Samuel Lenius   EE315   Fall 2017   HW2

1) a) T   b) F   c) T

2) a) See next

b) The output rails to 1 and holds there

The integrator continues to rise without bound, linearly increasing over time.

c) See next. Noise shaping evident in the upward slope of noise vs frequency.

d)

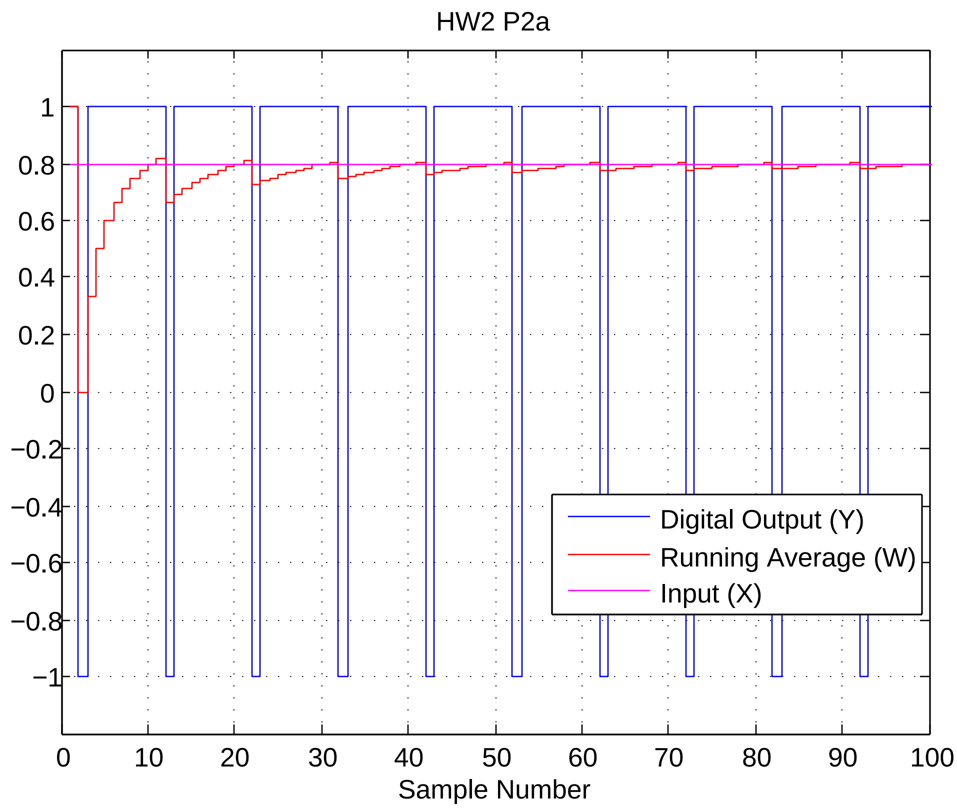
3) Thermometer MSB - 6bits $\sigma_u = 3\sigma$
Binary LSB - 6bits $\sigma_u = 4\sigma$

