THE UNIVERSITY OF BRITISH COLUMBIA
CPSC 121: MIDTERM 2 – Individual – March 20, 2017

Important notes about this examination

1. **You have 75 minutes to complete this exam.**
2. You are allowed up to three textbooks and (the equivalent of) a 3" 3-ring binder of notes as references. Otherwise, no notes or aids are allowed. No electronic equipment is allowed.
3. Good luck!
4. There are a total of marks available.

Full Name:______________________

Signature:______________________

UBC Student #: |_|_|_|_|_|_|_|_|

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Student Conduct during Examinations

1. Each examination candidate must be prepared to produce, upon the request of the invigilator or examiner, his or her UBCcard for identification.
2. Examination candidates are not permitted to ask questions of the examiners or invigilators, except in cases of supposed errors or ambiguities in examination questions, illegible or missing material, or the like.
3. No examination candidate shall be permitted to enter the examination room after the expiration of one-half hour from the scheduled starting time, or to leave during the first half hour of the examination. Should the examination run forty-five (45) minutes or less, no examination candidate shall be permitted to enter the examination room once the examination has begun.
4. Examination candidates must conduct themselves honestly and in accordance with established rules for a given examination, which will be articulated by the examiner or invigilator prior to the examination commencing. Should dishonest behaviour be observed by the examiner(s) or invigilator(s), pleas of accident or forgetfulness shall not be received.
5. Examination candidates suspected of any of the following, or any other similar practices, may be immediately dismissed from the examination by the examiner/invigilator, and may be subject to disciplinary action:
   i. speaking or communicating with other examination candidates, unless otherwise authorized;
   ii. purposely exposing written papers to the view of other examination candidates or imaging devices;
   iii. purposely viewing the written papers of other examination candidates;
   iv. using or having visible at the place of writing any books, papers or other memory aid devices other than those authorized by the examiner(s); and,
   v. using or operating electronic devices including but not limited to telephones, calculators, computers, or similar devices other than those authorized by the examiner(s)—(electronic devices other than those authorized by the examiner(s) must be completely powered down if present at the place of writing).
6. Examination candidates must not destroy or damage any examination material, must hand in all examination papers, and must not take any examination material from the examination room without permission of the examiner or invigilator.
7. Notwithstanding the above, for any mode of examination that does not fall into the traditional, paper-based method, examination candidates shall adhere to any special rules for conduct as established and articulated by the examiner.
8. Examination candidates must follow any additional examination rules or directions communicated by the examiner(s) or invigilator(s).

1 Pred Logic to Proof Structure [6 marks]

Consider the following theorem:

$$(\forall x \in A, P(x,x)) \rightarrow (\exists a \in A, \forall b \in A, P(a,b) \rightarrow \exists c \in A, b \neq c \land P(b,c))$$

There is **not enough information** about A and P to prove this theorem. Instead, lay out as much of the structure of a direct proof of this theorem as you can. **Whenever you choose a specific value for a variable, specify what (if anything) this choice can depend on.**

Please list each distinct step as its own bullet point so it's easy for us to follow your proof structure! We've started you with the first bullet point and completed step and the second bullet point.

Proof:

- Assume $\forall x \in A, P(x,x)$.

-

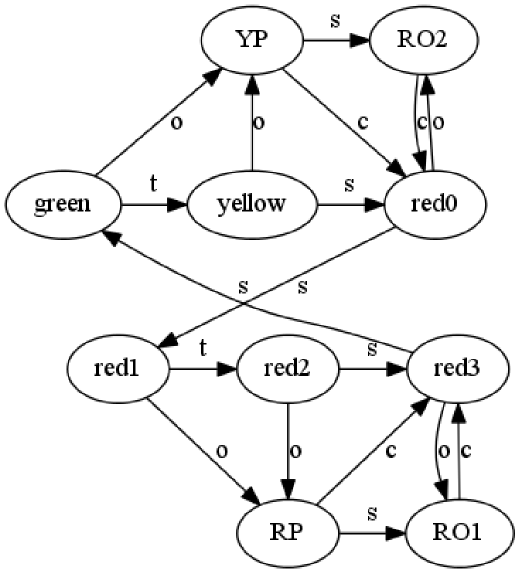
NOTE: you may need **much** less than the full page on this problem.

2 DFA to Circuit Layout [11 marks]

We now expand our traffic light DFA—as shown below—to handle an "ambulance override" that forces all directions to show red lights while an ambulance uses the intersection.

There are four new states: `RP` (red preparing for override, with the light red) and `RO1` (red override 1, with the light red) and `YP` (yellow preparing for override, with the light yellow) and `RO2` (red override 2, with the light red). There are two new inputs: override (`o`) starts the override and clear (`c`) ends it.

WARNING: To keep the diagram clean, we have omitted all "self-loops" from this DFA. That means there is no arrow in the DFA for any time an input letter would lead the DFA to transition **back to the same state**.



1. Imagine this DFA starts in the state `green` and processes the following input stream `tstssocs`. Indicate what state the DFA is in at the end of this input stream. **[2 marks]**

   REMEMBER that where an arrow is missing for an input at a particular state, that means the input causes the DFA to transition back to that state.

