Samuel Lenius EE313 FM HW 4

1) a) F

b) T

c) T

2) a) ① $Q_{in2,CT}(t) = \int_{t-D\cdot T_s}^{t} I_{out1d}(t')\,dt'$

$D =$ Duty Cycle $\quad T_s = 1/f_s \rightarrow$ (SC clock period)

$I_{out1d} =$ Differential output current from A_1

$Q_{in2CT} =$ Running windowed integral of I_{out1d}

I_{out1d} convolved with rectangle height 1, width $D \cdot T_s$

rectangle $\rightarrow$ $\updownarrow 1$

$\leftarrow DT_s \rightarrow$

$rect(x) = h \cdot \Pi\left(\frac{x-c}{b}\right)$ $\quad c = $ center $= \frac{D\cdot T_s}{2}$ $\quad b =$ width $= D \cdot T_s$

$h =$ height $= 1$ $\quad x = t - t'$

$$Q_{in2}CT(t) = \int_{-\infty}^{\infty} \Pi\left(\frac{t - \frac{D\cdot T_s}{2} - t'}{D\cdot T_s}\right)\cdot I_{out1d}(t')\,dt'$$

$$\int_{-\infty}^{\infty} x(t')\,dt' = x(t)$$

$$Q_{in_2CT}(t) = I_{out1d}(t)\cdot \Pi\left(\frac{t - \frac{DT_s}{2}}{DT_s}\right)$$

$$|Q_{in_2CT}(\omega)| = \left|\mathcal{F}\left(I_{out1d}(t)\cdot \Pi\left(\frac{t - DT_s/2}{DT_s}\right)\right)\right|$$

$$= |I_{out1d}(\omega)|\cdot\left|\int_{-\infty}^{\infty} e^{-j\omega t}\cdot \Pi\left(\frac{t - \frac{DT_s}{2}}{DT_s}\right)dt\right|$$

$$= |I_{out1d}(\omega)|\cdot\left|\int_{-DT_s}^{DT_s} e^{-j\omega t}\,dt\right| \Rightarrow \frac{-1}{j\omega}\left(e^{-j\omega DT_s} - e^{j\omega DT_s}\right).$$

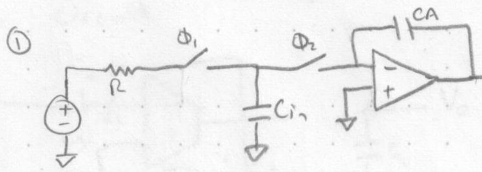
$$\sin(\omega) = \frac{1}{2j}\left(e^{j\omega} - e^{-j\omega}\right)$$

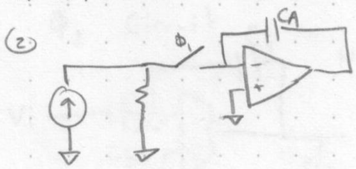
$$= \left|\frac{1}{j\omega}\left(e^{j\omega DT_s} - e^{-j\omega DT_s}\right)\right| = \left|\frac{2\cdot\sin(\omega DT_s)}{\omega}\right|\cdot|I_{out1d}(\omega)|$$

② $$\frac{|Q_{in2d}(\omega)|}{|I_{out1d}(\omega)|} = \left|\frac{2\cdot\sin(\omega DT_s)}{\omega}\right|$$

2) b) WEB of sinc prefilter 3.4× smaller than voltage sampling designed for 0.1% settling in DT_s

error @ $t = n\tau$ = 0.1% $\qquad n = -\ln(0.1\%) = 6.90$

① 

② 

$$ENBW_1 = \frac{kT}{C} \cdot \frac{1}{4kTR} = \frac{1}{4RC}$$

$$DT_s = n \cdot \tau = n \cdot R \cdot C \qquad RC = \frac{DT_s}{n}$$

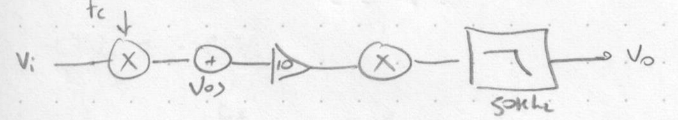
$$ENBW_1 = \frac{n}{4DT_s} =$$

② Boxcar sampler - $ENBW_2 = \frac{f_s}{2} = \frac{1}{2DT_s}$

$$\frac{ENBW_1}{ENBW_2} = \frac{\frac{n}{4DT_s}}{\frac{1}{2DT_s}} = \frac{n}{2} = \boxed{3.45}$$