$\sigma^2_{BIN} = (2^B - 1)\sigma_u^2 = 0.1008$ $\sigma_{BIN} = 0.3175$ LSB

$\sigma_{THERM} = \sigma_u = 0.03$

$\sigma_{DAC} = \sqrt{\sigma_{THERM}^2 + \sigma_{BIN}^2} = 0.3175$ LSB

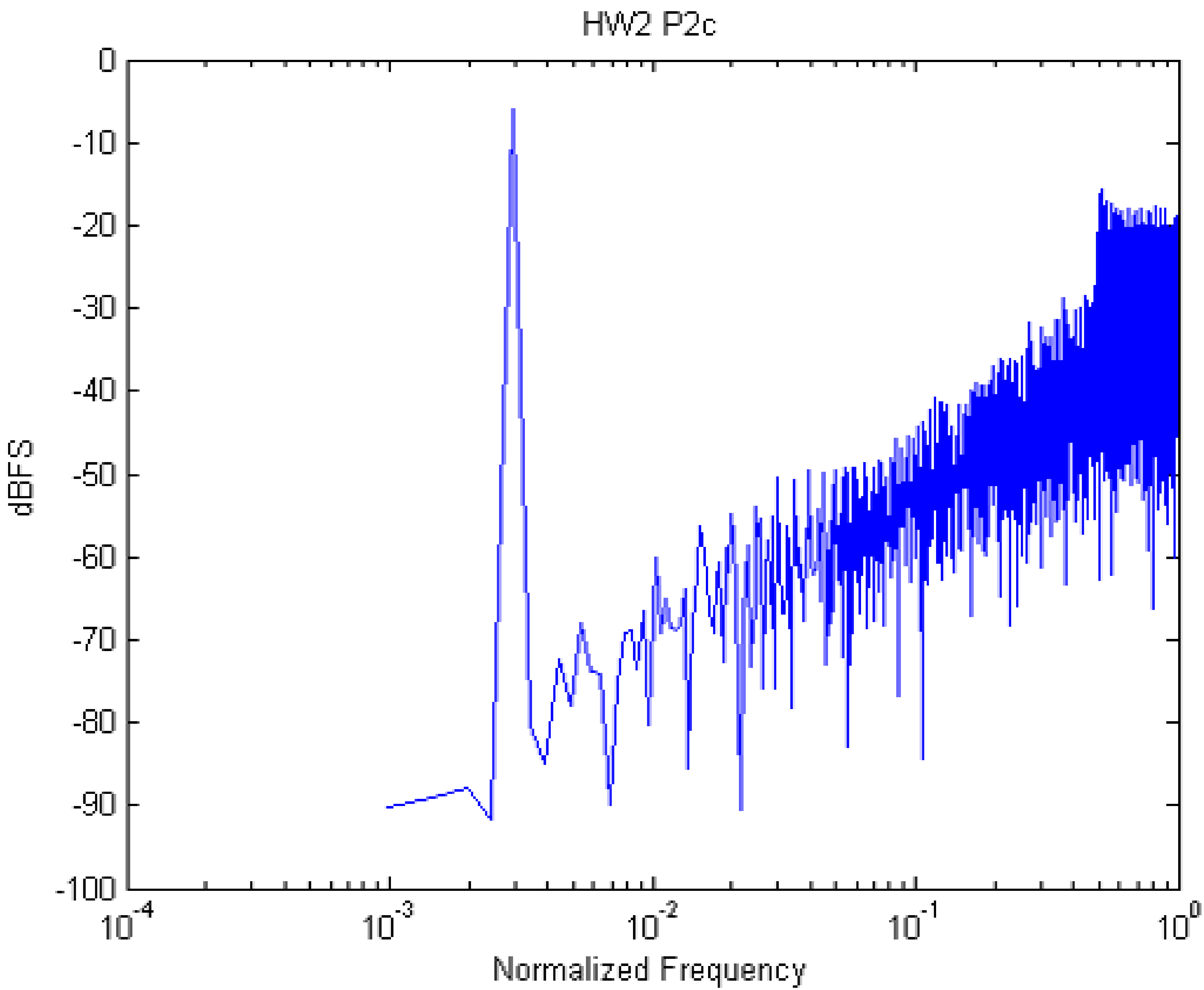
4) DAC designed for cable TV applications where you need to send a channel without disrupting adjacent channels through harmonic distortion



HW2 P2a
1
0.8
0.6
0.4
0.2
0
−0.2
-0.4
−0.6
−0.8
−1
0
10
20
30
40
50
60
70
80
90
100
Sample Number
Digital Output (Y)
Running Average (W)
Input (X)

```
% First-order, single-bit Delta-Sigma Modulator
% Code is incomplete.
clear all;
close all;

N = 2^12;
%X = 0.8;
samp = 1:N;
cycles = 50;
X = 0.5*sin(cycles*2*pi*samp./(N));
I = zeros(N,1);
Y = ones(N,1);

% Delta-Sigma Block
for i = 1:N-1
    if i > 1
        I(i) = I(i-1) + X(i) - Y(i);
    else
        I(i) = X(i) - Y(i);
    end
    if (I(i) < 0)
        Y(i+1) = -1;
    else
        Y(i+1) = 1;
    end
end

% Calculate Running Average (W) of output
% Enter Code here:
W = zeros(N,1);
for i = 1:N-1
    W(i) = mean(Y(1:i));
end


% Plot Output
figure(1);
stairs(Y, 'b-');
hold on;
stairs(W,'r-');
stairs(X, 'm-');
%stairs(I,'m-');
legend('Digital Output (Y)', 'Running Average (W)', 'Input (X)');
axis([0 N -1.2 1.2]);
xlabel('Sample Number')
grid on;
title('HW2 P2a');

s = abs(fft(Y));
s = s/(N/2);
s = s(1:N/2);
s_dbfs = 20*log10(s);

figure(2);
f = (1:N/2) ./ (N/2);
semilogx(f, s_dbfs, '-')
title('HW2 P2c');
```

```
ylabel('dBFS');
xlabel('Normalized Frequency')

%total_power = sum(s(1:N/(2*20)));
```


