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HOMEWORK ROUTE FORM

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Check One: ◉ Homework #: 5 ○ Midterm ○ Other

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Last modified October 27, 2008

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1) 9.2) Signal 150mV$_{rms}$ in series with noise voltage

a) SNR for 1mV$_{rms}$

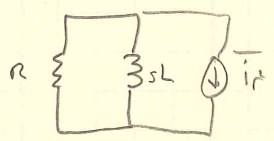
$$SNR = \frac{\overline{V_s^2}}{\overline{V_n^2}} = \frac{150^2}{1^2} = 22500 = \boxed{43.5\,dB}$$

b) 70dB, how small must noise be?

$$70\,dB = 20\log_{10}\left(\frac{V_{sig}}{V_{noise}}\right) \quad 10^{\frac{SNR}{20}} = \frac{V_{sig}}{V_{noise}} \quad V_{noise} = \frac{V_{sig}}{10^{SNR/20}} =$$

$\boxed{47.4\,\mu V_{rms}}$

2) 9.19)



$R \parallel L \Rightarrow Z = \frac{R \cdot sL}{R + sL}$ $\quad i_{no}(t) =$ current in inductor

Current divider $\quad i_L = \frac{R}{sL + R} \cdot i_T$

$$i_L = \frac{1}{1 + s\,L/R} \cdot i_t \Rightarrow f_o = \frac{1}{2\pi\,L/R}$$

$$\overline{I_n^2} = \frac{4kT\,f}{R}$$

ENBW

$$f_o = \frac{R}{2\pi L} \quad f_x = \frac{\pi}{2} f_o = \frac{R}{4L}$$

$$\overline{I_R^2} = \frac{4kT}{\cancel{R}} \cdot \frac{\cancel{R}}{4L} = \frac{kT}{2}$$

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3) $\gamma = 0.85$ $T = 300K$

$I_d = 571\mu A$ $g_m = 4.017mS$ $g_o = 123\mu S \Rightarrow r_o = 8.13k$

a) Calculate thermal noise

$R_L = 1K$ $R_S = 10k$ $R_L \| r_o = 890\Omega$

$A_{v_0} = g_m \cdot (R_L \| r_o) = 3.57$

$\overline{V^2_{T_{oR_L}}} = 4kTR_L = 16.56\, nV^2/Hz \rightarrow$ output

$\overline{V^2_{T_{iR_S}}} = 4kTR_S = 165.6\, nV^2/Hz \rightarrow$ input

$\overline{I^2_{d_0}} = 4kT\gamma g_m = 56.54\, pA^2/Hz \rightarrow$ output

$\overline{V^2_{d_0}} = \overline{I_d^2} \cdot (R_L \| r_o)^2 = 44.8\, nV^2/Hz \rightarrow$ output

Input referred

$\overline{V^2_{T_{i_{R_L}}}} = \overline{V^2_{T_{oR_L}}} / A_{v_0}^2 = 1.3\, nV^2/Hz$

$\overline{V_{Di}^2} = 3.51\, nV^2/Hz$

Total input referred noise

$\overline{V^2_{Ni_T}} = \overline{V^2_{t_{iR_L}}} + \overline{V^2_{t_{iR_S}}} + \overline{V^2_{di}} = 170.4\, nV^2/Hz$ half circuit

differential circuit $\overline{V^2_{Ni_T}} = 340.8\, nV^2/Hz = 18.46\, nV/\sqrt{Hz}$

b) Midband noise $= 339.2\, nV^2/Hz \rightarrow$ extremely good estimate
0.47% error at 10MHz

Low freq error due to flicker noise on gates

High Freq error due to capacitances in mosfets and load

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3)