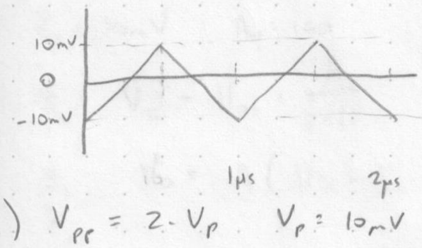
3) $A = 10 \quad BW = \infty \quad Z_{in} = \infty \quad Z_{out} = 0 \quad V_{os} = 20mV \quad f_c = 1MHz \quad f_{LPF} = 50kHz$

$f_c = 20 \cdot f_{LPF}$ on period of $\frac{1}{2}\tau_c$, step response looks like triangle wave



a) $V_i = 0$, plot V_o $\qquad V_{os} \cdot 10 = 200mV \quad$ @ 1MHz, 1 pole - gain $= \frac{1}{\sqrt{1 + \frac{1MHz}{50kHz}}} = 0.05$

$V_p = V_{os} \cdot 10 \cdot 0.05 = 0.01V$

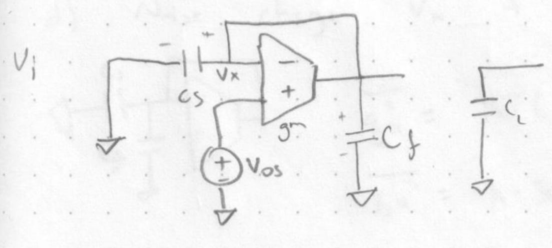


b) $V_{pp} = 2 \cdot V_p \quad V_p = 10mV \quad V_{pp} = 20mV$

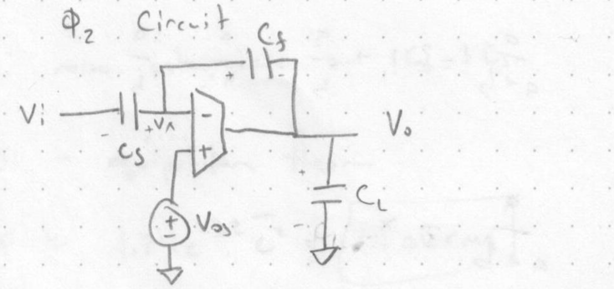
4) $f_{clk} = 10MHz$ $T_{\phi_1} = T_{\phi_2} = \frac{T_{clk}}{2}$ $C_s = C_L = 2pF$ $C_f = 1pF$ $G_m = 1mS$ $\gamma = 1$ $\alpha = 3$

a) Voltage gain from V_i to V_o

ϕ_1 circuit



Φ_2 Circuit



$Q = C \cdot V_c$ $\qquad$ $Q_L = C_L \cdot V_o$

$V_x \to$ Virtual gnd $\cong V_{os}$

ϕ_1, $V_{CS} = V_{Cf} = V_{os}$ $\quad Q_x = 0$

Φ_2, $V_{CS} = V_x - V_i$ $\quad V_{Cf} = V_x - V_o$ $\quad Q_x = 0$

$Q_x = 0 = C_s(V_x - V_i) + C_f(V_x - V_o) = C_sV_x - C_sV_i + C_fV_x - C_fV_o$

$C_fV_o = C_sV_x - C_sV_i + C_fV_x = V_x(C_s + C_f) - C_sV_i$

$V_x = V_{os} \sim 0$

$C_fV_o = -C_sV_i$

$\frac{V_o}{V_i} = -\frac{C_s}{C_f} = \frac{-2pF}{1pF} = -2$

b) $V_{os} = 10mV$ $\quad A_v = 100$

Φ_1 $V_c = V_{os} \cdot \frac{A}{1+A}$

Φ_2 $V_o = A(V_{in} - V_c + V_{os}) = A\left(V_{in} - V_{os}\frac{A}{1+A} + V_{os}\right)$

