CPSC 121 Midterm 1
Wednesday 31 May 2017

Name: _____ Student ID: _____

Signature: _____

Question	Marks
1	
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Total	

- You have 60 minutes (individual), 30 minutes (group) to write the 15 questions on this examination. A total of 34 marks are available.
- **Justify all of your answers.**
- You are allowed to bring in any written materials, and a non-programmable non-graphing calculator.
- Keep your answers short. If you run out of space for a question, you have written too much.
- The number in square brackets to the left of the question number indicates the number of marks allocated for that question. Use these to help you determine how much time you should spend on each question.
- Use the back of the pages for your rough work.
- **Good luck!**

UNIVERSITY REGULATIONS:

- Each candidate should be prepared to produce, upon request, his/her UBC card.
- No candidate shall be permitted to enter the examination room after the expiration of one half hour, or to leave during the first half hour of the examination.
- CAUTION: candidates guilty of any of the following, or similar, dishonest practices shall be immediately dismissed from the examination and shall be liable to disciplinary action.
    1. Having at the place of writing, or making use of, any books, papers or memoranda, electronic equipment, or other memory aid or communication devices, other than those authorised by the examiners.
    2. Speaking or communicating with other candidates.

3. Purposely exposing written papers to the view of other candidates. The plea of accident or forgetfulness shall not be received.

– Candidates must not destroy or mutilate any examination material; must hand in all examination papers; and must not take any examination material from the examination room without permission of the invigilator.

[1] 1. A Quick Start
Prove $\sqrt{16} \in \mathbb{Q}$.

[4] 2. I'm Feeling Irrational
Prove $\sqrt[4]{15} \notin \mathbb{Q}$.

[1] 3. This Exam is Divided Into Questions
Prove $3 \mid 18$.

[1] 4. Winter is Coming
If Jon Snow lives in Winterfell, then Jon Snow lives in the North. Jon Snow lives in the North. What do we know about whether Jon Snow lives in Winterfell?

[2] 5. More Division is Coming

Prove that if $18 \mid n$ for some $n \in \mathbb{Z}$, then $6 \mid n$.

[2] 6. Build Us a Circuit

Write *both* a logical expression and a circuit representation for the following function, which is given in truth-table format. (Note: no points are awarded for optimizing the circuit to minimize the number of gates.)

a	b	c	$f(a,b,c)$
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	0

[2] 7. How Many Segments?

In class, we created a seven-segment display to display ten different digits. But, such a design is not optimized, in that it uses more segments than necessary to display ten different outputs. Imagine our number system were base-43. How many different segments would we need, at a minimum, to display our different symbols? Justify your answer.

[2] 8. Which is It?

Is the following a tautology, a contradiction, or a contingency? Justify your answer.
$a \oplus b \leftrightarrow (a \wedge \sim b) \vee (\sim a \wedge b)$

[2] 9. A Circuit That Recognizes Two Things

Design a circuit that takes in two 3 bit unsigned numbers, and returns true if and only if: the two numbers are equivalent, or if the two numbers differ by 4 and are both even.

[1] 10. What Does this Word Mean?

Define the following two propositions:

- $r \equiv$ it is raining
- $b \equiv$ Ryan is going to the beach

Translate the following into propositional logic: it is raining, but Ryan is going to the beach.

[2] 11. A More General Multiplexer

In class, we designed a multiplexer that took three inputs: a and b, and a "choice" wire c to choose between a and b as the output. But, more generally, we could have a multiplexer with, e.g., 26 inputs, a through z, and multiple wires carrying a "choice" input. How many "choice" wires would you need to select from 26 inputs? Justify your answer.

[4] 12. Why We Use It

Assume we have 4 bits to represent a signed number, and we are using 2's-complement representation. Prove that the sum of any value v, and its negative value $-v$ in four-bit 2's-complement, equates to all-zero bits, as computed by a four-bit ripple-carry adder.

[2] 13. Fixed-Point Fun

Prove that you can represent $\frac{1}{8}$ in fixed-point base-16.

[4] 14. More Fixed-Point Fun

Prove that there are numbers representable in fixed-point base-6 that cannot be represented in fixed-point base-10.

[4] 15. The Fun Doesn't Stop

Prove that every number representable in fixed-point base-8 can also be represented in fixed-point base-16.