left((\sigma_0^2 + \sigma^2) x_0^2 - 2(\sigma_0^2 x_1 + \sigma^2 \mu_0) x_0 + \sigma_0^2 x_1^2 + \sigma^2 \mu_0^2\right)} dx_0$$

$$= \alpha_0 \alpha_1 \int e^{-\frac{\sigma_0^2 + \sigma^2}{2\sigma^2 \sigma_0^2} x_0^2 + \frac{\sigma_0^2 x_1 + \sigma^2 \mu_0}{\sigma^2 \sigma_0^2} x_0 - \frac{\sigma_0^2 x_1^2 + \sigma^2 \mu_0^2}{2\sigma^2 \sigma_0^2}} dx_0$$

$$According\ to\ Guassian\ Integration{:}\ \int e^{-ax^2+bx+c} dx = \sqrt{\frac{\pi}{a}} e^{\frac{b^2}{4a}+c}$$

$$P(X_1 = x_1)$$

$$= \alpha_0 \alpha_1 * \sqrt{\frac{2\sigma^2 \sigma_0^2 \pi}{\sigma_0^2 + \sigma^2}} e^{\frac{(\sigma_0^2 x_1 + \sigma^2 \mu_0)^2}{\sigma^2 \sigma_0^2} * \frac{1}{2(\sigma_0^2 + \sigma^2)} - \frac{\sigma_0^2 x_1^2 + \sigma^2 \mu_0^2}{2\sigma^2 \sigma_0^2}}$$

$$= \alpha_0 \alpha_1 * \sqrt{\frac{2\sigma^2 \sigma_0^2 \pi}{\sigma_0^2 + \sigma^2}} e^{\frac{(\sigma_0^2 x_1 + \sigma^2 \mu_0)^2}{\sigma^2 \sigma_0^2} * \frac{1}{2(\sigma_0^2 + \sigma^2)} - \frac{\sigma_0^2 x_1^2 + \sigma^2 \mu_0^2}{2\sigma^2 \sigma_0^2}}$$

$$= \frac{\alpha_0 \alpha_1 \sigma \sigma_0 \sqrt{2\pi}}{\sqrt{\sigma_0^2 + \sigma^2}} e^{-\frac{(x_1-\mu_0)^2}{2(\sigma_0^2 + \sigma^2)}}$$

$$\rightarrow \alpha = \frac{\alpha_0 \alpha_1 \sigma \sigma_0 \sqrt{2\pi}}{\sqrt{\sigma_0^2 + \sigma^2}}, \mu_1 = \mu_0, \sigma_1 = \sqrt{\sigma_0^2 + \sigma^2}$$