$V_o = Av_{in} + AV_{os}\left(1 - \frac{A}{1+A}\right) \Rightarrow A\left(1 - \frac{A}{1+A}\right) = A - \frac{A^2}{1+A} = 0.99$

$V_o = Av_{in} + 0.99 \cdot V_{os} - A\beta V_o$ $\qquad \beta = \frac{C_f}{C_f + C_s} = \frac{1}{3}$

$V_o + A\beta V_o = Av_{in} + 0.99V_{os}$

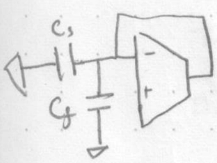
$V_o(1 + A\beta) = Av_{in} + 0.99V_{os}$ $\qquad V_o = \frac{Av_{in} + .99V_{os}}{1 + A\beta}$ $\qquad V_i = 0$, $V_o = \frac{V_{os} \cdot 0.99}{1 + A\beta}$

Input referred noise $\sim \frac{V_{os}}{A}$

4) c) V_{os} - slow drift 1Hz sinusoid A = 1mV

For $A \to \infty$, input referred amplitude = 1mV

d) Noise charge V_x → thermal noise integrated on C_f & C_s



$$\overline{q_x^2} = kT(C_s + C_f) \quad \text{- equipartition theorem}$$

$$\overline{q_x^2} = k \cdot 300K \cdot 3pF = 1.24e^{-32}\,\overline{C^2} = \boxed{111\ aCrms}$$

e) $$N_{A_0} = \frac{\alpha 4kT\gamma \Delta f}{G_m} \cdot A_{CL}^2 \qquad A_{CL} = \frac{A}{1+A\beta} \Rightarrow \frac{1}{\beta} \qquad \beta = \frac{C_f}{C_s+C_f} = \frac{1}{3}$$

$$K = -\frac{C_s}{C_s+C_f} \qquad A_{CL} = \frac{K}{\beta} = -\frac{C_f}{C_s}$$

$$N_A = \frac{\alpha 4kT\gamma\Delta f}{G_m} \cdot \left(-\frac{C_f}{C_s}\right)^2$$

$$R_o = \frac{1}{\beta G_m} \quad \overline{i_o^2} = \alpha \cdot 4kT\gamma G_m \Delta f$$

$$\frac{\overline{V_o^2}}{\Delta f} = \frac{\alpha 4kT\gamma}{\beta R} \cdot \left(\frac{R}{1+j\omega C_L}\right)^2$$

$$\overline{V_o^2} = \int_0^\infty \frac{\alpha 4kT\gamma}{\beta R}\left(\frac{R}{1+j\omega C_L}\right)^2 \Delta f\, df = \frac{\alpha kT\gamma}{\beta R} \cdot \int_0^\infty \frac{1}{R}\left(\frac{R}{1+j\omega C_L}\right)^2 \Delta f\, df$$

$$\overline{V_o^2} = \frac{\alpha kT\gamma}{\beta C_{Lt}} \qquad C_{Lt} = C_L \| C_f = C_L + C_f = 2pF + 1pF = 3pF$$

$$\overline{V_o^2} = \frac{1 \cdot k \cdot 300K \cdot 3}{\frac{1}{3} \cdot 3pF} = 12.42\,mV^2 = \boxed{111.5\ \mu Vrms}$$

f) $$\overline{V_{o_1}^2} = \frac{\overline{q^2}}{C_f^2} = \frac{1.24e^{-32}\,\overline{C^2}}{1pF^2} = 12.42\,mV^2 = 111.5\,\mu Vrms$$

$$\overline{V_{o_{tot}}^2} = \overline{V_{o_1}^2} + \overline{V_{o_2}^2} = 2\overline{V_o^2} = 24.84\,mV^2 = \boxed{157.6\ \mu Vrms}$$

All of the charge error in the circuit will be forced to the output by the feedback action of the amplifier, regardless of CDS action.

