CPSC 121,
2007/8 Winter Term 2
Section 203: Quiz 1

Name: SAMPLE SOLUTION   Student ID: _____

Signature: _____

- You have **15 minutes** to write the **3 questions** on this examination.
- A total of 18 marks are available. **You may want to complete what you consider to be the easiest questions first!**
- Justify all of your answers.
- No notes or electronic equipment are allowed.
- Keep your answers short. If you run out of space for a question, you have written too much.
- The number in square brackets to the left of the question number indicates the number of marks allocated to that question.
- Good luck!

Question	Marks
1	
2	
3	
Total	

UNIVERSITY REGULATIONS:
- Each candidate should be prepared to produce, upon request, his/her university-issued ID.
- No candidate shall be permitted to enter the examination room after the expiration of one half hour or to leave during the first half hour of the examination.
- CAUTION: candidates guilty of any of the following, or similar, dishonest practices shall be immediately dismissed from the examination and shall be liable to disciplinary action.
  1. Having at the place of writing, or making use of, any books, papers or memoranda, electronic equipment, or other memory aid or communication devices, other than those authorised by the examiners.
  2. Speaking or communicating with other candidates.
  3. Purposely exposing written papers to the view of other candidates. The plea of accident or forgetfulness shall not be received.
- Candidates must not destroy or mutilate any examination material; must hand in all examination papers; and must not take any examination material from the examination room without permission of the invigilator.

1.

(a) One of the following compound propositions is a tautology. Circle the tautology and prove that it is a tautology. [3 marks]

| (p ∧ (p ↔ q)) → q | p → (~(~p ∨ ~q) ∧ (q ∨ r)) |

(By the way, say you don't have time to try BOTH of these with truth tables. How do you find the tautology quickly? Quickly check the truth of each and quickly try to falsify each. For example:

If we know p and p <-> q, what does that mean? Well, we know q. And, p plus p <-> q implies q. So, it's a tautology!

On the right, if we know p, do we know both ~(~p v ~q) and (q v r)? The first is unclear, but the second sure doesn't follow from p! So, it's NOT a tautology. Alternatively: can I falsify it? To falsify x -> y, I need x true and y false. So, p has to be true. To falsify ~(~p v ~q) AND (q v r), I need to falsify either part. Falsifying (q v r) is easy if q = F and r = F. Done! **Not** a tautology.)

By truth table:

p	q	p ↔ q	p ∧ (p ↔ q)	(p ∧ (p ↔ q)) → q
T	T	T	T	T
T	F	F	F	T
F	T	F	F	T
F	F	T	F	T

By logical equivs:

```
(p ∧ (p ↔ q)) → q
[by def'n of <->]    ≡ (p ∧ (p → q) ∧ (q → p)) → q
[by def'n of ->]     ≡ (p ∧ (~p ∨ q) ∧ (~q ∨ p)) → q
[by absorption]      ≡ (p ∧ (~p ∨ q)) → q
[by distribution]    ≡ ((p ∧ ~p) ∨ (p ∧ q)) → q
[by negation]        ≡ (F ∨ (p ∧ q)) → q
[by identity]        ≡ (p ∧ q) → q
[by def'n of ->]     ≡ ~(p ∧ q) ∨ q
[by De Morgan's]     ≡ ~p ∨ ~q ∨ q
[by negation]        ≡ ~p ∨ T
[by domination]      ≡ T
```

(b) The following compound proposition is logically equivalent to one of the compound propositions from part a. Which one is it logically equivalent to? Prove that the two compound propositions are logically equivalent using the logical equivalences. [3 marks]

~p ∨ q

Rule of thumb: start from the more complex expression.

p → (~(~p ∨ ~q) ∧ (q ∨ r))

[def'n of cond.]	≡ ~p ∨ (~(~p ∨ ~q) ∧ (q ∨ r))
[De Morgan's]	≡ ~p ∨ ((p ∧ q) ∧ (q ∨ r))
[Absorption]	≡ ~p ∨ (p ∧ q)
[Distribution]	≡ (~p ∨ p) ∧ (~p ∨ q)
[Negation]	≡ T ∧ (~p ∨ q)
[Identity]	≡ ~p ∨ q

(c) Write a proposition that is equivalent to the following circuit. Try to express the proposition as closely to the existing circuit as possible. You should **not** simplify the proposition! [2 marks]

