# CPSC 121, Summer 2016: Midterm 1

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This exam is open-book, and you may use any written or printed materials to assist you. You may not use any electronic devies, though (e.g., calculators, phones, tablets, etc.). Please do not take any copies of these questions from the examination room.

1. Twins in Disguise? (2 marks)
   Are the two functions equivalent? Please justify your answer with truth tables, and explain why they are equivalent or not equivalent with reference to the two truth tables you draw.
   - $\sim (a \vee b)$
   - $\sim a \vee b$

2. Always, Always, or Sometimes (3 marks)
   Is the following statement a tautology, a contradiction, or neither? Justify your answer using a truth table. Hint: you probably want to use a multi-part truth table.

   $$(a \leftrightarrow b) \rightarrow (a \wedge b) \vee \sim (a \vee b)$$

3. The Same or Different? (3 marks)
   Is $\sim (a \wedge b)$ equivalent to $\sim a \wedge \sim b$? Provide a real-life (concrete) example, giving definitions to $a$ and $b$, to demonstrate that they are the same or that they are different.

4. Because I Say So (2 marks)
   Write a truth table that takes two inputs, $a$ and $b$, and outputs a tautology. You can name your tautological function with any letter or symbol that you please, except $a$ or $b$ (to avoid confusion).

5. A Feline Conundrum (2 marks)
   Let $b$ be the proposition that I am a black cat, and let $w$ be the proposition that I am a white cat. Write, in propositional logic, the fact that I am neither a black nor a white cat, using *nothing other than* negations, conjunctions, XANDs, XNORs, and implications.

6. Dragons and Trolls (1 mark)
   Everyone in this world is a dragon or a troll. Dragons always tell the truth, and trolls always lie. Alice says that she is a dragon, and Bob says that he is a dragon. Is Alice telling the truth? Justify your answer.

7. Game of Logic (1 mark)
   If I live in Winterfell, then I live in the North. I live in the North. What do you know about whether I live in Winterfell?

8. Generalization (2 marks)
Let $x$ be a proposition meaning that you are writing an exam (which you are, so $x$ is true). Define a proposition $y$ with some meaning you see fit, and write a true propositional statement using *at most* the propositions $x$ and $y$, conjunctions, disjunctions, and XORs.

9. Prove Me Wrong (2 marks)
Let $P(x)$ be the predicate that $x^2 > 2x + 5$. Prove that:

$$\forall x \in \mathbb{Z}, P(x)$$

is a false statement (where $\mathbb{Z}$ means the integers).

10. Prove Me Correct (4 marks)
Let $D$ be the domain $\{4, 6, 9\}$. Let $Q(x)$ be the predicate that $x$ is composite (that is, not prime). Prove that:

$$\forall x \in D, Q(x)$$

is a true statement.

11. Feeding my Pig (5 marks)
Consider the following propositions:

- If I have a pig, I wake up each morning to feed my pig;
- If I wake up each morning to feed my pig, then I set my alarm clock to get me up;
- I do not set my alarm clock;
- I have a pig, or I have a jar of cookies;
- If I have a jar of cookies, then I used up all my raisins.

Did I use up all my raisins? Justify your answer by defining appropriate propositions (e.g., "p" means "I have a pig"), and drawing the conclusion either that I used up all my raisins or that I did not use up all my raisins, using inference rules (note: you may use a truth table to help answer the question, but all of the available marks come from using the formal inference rules).

12. Living on the Prairies (1 mark)
If I live in the city of Edmonton, then I live in the province of Alberta. If I live in the city of Regina, then I live in the province of Saskatchewan. Assume that I do not live in Saskatchewan. Is it true that I live in Edmonton, false that I live in Edmonton, or unknown whether I live in Edmonton? Justify your answer.

13. Puppies and Kittens (3 marks)
Everything in ICCS 187 is a puppy or a kitten. The things in the room are Alice, Bob, Carol, Dave, Felix, Garrison, Hamlet, and Juliet (eight things). Define an "assignment" as a possible labelling of each of them as "kitten" or "puppy" (for example, one assignment could be: "Alice is a kitten, Bob is a puppy, Carol is a kitten, Dave is a kitten, Felix is a kitten, Garrison is a puppy, Hamlet is a kitten, and Juliet is a puppy").

