CPSC 121, Summer 2016: Midterm 2

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This exam is open-book, and you may use any written or printed materials to assist you. You may not use any electronic devies, though (e.g., calculators, phones, tablets, etc.). Please do not take any copies of these questions from the examination room.

1. How Many Moos? (2 marks)
   The number of cows in field A is prime. The number of cows in field B is prime. Must the number of cows, taken together (that is, the number of cows in field A plus the number of cows in field B) be even? Justify your answer in no more than two sentences.

2. Use Those Predicates (1 mark)
   Let $V(g)$ mean that g likes cheese, and let $R(a)$ mean that a likes running. Write, in predicate logic notation, that if u likes cheese (for some person u), then u likes running. Be sure to define a domain.

3. Puppies Are Silly (1 mark)
   Assume p is some puppy in the set of all puppies, P. Assume that p likes chasing its own tail. Let $C(x)$ mean that x likes chasing its own tail. Do we know that $\forall\ z \in P, C(z)$? Justify your answer in no more than two sentences.

4. What Becomes of This? (1 mark)
   Let $R = \emptyset$, $S = \emptyset$, and $L = \{cow, bunny, puppy\}$. What is $(R \cup S) \cup (L \backslash L)$?

5. Divide Me (1 mark)
   Create a partitioning of $\{bear, cow, puppy, kitten, cow\}$ into two subsets. Do not include any element more than once, in either or both of the two partitions.

6. The Truth of the Matter (2 marks)
   Determine if the two propositional statements are equivalent by drawing truth tables for each, and comparing the two truth tables, where m, a, and x are propositions (in your mind, give them any meaning that you feel appropriate, which you do not have to write down).

   - $\sim m \vee a \vee x \vee x$
   - a

7. Three's Company (2 marks)
   Assume n is an integer greater than zero, divisible by 3. Assume m is an integer greater than zero, divisible by 3. Assume c is a cow that likes grazing on grass. Assume p is a cute puppy. Prove that $m \cdot n$ is divisible by 9.

8. The Power of Everything (1 mark)
   In no more than two sentences, justify why $\emptyset \in \mathcal{P}(S)$ for any set S (where $\mathcal{P}$ means "the power set").

9. An Odd Issue, or Even Odds? (2 marks)
   Let $S = \langle s_i \rangle \; \forall i \in \mathbb{Z} \geq 1$ be a sequence. Let:

$$\begin{aligned} s_1 &= 13 \\ s_2 &= 3 \\ s_3 &= 2 \cdot s_2 + s_1 \\ s_4 &= 5 \\ s_n &= 5 \cdot s_{n-1} + 2 \; \forall n \geq 5 \end{aligned}$$

   Is s_n odd $\forall n \in \mathbb{Z} \geq 1$? Prove with induction, or disprove with a counterexample.

10. Grilling up Some Burgers (6 marks)
    Being from Alberta, Ryan likes to grill some burgers during the summer. The problem is, he can only buy beef in packs of 3 kg, 6 kg, or 7 kg. What is the largest amount of beef that he cannot buy, by combining different packs of beef? (E.g., he *can* purchase 23 kg of beef, by purchasing two 7 kg packages, a 6 kg package, and a 3 kg package).

11. Circles of Numbers (1 mark)
Is $\mathbb{Q} \subseteq \mathbb{R}$, $\mathbb{Q} \subsetneq \mathbb{R}$, neither, or both? Justify your answer in no more than two sentences.

12. What Comes Next? (1 mark)
Let S be a sequence, with $S = \langle s_i \rangle = \langle 1, 4, 9, 16, 25, \ldots \rangle \; \forall i \in \mathbb{Z} \geq 1$. What is the next element, s_6?

13. An Inductive Proof (6 marks)
Prove, $\forall n \in \mathbb{Z} \geq 1$, $\sum_{i=1}^{n} 2^i = 2^{n+1} - 2$.

14. Modular Thinking (2 marks)
Assume $r \in \mathbb{Z}$. Also assume $r \equiv 5 \bmod 9$. What is the smallest non-negative number to which $3r + 2 \bmod 3$ is equivalent? You can show your intermediate steps for partial marks if you are incorrect, or show the final correct answer for full marks.

15. The Powerful NAND (6 marks)
    We know that the combination of conjunction, disjunction, and negation can be used to express any truth table. That is, these three operations, taken together, are universal. We also learned, from an earlier assignment, that NORs are universal (using only NORs, you can express any truth table).

    As a matter of fact, so too are two-input NANDs (we can express any truth table using just two-input NANDs).

    The way to prove this fact would be either (a) to use only NANDs to create conjunction, disjunction, and negation; or (b) to use only NANDs to create a NOR. Choose either approach, and show how it could be done.

    You can use either a circuit diagram or propositional logic notation.