c) 100 Hz - 100 MHz at output

$340.8\ nV^2/Hz \cdot A_{v_o}^2 = 4.343\ \mu V^2/Hz = \overline{V_{T_o}^2}$

$\overline{V_{T_o}^2} \cdot \Delta f = 434\mu V^2 = 659\ \mu V rms$

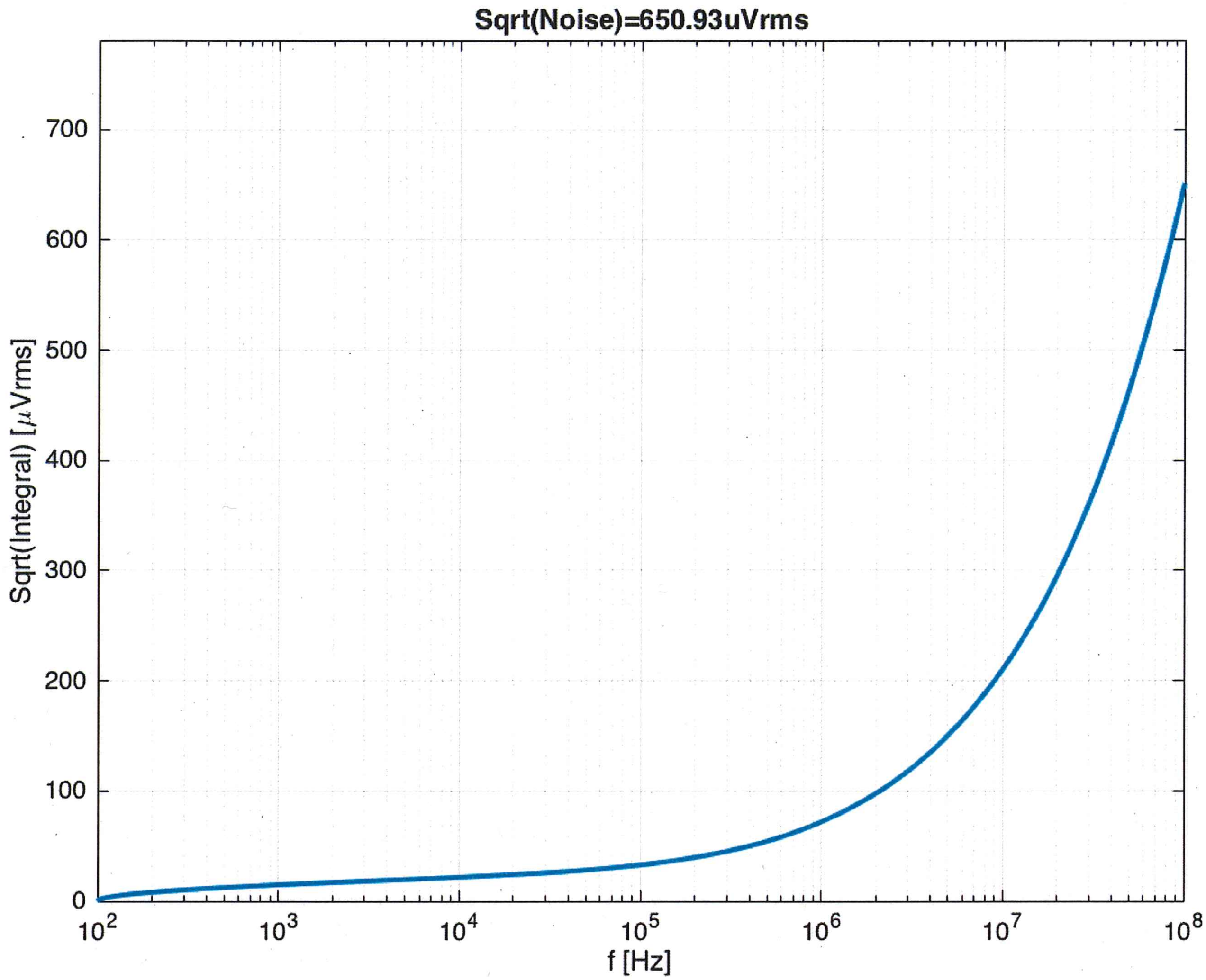
d) Calculated noise $650.9\ \mu V rms$ error: 1.24%