HW2 P2c
dBFS
0
-10
-20
-30
-40
-50
-60
-70
-80
-90
-100
10^-4
10^-3
10^-2
10^-1
10^0
Normalized Frequency

5) a) $B = 12$ bits

$INL < 0.5\ LSB$ $\quad DNL < 0.5\ LSB$ $\quad 95\%$ Yield $\rightarrow C = 2$

$A_{decode} = 2^{B_t} \cdot 2000\mu m^2$ $\quad A_{tot} = 2^{B} \cdot A_{unit} + A_{decode}$

$K_u = 6\%\mu m$ $\quad \sigma_u = \frac{K_u}{\sqrt{A_{unit}}}$

A_{unit} = area for DNL of thermometer = 0.5LSB

$\sigma_u = \frac{K_u}{\sqrt{A_{unit}}} \Rightarrow A_{unit} = \left(\frac{K_u}{\sigma_u}\right)^2$

$Yield_{unit} = Yield_{chip}^{\frac{1}{2^{B_t}}}$

$C = \sqrt{2} \cdot erfinv(Yield_{unit})$

$DNL_t = C \cdot \sigma_{unit}$ $\quad \sigma_{unit} = \frac{DNL_t}{C} = \frac{0.5}{C}$

$A_{unit} = \left(\frac{6\% um}{\sigma_{unit}}\right)^2$

$B = B_t + B_b$

$INL = \frac{1}{2}\sigma_u\sqrt{2^B} = \frac{k_u\sqrt{2^B}}{2\sqrt{A_{unit}}}$ $\quad \cancel{A_{INL} = \left(\frac{K_u\sqrt{2^B}}{2 \cdot INL}\right)^2}$

$INL = \frac{1}{2}\sqrt{2^B}\sigma_u = 32\sigma_u$ $\quad Area(INL = 0.5) = 2$

$A_{unit} = \frac{1}{\sigma_u^2}$ $\quad 0.5 = 32 \cdot \sigma_{INL}$ $\quad \frac{1}{64} = \sigma_{INL}$

$A_{INL} = \frac{A_{unit}}{\left(\frac{1}{64}\right)^2} = 4096 \cdot A_{unit}$

$DNL_{Binary} = \sigma_u\sqrt{2^B - 1} = \sigma_{DNL_{Bin}} 64$ $\quad 0.5 = 64\,\sigma_{DNL_{Bin}}$ $\quad \frac{1}{128} = \sigma_{DNL_{Bin}}$

$A_{DNL_{BIN}} = \frac{A_{unit}}{\left(\frac{1}{128}\right)^2} = 16384 \cdot A_{unit}$

$DNL_{seg} \approx \sigma_{seg}\sqrt{2^{B_b+1}}$ $\quad \frac{0.5}{\sqrt{2^{B_b+1}}} = \sigma_{seg}$ $\quad A_{DNL_{seg}} = \frac{A_{unit}}{\left(\frac{0.5}{\sqrt{2^{B_b+1}}}\right)^2}$

5) a)

Segmented

$\sigma_{DNL} = \sigma_\mu \sqrt{2^{B_b+1} - 1}$

$\sigma_{INL} = \frac{1}{2}\sigma_u \sqrt{2^B}$

$\sigma_{u_{DNL}} = \dfrac{\sigma_{DNL}}{\sqrt{2^{B_b+1} - 1}}$

$A_{unit} = \left(\dfrac{K_u}{\sigma_u}\right)^2$

$\sigma_{u_{INL}} = \dfrac{2\sigma_{INL}}{\sqrt{2^B}}$

$C = \sqrt{2} \cdot \mathrm{erfinv}\left(\mathrm{yield}^{\left(\frac{1}{2^{b_t}}\right)}\right)$

