1 Bogoliubov Tranformation for the Kondo Lattice

We start from the Hamiltonian for the Kondo lattice

$$H = -\sum_{i,\sigma} t_{ij} c_{i\sigma}^{\dagger} c_{j\sigma} + J \sum_{r} \mathbf{S}_{r} \cdot \mathbf{s}_{r} + \sum_{\langle r,s \rangle} I_{rs} \mathbf{S}_{r} \cdot \mathbf{S}_{s}$$

where \mathbf{S}_i , \mathbf{s}_i are the spin operators for the localized f-electrons and the itinerant d-electrons. Next, we use that

$$\mathbf{S}_r = \mathbf{S}_r^f + \mathbf{S}_r^b$$

representing the fermionic and bosonic components of the spin. For the fermionic component, we use

$$\mathbf{S}_{r}^{f} = rac{1}{2} \sum_{lpha,eta} f_{r,lpha}^{\dagger} \sigma_{lphaeta} f_{r,eta}$$

$$\mathbf{s}_r = \frac{1}{2} \sum_{\alpha,\beta} c_{r,\alpha}^{\dagger} \sigma_{\alpha\beta} c_{r,\beta}$$

We will assume that the bosonic part of the spins orders antiferromagnetically, and hence use the Holstein-Primakoff transformation on the $\cal A$ sublattice

$$S_r^z = S - a_r^{\dagger} a_r$$

$$S_r^+ = \sqrt{2S} \sqrt{1 - \frac{a_r^{\dagger} a_r}{2S}} a_r$$

$$S_r^- = \sqrt{2S} a_r^{\dagger} \sqrt{1 - \frac{a_r^{\dagger} a_r}{2S}}$$

and on the B sublattice

$$S_r^z = -S + a_r^{\dagger} a_r$$

$$S_r^- = \sqrt{2S} a_r^{\dagger} \sqrt{1 - \frac{a_r^{\dagger} a_r}{2S}}$$

$$S_r^+ = \sqrt{2S} \sqrt{1 - \frac{a_r^{\dagger} a_r}{2S}} a_r$$

We obtain

$$H_{MF} = \sum_{k} \begin{pmatrix} c_{k,\sigma}^{\dagger} & f_{k,\sigma}^{\dagger} & c_{k+Q,\sigma}^{\dagger} & f_{k+Q,\sigma}^{\dagger} \end{pmatrix} \begin{pmatrix} \varepsilon_{k} & -V & U_{c} & 0 \\ -V & \chi_{k} & 0 & U_{f} \\ U_{c} & 0 & \varepsilon_{k+Q} & -V \\ 0 & U_{f} & -V & \chi_{k+Q} \end{pmatrix} \begin{pmatrix} c_{k,\sigma} \\ f_{k,\sigma} \\ c_{k+Q,\sigma} \\ f_{k+Q,\sigma} \end{pmatrix}$$

$$\hat{H}_{k} = \begin{pmatrix} \varepsilon_{k} & -V & U_{c} & 0 \\ -V & \chi_{k} & 0 & U_{f} \\ U_{c} & 0 & \varepsilon_{k+Q} & -V \\ 0 & U_{f} & -V & \chi_{k+Q} \end{pmatrix}$$

Where

$$\varepsilon_k = -2 \cdot t(\cos(k_x) + \cos(k_y)) - \mu
\chi_k = -2 \cdot \chi_0(\cos(k_x) + \cos(k_y)) - \varepsilon_f
V = \frac{JN}{4} \sum_{q,\alpha} \langle f_{q\alpha}^{\dagger} c_{q\alpha} \rangle
U_c = \frac{JS}{2} sgn(\sigma)
U_f = \frac{IS}{2} sgn(\sigma)$$

Then the Green's function matrix is defined as

$$\hat{G}_R(k,\omega_n+i\delta) = [(\omega_n+i\delta)\hat{I} - \hat{H}_k]^{-1}$$

From which we calculate the QPI spectrum

$$g(q,\omega) = \int \frac{d^2k}{(2\pi)^2} Im[[\hat{G}_R(k,\omega)]_{11}[\hat{G}_R(k+q,\omega)]_{11}]$$