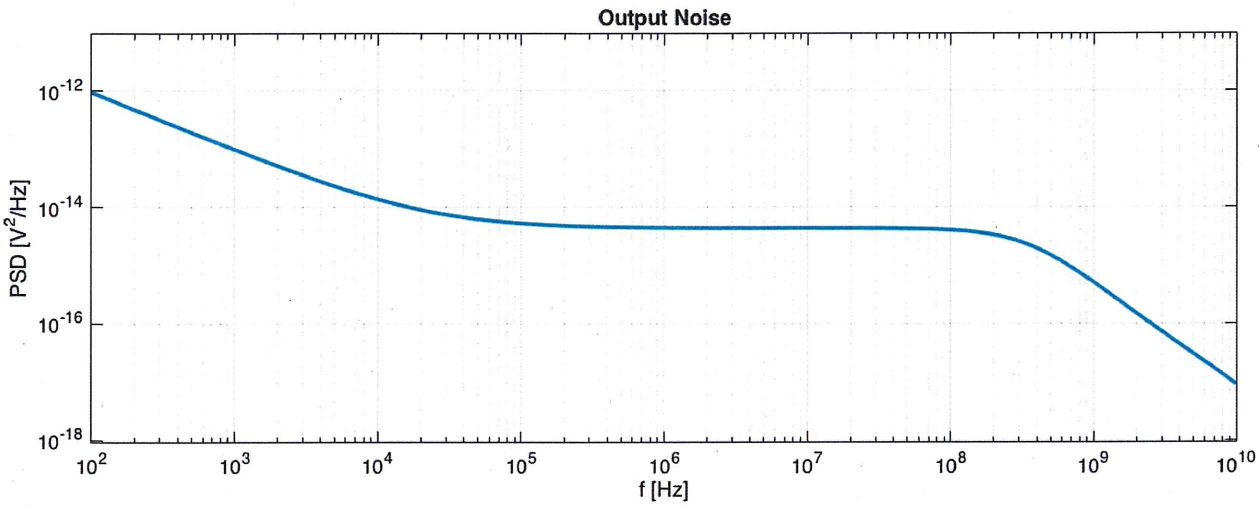
Plot attached

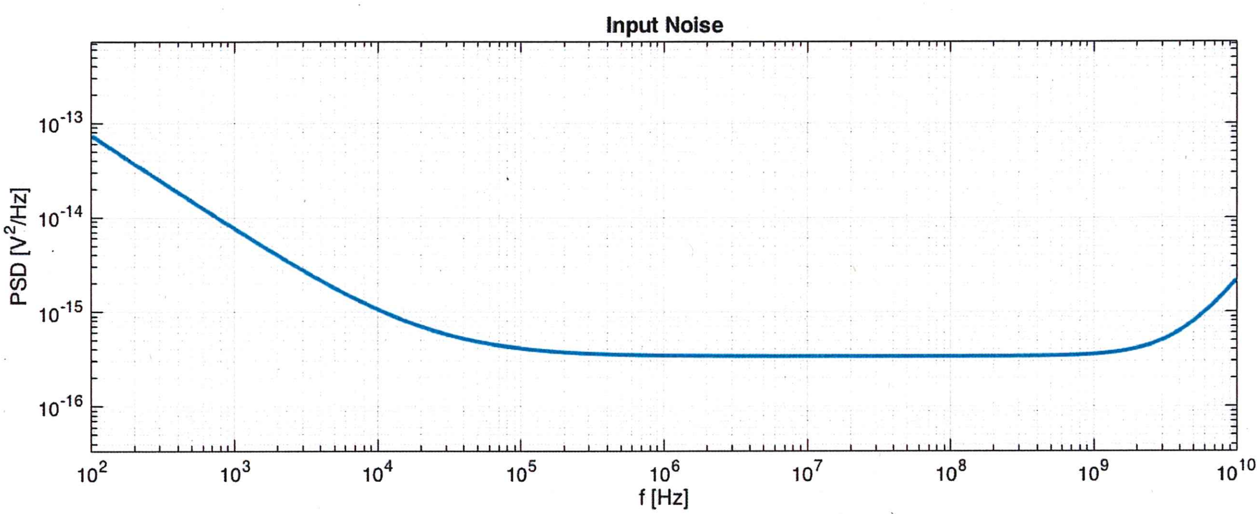
flicker noise is completely insignificant

e) Signal power $= \sqrt{2}\ \overline{V_A} = 14.14\ mVrms$

$SNR = 20 \log_{10}\left(\frac{\text{RMS signal}}{\text{RMS noise}}\right) = 26.7 dB$


Sqrt(Noise)=650.93uVrms
Sqrt(Integral) [μVrms]
0
100
200
300
400
500
600
700
10^2
10^3
10^4
10^5
10^6
10^7
10^8
f [Hz]


Output Noise
PSD [V^2/Hz]
10^-12
10^-14
10^-16
10^-18
10^2
10^3
10^4
10^5
10^6
10^7
10^8
10^9
10^10
f [Hz]


Input Noise
PSD [V^2/Hz]
10^-13
10^-14
10^-15
10^-16
10^2
10^3
10^4
10^5
10^6
10^7
10^8
10^9
10^10
f [Hz]