$\sigma_u = \min(\sigma_{u_{DNL}}, \sigma_{u_{INL}}) / C$

i) Minimized at $B_t = 3$, $A_{unit} = 109.6\,\mu m^2$ Total Area $= 0.465\,mm^2$

ii) Paper optimum at $B_t = 9$, $A_{unit} = 223.1\,\mu m^2$ Total Area $= 1.938\,mm^2$

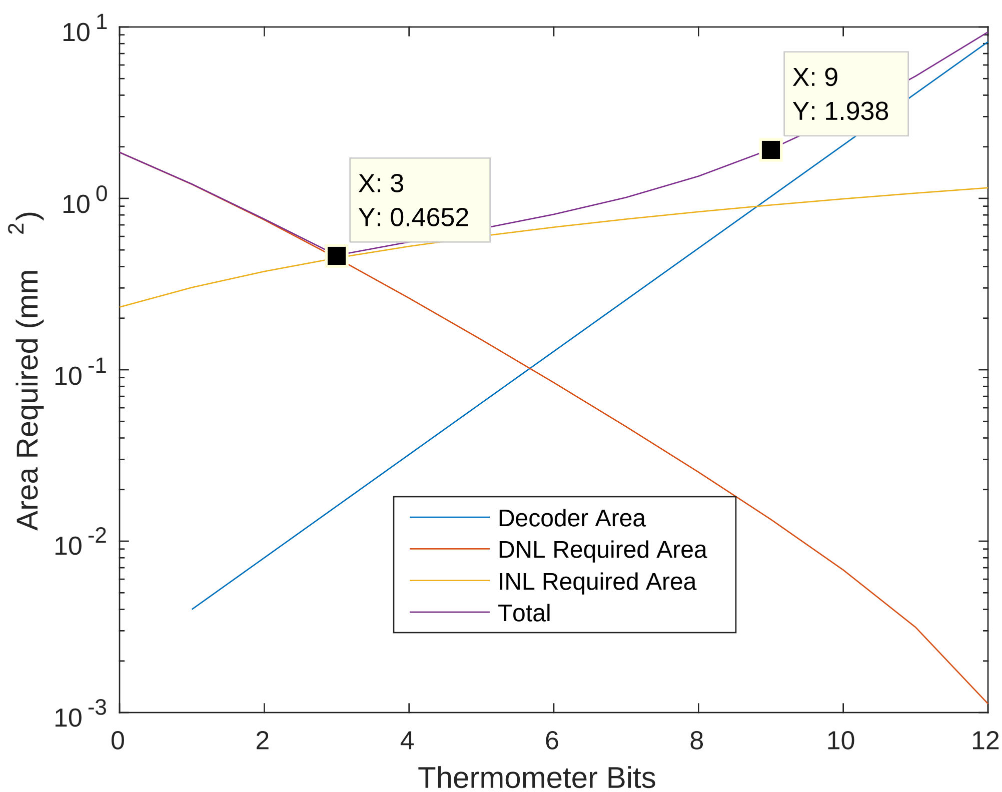
b) See next page

6) a) See next

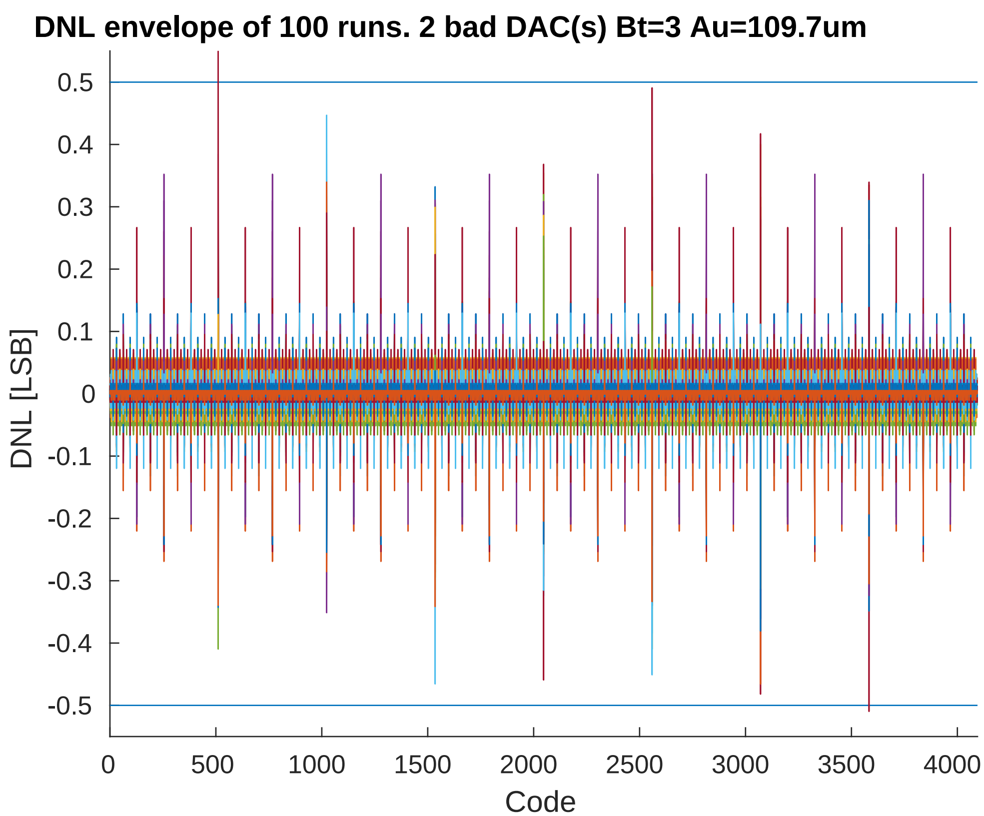
b) See next Using $B_t = 3$ $A_{unit} = 109.6$

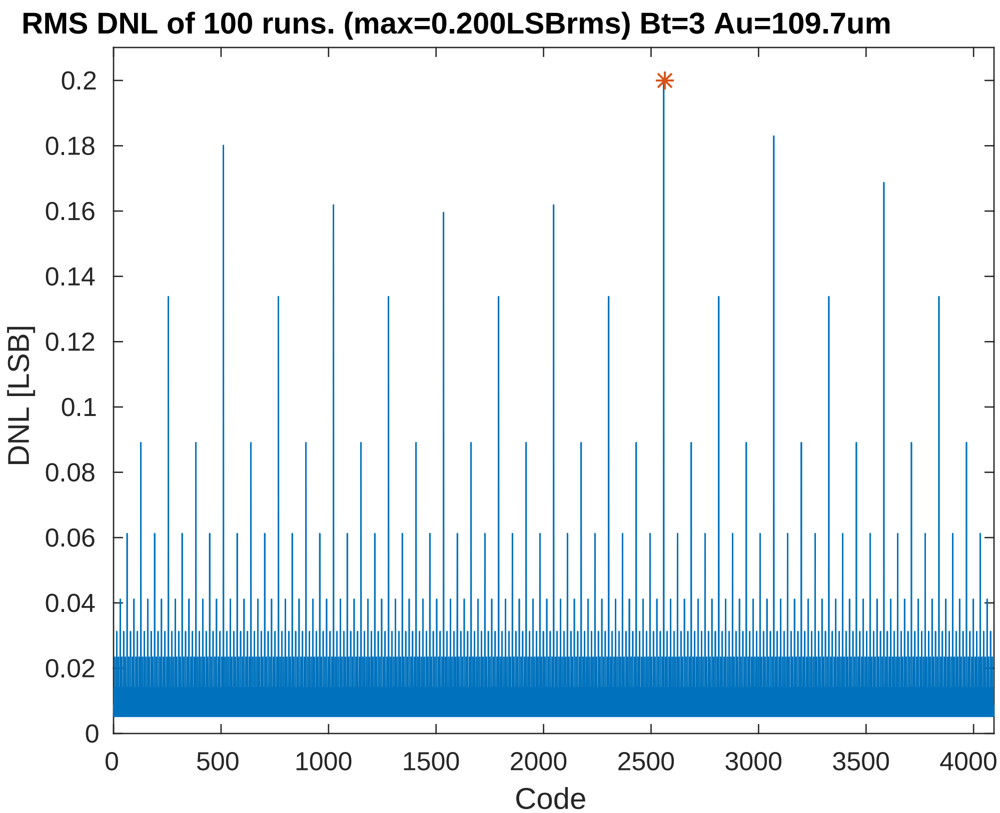
c) Yes, just barely (Fails = 5)