(q ∧ p ∧ r) ∨ (~p ∧ r)

q ∧ p ∧ r

~p

out

((q ∧ p ∧ r) ∨ (~p ∧ r)) ∧ ~r

~p ∧ r

~r
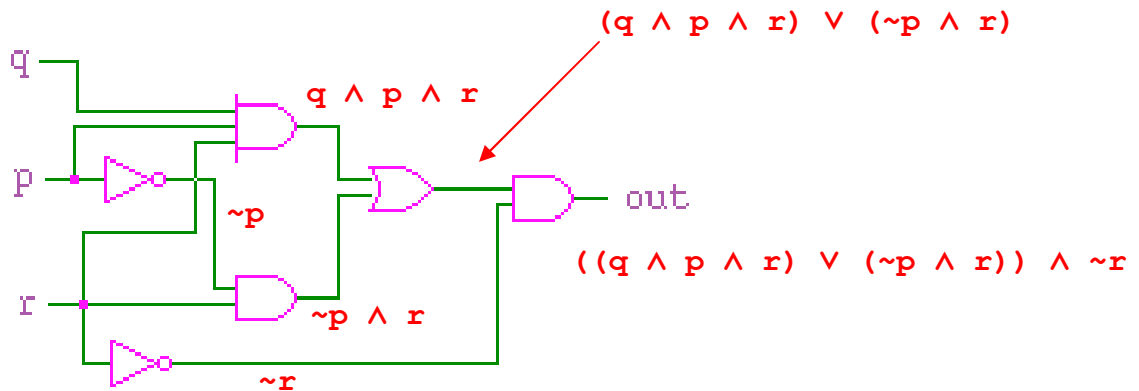
Just read the proposition off the circuit. A good way to work through this is to label each wire as you go. I've done that above, with the solution next to "out".

2. There are **two** erroneous steps in the following proof. Identify both incorrect steps and correct the errors. If an error cannot be corrected, briefly explain what is wrong with the step. (The errors are **not** in the premises!) [5 marks]

(1) `t`   premise
(2) `t → ~p ∧ s`   premise
(3) `~r ∨ p`   premise
(4) `q → r ∧ s`   premise
(5) `~p ∧ s`   modus ponens, (1), (2)

(6) `~p`   simplification, **(5)**
(7) `~r`   disj syll, (6), (3)
(8) `s`   simplification, (5)
(9) `~r ∧ s`   conjunction, (7), (8)
(10) `~q`   modus tollens, (9), (4)   Incorrect use of modus tollens:
                                                              `~r ∧ s` is not equivalent to
                                                              `~(r ∧ s)`!

Note: we did **not** intend for you to fix step (10), but if you did, good work!   We should have written "If an error can be corrected without additional steps…"   Oops!

Here's how you would fix it:

(10) `~r ∨ ~s`   addition, (7)
(11) `~(r ∧ s)`   De Morgan's, (10)
(12) `~q`   modus tollens, (11), (4)

3. Translate the following statement into propositional logic. Be sure to define each proposition you use! (For brevity, you can define a proposition by circling its text in the statement and labelling it with a letter.) [5 marks]

                          d
Detecting a wall is a sufficient condition for the robot to halt awaiting further instructions or swerve to avoid collision.
                              s   **h**

`d → h ∨ s`

(XOR would also be totally acceptable.)

Rule of inference	Name
p ∴ p ∨ q	Addition
p q ∴ p ∧ q	Conjunction
p ∧ q ∴ p	Simplification
P p → q ∴ q	Modus ponens
~q p → q ∴ ~p	Modus tollens
p → q q → r ∴ p → r	Hypothetical syllogism
p ∨ q ~p ∴ q	Disjunctive syllogism

Law Name	Equivalences
Identity	p ∧ T ≡ p p ∨ F ≡ p
Domination	p ∧ F ≡ F P ∨ T ≡ T
Distributive	p ∧ (q ∨ r) ≡ (p ∧ q) ∨ (p ∧ r) p ∨ (q ∧ r) ≡ (p ∨ q) ∧ (p ∨ r)
Absorbtion	p ∨ (p ∧ q) ≡ p p ∧ (p ∨ q) ≡ p
Negation	p ∧ ~p ≡ F p ∨ ~p ≡ T
De Morgan's	~(p ∧ q) ≡ (~p) ∨ (~q) ~(p ∨ q) ≡ (~p) ∧ (~q)
Definition of conditional	p → q ≡ ~p ∨ q
Contrapositive	p → q ≡ ~q → ~p