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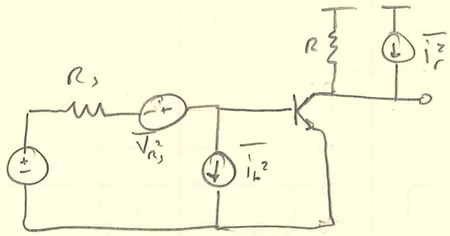
4) a)



$$\frac{\overline{V_s^2}}{\Delta f} = 4kT\left(R_s + \frac{g_m R_s^2}{2\beta} + \frac{R}{g_m^2 R^2} + \frac{1}{2g_m}\right)$$

Collector current that minimizes noise

$$g_m = \frac{I_c}{V_t} \qquad V_t = \frac{kT}{q} \Rightarrow g_m = \frac{I_c q}{kT}$$

$$4kTR_s + \frac{4kT\, I_c R_s^2}{V_t \cdot 2\beta} + \frac{4kTR}{\left(\frac{I_c}{V_t}R\right)^2} + \frac{4kT}{2\frac{I_c}{V_t}}$$

$$4kTR_s + \frac{2q I_c R_s^2}{\beta} + \frac{4kTR}{\frac{I_c^2 q^2}{k^2T^2}} + \frac{2kT}{\frac{I_c q}{kT}}$$

$$4kTR_s + I_c\left(\frac{2q R_s^2}{\beta}\right) + \frac{1}{I^2}\left(\frac{4k^3T^3R}{q^2}\right) + \frac{1}{I}\left(\frac{2k^2T^2}{q}\right)$$

$$a = 4kTR_s \qquad b = \frac{2qR_s^2}{\beta} \qquad c = 4V_T^2 kTR \qquad d = 2V_t kT$$

$$\frac{\overline{V_s^2}}{\Delta f} = a + I_c \cdot b + I_c^{-2} \cdot c + I_c^{-1} \cdot d$$

$$\frac{\partial}{\partial I_c}\frac{\overline{V_s^2}}{\Delta f} = b - 2I_c^{-3} \cdot c - I_c^{-2} \cdot d$$