5) a)

1) Total noise = $1.862e^{-16}$ Crms

98% of total noise = $1.8247e^{-16}$ $\bar{C}^2$ occurs at 773 MHz

2) swnoise = 0

Total noise = $1.813e^{-16}$ Crms

Drop of 2.6% compared to baseline

3) to_phase = 2 swnoise = 1 node /vo

Total noise 118.7 μVRMS

4) Total output noise $= \sqrt{\frac{\overline{q^2}}{Cs^2} + \overline{V_{o2}^2}} = \sqrt{\left(\frac{1.862e^{-16}}{1pF}\right)^2 + \left(118.7e^{-6}V\right)^2}$

$= 220.8 \mu Vrms$

This is 40% higher than the noise calculated in step f of problem 4

b) 1) flicker_on = 0 cds_en = 1

Total integrated noise 214.1 μVRMS

2) flicker_on = 1 cds_en = 1

Total integrated noise 216.2 μVRms

No flicker noise evident in plot

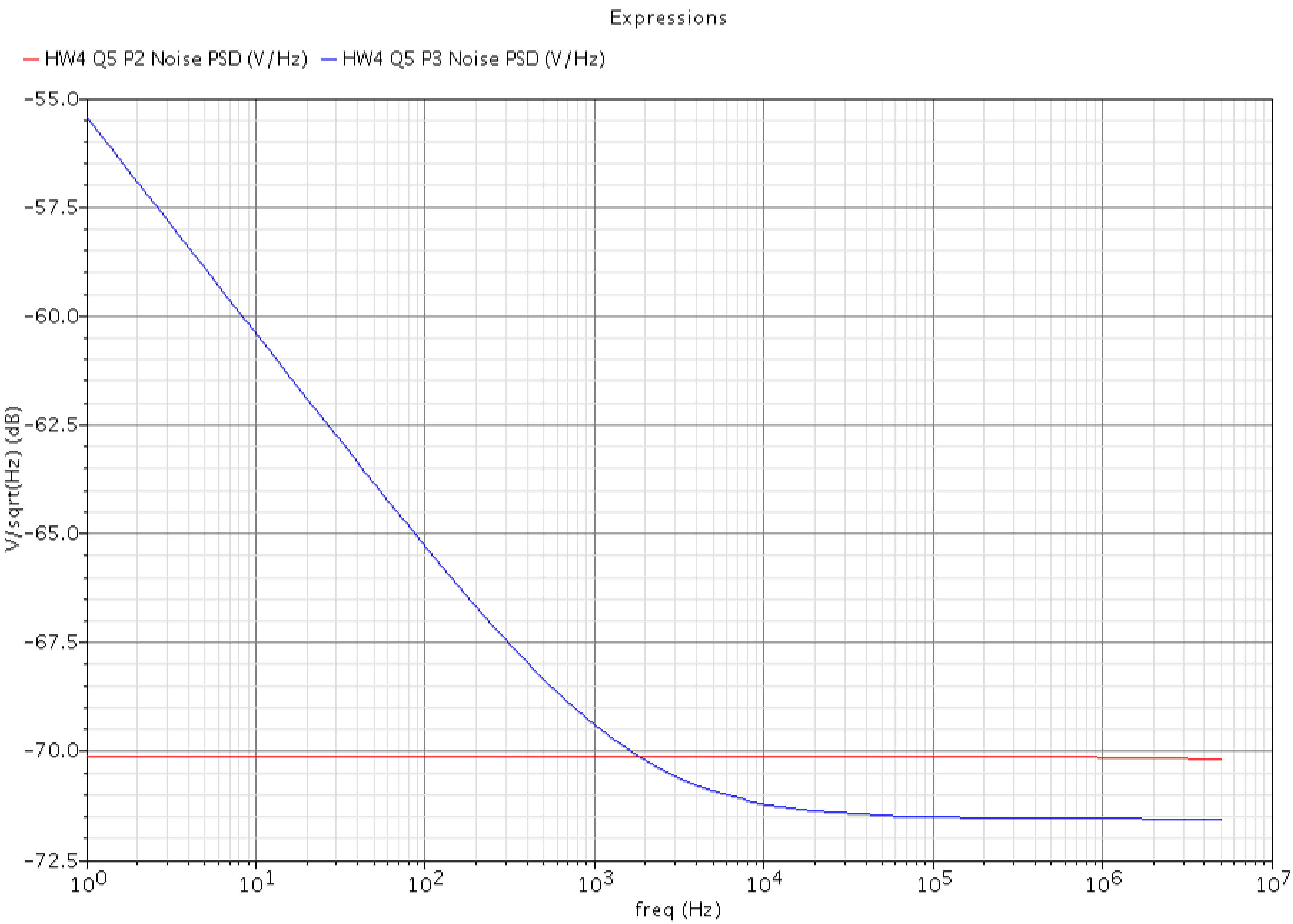
3) flicker_on = 1 cds_en = 0

Total integrated noise 151.2 μVRMS

Flicker noise very evident in PSD

4) flicker_on = 0 cds_on = 0

Total integrated noise 156 μVRMS



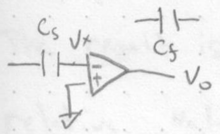
Expressions
HW4 Q5 P2 Noise PSD (V/Hz)
HW4 Q5 P3 Noise PSD (V/Hz)
-55.0
-57.5
-60.0
-62.5
-65.0
-67.5
-70.0
-72.5
V/sqrt(Hz) (dB)
10^0
10^1
10^2
10^3
10^4
10^5
10^6
10^7
freq (Hz)

6) Calculate V_o at $t = t_{end}$

$A_{OL} = \infty$ switches ideal

Cases:	S_2	S_4	S_{13}	S_{14}	S_{23}	S_{124}
0	0	0	0	0	0	0
1	0	0	1	1	0	1
2	0	0	0	0	0	0
3	1	0	0	0	1	1
4	0	0	0	0	0	0