d) Met Spec - not iterating

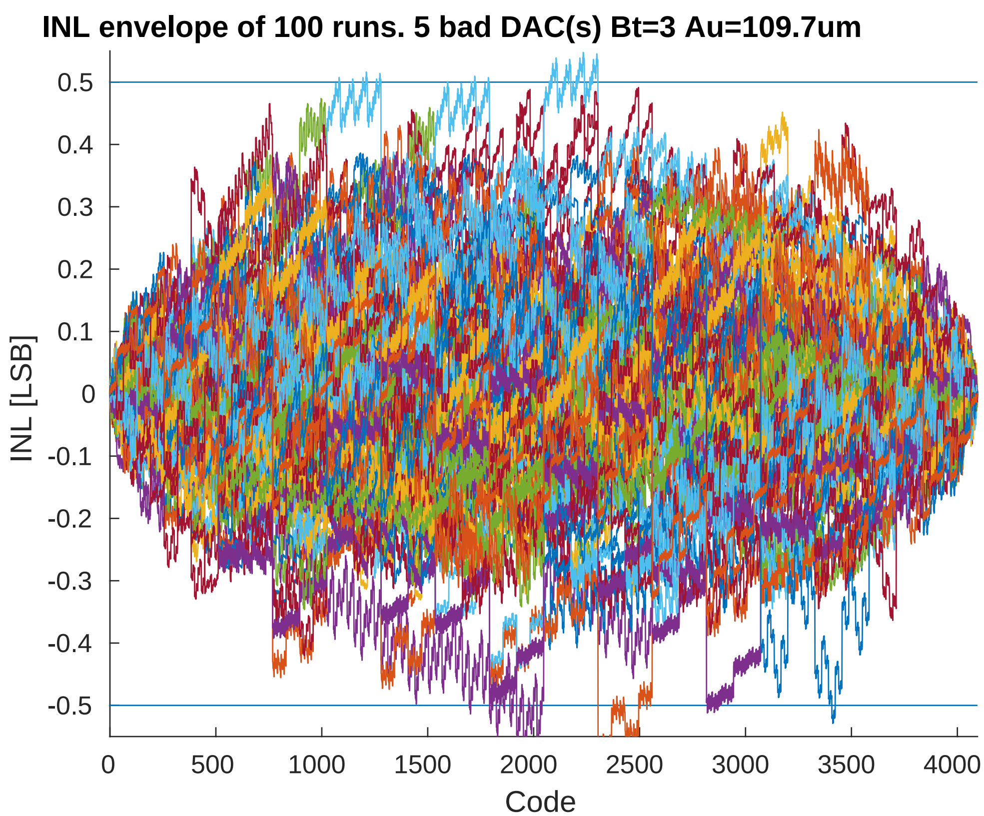


X: 3
Y: 0.4652
X: 9
Y: 1.938
Decoder Area
DNL Required Area
INL Required Area
Total
Area Required (mm²)
Thermometer Bits
10^1
10^0
10^-1
10^-2
10^-3
0
2
4
6
8
10
12