Minimum noise will be at a root of this equation

$$0 = \frac{2qRs^2}{\beta} - \frac{8V_t^2 kTR}{I_c^3} - \frac{2V_t kT}{I_c^2}$$

$$\frac{2qR_s^2}{\beta} = \frac{8V_t^2 kTR}{I^3} - \frac{2V_t kT}{I_c^2}$$

4) a) cont'd

$$\frac{I_c^3 \, 8 \, R_s^2}{\beta} = 4 V_t^2 kTR - I_c V_t kT$$

$$\frac{I_c^3 \, 8 \, R_s^2}{\beta} + I_c V_t kT = 4 V_t^2 kTR$$

$$\frac{I_c^3 R_s^2}{\beta} + I_c V_t^2 = 4 V_t^3 R$$

$$I_c^3 R_s^2 + I_c V_t^2 \beta = 4 V_t^3 R \beta$$

neglecting R

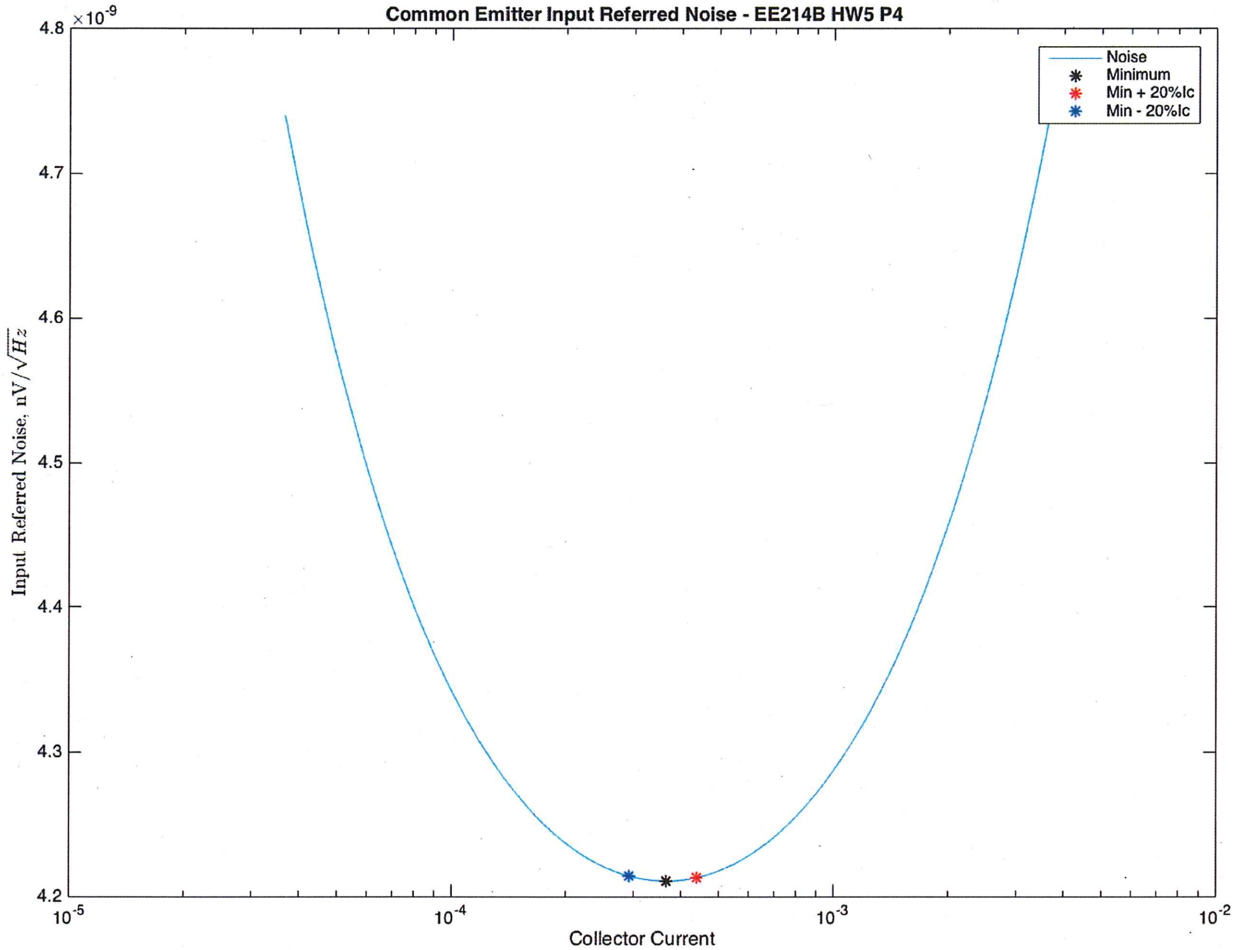
$$I_c^3 R_s^2 + I_c V_t^2 \beta = 0$$

$R_s = 1K \quad \beta = 200 \Rightarrow I_c = 0$?

b) plot attached

Error I_c $+20\%$ -0.055%

Error I_c -20% -0.082%


Common Emitter Input Referred Noise - EE214B HW5 P4
Noise
Minimum
Min + 20%Ic
Min - 20%Ic
Input Referred Noise, $nV/\sqrt{Hz}$
Collector Current
$\times 10^{-9}$
4.8
4.7
4.6
4.5
4.4
4.3
4.2
10^{-5}
10^{-4}
10^{-3}
10^{-2}

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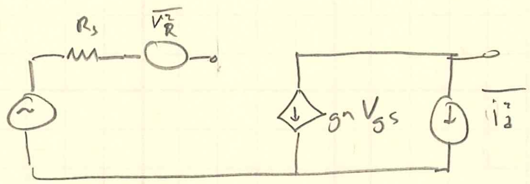
5) CE & CS Stages

Noise due to transistor and R_s

a) Input source referred voltage noise

$$\frac{\overline{V_s^2}}{\Delta f} = 4kT\left(R_s + r_b + g_m\frac{(R_s+r_b)^2}{2\beta} + \frac{1}{2g_m}\right)$$

b)



$$\frac{\overline{V_R^2}}{\Delta f} = 4kTR_s \qquad \frac{\overline{i_d^2}}{\Delta f} = 4kT\gamma g_m$$

$$\frac{\overline{V_s^2}}{\Delta f} = \frac{\overline{V_R^2}}{\Delta f} + \frac{\overline{V_{di}^2}}{\Delta f} \qquad \frac{\overline{V_d^2}}{\Delta f} = \frac{\overline{i_d^2}}{\Delta f}\cdot\frac{1}{g_m^2}$$

$$\frac{\overline{V_s^2}}{\Delta f} = 4kTR_s + \frac{4kT\gamma}{g_m} = \boxed{4kT\left(R_s + \frac{\gamma}{g_m}\right)}$$

c) $R_s = 10-100k$ $\quad I_c = 1mA$ $\quad r_b = 0$ $\quad \beta = 300$ $\quad T = 300$

$I_d = 1mA$ $\quad g_m/I_d = 13$ $\quad \gamma = .85$

find value of R_s for BJT > MOSFET noise.