   (a) This part of the question is worth 1 of the 3 marks
   How many possible assignments of puppy and kitten are there to the eight things in the room? You do not have to simplify your exponents to get full marks.

(b) This part of the question is worth 2 of the 3 marks
Express the number of assignments in unsigned binary (i.e., in base-2, with an unsigned number). Additionally, re-express the binary number as a sum of powers of two (i.e., $a \cdot 2^0 + b \cdot 2^1 + \ldots$).

14. A Base for this Problem (2 marks)
Express the number $258_{10}$ in base-5 (hint: $5^0 = 1$, $5^1 = 5$, $5^2 = 25$, and $5^3 = 125$). It is recommended that you show your intermediate steps.

15. A Few Inputs (2 marks)
Provide a propositional statement that is equivalent to the following function $f$, expressed in this truth table. Hint: use a disjunction of multiple conjunctions / negations — you may be able to devise a more efficient or more clever solution, but there are no more marks awarded for doing so.

| $a$ | $b$ | $c$ | $d$ | $f(a,b,c,d) = ?$ |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 1 | 1 |
| 0 | 0 | 1 | 0 | 0 |
| 0 | 0 | 1 | 1 | 0 |
| 0 | 1 | 0 | 0 | 0 |
| 0 | 1 | 0 | 1 | 0 |
| 0 | 1 | 1 | 0 | 0 |
| 0 | 1 | 1 | 1 | 1 |
| 1 | 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 1 | 0 |
| 1 | 0 | 1 | 0 | 0 |
| 1 | 0 | 1 | 1 | 0 |
| 1 | 1 | 0 | 0 | 1 |
| 1 | 1 | 0 | 1 | 0 |
| 1 | 1 | 1 | 0 | 0 |
| 1 | 1 | 1 | 1 | 0 |

16. Not Feeling Negative About this Exam (2 marks)
Assume you have an eight-bit two's-complement (i.e., signed) binary number, 11111101. What number does that represent in base-ten signed numbers (i.e., the positive or negative numbers we use everyday)? It is recommended that you show your intermediate steps.

17. A Small Space (1 mark)
Assume you have two bits to store a two's-complement (i.e., signed) binary number. What is the largest (i.e., most positive) number you can store, expressed in base-10?

18. Of All the Questions in My Domain (1 mark)
Let $A(x)$ be a predicate over the domain of all people $x \in P$, where $P$ is the set of all people. Let $A(x)$ mean that $x$ likes running, enjoys walking on the beach, and thinks that pepperoni pizza is the best flavour of pizza. What is a potential codomain of $A$?

19. Forgetting the Symbol (4 marks)
Consider a circuit that takes two inputs (call them $a$ and $b$) and computes an output $a \oplus b$.

(a) This part of the question is worth 2 of the 4 marks
Draw a circuit that computes the output $a \oplus b$, without using the XOR circuit symbol. Hint: design the circuit using only conjunction, disjunction, and negation.

(b) This part of the question is worth 2 of the 4 marks
Write the propositional expression, using variables $a$ and $b$, that computes this output using only conjunction, disjunction, and negation.

20. Possibilities (2 marks)
If every dog who likes chasing cars has brown fur, must there be a dog who likes chasing cars? Justify your answer.

21. Different Sets (4 marks)
Construct a predicate $P$, such that $P$ has a different domain, codomain, image, and truth set. Tell us what your predicate, domain, codomain, image, and truth set are. Even if you do not get all four to be different, do not be afraid to put down a partial answer.

22. Ordering Qualifiers (3 marks)
Let $S$ be the set of all students, and $C$ be the set of all chairs in the classroom. In no more than two sentences, describe the difference between

$$\forall s \in S, \exists c \in C \text{ s.t. } O(s, c)$$

and

$$\forall c \in C, \exists s \in S \text{ s.t } O(s, c),$$

where $O(x, y)$ means that $x$ occupies $y$.

23. A Fixed Number (2 marks)
Let $W(x)$ be a predicate that means: $x$ wishes this test were over. In set-notation (i.e., utilizing the truth set of $W$), write a mathematical statement meaning that no less than 15, but no more than 35, students wish that the test were over, where $S$ is the domain of all students.

24. An Impossible Problem (1 bonus mark)
Give an example of a problem that a computer provably cannot solve. You can give the name of the problem in English or (should you recall one problem we discussed in class) in German. If you don't remember the name of the problem, you may describe the problem for full credit as well.