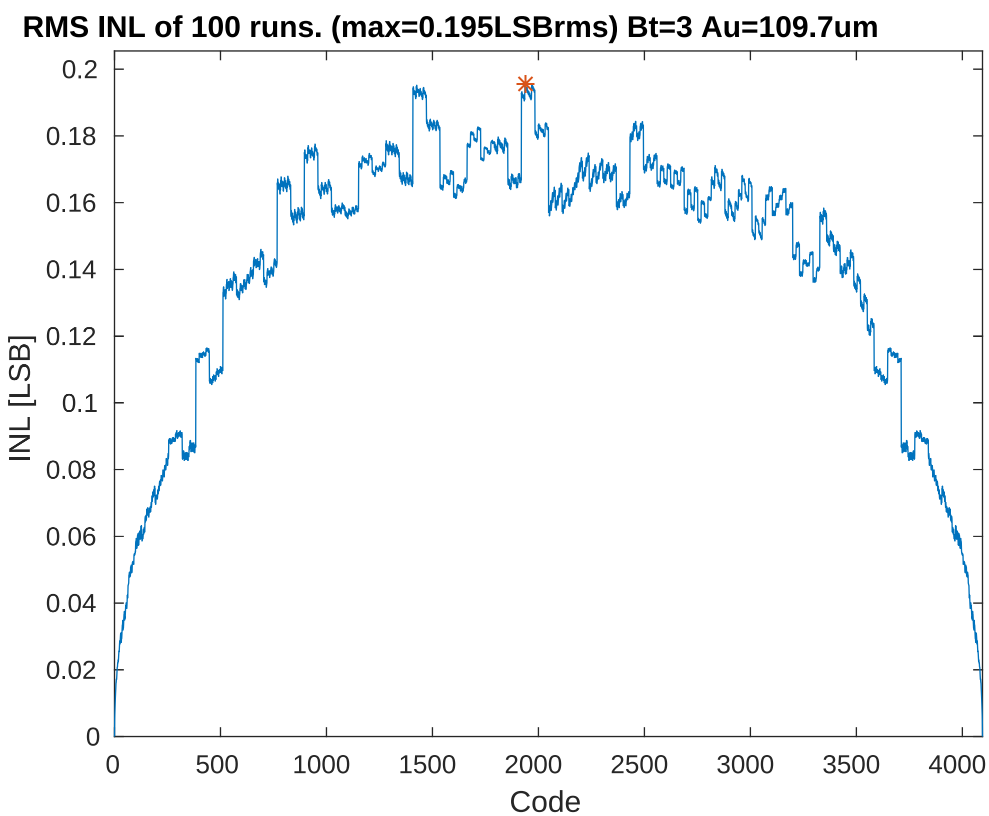

DNL envelope of 100 runs. 2 bad DAC(s) Bt=3 Au=109.7um
DNL [LSB]
0.5
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4
-0.5
0
500
1000
1500
2000
2500
3000
3500
4000
Code



RMS DNL of 100 runs. (max=0.200LSBrms) Bt=3 Au=109.7um
DNL [LSB]
Code
0
0.02
0.04
0.06
0.08
0.1
0.12
0.14
0.16
0.18
0.2
0
500
1000
1500
2000
2500
3000
3500
4000



INL envelope of 100 runs. 5 bad DAC(s) Bt=3 Au=109.7um
0.5
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4
-0.5
INL [LSB]
0
500
1000
1500
2000
2500
3000
3500
4000
Code


RMS INL of 100 runs. (max=0.195LSBrms) Bt=3 Au=109.7um
INL [LSB]
Code
0
0.02
0.04
0.06
0.08
0.1
0.12
0.14
0.16
0.18
0.2
0
500
1000
1500
2000
2500
3000
3500
4000

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Your code goes here

%   only turn in the code below (no need to turn in the given code above)

%   caculate dnl
dnl = zeros(r,(2^B)-1);
%step_avg = zeros(r,1);
for m=1:r
  d = diff(code(m,:));
  step_avg(m) = mean(d);
  dnl(m,:) = (d - step_avg(m)) ./ step_avg(m);
  %avg_width = (code(m, end)-code(m, 1))/(2^B-1);
  %inl(m, :)=(code(m,:)-avg_width*[0:2^B-1])./avg_width;
end

%   dnl scatter plot

%   dnl rms plot

% dnl scatter plot
figure(3); clf; hold on;
xlabel('Code');
ylabel('DNL [LSB]');
axis([0 2^B-1 -(dnlspec+0.05) (dnlspec+0.05)]);
line([0 2^B-1], [dnlspec dnlspec]);
line([0 2^B-1], [-dnlspec -dnlspec]);

bad_dacs_dnl=0;
for m=1:r
  figure(3);
  plot(0:2^B-2, dnl(m,:));
  if find(abs(dnl(m,:))>dnlspec)
     bad_dacs_dnl=bad_dacs_dnl+1;
  end
end
figure(3); hold off;
title( sprintf('DNL envelope of %d runs. %d bad DAC(s) Bt=%d Au=%0.1fum^2', r, bad_dacs_dnl, Bt, Aunit));

% dnl rms plot
dnl_rms = sqrt(sum( dnl.^2, 1 ) ./r);
[maxdnlrms dmax] = max(dnl_rms);
h = figure();
plot(0:2^B-2, dnl_rms, dmax, maxdnlrms, '*');
xlabel('Code');
ylabel('DNL [LSB]');
title( sprintf('RMS DNL of %d runs. (max=%1.3fLSBrms) Bt=%d Au=%0.1fum^2', r, maxdnlrms, Bt, Aunit));
axis([0 2^B-1 0 maxdnlrms+0.01]);
```