5	0	0	1	0	1	0
6	0	0	0	0	0	0
7	0	1	0	1	0	1

Circuit 0, 2, 4, 6



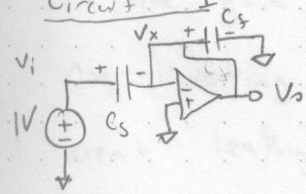
$Q = V \cdot C$ $\quad V_f = V(C_f)$ $\quad V_s = V(C_s)$

$Q_f = V_f \cdot C_f$ $\quad Q_s = V_s \cdot C_s$

Case 1

$V_i = 1V$ $\quad V_x = 0V$

Circuit 1



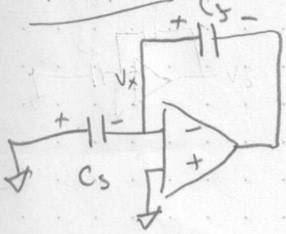
$V_f = 0$ $\quad V_s = V_i - V_x = V_i$

$Q_{f_1} = 0$ $\quad Q_{s_1} = V_i \cdot C_s$

Case 3

$Q_{f_1} = 0$ $\quad Q_{s_1} = V_i C_s$

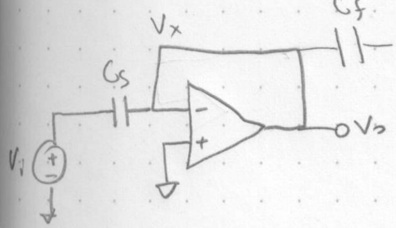
Circuit 3



$V_x \Rightarrow 0,$ $\quad Q_{f_3} = Q_{s_1} = V_i C_s$

6) Cont'd

Circuit 5

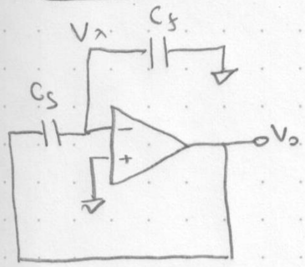


$Q_{f3} = V_i C_s$ $Q_{s3} = 0$

$V_x = 0$ $V_s = V_i - V_x = V_i$

$Q_{s5} = V_i C_s$ $Q_{f5} = Q_{f3} = V_i C_s$

Circuit 7



$Q_{f5} = V_i C_s$ $Q_{s5} = V_i C_s$

$V_x \Rightarrow 0$, $Q_f \rightarrow C_s$

$Q_s = 2 \cdot V_i C_s$ $Q = V \cdot C \Rightarrow V = \frac{Q}{C}$

$V_o = \frac{Q_s}{C_s} = \frac{2 V_i C_s}{C_s} = 2 \cdot V_i$

The result is interesting because first, it's not dependent on the relative values of C_f and C_s, that is C_s and C_f don't appear in the gain equation. Second it's interesting because it's non-inverting. If the switches are good and the capacitors aren't leaky this is a neat way to get a precision gain as the component values don't matter.