BJT inferior to MOSFET at $\boxed{R_s = 813\Omega}$

d) New value: $\boxed{R_s = 511\Omega}$

e) $$R_s + r_b + \frac{g_m(R_s+r_b)^2}{2\beta} + \frac{1}{2g_m} = R_s + \frac{\gamma}{g_m}$$

$R_s \to 0$

$$r_b + g_{m_B}/2\beta \cdot r_b^2 + \frac{1}{2g_{m_B}} = \frac{\gamma}{g_{m_{mos}}}$$

$$r_b^2\frac{g_{m_B}}{2\beta} + r_b + \left(\frac{1}{2g_{m_B}} - \frac{\gamma}{g_m}\right) = 0$$

$a = \frac{g_{m_B}}{2\beta}$ $\quad b = 1$ $\quad c = \frac{1}{2g_{m_B}} - \frac{\gamma}{g_{m_m}}$

$$\boxed{r_b = 43.6\Omega}$$

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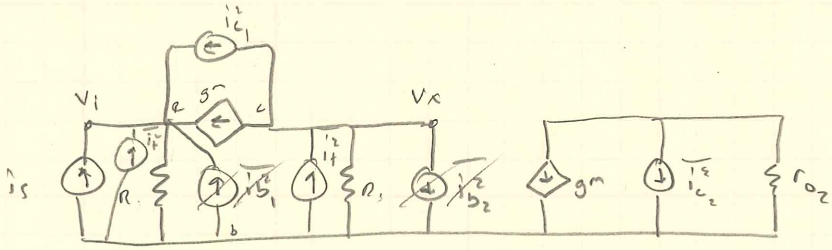
6) $I_{C_1} = I_{C_2} = 1mA$ $\beta = \infty \Rightarrow r_\pi = \infty$

$r_{b_1} = r_{b_2} = 0$ $R_S = 5k = R$



$g_m = \frac{I_c}{V_t} \approx 38mS$ as $\beta \to \infty$ $i_b \to 0$ hence $\overline{i_b^2} \to 0$

$1/g_m \Rightarrow 26\Omega$

$\overline{i^2_{t_R}} = \frac{4kT}{R}$ $\overline{i_c^2} = 2q I_c$ $\overline{i_b^2} = 0$ output gain $= g_m r_{o_2} = \frac{g_m V_A}{I_c}$

M_1 input impedance $= \frac{1}{g_m}$ M_1 output impedance $= r_o \Rightarrow \infty$

$\frac{V_1}{R} = i_s + g_m V_1$

V_x referred $\overline{i_c^2}$ noise $= \frac{\overline{i^2_{c_1}}}{g_m^2}$

First stage gain A_{v_0} $A_{I_0} = 1$

$V_x = R \cdot I_{c_1}$ $I_{c_1} \to$ current divider $I_c = \frac{R}{\frac{1}{g_m} + R} I_s$

$V_x = \frac{R^2}{\frac{1}{g_m} + R} I_s \cong R I_s$

i_s referred v_x noise

$\overline{i^2_{x_i}} = \frac{\overline{V_x^2}}{R^2}$

$\overline{i^2_{r_{T,i}}} = 3.312 pA^2/Hz$ $\overline{i^2_{r_{T_{2i}}}} = \frac{\overline{i^2_{r_{T_2}}}}{R^2} \cong 0$

$\overline{i^2_{c_1}} = 320 pA^2/Hz$

$\overline{i^2_{c_{2i}}} = \frac{\overline{i^2_{c_2}}}{g_m^2 R^2} = 8.87 aA^2/Hz \cong 0$

total input referred noise $= \sqrt{\overline{i^2_{c_1}} + \overline{i^2_{r_1}}} = 13 pA/\sqrt{Hz}$

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6 b) SNR = 20db   BW = 150MHz

noise power = 220 nA$_{rms}$

$$20dB = 20 \log\left(\frac{\text{rms signal}}{220nA_{rms}}\right) \Rightarrow 10 = \frac{S_{is}}{220nA_{rms}}$$

signal power = 2200 nA$_{rms}$