

$k(x, x') = x^T \text{rev}(x')$  where  $\text{rev}(x)$  reverses the order of the components in vector  $x'$

*Counterexample:*

Let Gram matrix with  $k$  be  $K$ ,

$$X = \begin{bmatrix} -1 & 1 \\ 2 & 3 \end{bmatrix}$$

nonzero vector  $v = [1, 0]$

$$v^T K v$$

$$= (1, 0) \begin{pmatrix} k(x_1, x_1) & k(x_1, x_2) \\ k(x_2, x_1) & k(x_2, x_2) \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$= (1, 0) \begin{pmatrix} [-1, 1] \begin{bmatrix} 1 \\ -1 \end{bmatrix} & [-1, 1] \begin{bmatrix} 3 \\ 2 \end{bmatrix} \\ [2, 3] \begin{bmatrix} 1 \\ -1 \end{bmatrix} & [2, 3] \begin{bmatrix} 3 \\ 2 \end{bmatrix} \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$= (1, 0) \begin{pmatrix} -2 & -1 \\ -1 & 12 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$= -2 < 0$$

Thus,  $k(x, x') = x^T \text{rev}(x')$  is not a valid kernel.