2. Briefly justify why this DFA will require four D Flip-Flops to translate it into a circuit. **[2 marks]**

3. Fill in the blanks in the following partial design of a circuit corresponding to this DFA. You are working **only** on truth table rows where the current state is `green` or `yellow` and the part of the circuit that updates the leftmost bit of the next state based on those two cases. We use the following representations for states and inputs:

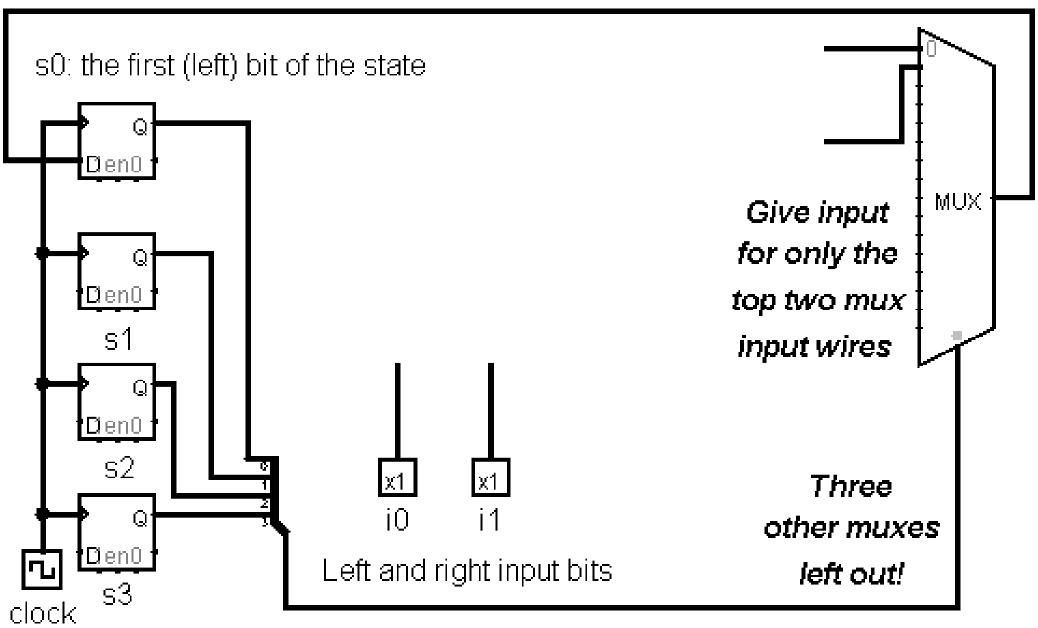
State:	`green`	`yellow`	`red0`	`red1`	`red2`	`red3`	`RP`	`RO1`	`YP`	`RO2`
Representation:	0	1	2	3	4	5	6	7	8	9

Input	`s`	`c`	`o`	`t`
Representation	0	1	2	3

Here is a partial truth table for this circuit. *Suggestion:* fill in the next state **names** first. **[4 marks]**

State	Input	State (as s0 s1 s2 s3)	Input (as i0 i1)	Next state (in binary)	Next state (name)
`green`	`s`	0 0 0 0	0 0		
`green`	`c`	0 0 0 0	0 1		
`green`	`o`	0 0 0 0	1 0		
`green`	`t`	0 0 0 0	1 1		
`yellow`	`s`	0 0 0 1	0 0		
`yellow`	`c`	0 0 0 1	0 1		
`yellow`	`o`	0 0 0 1	1 0		
`yellow`	`t`	0 0 0 1	1 1		

Here is a partial circuit. You should follow the instructions in the circuit to provide input for only the top two wires of the one multiplexer shown: **[3 marks]**



3 Proof Structure to Pred Logic [7 marks]

The following is a portion of a proof remaining on a ~~blood-stained~~ peanut butter-stained sheet of paper after a hungry wolf attack. (Happily, the proof is double-spaced in case you want to take notes.)

> Without loss of generality, let s be an arbitrary snack. Assume that s is not peanut butter and that it masses at least 0.1 kilograms. We choose animal a to be... *the choice of A is unreadable.*
>
> Note that a likes s, as required. We then show... *the rest of the proof is unreadable*

Write down as much as possible of the original predicate logic theorem that is being proven.

If you need them, use the sets A of animals (such as `wolf`) and S of snacks (such as `PB` for peanut butter), and that you have the predicate $L(x, y)$ meaning x likes y and the function $m(x)$ that gives the mass in kilograms of x (which can be an animal or snack).

NOTE: you may need **much** less than the full page on this problem.

4 Direct Proof [7 marks]

Consider the following function: $f(n) = a^n - \frac{b^n}{3}$.

Prove the following theorem directly:

> There is a negative real number b such that for all positive real numbers a and all natural numbers n, if $f(n)$ is an integer, then $f(n)$ is also the result of rounding a^n to the nearest integer.

Substantial partial credit is available for the structure of your proof. To finish the proof, here's a hint: you'll want $\frac{b^n}{3}$ to be less than $\frac{1}{2}$ and greater than $-\frac{1}{2}$. There's a **simple**, **integer** value for b that makes this work.

NOTE: you may need **much** less than the full page on this problem.

5 Proof by Contradiction [5 marks]

Prove the following theorem **using a proof by contradiction**.

(**NO** credit is available for other styles of proof besides contradiction. Substantial partial credit is available for the structure of the proof. Don't overthink this problem! Your proof should be about 2–3 lines long.)

Theorem: For any pair of integers x and y, $5x + 10y \neq 1$.

NOTE: you may need **much** less than the full page on this problem.

6 Induction [14 marks]

Prove by induction that $4^n + 5$ is divisible by 3 for all natural numbers $n \geq 0$. (That is, prove that $4^n + 5 = 3k$ or $4^n = 3k - 5$ for some integer k.)

(**NO** credit is available for other styles of proof besides induction. Substantial partial credit is available for the structure of the proof. Remember to clearly label the parts of your proof and to briefly justify any step you take that is not a basic arithmetic step. The marks in this problem are roughly divided into **4 marks** for the base case, **6 marks** for the structure of the inductive step/induction hypothesis, and **4 marks** for completing the proof.)

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If you write answers here, you must CLEARLY indicate on this page what question they belong with AND on the problem's page that you have answers here.