1 Bogoliubov Tranformation for the Kondo Lattice

We start from the Hamiltonian for the Kondo lattice

$$H = -\sum_{i,\sigma} t_{ij} c^\dagger_{i\sigma} c_{j\sigma} + J \sum_r \mathbf{S}_r \cdot \mathbf{s}_r + \sum_{\langle r,s \rangle} I_{rs} \mathbf{S}_r \cdot \mathbf{S}_s$$

where $\mathbf{S}_i, \mathbf{s}_i$ are the spin operators for the localized f-electrons and the itinerant d-electrons. Next, we use that

$$\mathbf{S}_r = \mathbf{S}^f_r + \mathbf{S}^b_r$$

representing the fermionic and bosonic components of the spin. For the fermionic component, we use

$$\mathbf{S}^f_r = \frac{1}{2} \sum_{\alpha,\beta} f^\dagger_{r,\alpha} \sigma_{\alpha\beta} f_{r,\beta}$$

$$\mathbf{s}_r = \frac{1}{2} \sum_{\alpha,\beta} c^\dagger_{r,\alpha} \sigma_{\alpha\beta} c_{r,\beta}$$

We will assume that the bosonic part of the spins orders antiferromagnetically, and hence use the Holstein-Primakoff transformation on the A sublattice

$$S^z_r = S - a^\dagger_r a_r$$

$$S^+_r = \sqrt{2S} \sqrt{1 - \frac{a^\dagger_r a_r}{2S}} a_r$$

$$S^-_r = \sqrt{2S} a^\dagger_r \sqrt{1 - \frac{a^\dagger_r a_r}{2S}}$$

and on the B sublattice

$$S^z_r = -S + a^\dagger_r a_r$$

$$S^-_r = \sqrt{2S} a^\dagger_r \sqrt{1 - \frac{a^\dagger_r a_r}{2S}}$$

$$S^+_r = \sqrt{2S} \sqrt{1 - \frac{a^\dagger_r a_r}{2S}} a_r$$

We obtain

$$H_{MF} = \sum_k \begin{pmatrix} c^\dagger_{k,\sigma} & f^\dagger_{k,\sigma} & c^\dagger_{k+Q,\sigma} & f^\dagger_{k+Q,\sigma} \end{pmatrix} \begin{pmatrix} \varepsilon_k & -V & U_c & 0 \\ -V & \chi_k & 0 & U_f \\ U_c & 0 & \varepsilon_{k+Q} & -V \\ 0 & U_f & -V & \chi_{k+Q} \end{pmatrix} \begin{pmatrix} c_{k,\sigma} \\ f_{k,\sigma} \\ c_{k+Q,\sigma} \\ f_{k+Q,\sigma} \end{pmatrix}$$

$$\hat{H}_k = \begin{pmatrix} \varepsilon_k & -V & U_c & 0 \\ -V & \chi_k & 0 & U_f \\ U_c & 0 & \varepsilon_{k+Q} & -V \\ 0 & U_f & -V & \chi_{k+Q} \end{pmatrix}$$

Where

$$\begin{aligned} \varepsilon_k &= -2 \cdot t(\cos(k_x) + \cos(k_y)) - \mu \\ \chi_k &= -2 \cdot \chi_0(\cos(k_x) + \cos(k_y)) - \varepsilon_f \\ V &= \frac{JN}{4} \sum_{q,\alpha} \langle f^\dagger_{q\alpha} c_{q\alpha} \rangle \\ U_c &= \frac{JS}{2} sgn(\sigma) \\ U_f &= \frac{IS}{2} sgn(\sigma) \end{aligned}$$

Then the Green's function matrix is defined as

$$\hat{G_R}(k, \omega_n + i\delta) = [(\omega_n + i\delta)\hat{I} - \hat{H}_k]^{-1}$$

From which we calculate the QPI spectrum

$$g(q, \omega) = \int \frac{d^2x}{(2\pi)^2} [\hat{G_R}(k, \omega)]_{11} [\hat{G_R}(k+q, \omega)]_{11}$$