CPSC 121 Midterm 2
Wednesday 14 June 2017

Name: _____ Student ID: _____

Signature: _____

- You have 60 minutes (individual), 30 minutes (group) to write the 13 questions on this examination. A total of 45 marks are available.
- **Justify all of your answers.**
- You are allowed to bring in any written materials, and a non-programmable non-graphing calculator.
- Keep your answers short. If you run out of space for a question, you have written too much.
- The number in square brackets to the left of the question number indicates the number of marks allocated for that question. Use these to help you determine how much time you should spend on each question.
- Use the back of the pages for your rough work.
- **Good luck!**

Question	Marks
1	
2	
3	
4	
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7	
8	
9	
10	
11	
12	
13	
Total	

UNIVERSITY REGULATIONS:

- Each candidate should be prepared to produce, upon request, his/her UBC card.
- No candidate shall be permitted to enter the examination room after the expiration of one half hour, or to leave during the first half hour of the examination.
- CAUTION: candidates guilty of any of the following, or similar, dishonest practices shall be immediately dismissed from the examination and shall be liable to disciplinary action.
  1. Having at the place of writing, or making use of, any books, papers or memoranda, electronic equipment, or other memory aid or communication devices, other than those authorised by the examiners.
  2. Speaking or communicating with other candidates.
  3. Purposely exposing written papers to the view of other candidates. The plea of accident or forgetfulness shall not be received.

– Candidates must not destroy or mutilate any examination material; must hand in all examination papers; and must not take any examination material from the examination room without permission of the invigilator.

[4] 1. Oh, a Question
Prove $5n^5 + 3n^2 \in \mathcal{O}(n^5)$.

[4] 2. Oh No, a Question
Prove $3n^2 \notin \mathcal{O}(n)$.

[1] 3. How Many Moos?
The number of cows in field A is prime. The number of cows in field B is prime. Must the number of cows, taken together (that is, the number of cows in field A plus the number of cows in field B) be even? Justify your answer in no more than three sentences.

[1] 4. What Becomes of This?
Let $R = \emptyset$, $S = \emptyset$, and $L = \{cow, bunny, puppy\}$. What is $(R \cup S) \cup (L \backslash L)$?

[6] 5. Predicates Abound

Consider predicates $F(x)$, $G(x)$, and $H(x, y)$, on domain D (you can think of these three predicates and the domain in the abstract — they can have any meaning that you please). We know the following three statements are true:

- 1. $\exists a \in D$ s.t. $F(a)$
- 2. $\forall b \in D, \sim G(b) \vee H(b, b)$
- 3. $\forall c \in D, \forall d \in D, F(c) \rightarrow G(d) \wedge H(c, d)$

Using these known facts, prove $\exists e \in D$ s.t. $H(e, e)$. Explicitly state the rule used in each step of your proof, referring to previous line numbers in your proof as "inputs" to the rule.

[12] 6. A Little Inductive Fun, With Lemmas

In this question, you will prove a statement in three stages. You are free to invoke the conclusion of an earlier part in a later part, even if you did not successfully prove the earlier part. That is, even if you did not solve Part "a", you are free to use the statement you are trying to prove in Part "a" in your answer to Part "b" or Part "c".

[4] a. Prove, via induction, that $2^n \geq 2$ for all $n \in \mathbb{Z}$ where $n \geq 1$.

[4] b. Next, prove, via induction, that $2^n \geq 2n + 1$ for all $n \in \mathbb{Z}$ where $n \geq 3$.

[4] c. Finally, prove, via induction, that $2^n \geq n^2$ for all $n \in \mathbb{Z}$ where $n \geq 4$.

[3] 7. Think Backwards About This One

Prove that, for all $n \in \mathbb{Z}$, if $n^2 + 4n - 3$ is even, then n is odd.

[1] 8. Puppies Are Silly

Assume p is some puppy in the set of all puppies, P. Assume that p likes chasing its own tail. Let $C(x)$ mean that x likes chasing its own tail. Do we know that $\forall\ z \in P, C(z)$? Justify your answer in no more than three sentences.

[1] 9. Powerful Power Sets

What is $\mathcal{P}\big(\{\emptyset\}\big)$?

[2] 10. What a Nice Compliment

Assume you have an 8-bit signed (i.e., 2's-complement) number. The number stored is 0xE9. What value, in base-10, does this number represent.

[4] 11. A Bijective Function

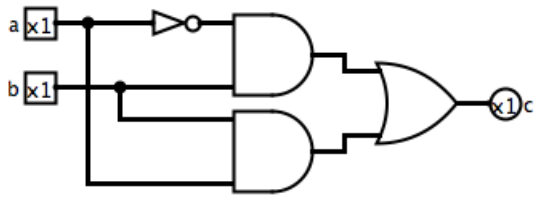
Let $f : \mathbb{R} \to \mathbb{R}$ be defined as $f(x) = x^5 + 8$. Prove that f is bijective.

[2] 12. A Tricky Conversion

Convert 320_7 into base-12.

[4] 13. A Subtle Glitch

Consider the following circuit. Denote the inputs to the circuit as a and b, and the output of the circuit as c.



[2] a. The circuit has a glitch in it, such that its output is not stable. That is, there exist inputs a_1 and b_1 that produce output c', and there exists other inputs a_2 and b_2 that also produce output c'.

However, each gate takes time to change its output after its input changes. Because of this delay, when you change the inputs to the circuit from a_1 and b_1, to a_2 and b_2, the circuit may briefly output the opposite of c'.

Identify inputs a_1, b_1, a_2, and b_2 that trigger this glitch.

[2] b. Assume each gate takes 5 ns to operate (that is, to change its output when its input changes). For what period of time will the circuit be outputting the incorrect output? Justify your answer by describing the timeline.

(Denote the moment when inputs a_1 and b_1 change to a_2 and b_2 as $t = 0$ ns.)

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